BOUNDARY INTEGRAL EQUATIONS FOR MODELING ARBITRARY FLAW GEOMETRIES IN ELECTRIC CURRENT INJECTION NDE

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INTRODUCTION

The Electric Current Injection (ECI) method of nondestructive evaluation is applied to materials that are electrically conductive but not magnetically permeable, such as aluminum, magnesium, and titanium. It consists of detecting current-flow anomalies due to voids, nonmetallic inclusions and open cracks in the conducting material, through distortions introduced in the magnetic field generated by the sample [l].

Several 2-D analytical solutions have been derived to simulate the magnetic field produced by a flaw in a conductor for direct current injection [2][3][4]. Scans of standard flaw specimens have validated these models experimentally. However, these solutions are limited to only a few problems with very simple geometries. This paper presents a boundary integral equation (BIE) formulation, which allows arbitrary two-dimensional plate and flaw shapes to be modeled, providing a much greater flexibility to the measurement model. Also, since only l-D boundary elements are required, this approach has a significant computational advantage over finite element methods (FEM) for solving problems that can be regarded as two-dimensional

The next section describes the BIE formulation, followed by a sample calculation for a square aluminum plate, and a comparison with the results given by a commercial finite element method software. Also, the procedure needed to simulate a thick conductive plate is described.

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BOUNDARY INTEGRAL EQUATION FORMULATION

The vector electrical field $\vec{E}$ for the steady electrical conduction problem is taken to be the negative gradient of the scalar potential $\vec{V}$, and the current density $\vec{J}$ is scaled from $\vec{E}$ by the conductivity σ, considered to be constant for the body, i.e.,

$$\left.\begin{array}{l}\vec{E}=-\vec{\nabla}V\\ \vec{J}=\sigma\cdot\vec{E}\end{array}\right\} \rightarrow \vec{J}=-\sigma\cdot\vec{\nabla}V. \tag{1}$$

As V is harmonic, it must satisfy Laplace's equation,

$$\nabla^2 V=0. \tag{2}$$

From Green's second identity, a boundary integral equation that solves this harmonic potential problem can be derived [6]

$$\int_S [V(Q)-V(P)]\frac{\partial\psi(P,Q)}{\partial n(Q)}dS(Q)=\int_{S_N}\psi(P,Q)\frac{\partial V(Q)}{\partial n(Q)}dS(Q). \tag{3}$$

In the above equation, both P and Q are points located at the boundaries of the plate and flaw(s), and the integral equation is solved for the potential values $V(P)$. $\psi(P,Q)$ is the fundamental solution of Laplace's equation in two dimensions, given by [5]

$$\psi(P,Q)=\frac{1}{2\pi}\mathrm{Re}\left\{\log\left[\left(x_P-x_Q\right)+i\left(y_P-y_Q\right)\right]\right\}. \tag{4}$$

In equation (3), S and S_N are the integration regions, S being the entire boundary surface, and S_N the portion of the boundary to which are assigned Neumann boundary conditions, that is, boundary conditions referring to the normal derivative of the desired solution. From equation (1), such conditions can be written as

$$\frac{\partial V}{\partial n}=-\frac{J_n}{\sigma}. \tag{5}$$

So, one just has to define the plate edge regions where the dc current is injected and removed. Once calculated, the potential values at the boundaries of the plate and flaw(s) can be used to determine the magnetic field at a measurement point c by means of the law of Biot-Savart

$$\vec{B}(c)=\int_S\frac{\vec{\nabla}V(Q)\ \hat{t}}{r(c,Q)}dS(Q) \tag{6}$$

A program has been developed to calculate the magnetic field, using equation (6), for the measurement points desired, usually over a plane parallel to the surface of the plate.

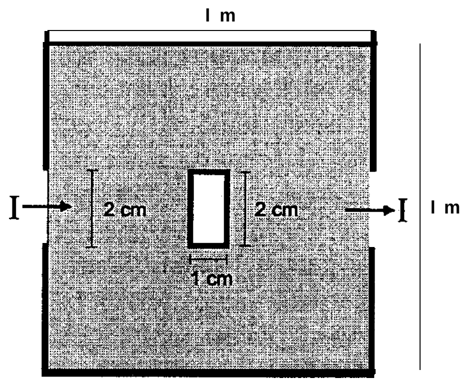

Figure 1. Geometry for the sample problem. The solid boundary is insulating.

SAMPLE APPLICATION

Figure 1 above shows one of the sample problems used to test the BIE model. It consists of a 1 m x 1 m x 0.1 mm aluminum plate, with a 1 cm x 2 cm rectangular hole in its center. A dc current of 15 A is injected and removed in the direction orthogonal to the major side of the rectangle, through 2 cm sections of the plate edges. The BIE model has 22 linear segments defining the plate perimeter, and 20 linear segments defining the rectangular hole.

Using the measurement model developed, the magnetic flux density was calculated on a plane parallel to the plate at a 5 mm liftoff distance. Figure 2a shows the contour plot of the simulated results. The magnetic flux density peak value was 63.25 mGauss.

The geometry shown in Fig. 1 was implemented using a commercial finite element software [6], and the magnetic flux density was calculated over the same plane, with the same liftoff distance. Figure 2b shows the finite element results, after optimizing the finite element mesh to the maximum extent possible, and the peak value was 63.0 1 mGauss.

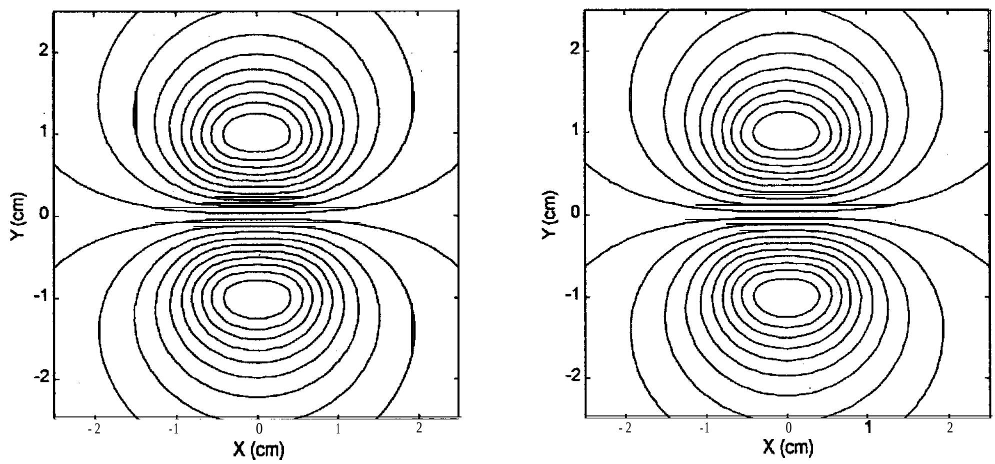

Figure 2. (a) Magnetic field calculated using the BIE formulation, for a liftoff of 5mm. The peak value is 63.25 mGauss. (b) Magnetic field calculated using the finite element analysis, for a liftoff of 5mm. The peak value is 63.01 mGauss.

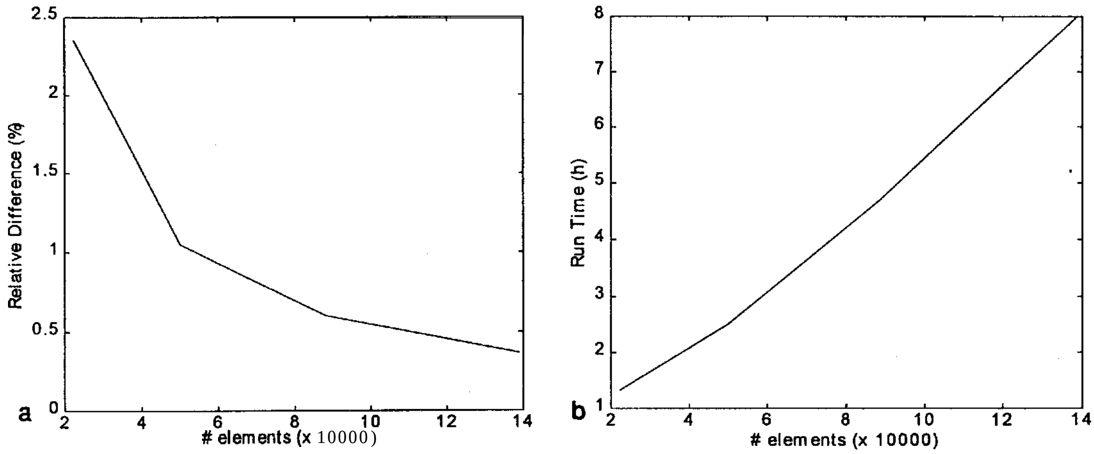

Figure 3. a) Plot of the relative difference between the BIE model and the FEM model, according to the number of finite elements. b) Plot of the run time needed by the FEM model, according to the number of finite elements.

It can be seen from Fig. 2 that the magnetic field calculated by the two methods is almost the same, The relative difference between the peak values of the BIE and FEM results was 0.38%. In fact, such difference is highly dependent on the number of finite elements, as can be seen in the plot shown in Fig. 3a.

The major advantage of the BIE model over the FEM model, for planar geometries, is the time demanded to calculate the magnetic field. For the situation considered, the BIE code spent approximately 30 seconds to run, while the FEM software needed a run time, dependent on the number of finite elements, that was substantially higher, ranging from 1 to 8 hours as can be seen in the plot shown in Fig. 3b.

APPLICATION OF THE BIE MODEL TO THICK PLATES

The boundary integral equation solves for the potentials in a 2-D (infinitely thin plate) problem. For the sample calculation performed in the previous section using very thin plates (0.1 mm), this is a valid approximation. To use the BIE model to calculate the magnetic field associated with a thick plate, a pseudo-integration can be done by first dividing the thickness of the plate into stacked thin sheets with 0.1 mm thickness and then by calculating the sum of the magnetic fields generated by each sheet, at incremented liftoffs. Although this pseudo-integration is an approximation, since it does not consider current flows perpendicular to the sheets, it produces acceptable results in many cases of interest.

As an example, consider a 1 mm thick aluminum plate, with the same geometry shown in Fig. 1. The plate is subdivided into IO planar sheets with a thickness of 0.1 mm, and the magnetic field is calculated for each of such planar sheets, varying the liftoff. For an overall liftoff of 5 mm, the liftoff distances used in the calculation shall range from 5.05 mm (top sheet) to 5.95 mm (bottom sheet). The peak value found for the 1 mm thick plate using the BIE model was 58.23 mGauss, compared to a FEM result of 58.0 mGauss, which corresponds to a difference of 0.39%.

It is important to point out that the boundary integral equation, which solves for the tangential derivatives of the electric scalar potential, needs to be calculated only once for multiple sheets. Once these are calculated, the magnetic field can be determined for various configurations (e.g., different liftoffs), from the single set of BIE results.

As regards the run time, the FEM model took approximately the same as before, about 8 hours, as the number of elements remained the same (about 140,000), and just their geometry had to be changed. For the BIE model, there is an increase in the run time, as the magnetic field has to be calculated for each of the 10 thin sheets to calculate the final field. Even so, the total time was about 5 minutes, which is still much less than the time spent by the FEM method.

CONCLUSION

A BIE-based model was developed to simulate the magnetic field generated by a flawed plate carrying a dc current. Arbitrary shaped boundaries can be modeled with 1-D boundary elements giving a significant computational advantage over finite element methods for solving problems that can be regarded as two-dimensional. A sample calculation has been performed, and a systematic comparison with a FEM calculation has been made. The relative difference found between the FEM and BIE results was 0.38 %, but the FEM simulation demanded a run time almost three orders of magnitude longer than the BIE code.

Future work is planned to experimentally validate the BIE method, using SQUIDs and fluxgate sensors.

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