Boundary Integral Equation Measurement Model for the Electric Current Injection Method of Nondestructive Evaluation

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Abstract-**A measurement model was developed to simulate the magnetic field obtained by scanning a magnetometer over a flawed plate carrying a dc current. By using a boundary integral equation formulation, arbitrary plate/flaw shapes can be modeled, which provides a flexibility not given by analytical solutions, which are limited to only a few geometries. Also, as only 1-D boundary elements are required, this approach has a significant computational advantage over finite element methods. An experimental validation has been accomplished, and the 6% difference between the peak-to-peak values for the experimental and modeled results is within the expected experimental error.**

Introduction

Nondestructive evaluation (NDE) is the monitoring of any structural changes and defect initiation in materials or structures, in a way that does not disturb the physical properties and overall performance of the object under test [1][2][3].

The Electric Current Injection (ECI) method of nondestructive evaluation is applied to materials that are electrically conductive but not magnetically permeable, such as aluminum, magnesium, and titanium. It consists of detecting current-flow anomalies due to voids, nonmetallic inclusions and open cracks in the sample, through distortions introduced in the magnetic field generated by the plate [1].

Several 2-D analytical solutions have been derived to simulate the magnetic field produced by a flaw in a conductor for direct current injection [4][5]. Scans of standard flaw specimens have validated these models experimentally. However, these solutions are limited to only a few very simple geometries. By using a boundary integral equation (BIE) formulation, arbitrary plate and flaw shapes can be modeled, which provides a much greater flexibility to the measurement model.

The next section describes the BIE formulation, followed by a sample calculation for a square aluminum plate, and the experimental validation of the calculated field using a fluxgate magnetometer.

Boundary Integral Equation Formulation

The vector electrical field $\vec{E}$ for the steady-electrical conduction problem is taken to be the negative gradient of a harmonic function, and the current density $\vec{J}$ is scaled from $\vec{E}$ by the conductivity σ, taken to be constant for the body:

$$\left.\begin{aligned}\vec{E}&=-\vec{\nabla}V\\ \vec{J}&=\sigma\cdot\vec{E}\end{aligned}\right\}\rightarrow \vec{J}=-\sigma\cdot\vec{\nabla}V . \tag{1}$$

As V is harmonic, it must satisfy Laplace's equation,

$$\nabla^2 V=0. \tag{2}$$

From Green's second identity, a boundary integral equation that solves this harmonic potential problem can be derived [6]:

$$\begin{aligned}&\int_S\left[V(Q)-V(P)\right]\frac{\partial\psi(P,Q)}{\partial n(Q)}dS(Q)=\\ &\int_{S_N}\psi(P,Q)\frac{\partial V(Q)}{\partial n(Q)}dS(Q).\end{aligned} \tag{3}$$

In the above equation, both P and Q are points located at the boundaries of the plate and flaw(s), and the integral equation is solved for the potential values $V(P)$. $\psi(P,Q)$ is the fundamental solution of Laplace's equation in two dimensions, given by

$$\psi(P,Q)=\frac{1}{2\pi}\mathrm{Re}\left\{\log\left[\left(x_P-x_Q\right)+i\left(y_P-y_Q\right)\right]\right\}. \tag{4}$$

S is the entire boundary surface, while S_N is the portion of the boundary to which are assigned Neumann boundary conditions, that is, boundary conditions referring to the normal derivative of the desired solution. From equation (1):

$$\frac{\partial V}{\partial n}=-\frac{J_n}{\sigma}. \tag{5}$$

So, one just has to define the plate edge regions where the dc current is injected/removed. Once calculated, the potential values at the boundaries of the plate/flaw(s) can be used to determine the magnetic field at a measurement point c by means of Biot-Savart law [6]:

$$\vec{B}(c) = \int_S \frac{\vec{\nabla}V(Q)\cdot\hat{t}}{r(c,Q)} dS(Q) \qquad (6)$$

EXPERIMENTAL VALIDATION

Fig. 1 shows one of the sample problems used to validate the BIE model. It consists of a 1 m × 1 m × 1.1 mm aluminum plate, with a 1 cm × 2 cm rectangular hole in its center. A dc current of 10 A was injected and removed in the direction orthogonal to the major side of the rectangle, through 1 cm sections of the plate edges. The model is composed of 22 linear segments defining the plate perimeter, and 20 linear segments defining the rectangular hole.

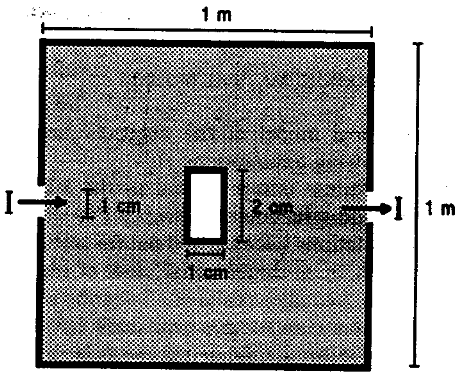

Fig. 1: Geometry for the sample problem.

Using the measurement model developed, the magnetic flux density was calculated on a plane parallel to the plate at a distance of 6 mm. Fig. 2 shows the surface plot of the simulated results.

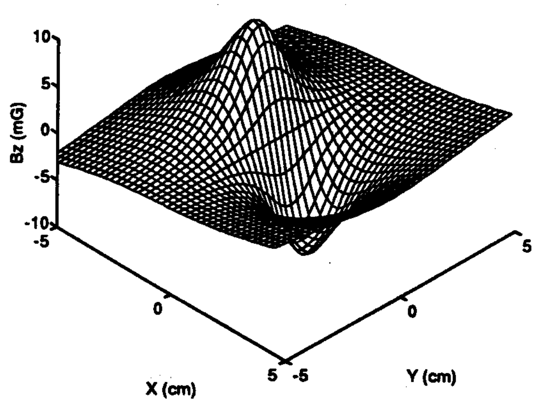

Fig. 2: Magnetic field calculated using the BIE formulation.

In order to validate the simulated results, the geometry shown in Fig. 1 was built, and the magnetic flux density was measured with a fluxgate sensor. Fig. 3 shows the experimental result for the sample plate.

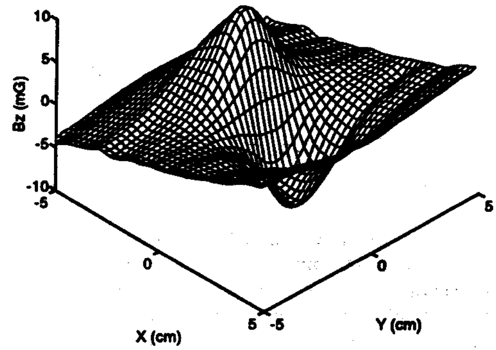

Fig. 3: Magnetic field measured with a fluxgate magnetometer.

The error between the peak-to-peak values of the experimental and modeled results was 6%, which can be attributed to imprecisions in the experimental setup, since an variation of 1 mm in the liftoff distance can lead to errors in the calculated field up to 10%.

CONCLUSION

A BIE-based model was developed to simulate the magnetic field generated by a flawed plate carrying a dc current, with arbitrary shapes. Only 1-D boundary elements are required, which gives a significant computational advantage over finite element methods. An experimental validation was accomplished, using a fluxgate magnetometer on a 1 m × 1 m × 1.1 mm aluminum plate with a 1 cm × 2 cm rectangular hole, carrying a 10 A dc current.

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