Uncertainty in Crisis Bargaining with Multiple Policy Options*

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Abstract

There are many forms of coercion—including supporting rebel groups, sanctions, and cyberattacks—but formal models commonly characterize interstate bargaining as dichotomous, ending in either war or peace. How does the availability of intermediate policy options affect the incidence of war and peace? We present a game-free analysis of crisis bargaining models with flexible policy options that challenges conventional results about the relationship between private information and negotiation outcomes. Unlike in traditional crisis bargaining models, we find that greater private war payoffs may be associated with a lower probability of war or worse settlement values. Our results are not specific to any particular game form, but instead emerge regardless of the precise negotiating protocol. By combining the substantive richness of flexible policy responses with the generality of a mechanism design analysis, we derive robust explanations of how private information influences international conflict.

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"Our traditional approach is either we're at peace or at conflict. And I think that's insufficient to deal with the actors that actually seek to advance their interests while avoiding our strengths."

—General Dunford (2016), Chairman of the U.S. Joint Chiefs of Staff.

The central question in the study of international conflict is why wars occur even though they are costly to all sides. It is puzzling that states seek political gains through violence when the underlying stakes could be divvied up costlessly at the bargaining table (Fearon 1995). Perhaps the most prominent explanation for this puzzle is that states have incomplete information about each other's capabilities or intentions (Jervis 1976), leading them to make incompatible demands about the shape of a negotiated settlement. To understand why inefficient conflict takes place, we must understand how private information shapes states' incentives and actions.

A long tradition of scholars have developed game-theoretical models of international crisis bargaining to understand the role of incomplete information in the outbreak of conflict. Building from foundational theories of bargaining under uncertainty (Morrow 1989; Fearon 1995; Powell 1999; Gartzke 1999), an impressive line of recent work has used crisis bargaining models to study the role of incentives, information, and capabilities in determining when and why states go to war (Leventoğlu and Tarar 2008; Fey, Meirowitz and Ramsay 2013; Acharya and Grillo 2015; Bas and Schub 2017; Spaniel and Malone 2019; Morrow and Sun 2020). These crisis bargaining models have made valuable progress in the theoretical study of interstate war, but they are limited by a common simplifying assumption about the nature of state interactions—namely, that states either reach a peaceful and efficient bargain or else engage in a decisive full-scale war. In reality, states in crises possess a vast array of coercive intermediate policy options besides war and peace: implementing sanctions or tariffs (Bapat and Kwon 2015; McCormack and Pascoe 2017; Di Lonardo and Tyson 2022*a*), offering third-party support to an adversary's enemies (Bapat 2006; Schultz 2010; Berman and Lake 2019; Qiu 2022*b*; Chinchilla 2021), engaging in cyberattacks (Gartzke and Lindsay 2015; Baliga, Bueno de Mesquita and Wolitzky 2020), conduct drone strikes (Lin-Greenberg 2022*b*), or engaging in low-level conflict, hybrid warfare, or "hassling" (Lanoszka 2016; Gurantz and Hirsch 2017; Schram 2021; Lin-Greenberg 2022*a*). While simplifying assumptions are useful in allowing models to abstract away from irrelevant features and focus in on key mechanisms (Paine et al. 2020), the war-or-peace dichotomy in crisis bargaining models is overly reductive and far more consequential. As we show below, this assumption systematically limits our predictions about the relationship between private information and the outcomes of international disputes.

Crisis bargaining models that end either in peaceful settlement or a decisive war offer a strikingly consistent portrayal of the conditions that lead to conflict. Asymmetric information is modeled as a private signal that a state receives about its ability, willingness, or political motivations to wage war, often called the state's "type." In models of crisis bargaining with private information, states with stronger private signals are more likely to take actions that result in war, and they receive greater payoffs from bargaining even when the game ends peacefully. These relationships between private type and crisis outcomes are not specific to any particular model, but rather occur in *every* equilibrium of *any* crisis bargaining game where the only possible outcomes are war or peace (Banks 1990; Fey and Ramsay 2011).[1] Consequently, traditional crisis bargaining models provide a consistent answer to the question of when states are most likely to resort to violence: when at least one state has a strong private signal of its battlefield ability or willingness to fight.

We reexamine the relationship between private information and war in *flexible-response crisis bargaining games* that allow for forms of costly conflict short of all-out war. Once we allow for multiple forms of conflict, the traditional models' monotonic relationships between private type and bargaining outcomes begin to break down. States with better private signals about their ability to wage war may be less likely to fight and may have lower payoffs in equilibrium. These exceptions to the traditional patterns arise in the presence of multiple policy options because the factors that influence a state's ability to wage war may also shape its facility with alternative coercive instruments. Importantly, we do not simply present a single counterexample to established theories. Instead, we conduct a game-free analysis along the lines of previous mechanism design research (Banks 1990; Fey and Ramsay 2011; Akçay et al. 2012; Spaniel 2020; Liu 2021), showing that the breakdown in standard patterns can arise in *any* interaction with flexible policy options, regardless of the specifics of the bargaining protocol. Our approach combines the methodological generality of previous game-free analyses with the substantive richness of newer models of coercive policies short of war, which allows us to characterize broad influences on the likelihood of conflict in more realistic strategic environments.

The key driver of our results is that the same aspects that affect a state's war payoffs might also affect its effectiveness in using alternative policy choices, for better or worse. The relationship between policy choices and war payoffs is important because it means the private signal that a state receives about its wartime strength contains information about its ability to use other coercive policies as well. For example, if a state has a wide range of

[1] These monotonicity results may not hold for models with alternative forms of uncertainty (e.g., Slantchev 2011; Spaniel and Malone 2019).

privately known cyber-exploits, then the state knows that it could not only perform well in a conventional war that uses cyberattacks but also perform well in a precise cyberattack against a target's infrastructure. Because this state possesses strong private cyber-capabilities, it is advantaged in both war and low-level conflict. Alternatively, if a state's leaders are privately concerned about losing popular domestic support, then they may be more willing to fight a war to create a rally-around-the-flag effect than they would be to choose intermediate options such as tariffs or low-level attacks (Baker and Oneal 2001; Chapman and Reiter 2004). Here, a strong private desire to garner domestic support could increase a state's utility from war while making less drastic options less attractive.[2]The question of whether private war payoffs are associated with greater or lower payoffs from alternative policy options is an empirical one, and its answer varies across cases and contexts—but these linkages undoubtedly exist, and their effects on the outcomes of crisis bargaining have not been systematically examined. We show that knowing the basic relationship between a state's signal of wartime strength and its ability to use coercive alternatives (i.e., whether types with higher war payoffs are advantaged or disadvantaged in the use of other instruments) provides immense leverage in predicting how private information affects the likelihood of conflict.

We provide a general characterization of the relationship between private war payoffs, the likelihood of conflict, and equilibrium payoffs in all flexible-response crisis bargaining games. The relationship between private signals about war payoffs and the effectiveness of low-level conflict is critical and refines existing understandings in several ways. First, the likelihood of war increases with the strength of a state's private ability or willingness to fight a war only when states with higher war payoffs also have less capacity for low-level responses or when low-level capabilities only increase slightly with war payoffs—for example, as in the rally-round-the-flag scenario above. Second, a state's equilibrium payoff is only guaranteed to increase with its privately known strength if greater war payoffs are associated with higher ability to execute flexible responses—for example, as in the cyber-conflict scenario. Third, unlike in traditional models, states with stronger private signals of wartime prowess do not necessarily receive better settlements when the interaction ends short of war. The value of settlement instead depends on the extent to which the state employs flexible coercive instruments.

Building on our formal analysis, we also offer new substantive insights into the value of specific military technological advancements. Existing research examines the value of certain coercive capabilities within specific military contexts, such as the value of airstrike

[2]Put another way, a leader's private motivations for going to war could also make that leader privately less inclined to engage in lower-level options.

capabilities in low-level conflicts and war (Pape 1996; Horowitz and Reiter 2001; Kreps and Fuhrmann 2011; Allen and Martinez Machain 2019). Most, though not all,[3] of the previous research in this area considers only how these technologies fare within an active conflict or after a challenger has already transgressed. However, it is also valuable to know how these capabilities shape outcomes at the bargaining table. That is where our analysis comes in. Once we identify whether improved private wartime capabilities improve hassling capabilities or are detrimental to them, then we are able to make clear predictions about how changes in wartime capabilities affect the likelihood of war, as well as distributive outcomes at the bargaining table. Armed with our results, scholars can leverage existing empirical and public policy research to identify how improving specific capabilities or altering the factors that increase resolve can affect the outcomes in any crisis setting where states have options short of all-out war.

Our work builds on a recent line of research on crisis bargaining and deterrence in which states are assumed to have multiple coercive options available to respond to a threat (Schultz 2010; Powell 2015; McCormack and Pascoe 2017; Coe 2018; Spaniel and Malone 2019; Qiu 2022*b*; Baliga, Bueno de Mesquita and Wolitzky 2020; Schram 2021; Di Lonardo and Tyson 2022*a*). This paper is the first to systematically examine how the spillover effects of improvements in one kind of conflict capability can also affect other response options, allowing us to offer novel insights into a wide range of previously neglected substantive settings. The most closely related work to ours is by Schram (2022), who considers a deterrence game with multiple conflict options and a publicly observed type that determines payoffs from conflict. Our key innovation is to assume that a state's private type affects both its war payoff and its low-level conflict costs, which drives our findings breaking prior monotonicity results (Banks 1990; Fey and Ramsay 2011).[4]

1 Flexible Responses in International Crises

Our goal is to understand crisis bargaining in cases where states have intermediate coercive options short of all-out war. It is helpful to start with an example. In 2006, Israel discovered that Syria was building a nuclear reactor. Internally, Israeli decision-makers viewed the possibility of a nuclear-armed Syria as an "existential threat" to the Israeli state (Opall-Rome 2018). Additionally, Israel possessed a covert capability at it's disposal: an electronic

[3]See Post (2019) as an exception.

[4]Methodologically, our paper is also similar to a class of work on political science topics that uses mechanism design to study treaty-making (Morrow and Cope 2021), bureaucracies and delegation (Ashworth and Sasso 2019), firm regulation (Baron and Besanko 1987, 1992), and legislation and policy-making (Meirowitz 2006).

warfare attack that could (at least temporarily) disable Syria's integrated air defense system, impairing Syria's ability to track incoming aircraft and act (Fulghum, Wall and Butler 2007). Traditional crisis bargaining models—where high private willingness or private capabilities to fight a war always leads to a greater likelihood of war (Banks 1990; Fey and Ramsay 2011)—would portray this as the exact setting in which we might expect Israel to resort to war. But instead, Israel used the electronic warfare attack as part of a limited airstrike on the reactor (Katz 2010). Israel's airstrike, known as Operation Outside the Box, successfully destroyed a critical component of the Syrian nuclear program while negating the need for a more expansive response.

Flexible-response crisis bargaining models capture political interactions like those surrounding the Syrian reactor. First, a challenger state may undertake some opportunistic and costly action, which we will call a "transgression," against a defender. Transgressions are beneficial to the challenger but harm the defender's interests. In our example, Syria's construction of the reactor was a transgression that could have eventually led to Syria possessing a nuclear bomb, thus strengthening its future leverage against Israel. Transgressions like this arise in theories of enforcement problems in bargaining (Schultz 2010), deterrence (Fearon 1997; Gurantz and Hirsch 2017), and endogenous power shifts (Debs and Monteiro 2014).[5] Examples of transgressions include states investing in new military technologies (Debs and Monteiro 2014; Gartzke and Lindsay 2017; Spaniel 2019), forming alliances (Benson and Smith 2022), or securing geopolitically valuable territory (Powell 2006).

In response to the transgression, the defender may settle the issue peacefully through negotiations, go to war to resolve the issue decisively, or engage in some low-level action to undercut the transgression. The first two options—settlement or war—are the two standard outcomes in models of crisis bargaining and deterrence. We will refer to the the defender's low-level response option as "hassling." As originally defined in Schram (2021), hassling is the use of limited conflict to degrade a challenger's rise. Our use of the term here is consistent with this definition, but expands it to include any actions by the defender that are costly, coercive, and fall outside of war, like sanctions, low-level conflict, limited airstrikes, or cyberattacks. In the Syrian case, Israel detected Syria's nuclear reactor and destroyed it. This is a form of hassling because it was a destructive blow to Syria's nuclear program, but it was not a decisive military move that would prevent the Assad regime from ever possessing a nuclear weapon. By contrast, the 2003 U.S. invasion of Iraq decisively prevented the Ba'athist regime from ever attaining nuclear weapons by overthrowing it, consistent with our

[5] The "transgression" here is similar to the challenger's first move in a deterrence game (e.g., Chassang and Miquel 2010; Di Lonardo and Tyson 2022*b*; Kydd and McManus 2017; Baliga and Sjöström 2020).

treatment of war. Hassling can take the form of limited airstrikes (Reiter 2005; Fuhrmann and Kreps 2010), hybrid conflict (Lanoszka 2016), aspects of gray-zone conflict (Mazarr 2015; Gannon et al. 2020), (limited) preventive war (Levy 2011), *fait accompli* (Tarar 2016), sanctions (McCormack and Pascoe 2017), or arming (Coe 2018).[6]

We assume the defender possesses a private type that influences both war and hassling capabilities. As one example, suppose a state possessed a covert capability where it could use an electronic warfare attack or a cyberattack to disable a rival's air defense systems. Based on these private capabilities, this state might be especially willing to conduct a war—but also willing to implement a low-level hassling attack to weaken a target. As a second example, suppose a different state is privately very concerned about the domestic political costs from fighting a war. Based on these private preferences, this state may be much more willing to turn to hassling techniques—like drone strikes—within a crisis.

Ultimately, the question of whether better private capabilities or the factors that result in a greater willingness to go to war also make low-level options more or less appealing is an empirical one, as we discuss below following the main results.

2 Why Flexible Responses Undermine Standard Crisis Bargaining Results

Before we present our general results, we consider two stylized examples of how introducing intermediate conflict options can undermine the canonical relationships between private information and outcomes in crisis bargaining games. These games are intentionally simple: our analysis below encompasses a wide range of bargaining protocols and possible moves, including offers, counteroffers, accept-reject plans, and costly or cheap talk signals. Both games are between a challenger (C) and defender (D). Nature moves first and decides whether D's war payoff is low ($\theta = \underline{\theta}$) or high ($\theta = \bar{\theta}$), with each outcome having positive probability.

D observes their private type, while C only knows the prior probability that D is a high or low type. This type represents D's privately known abilities, willingness, or political motivations for waging a war against C. Next, C selects whether to transgress ($t = 1$) or not ($t = 0$). Finally, D can accept the transgression, go to war over it, or conduct some limited response via hassling (where hassling could refer to sanctions, cyberattacks, gray-zone conflict, etc).

[6] In some ways, hassling resembles a restrained, "instrumental" style of coercive action conducted for the pursuit of limited goals and with the intent to not violate important escalation thresholds; in many ways, this echoes discussions in Kahn (2017) about how conflict will evolve in the Twenty-first Century.

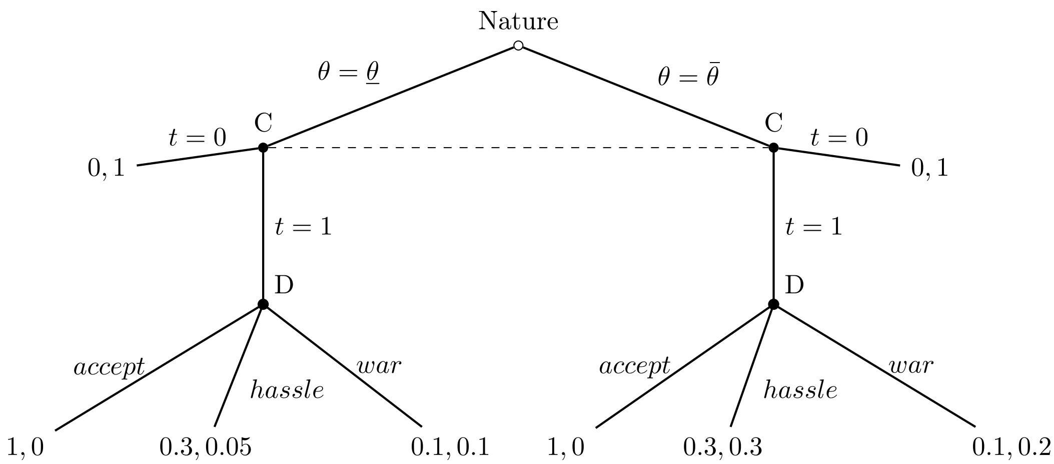

Figure 1: Greater private type θ implies less war.

C's payoffs are listed first. Note here that $\bar{\theta}$ has both greater wartime payoffs and hassling payoffs relative to $\underline{\theta}$. In equilibrium, C will transgress ($t = 1$), $\underline{\theta}$ D's will go to war and $\bar{\theta}$ D's will hassle.

Figure 1 illustrates a setting where only the weak type of D goes to war, contrary to the standard result that the probability of conflict increases with D's private value of war (Banks 1990; Fey and Ramsay 2011). This happens because the type with a greater war payoff is also more effective at hassling, as in the cyber-capability example above. Specifically, while war outcomes improve with θ, hassling outcomes improve even more. In equilibrium, the stronger type opts not to fight a war because hassling is better. Note that if the hassling option were not available, then the equilibrium would conform to the usual pattern, with both types going to war.

We next consider an interaction in which the stronger type of D receives a lower payoff, illustrated in Figure 2. This stands in contrast to the standard result that actors with higher private war payoffs end up better off (Banks 1990; Fey and Ramsay 2011). In this example, greater private payoffs from fighting a war corresponds to a lower hassling payoffs, as in the rally-round-the-flag example above. In equilibrium, $\underline{\theta}$ hassles and $\overline{\theta}$ goes to war. The improvement in the stronger type's war payoff is not enough to offset the decrease in its hassling payoff. Once more, if the hassling option were unavailable, we would return to the standard pattern: both types would go to war in equilibrium, with the stronger one yielding a greater utility.

These two games clearly illustrate how introducing multiple related conflict options can undermine the patterns observed in standard crisis bargaining models. However, because the models are so simple, one might wonder how much they tell us about crisis bargaining

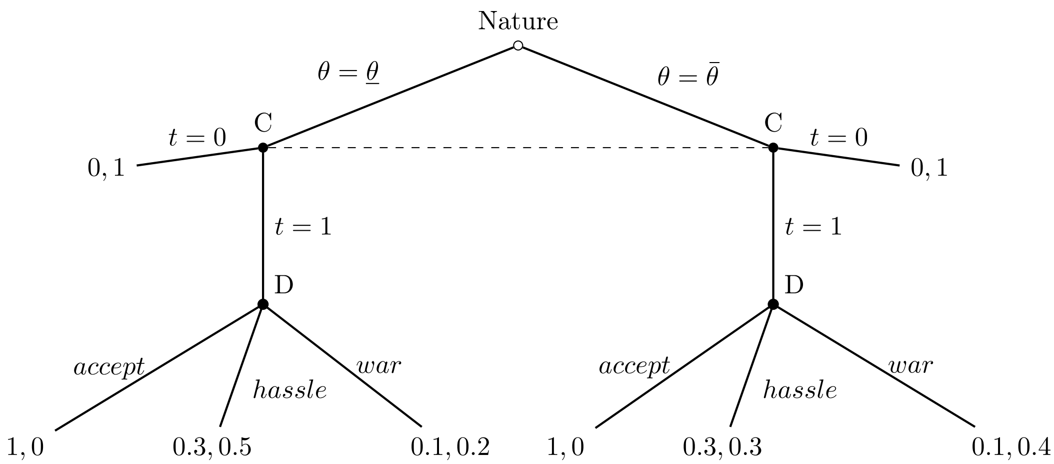

Figure 2: Greater private type θ implies lower utility

C's payoffs are listed first. Here we assume that $\bar{\theta}$ has greater wartime payoffs and lower hassling payoffs than $\underline{\theta}$. In equilibrium, C will transgress ($t = 1$), $\underline{\theta}$ D's will hassle and $\bar{\theta}$ D's will go to war.

with flexible responses in general. More broadly, the anarchic nature of international politics means there is no one "correct" game form for crisis bargaining—states themselves choose how to negotiate (Waltz 1979; Axelrod and Keohane 1985; Mearsheimer et al. 2001). To ensure that our findings are as broad as possible, we employ the game-free methodology of Banks (1990) and Fey and Ramsay (2011), identifying properties that arise from foundational requirements of equilibrium rather than idiosyncratic features of any given game tree. We find, in fact, that the outcomes in Figures 1 and 2 are typical of crisis bargaining with flexible responses.

3 Formal Framework

We consider a class of models where crisis negotiations may end in pure peace, total war, or some costly non-war outcome. We assume that a state's private information may affect its payoffs from both war and the costly intermediate option, and we use the tools of Bayesian mechanism design to obtain general results about the relationship between private type and the equilibrium outcomes.

3.1 Structure of the Interaction

The players are a *C*hallenger and *D*efender in dispute over a prize of size 1. At the outset of the game, Nature assigns D's private type, $\theta \in \mathbb{R}$. D's war payoff increases with θ, while D's cost of hassling may increase, decrease, or neither. The realized value of θ is known only to D, but its prior distribution is common knowledge. Let F denote the cumulative distribution function of this prior distribution, and let Θ denote its support.

The interaction takes the familiar form of a crisis negotiation, except each state may engage in activity that affects outcomes short of war. First, C selects a transgression $t \in \mathcal{T} \subseteq \mathbb{R}_+$. After this transgression, C and D partake in a bargaining process that may end in war or in some hassling response by D. We place no particular structure on the bargaining process; we only assume the choices here determine whether the game ends in war and, if not, how the prize is divided. Let $b_C \in \mathcal{B}_C$ denote C's bargaining strategy (offers, counteroffers, accept-reject plans, costly or cheap talk signals, etc.). D's strategy consists of an analogous bargaining strategy $b_D \in \mathcal{B}_D$, as well as a level of hassling, $h \in \mathcal{H} \subseteq \mathbb{R}_+$.[7]

A game form G consists of the bargaining action spaces, $\mathcal{B}_A$ and $\mathcal{B}_D$, along with an outcome function g that maps the choices (t, h, b_C, b_D) into the set of possible crisis bargaining outcomes.[8] We decompose the outcome function g into three components: whether war occurs, what C receives from bargaining, and what D receives from bargaining.[9] Whether war occurs depends solely on actions taken in bargaining. Let $\pi^g(b_C, b_D) \in \{0, 1\}$ be an indicator for whether the interaction ends without war.[10] Conditional on war not occurring, each player's payoff depends on the bargaining behavior, C's choice of transgression, and D's selection of hassling. Let $V_C^g(t, h, b_C, b_D)$ and $V_D^g(t, h, b_C, b_D)$ denote the benefits that C and D receive, respectively, if war is avoided.

D's private type and the selected transgression and hassling levels shape the final utilities in the game. If war occurs, war payoffs depend on D's private information, but they do not depend on any of the endogenous choices in the game, including transgressions and hassling. We therefore write war payoffs as $W_C(\theta)$ and $W_D(\theta)$. We assume W_D is strictly increasing, so higher types of D can be interpreted as stronger in wartime. If war is avoided, each player receives their division of the spoils but must pay the cost of their transgression or hassling. Let $K_C(t)$ denote the cost to C, and let $K_D(h, \theta)$ denote the cost to D.[11] We assume that

[7]We place no restriction on whether hassling is chosen before, during, or after the bargaining process—all that matters is that the cost of any given h is independent of b_C and b_D.

[8]The game form represents the elements of the model that are specific to a particular bargaining protocol. Thus, we treat the type space, prior distribution, transgression and hassling action sets, cost functions, and war payoff functions as primitives of the interaction rather than features of a specific game form.

[9]Unlike Banks (1990) and Fey and Ramsay (2011), we allow for inefficient settlements. This means that D's value from bargaining cannot be immediately deduced from C's, and vice versa.

[10]By ruling out $\pi^g \in (0, 1)$, we are implicitly assuming that the bargaining process has no exogenous random components (see Fey and Kenkel 2021).

[11]The players' payoffs from any non-war outcome do not depend on D's private information, except insofar as it affects D's hassling cost.

K_C is strictly increasing in t, and we assume that K_D is strictly increasing in h. For now, we are agnostic whether K_D is increasing or decreasing in θ. We let $h = 0$ denote no hassling, which entails assuming that $0 \in \mathcal{H}$ and $K_D(0, \theta) = 0$ for all θ. Putting these together, the players' utility functions in a given game form are:

$$u_C^g(t, h, b_C, b_D \mid \theta) = (1 - \pi^g(b_C, b_D))W_C(\theta) + \pi^g(b_C, b_D)[V_C^g(t, h, b_C, b_D) - K_C(t)],$$
$$u_D^g(t, h, b_C, b_D \mid \theta) = (1 - \pi^g(b_C, b_D))W_D(\theta) + \pi^g(b_C, b_D)[V_D^g(t, h, b_C, b_D) - K_D(h, \theta)].$$

We restrict our attention to game forms in which neither player can force a settlement on the other. This assumption reflects the anarchic nature of international politics, in which states always have the option to resort to force. A sufficient condition is that each player has an action $b_i' \in \mathcal{B}_i$ to guarantee war: $\pi^g(b_i', b_j) = 0$ for all $b_j \in \mathcal{B}_j$. As we show below, this condition places important limits on what kinds of outcomes are sustainable as equilibria.

The relationship between D's private type and hassling ability is central to our analysis. We will show that the effects of θ on equilibrium outcomes depend critically on whether greater payoffs from war are associated with higher or lower costs of hassling. In general, when comparing types θ' and θ'', we say that θ'' has greater hassling effectiveness than θ' if $K_D(h, \theta'') < K_D(h, \theta')$ for all $h > 0$. We say that θ improves hassling effectiveness if higher types always have greater hassling effectiveness than lower types. In the opposite case, when K_D strictly increases with D's type, we say θ degrades hassling effectiveness.

3.2 Solution Concept and Direct Mechanisms

We restrict attention to pure strategy perfect Bayesian equilibria of each flexible-response crisis bargaining game. Depending on the bargaining protocol and the equilibrium selected, the equilibrium path may be very complex, involving numerous offers and counteroffers before concluding, or it may be simple, ending quickly in war or a settlement. We will not dwell on the details of bargaining itself, as our primary concern is the outcome of the interaction: whether war occurs, and if not, what each party receives from a bargained outcome.

We will focus on the incentives of D, the player with private information. Given an equilibrium of a flexible-response crisis bargaining game, we can summarize the outcome of the game for each type of D with three functions:[12]

- Their hassling level, $h(\theta)$.

[12] In the Appendix, we formally define an equilibrium and describe how a direct mechanism can be derived from it. See also the discussion in Fey and Ramsay (2011).

- Whether a bargained outcome prevails, $\pi(\theta)$.
- Their settlement value in case of a bargained resolution, $V_D(\theta)$.

A direct mechanism for D consists of these functions, (h, π, V_D). If type θ of D were to follow the equilibrium bargaining strategy of type θ', D's expected utility from doing so would be:

$$\Phi_D(\theta' \mid \theta) = \underbrace{(1-\pi(\theta'))W_D(\theta)}_{\text{war}} + \underbrace{\pi(\theta')[V_D(\theta') - K_D(h(\theta'), \theta)]}_{\text{bargained outcome, possibly with hassling}}.$$

While mimicking another type's strategy may change the hassling level, the occurrence of conflict, and the settlement value, it does not change D's war payoff, nor the cost D pays for any given hassling level.[13] The key requirement of Bayesian equilibrium is that no type can increase its payoff by mimicking another type's bargaining strategy. We can phrase this requirement as an incentive compatibility condition on the direct mechanism. Let $U_D(\theta)$ denote each type's expected utility along the path of play, so that $U_D(\theta) = \Phi_D(\theta \mid \theta)$.

Definition 1. A direct mechanism (h, π, V_D) is incentive compatible if

$$U_D(\theta) \geq \Phi_D(\theta' \mid \theta) \qquad \text{for all } \theta, \theta' \in \Theta. \tag{IC}$$

We rely on the revelation principle: for any Bayesian Nash equilibrium of a particular game form, there is an incentive-compatible direct mechanism that yields the same outcome (Myerson 1979). Logically, this means that if we find that some property holds for all incentive-compatible direct mechanisms, than it is true of all equilibria of all flexible-response crisis bargaining games. Without bogging ourselves down in the particulars of how crisis bargaining plays out in any particular game, we are able to characterize robust properties of the outcomes of any flexible-response crisis bargaining game.

Recall that we only consider game forms in which neither player can impose a settlement on the other. This condition ensures that no type of D may receive less than its war payoff in equilibrium—if a settlement would yield less, then it would be profitable for D to deviate to fighting a war. In the language of mechanism design, this requirement amounts to a participation constraint, or what Fey and Ramsay (2011) call voluntary agreements in the crisis bargaining context.

[13] In contrast with the inefficient settlements studied by Fey and Ramsay (2009), we assume the cost of hassling is a function of private type, meaning different types may value the same non-war outcome differently.

Definition 2. A direct mechanism (h, π, V_D) has voluntary agreements if

$$\pi(\theta)[V_D(\theta) - K_D(h(\theta), \theta)] \geq \pi(\theta) W_D(\theta) \qquad \text{for all } \theta \in \Theta. \tag{VA}$$

Naturally, the voluntary agreements condition is automatically satisfied for those types that go to war in equilibrium. The constraint only applies to the types that settle—the settlement must yield at least as much as their war payoff, even when accounting for the costs of the hassling. Throughout the analysis, we will restrict attention to direct mechanisms that satisfy both (IC) and (VA), as any equilibrium of a flexible-response crisis bargaining game with voluntary agreements must be outcome-equivalent to some such mechanism (Fey and Ramsay 2011).

4 Private Type and the Probability of War

In crisis bargaining games without flexible responses, private signals of high strength or resolve are associated with a greater equilibrium probability of war (e.g., Morrow 1989; Fearon 1995; Schultz 1999; Tarar 2021). A simple intuition drives this result. If some type finds it worthwhile to risk war to receive a better deal at the bargaining table, then all stronger types must be willing to run at least as great a risk—after all, by mimicking weaker types, strong types could attain the same bargained settlement without incurring as high of costs from the fail case of fighting.

For equilibria with no hassling, our analysis recovers the classic positive relationship between private type and the likelihood of conflict. As long as $h(\theta) = 0$ for all types, weaker types never have a greater chance of conflict than higher types. The following result recovers Lemma 1 of Banks (1990) as a special case in our environment.[14]

Lemma 1. *If $h = 0$ and $\theta' < \theta''$, then $\pi(\theta') \geq \pi(\theta'')$.*

This result confirms that the exceptions we find to the classic monotonicity results are due exclusively to our introduction of flexible responses, not to any other feature of our formal framework. Once states begin to employ alternative instruments for altering the balance of bargaining power, this straightforward relationship between private strength and the risk of war holds only under special conditions.

[14]Though we restrict to deterministic outcomes and thus $\pi(\theta) \in \{0, 1\}$ in the bulk of our analysis, the proof of Lemma 1 holds even if we allow for probabilistic outcomes. All proofs are in the Appendix.

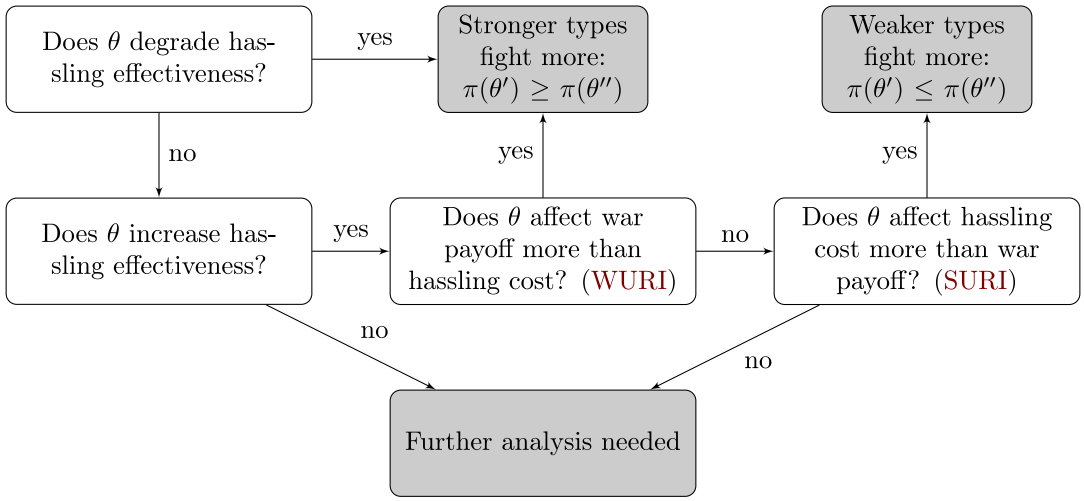

Figure 3: The relationship between type ($\theta' < \theta''$) and war likelihood in all flexible-response crisis bargaining games.

The flow chart in Figure 3 summarizes our findings on the relationship between private information and the occurrence of conflict. If private strength degrades hassling effectiveness, then stronger types are more likely to fight a war, just as in models without flexible responses. The same is true if private strength increases hassling effectiveness and has a stronger effect on war payoffs than on the cost of hassling (WURI condition, formally defined below). However, we find that weaker types are more likely to fight a war—the opposite of the traditional result—when private strength improves hassling effectiveness but affects war payoffs less than hassling costs (SURI, also defined below).

When hassling takes place, the effect of private type on the chance of war depends on whether a state's private war capability improves or degrades its hassling effectiveness. If private type degrades hassling effectiveness—i.e., if types with greater war payoffs also have greater costs of hassling—then higher types are weakly more likely to go to conflict.

Proposition 1. *Assume θ degrades hassling effectiveness. If $\theta' < \theta''$, then $\pi(\theta') \geq \pi(\theta'')$.*

Because the value of an efficient settlement without hassling is the same regardless of θ, higher types have the most incentive to choose war over an efficient settlement. If private type degrades hassling effectiveness, then this logic carries over to settlements involving hassling as well. If some type of D prefers war over a settlement with hassling level $h \geq 0$, then all stronger types must have the same preference: they have an even higher war payoff and would pay a greater cost from the same level of hassling.

When private type is instead associated with lower costs of hassling, we need more conditions to characterize its effect on the likelihood of war. In this case, not only is war more attractive to stronger types of D, but so is any given settlement with hassling. Because war payoffs and settlement payoffs are now moving in the same direction as D's type increases, the critical question for our purposes is which rate of increase is quicker. The probability of conflict increases with D's private strength in equilibrium if θ has a stronger effect on war payoffs than on the cost of hassling. On the other hand, if the effect of θ on hassling cost is dominant, then weaker types are more likely to fight a war in equilibrium—the opposite of the pattern found in traditional crisis bargaining games.

To prove these claims, we must formally state what it means for θ to affect war payoffs more than the cost of hassling, or vice versa. We say that the war utility is relatively increasing (WURI) when θ has a greater marginal effect on war payoffs than on the cost of hassling. In the opposite case, we say the settlement utility is relatively increasing (SURI).

Definition 3. In a direct mechanism, the *war utility is relatively increasing* if

$$\begin{aligned} W_D(\theta'') - W_D(\theta') &> K_D(h(\theta''), \theta') - K_D(h(\theta''), \theta'') \\ &\text{for all } \theta', \theta'' \in \Theta \text{ such that } \theta' < \theta'' \text{ and } \pi(\theta'') = 1. \end{aligned} \tag{WURI}$$

The *settlement utility is relatively increasing* if

$$\begin{aligned} W_D(\theta'') - W_D(\theta') &< K_D(h(\theta'), \theta') - K_D(h(\theta'), \theta'') \\ &\text{for all } \theta', \theta'' \in \Theta \text{ such that } \theta' < \theta'' \text{ and } \pi(\theta') = 1. \end{aligned} \tag{SURI}$$

If either of these holds and private strength increases hassling effectiveness, then we can pin down the type's effect on the equilibrium chance of conflict.

Proposition 2. *Assume θ improves hassling effectiveness, and let $\theta' < \theta''$. If (WURI) holds, then $\pi(\theta') \geq \pi(\theta'')$. If (SURI) holds, then $\pi(\theta') \leq \pi(\theta'')$.*

The result shows that the conventional relationship between private information and the likelihood of conflict is not robust to the introduction of hassling that affects payoffs from bargaining. Assuming that types with greater war effectiveness are also more effective at hassling activities, the relationship between θ and the likelihood of conflict depends critically on the technology of hassling. If the marginal effect of D's type on the costs of hassling always outweighs its effect on the war payoffs, then we have the opposite of the usual result, with stronger types less likely to fight a war on the path of play.

The two conditions we have outlined here are mutually exclusive (except in the trivial case where all types end up going to war in equilibrium), but they are not mutually exhaustive. Depending on the functional forms of W_D and K_D, it is possible for the marginal effect of θ on the war payoff to be relatively strong for some types and relatively weak for others. In this scenario, we cannot generally characterize the relationship between D's private type and which outcome prevails in equilibrium.

While Proposition 2 is useful for understanding how private information affects the occurrence of war in flexible response crisis bargaining games, its practical applicability is somewhat limited. Ideally, we would be able to say on the basis of the model primitives—the war payoff and hassling cost functions—whether stronger types will be associated with a greater likelihood of conflict in any given strategic environment. However, the WURI and SURI conditions refer to the levels of hassling chosen on the path of play. This raises the possibility that the relationship between D's private type and the likelihood of conflict may vary depending on the exact bargaining protocol.

With additional conditions on the model primitives, we can ensure that the war utility is increasing relative to the hassling utility, meaning that the likelihood of conflict increases with D's type. In particular, we need the hassling cost function to have decreasing differences, which means that types with lower absolute costs also have lower marginal costs. This condition ensures that D's settlement utility has the single-crossing property, allowing us to characterize monotone comparative statics without imposing specific functional forms (Ashworth and Bueno de Mesquita 2006).

Definition 4. The cost function K_D has decreasing differences in h and θ if

$$\begin{aligned} &\theta' \text{ has greater hassling effectiveness than } \theta \Rightarrow \\ &\quad K_D(h', \theta') - K_D(h, \theta') < K_D(h', \theta) - K_D(h, \theta) \text{ for all } h < h'. \end{aligned} \tag{DD}$$

In addition to the cost function having decreasing differences, we also need the marginal effect of θ on the war payoff to always exceed its marginal effect on the hassling cost when h is at its upper bound. Under these conditions, higher types are more likely to end up at war regardless of the exact negotiating protocol employed.

Lemma 2. *Assume θ improves hassling effectiveness,* (DD) *holds, and* $\max \mathcal{H} = \bar{h} < \infty$. *If*

$W_D(\theta'') - W_D(\theta') > K_D(\bar{h}, \theta') - K_D(\bar{h}, \theta'')$ for all $\theta', \theta'' \in \Theta$ such that $\theta' < \theta''$, then (WURI) holds.

There is not an analogous sufficient condition for the settlement utility to be relatively increasing. The obstacle here is our assumption that $h = 0$ has the same cost (zero) for all types. This means the marginal effect of θ on hassling costs is zero for $h = 0$, ruling out any way for the marginal effect of θ on the hassling cost to always exceed its effect on the war payoff. At most, if we assume decreasing differences in the cost of hassling, we can make the SURI condition slightly less onerous to check. In this case, letting $\underline{h}$ denote the minimal hassling among types that end up in a bargained resolution, a sufficient condition is that $W_D(\theta') - W_D(\theta) < K_D(\underline{h}, \theta) - K_D(\underline{h}, \theta')$ for all $\theta < \theta'$. In any equilibrium where this condition holds, we cannot have low types settle while high types go to war.

5 Private Type and Payoffs

A second key regularity in crisis bargaining games without flexible responses is that a better private war capability corresponds to a better expected equilibrium payoff, even when war does not occur (Banks 1990; Fey and Ramsay 2011). In this class of models, greater private strength increases an actor's willingness to run the risk of war, allowing the actor to extract more from negotiations if the interaction does not ultimately end in war.

We recover the same relationship between private strength and expected equilibrium payoffs in the flexible response environment when hassling does not take place along the path of play. The following result is our analogue of Lemma 4 from Banks (1990).

Lemma 3. *If $h = 0$ and $\theta' < \theta''$, then $U_D(\theta') \leq U_D(\theta'')$.*

Once we introduce additional ways for states to operate outside of war and peaceful negotiations, the tight relationship between private war capability and equilibrium payoffs no longer holds. The flowchart in Figure 4 summarizes the relationship between private strength and utility in flexible-response crisis bargaining games. Obviously, among types that end up going to war in equilibrium, stronger types are always better off. Outside of that case, however, private strength is only guaranteed to increase payoffs when it is also associated with greater hassling capability. If private strength instead degrades hassling capability, we find the opposite: types with lower private strength have greater payoffs when war does not occur.

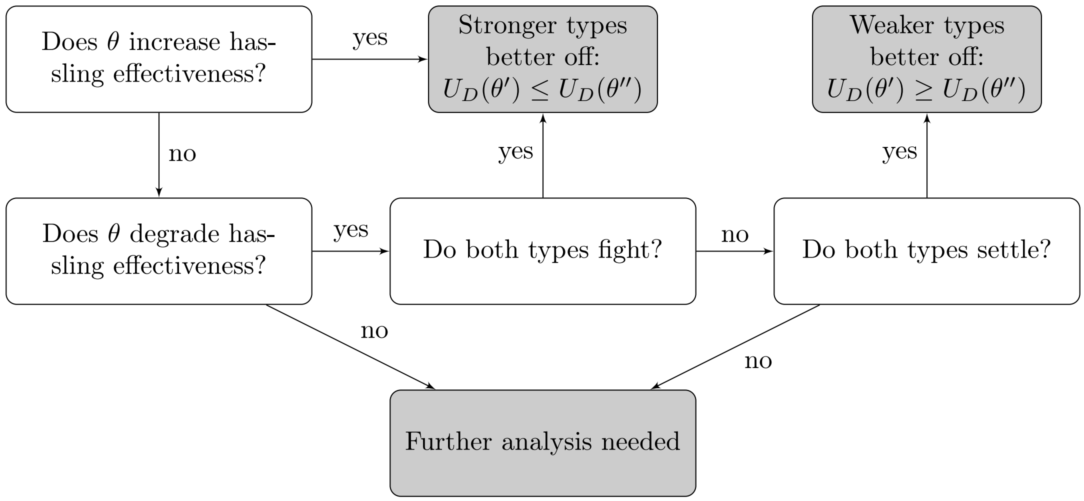

Figure 4: The relationship between type ($\theta' < \theta''$) and equilibrium payoffs in all flexible-response crisis bargaining games.

We recover the positive relationship between private strength and equilibrium payoffs when θ improves hassling effectiveness. In this case, stronger types have an advantage in both channels of bargaining leverage—hassling and the threat of war—and therefore never come away worse off at the bargaining table. In fact, a stronger type has a strictly greater payoff than all weaker types whenever it goes to war in equilibrium or engages in non-zero hassling.

Proposition 3. *Assume θ improves hassling effectiveness. If $\theta' < \theta''$, then $U_D(\theta') \leq U_D(\theta'')$. The inequality is strict if $\pi(\theta') = 0$ or $h(\theta') > 0$.*

A simple incentive compatibility logic is behind this result. Consider the outcomes for two types of D, one weaker and one stronger. If the weaker one goes to war, then obviously the stronger one could do better by going to war as well. Conversely, if the weaker one settles, the stronger type could get the same terms of settlement as the weaker one by choosing the same hassling level and bargaining actions. Moreover, under the condition that private type improves hassling capabilities, the stronger type's hassling cost would be no greater than that of the weaker type; in fact, it would be strictly less if $h > 0$. No matter what the outcome for the weaker type is, the stronger type would receive a weakly greater payoff from the same strategy. Consequently, the stronger type's expected equilibrium payoff must be no lower than the weaker one's.

If private strength is instead associated with lower hassling effectiveness, then we find an exception to the traditional positive relationship between private type and equilibrium payoff. In this case, the relationship is U-shaped. Low types, which have poor war effectiveness but relatively low costs of hassling, choose to settle rather than to fight a war in equilibrium. Among these types, lower private strength is associated with greater hassling ability, and thus a greater equilibrium payoff. At a certain level of strength, however, it becomes profitable to fight a war rather than settle. After this point, greater military strength leads to a greater payoff.

Proposition 4. *Assume θ degrades hassling effectiveness. There exists $\hat{\theta}$ such that $\pi(\theta) = 1$ for all $\theta < \hat{\theta}$ and $\pi(\theta) = 0$ for all $\theta > \hat{\theta}$. If $\theta' < \theta'' < \hat{\theta}$, then $U_D(\theta') \geq U_D(\theta'')$ (strictly if $h(\theta'') > 0$). If $\hat{\theta} < \theta' < \theta''$, then $U_D(\theta') < U_D(\theta'')$.*

This result illustrates the new sources of bargaining leverage that arise in crisis bargaining games with flexible responses. In ordinary crisis bargaining games, a state's sole source of bargaining power is its threat to resort to war. In our framework, hassling provides another means of shifting the balance of spoils. There is a direct effect of hassling efficiency, where more effective types end up better off because they can afford to hassle more, thereby shifting the balance of goods in their favor. There is also an indirect effect: more effective types can pay a lower cost to undertake the same amount of hassling. Both effects contribute to the negative relationship between private strength and payoffs from non-war outcomes when θ degrades hassling effectiveness.

In analyzing private strength's relationship with both the probability of war and equilibrium payoffs in flexible-response crisis bargaining games, we have identified some conditions under which our results diverge with the traditional patterns and other conditions under which they agree. Interestingly, we find only one case in which *both* the chance of conflict and the equilibrium utility increase with private strength, as in the traditional pattern—namely, when θ increases hassling effectiveness but has an even stronger effect on war payoffs (WURI). The ordinary crisis bargaining framework may, in a sense, be considered a special case of these more general conditions. If private strength increases hassling effectiveness while having a relatively low effect on war payoffs (SURI), then equilibrium payoffs increase with θ as in traditional models, but the probability of war decreases. We see the converse pattern if private strength degrades hassling effectiveness: the probability of war increases with θ as usual, but the equilibrium utility is U-shaped.

6 Terms of Settlement

In ordinary crisis bargaining games, the only way to get a better deal at the bargaining table is to run a greater risk of war (Banks 1990, Lemma 3). As we observed in the previous section, flexible responses like hassling introduce a new means for states to obtain better terms from a settlement. Even with little or no threat of war, a state may use transgressions or hassling to shift the balance of bargaining power.

But because hassling is costly, any increase in hassling must come with a commensurate benefit in the terms of settlement. Consequently, whenever we compare two types that both end up avoiding war in the equilibrium of a flexible response crisis bargaining game, the one that hassles more must get a better deal. If both hassle the same amount, then they should receive identical settlements—just like states that run the same risk of war in ordinary crisis bargaining games.

Proposition 5. *If $\pi(\theta) = \pi(\theta') = 1$ and $h(\theta) \leq h(\theta')$, then $V_D(\theta) \leq V_D(\theta')$. Furthermore, if $h(\theta) < h(\theta')$, then $V_D(\theta) < V_D(\theta')$.*

Placing additional structure on the model primitives allows us to be even more specific about the relationship between the extent of hassling and the value of settlement. First, we will assume D's type is drawn from an interval, $\theta \in [\underline{\theta}, \bar{\theta}]$. This requirement effectively allows us to strengthen the incentive compatibility conditions for equilibrium, as we can now say that every type of D must find it unprofitable to mimic the strategy of a marginally stronger or weaker type. Second, we will assume a degree of differentiability (and thus continuity) in the relationship between private type and war payoffs, as well as that between private type, hassling amount, and the cost of hassling.[15] These assumptions allow us to characterize local incentive compatibility conditions—the lack of incentive to mimic a slightly lower or higher type—in terms of derivatives of the war payoff and hassling cost functions. We refer to the collection of these assumptions as bounded variation conditions, or (BV).

[15]Specifically, we assume W_D and K_D are Lipschitz continuous, a weaker requirement than continuous differentiability.

Definition 5. The model has bounded variation if W_D and K_D are differentiable and

$$\left.\begin{aligned} \Theta = [\underline{\theta}, \bar{\theta}] \qquad & \text{where } \underline{\theta} < \bar{\theta}, \\ |W_D(\theta) - W_D(\theta')| \leq M_W |\theta - \theta'| \qquad & \text{for all } \theta, \theta' \in \Theta, \text{ where } M_W < \infty, \\ |K_D(h, \theta) - K_D(h', \theta')| \leq \qquad & \text{for all } h, h' \in \mathcal{H} \\ M_D \|(h, \theta) - (h', \theta')\| \qquad & \quad \text{and } \theta, \theta' \in \Theta, \text{ where } M_D < \infty, \end{aligned}\right\} \tag{BV}$$

where $M_W \geq 0$ and $M_D \geq 0$ are real constants.

The bounded variation conditions allow us to apply the "envelope theorem" commonly employed in mechanism design analyses of crisis bargaining models (Banks 1990; Fey and Ramsay 2011).[16] Given just a few endogenous elements of the equilibrium, we can determine every type's equilibrium payoff, which in turn will allow us to back out the precise terms of settlement for each type that ends up avoiding war. All we need to know are the lowest type's equilibrium utility,[17] whether each type ends up at war, and the extent of hassling carried out by those types that end up avoiding war. The following proposition gives a precise statement of $U_D(\theta)$ as a function of these equilibrium quantities.

Proposition 6. *Assume* (BV) *holds. For all* $\theta_0 \in \Theta$,

$$U_D(\theta_0) = U_D(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_0} (1 - \pi(\theta)) \frac{dW_D(\theta)}{d\theta} \, d\theta - \int_{\underline{\theta}}^{\theta_0} \pi(\theta) \left. \frac{\partial K_D(h, \theta)}{\partial \theta} \right|_{h = h(\theta)} d\theta.$$

This complex expression boils down to two essential facts about the relationship between private type and equilibrium payoffs. First, for types that go to war, the marginal increase in utility as θ increases is, naturally, the same as the marginal increase in war payoff. Second, among those that settle in equilibrium, the marginal change in equilibrium payoff depends exactly on the marginal effect of private type on the cost of hassling. This second fact is what allows us to pin down the value of settlement once we know which types settle and how much they spend on hassling. Suppose there is an interval of types $[\theta', \theta''] \subseteq \Theta$ which all choose to settle in equilibrium. We can use Proposition 6 to characterize how the terms

[16] The M_W and M_D terms are simply real-valued constants.

[17] In fact, all that is necessary is to know the equilibrium payoff of a single type, not necessarily that of $\underline{\theta}$.

of the bargain differ between the poles of this interval:

$$V_D(\theta'') - V_D(\theta') = \underbrace{K_D(h(\theta''), \theta'') - K_D(h(\theta'), \theta')}_{\text{cost difference}} - \underbrace{\int_{\theta'}^{\theta''} \left. \frac{\partial K_D(h, \theta)}{\partial \theta} \right|_{h=h(\theta)} d\theta}_{\text{effectiveness premium}}.$$

The first term here, the cost difference, is a baseline: incentive compatibility means the terms of settlement must adjust at least roughly in accordance with the cost paid for hassling. If not for the second term, the effectiveness premium, then each type $\theta \in [\theta', \theta'']$ would have the same equilibrium payoff. The effectiveness premium represents the additional benefit that states with greater hassling effectiveness can extract from bargaining. For example, suppose θ improves hassling capability, so K_D is decreasing in θ. Then the effectiveness term will be positive, more so if there is a steep relationship between private type and the marginal cost of hassling. Conversely, there will be no benefit from the effectiveness premium if hassling does not take place. If $h(\theta) = 0$ for all $\theta \in [\theta', \theta'']$, then the cost difference and effectiveness premium are both zero, and all types in this interval receive the same settlement. We are then back in the world of ordinary crisis bargaining games, where the only source of bargaining leverage is the threat of war.

In the special case where the model has bounded variation and θ degrades hassling capability, we can further pin down the value of settlement. We know from Proposition 1 that any equilibrium in this case will be characterized by a cutpoint $\hat{\theta} \in \Theta$, with all types below $\hat{\theta}$ settling in equilibrium and all types above it going to war. As we show in Corollary A.1 in the Appendix, we can then characterize the settlement value for all types below the cutpoint in terms of the cutpoint type's war payoff and the choice of hassling by each intermediate type.

7 Additional Analysis

We have focused on key points of distinction between ordinary crisis bargaining models and those with flexible responses—namely, in the relationships between private type and the probability of war, the equilibrium payoffs, and the terms of settlement. Our game-free approach provides us with further characterization of choices and outcomes in flexible-response crisis bargaining games. Here we briefly summarize these additional findings, leaving the formal details and results to Appendix B.

Amount of hassling. We have shown that greater hassling is the only way for a state to increase the share of the prize it receives from a negotiated settlement (Proposition 5). This raises the question of whether we can predict which types will hassle more in equilibrium, as these types will get the most favorable terms at the bargaining table. As long as the cost of hassling has decreasing differences (DD), incentive compatibility implies that the equilibrium level of hassling weakly increases with D's effectiveness at doing so.[18] This holds because the benefits of hassling are the same for all types, but the costs are lower for more effective types. The equilibrium degree of hassling, $h(\theta)$, therefore increases with θ if wartime strength enhances hassling capability and decreases with θ if strength instead degrades hassling capability.

Working with our game-free methodology, we cannot go much further than to rule out equilibria in which less effective types engage in greater hassling. In fact, if private strength degrades hassling capability, then virtually any pattern of hassling that decreases with θ can be supported as the equilibrium of some crisis bargaining game.[19] In other words, different bargaining protocols can lead to radically different amounts of hassling in equilibrium, even when the underlying model primitives (war payoffs, costs of hassling, etc.) are the same.

Possibility of peace. Is conflict an inevitable result of crisis negotiations between certain states, or can it be avoided with the right choice of bargaining protocol? As in previous game-free analyses of crisis bargaining (Fey and Ramsay 2009, 2011), our approach is well suited to address whether any bargaining game could result in peace in equilibrium. Even if one particular game form ends inefficiently—whether through war or through high levels of transgressions and hassling—that does not mean such an outcome is inevitable, as states interacting under anarchy are not bound to follow any particular bargaining protocol.

We analyze when there is at least one game form that ends in a settlement with zero transgressions or hassling regardless of D's type. Such an equilibrium would essentially entail all types of D pooling on a bargaining strategy, so the conditions for its existence depend on C's expected war payoff $\mathbb{E}[W_C(\theta)]$, where the expectation is taken over the prior distribution of θ. As long as the value of the prize is greater than the sum of C's expected war payoff and the strongest type of D's war payoff, there is a game form that ends with no war, transgressions, or hassling.[20] This is almost identical to the condition for a peaceful solution to the ordinary crisis bargaining problem (Fey and Ramsay 2009, 2011). Additionally, our condition for peaceful outcomes to flexible-response crisis bargaining is always satisfied when

[18] Proposition B.1 in the Appendix.
[19] Proposition B.2 in the Appendix.
[20] Proposition B.3 in the Appendix.

D's type θ only affects $W_D(\theta)$ (e.g., because it represents the cost of war rather than the balance of capabilities).[21]

Separate war payoffs and hassling costs. So far, we have treated D's war payoffs and hassling costs both as functions of the same private type, θ. We extend the framework to make these separate components of D's type, so that there is no longer necessarily a one-to-one correspondence between war payoffs and hassling effectiveness. The main results of our analysis continue to hold in this setting. An increase in D's war payoff combined with a decline in its hassling effectiveness can only increase the chance of war in equilibrium.[22] However, the effect of a simultaneous increase in D's war payoff and hassling effectiveness depends on the relative magnitudes of the differences.[23] Finally, D's equilibrium payoff increases with its war payoff and decreases with the cost of hassling, all else equal.[24]

8 Empirical Implications

How does this theory update our understanding of conflict? First and most simply, our analysis offers a new perspective on what conditions lead to war. We find that it is inadequate to only consider private wartime payoffs to identify when a crisis will end in war. As discussed previously, in the lead-up to Operation Outside the Box, Israel possessed both a high private willingness to go to war with Syria over their reactor and a robust private capacity to conduct an attack. If we only consider the lessons from traditional crisis bargaining models, this is the set of circumstance lends itself to a greater probability of war. Instead, the same capabilities that would have made Israel effective in a war also made Israel effective in the low-level attack they eventually carried out. Our theory suggests that when private war and hassling capabilities move in tandem, we may not expect states that are better at war to be more likely to go to war—instead, they may switch into low-level conflict. To that end, we offer a new, general result on when we should expect war to occur.

Second, our analysis offers insight into how developing military capabilities shape international affairs (i.e. when crises will end in war) and determine how states fare in international crisis bargaining. Consider the ongoing debate over the usefulness of aerial bombing in conflict. Earlier studies have shown that aerial bombing capabilities can be useful in hassling operations (Kreps and Fuhrmann 2011) or in a conventional war (Pape 1996; Horowitz and

[21] Corollary B.2 in the Appendix.
[22] Proposition B.4 in the Appendix.
[23] Proposition B.5 in the Appendix.
[24] Proposition B.6 in the Appendix.

Reiter 2001; Allen and Martinez Machain 2019), but may be less effective in counterinsurgency (Lyall 2013; Dell and Querubin 2018). These findings, while important in their own right, do not address how airstrike technology shapes deterrence or bargaining.[25] Each of these papers evaluate the utility of aerial weapons conditional on bargaining having already failed. It would be valuable to know how these weapons influence decision making in the lead-up to crises, how they affect the chance of war, and whether they produce windfalls for states that develop them. These questions are difficult to answer empirically: we cannot observe the counterfactual where a given state does not develop airstrike capabilities, and it is difficult to identify how these capabilities affect crisis initiation or behavior. Our theory is well suited to address these questions. As shown in Figures 3 and 4, once we identify whether private wartime capabilities improve or degrade hassling capabilities, then we can pin down how changes in wartime capabilities affect crisis outcomes. Put another way, our results can leverage existing research on how weapons systems function in war and hassling to better speak to how international crises play out, ending in peaceful negotiations, a war, or a continuum of low-level conflict options.

As an illustration, consider a crisis where:

(a) A defender state has private information about its ability to conduct airstrikes.

(b) The most feasible low-level conflict option is an airstrike (e.g., Operation Desert Fox).

(c) The war option would be a conventional war (e.g., Operation Desert Storm).

Because airstrike technology is dual-use for both hassling and war, then private capabilities would have a positive effect on both war and hassling outcomes (noted by + symbols in Table 1). Thus, if (a)–(c) held, then any model formalizing this case would find that the defender's privately known ability to conduct airstrikes would improve its overall payoffs (Figure 4), but would not necessarily increase the chance of conventional war (Figure 3).

On the other hand, consider a crisis where (a) and (b) held, but instead of (c):

(d) The war option would be a protracted counterinsurgency.

Based on the research cited above, airstrike capabilities would now be relatively ineffective in war (the – symbol in Table 1).[26] If (a), (b), and (d) held, then a model formalizing this case would find that a defender state with a better private ability or willingness to conduct

[25] One notable exception is Post (2019), which analyzes airpower events as signals in compellence.

[26] If new research emerges (or alternate research exists) showing that airstrike capabilities are effective in protracted counterinsurgencies, then our theory can still function, just with the direction of the results flipped.

airstrikes would be less likely to enter into a war (Figure 3), and may or may not end up with greater payoffs (Figure 4).

A number of other conflict capabilities can be dual-use for hassling and war. During Operation Outside the Box (2007), Israel disabled Syrian air defenses with an electronic warfare attack. While the full details of the electronic warfare attack have not been disclosed, any attack that allowed multiple Israeli aircraft to enter Syria and conduct a raid without harassment plausibly could have also been used to conduct a more extensive conventional attack (Katz 2010). Additionally, cyberattacks have been used as part of a conventional war (Russia-Georgia War, 2008), in hassling as an independent act (Stuxnet and Estonian cyber attacks), and in hassling as part of a cluster of other operations (the NotPetya attacks targeting Ukraine) (Buchanan 2020; Gannon et al. 2020). Similarly, developments in anti-satellite technologies have opened the possibility for disruption of GPS signals; low-level disruptions could be used for hassling, and more serious disruptions could create problems for modern air and sea warfare (Harrison et al. 2020).

While we do not have the space to discuss every dual-use technology case at length, existing research suggests that a number of other capabilities positively affect both hassling and war. Airlift capabilities can facilitate special operations for use in low-level conflict, conventional war, and irregular warfare (Bolkcom 2007; Pietrucha and Renken 2019). Support for violent non-state actors has been used for both hassling and war (Schultz 2010; Schram 2021). Control of the information environment and intelligence operations have been used in both hassling operations (Russia in Eastern Ukraine, 2015–2021) and in hearts-and-minds counterinsurgencies (Shapiro and Weidmann 2015).

Of course, some capabilities are only effective in certain forms of conflict. Airstrike capabilities may be less useful in winning protracted counterinsurgencies (Lyall 2013; Dell and Querubin 2018). Furthermore, if states invest in electronic warfare capabilities to disrupt air defense systems, cyberwarfare tools to disrupt industrial or nuclear processes, or anti-satellite tools, they may invest less in the capabilities needed to win population-centric counterinsurgencies (Berman, Shapiro and Felter 2011).[27] Similarly, civilian information operations may be critical to certain kinds of hassling or counterinsurgency contexts, but may be less important in conflicts that rely heavily on conventional firepower.[28] Finally, many conventional technologies are unsuited to combat hassling. For example, the U.S. Navy, Marine Force, and Coast Guard jointly issued a report suggesting that conventional naval vessels may not

[27] There are naturally some notable exceptions, (see Shachtman 2011).

[28] For example, Operation Desert Storm (a conventional conflict) placed less emphasis on "winning hearts and minds" than Operation Iraqi Freedom (a protracted counterinsurgency campaign) did (Nagl et al. 2008).

Capability/Willingness Mechanism	**Hassling Type**	**War Type**	**Rationale**
Airstrikes/Drones	Targeted strikes against military facilities (+)	Conventional War (+)	Dual-Use Technology
Electronic/Cyber/Anti-Satellite Attacks	Disruption of military systems or weapons facilities (+)	Conventional War (+)	Dual-Use Technology
Airlift capabilities	Special operations (+)	Conventional/ Irregular War (+)	Dual-Use Technology
Militants on Retainer	Supporting low-level conflict/insurgency (+)	Conventional/ Irregular War (+)	Dual-Use Technology
Domestic Information Operations	Undermining domestic authority/promoting discord (+)	Irregular War (+)	Dual-Use Technology
Domestic Information Operations	Undermining domestic authority/promoting discord (+)	Conventional War (–)	Specialized Technology/Budgetary Issues
Air, Electronic, Cyber, Anti-Satellite Attacks	Targeted strikes against military facilities (+)	Irregular War/COIN (–)	Specialized Technology/Budgetary Issues
Conventional Navy gray hulls optimized for high-end naval warfighting	Countering naval gray zone operations (–)	Conventional War (+)	Specialized Technology/Budgetary Issues
Domestic Electoral Considerations	Sanctions, hassling, or any other low-level response (–)	Conventional War (+)	Rally 'round the Flag
Domestic Political Economy Considerations	Sanctions or low-level conflict (+/–)	Conventional War (+/–)	Political Elite Disconnect

Table 1: How various mechanisms affect hassling and war capacities. The + symbol indicates that the mechanism improves the corresponding hassling operations or war type; the – symbol denotes a detriment.

be as effective in deft handling of gray zone attacks in the Pacific theater (Berger, Gilday and Schultz 2020; Owen 2021).

Factors besides military capabilities could also alter leaders' willingness to opt for certain forms of conflict. Analyses of the "rally-round-the-flag" phenomenon suggest that larger military operations, especially wars, generate an increase in public support for domestic leadership (Baker and Oneal 2001; Chapman and Reiter 2004). This effect could alter a leader's preferences, causing them to prefer war over hassling operations. Alternatively, domestic political economy considerations can also shape leaders' incentives. For example, in 1954, the United States provided arms, funds, and training to Guatemalan rebels who overthrew Jacobo Árbenz and installed right-wing dictator Castillo Armas—a move that benefited the politically connected United Fruit Company (Kinzer 2007, 125–147) in ways that sanctions would not. Similarly, while "blood for oil" may not fully explain the 2003 Iraq invasion (Paul 2003; Stokes 2007), oil market considerations could have still increased leaders' private willingness to go to war rather than impliment half-measures and hassle. Conversely, relevant domestic actors might prefer low-level action over war, such as those who would benefit from economic sanctions or tariffs.

9 Conclusion

Why does war occur? Using crisis bargaining models, a broad set of influential game theoretic research has identified a remarkably consistent answer to this foundational question: when one state possesses a greater private ability or willingness to go to war, the game is more likely to end in war. However, crisis bargaining models make a common simplifying assumption: that the game ends in either war or in peace. In practice, we know that this is not true, as states face a broad menu of policy options when in a crisis. The question, then, is whether this simplifying assumption is innocuous, or if this simplifying assumption shapes how these models answer the war puzzle.

To address this, we put forward a new class of models—flexible-response crisis bargaining models—and conducted a comprehensive analysis of them using the tools of mechanism design. These flexible-response crisis bargaining models represent a useful adaptation of the standard, dichotomous crisis bargaining framework, where war and peace are the only possible outcomes. In our framework, states can engage in a continuum of conflict operations; this better captures the conditions of actual international crises, in which states select from an array of options like sanctions or gray zone operations. Rather than solve a single game form, we have identified the properties shared by all equilibria in the full class of flexible-response crisis bargaining games. This general analysis allows us to be confident that our results are not driven by specifics of the game form or a specific equilibrium, but are generalizable to all flexible-response crisis bargaining models.

Our most surprising results are those that differ from the Banks (1990) and Fey and Ramsay (2011) monotonicity results. While existing research has shown that improved private war capabilities or a greater private willingness to go to war can never decrease the likelihood of war, we show that this relationship is more nuanced when war capabilities can also benefit low-level conflict capabilities. Similarly, while Banks (1990) and Fey and Ramsay (2011) have shown that an improved private ability to conduct war *always* produces a greater utility, we find that these results do not necessarily hold when a robust ability to go to war can hurt an actor's ability to effectively sanction or hassle.

Though the primary application of our analysis is to interstate conflict, our analysis can be applied to other political economy settings with negotiation between actors who possess multiple outside options. For example, when insurgent groups or drug cartels compete, they regularly engage in restrained and precise violent acts (Shapiro 2013; Schram 2019; Cruz and Durán-Martínez 2016). Alternatively, government strategies of repression (Ritter 2014) or economic extraction (Acemoglu and Robinson 2012) can also function as low-level pressure outside of a greater conflict. Similarly, firms engage in a range of conflict-type behaviors with rivals, including price wars, lawsuits, and predatory hiring.

There is more work to be done to understand how hassling and other flexible policy tools affect the outcomes of international crises. First, one could extend our flexible-response crisis bargaining framework to scenarios where observations of transgressions or hassling decisions are noisy. Such an extension could capture, for example, settings where a transgression is imperfectly observed or where attribution is hard. These include cyberwarfare with attribution problems (Baliga, Bueno de Mesquita and Wolitzky 2020), as well as when the hidden development of technological capabilities is itself the transgression (Meirowitz and Sartori 2008; Baliga and Sjöström 2008; Schultz 2010; Debs and Monteiro 2014; Bas and Coe 2016; Spaniel 2019; Meirowitz et al. 2019). Second, our initial work here has treated the transgression and hassling options as choices along a single dimension. In practice, states may choose among many distinct instruments of flexible responses, and private strength in one area may be associated with improvement or degradation in others. Third, we treat hassling and war as distinct policy options. Future game-form free analyses can do more to treat distinct policy options as related—for example, hassling choices in bargaining may shape war payoffs (like in Qiu (2022*a*)), or forms of low-level conflict may increase or decrease the likelihood of an elevated response (as formalized in Powell (2015) and described in Schelling (1980) and Kahn (2017)). By building out a more sophisticated framework with multidimensional flexible responses, future research could provide even more accurate descriptions of policy choices in international crises.

References

Acemoglu, Daron and James A. Robinson. 2012. *Why Nations Fail: The Origins of Power, Prosperity, and Poverty.* Crown Books.

Acharya, Avidit and Edoardo Grillo. 2015. "War with Crazy Types." *Political Science Research and Methods* 3(2):281–307.

Akçay, Erol, Adam Meirowitz, Kristopher W. Ramsay and Simon A. Levin. 2012. "Evolution of Cooperation and Skew Under Imperfect Information." *Proceedings of the National Academy of Sciences* 109(37):14936–14941.

Allen, Susan Hannah and Carla Martinez Machain. 2019. "Understanding the Impact of Air Power." *Conflict Management and Peace Science* 36(5):545–558.

Ashworth, Scott and Ethan Bueno de Mesquita. 2006. "Monotone comparative statics for models of politics." *American Journal of Political Science* 50(1):214–231.

Ashworth, Scott and Greg Sasso. 2019. "Delegation to an Overconfident Expert." *Journal of Politics* 81(2):692–696.

Axelrod, Robert and Robert O Keohane. 1985. "Achieving cooperation under anarchy: Strategies and institutions." *World politics* 38(1):226–254.

Baker, William D. and John R. Oneal. 2001. "Patriotism or Opinion Leadership? The Nature and Origins of the "Rally 'Round the Flag" Effect." *Journal of Conflict Resolution* 45(5):661–687.

Baliga, Sandeep, Ethan Bueno de Mesquita and Alexander Wolitzky. 2020. "Deterrence with Imperfect Attribution." *American Political Science Review* 114(4):1155–1178.

Baliga, Sandeep and Tomas Sjöström. 2008. "Strategic Ambiguity and Arms Proliferation." *Journal of Political Economy* 116(6):1023–1057.

Baliga, Sandeep and Tomas Sjöström. 2020. "The Strategy and Technology of Conflict." *Journal of Political Economy* 128(8):3186–3219.

Banks, Jeffrey S. 1990. "Equilibrium Behavior in Crisis Bargaining Games." *American Journal of Political Science* 34(3):599–614.

Bapat, Navin A. 2006. "State Bargaining with Transnational Terrorist Groups." *International Studies Quarterly* 50(1):213–229.

Bapat, Navin A and Bo Ram Kwon. 2015. "When are sanctions effective? A bargaining and enforcement framework." *International Organization* 69(1):131–162.

Baron, David P and David Besanko. 1987. "Commitment and Fairness in a Dynamic Regulatory Relationship." *Review of Economic Studies* 54(3):413–436.

Baron, David P. and David Besanko. 1992. "Information, Control, and Organizational Structure." *Journal of Economics & Management Strategy* 1(2):237–275.

Bas, Muhammet A. and Andrew J. Coe. 2016. "A Dynamic Theory of Nuclear Proliferation and Preventive War." *International Organization* 70(4):655–685.

Bas, Muhammet A and Robert Schub. 2017. The Theoretical and Empirical Approaches to Uncertainty and Conflict in International Relations. In *Oxford Research Encyclopedia of Politics*.

Benson, Brett V. and Bradley C. Smith. 2022. "Commitment Problems in Alliance Formation." *American Journal of Political Science* .

Berger, David H., Michael M. Gilday and Karl L. Schultz. 2020. *Advantage at Sea: Prevailing with Integrated All-Domain Naval Power*. United States Department of Defense.

Berman, Eli and David A. Lake. 2019. *Proxy Wars: Suppressing Violence through Local Agents*. Cornell University Press.

Berman, Eli, Jacob N Shapiro and Joseph H Felter. 2011. "Can hearts and minds be bought? The economics of counterinsurgency in Iraq." *Journal of Political Economy* 119(4):766–819.

Bolkcom, Christopher. 2007. Military Airlift: C-17 Aircraft Program. Library of Congress.

Buchanan, Ben. 2020. *The Hacker and the State: Cyber Attacks and the New Normal of Geopolitics*. Harvard University Press.

Chapman, Terrence L. and Dan Reiter. 2004. "The United Nations Security Council and the Rally 'Round the Flag Effect." *Journal of Conflict Resolution* 48(6):886–909.

Chassang, Sylvain and Gerard Padró I Miquel. 2010. "Conflict and Deterrence under Strategic Risk." *Quarterly Journal of Economics* 125(4):1821–1858.

Chinchilla, Alexandra Cecylia. 2021. "Advising War: Limited Intervention in Conflict.". Typescript, University of Chicago.
URL: https://www.alexandrachinchilla.com/uploads/1/2/0/4/120443671/chinchilla_jmp_9.28.pdf

Cobzaş, Ştefan, Radu Miculescu and Adriana Nicolae. 2019. *Lipschitz Functions.* Springer.

Coe, Andrew J. 2018. "Containing Rogues: A Theory of Asymmetric Arming." *Journal of Politics* 80(4):1197–1210.

Cruz, José Miguel and Angélica Durán-Martínez. 2016. "Hiding Violence to Deal with the State: Criminal Pacts in El Salvador and Medellin." *Journal of Peace Research* 53(2):197–210.

Debs, Alexandre and Nuno P. Monteiro. 2014. "Known Unknowns: Power Shifts, Uncertainty, and War." *International Organization* 68(01):1–31.

Dell, Melissa and Pablo Querubin. 2018. "Nation Building through Foreign Intervention: Evidence from Discontinuities in Military Strategies." *Quarterly Journal of Economics* 133(2):701–764.

Di Lonardo, Livio and Scott Tyson. 2022*a*. "Deterrence and Preventive Sanctions." *Working Paper* .

Di Lonardo, Livio and Scott Tyson. 2022*b*. "Political Instability and the Failure of Deterrence." *Journal of Politics* 84(1):180–193.

Dunford, Joseph. 2016. "Gen. Dunford's Remarks and Q&A.". Center for Strategic and International Studies.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379–414.

Fearon, James D. 1997. "Signaling Foreign Policy Interests: Tying Hands versus Sinking Costs." *Journal of Conflict Resolution* 41(1):68–90.

Fey, Mark, Adam Meirowitz and Kristopher W Ramsay. 2013. "Credibility and commitment in crisis bargaining." *Political Science Research and Methods* 1(1):27–52.

Fey, Mark and Brenton Kenkel. 2021. "Is an Ultimatum the Last Word on Crisis Bargaining?" *Journal of Politics* 83(1):87–102.

Fey, Mark and Kristopher W. Ramsay. 2009. "Mechanism Design Goes to War: Peaceful Outcomes with Interdependent and Correlated Types." *Review of Economic Design* 13(3):233.

Fey, Mark and Kristopher W Ramsay. 2011. "Uncertainty and Incentives in Crisis Bargaining: Game-Free Analysis of International Conflict." *American Journal of Political Science* 55(1):149–169.

Fuhrmann, Matthew and Sarah E Kreps. 2010. "Targeting Nuclear Programs in War and Peace: A Quantitative Empirical Analysis, 1941–2000." *Journal of Conflict Resolution* 54(6):831–859.

Fulghum, David, Robert Wall and Amy Butler. 2007. "Cyber-Combat's First Shot." *Aviation Week & Space Technology.* 167.

Gannon, Andres J., Erik Gartzke, Jon R. Lindsay and Peter Schram. 2020. "The Shadow of Deterrence." Typescript, University of California, San Diego.
URL: https://jandresgannon.com/research

Gartzke, Erik. 1999. "War Is in the Error Term." *International Organization* 53(3):567–587.

Gartzke, Erik and Jon R. Lindsay. 2015. "Weaving Tangled Webs: Offense, Defense, and Deception in Cyberspace." *Security Studies* 24(2):316–348.

Gartzke, Erik and Jon R. Lindsay. 2017. "Thermonuclear Cyberwar." *Journal of Cybersecurity* 3(1):37–48.

Gurantz, Ron and Alexander V. Hirsch. 2017. "Fear, Appeasement, and the Effectiveness of Deterrence." *Journal of Politics* 79(3):1041–1056.

Harrison, Todd, Kaitlyn Johnson, Thomas G. Roberts, Tyler Way and Makena Young. 2020. *Space threat assessment 2020.* Center for Strategic and International Studies.

Horowitz, Michael and Dan Reiter. 2001. "When Does Aerial Bombing Work? Quantitative Empirical Tests, 1917–1999." *Journal of Conflict Resolution* 45(2):147–173.

Jervis, Robert. 1976. *Perception and Misperception in International Politics.* Princeton University Press.

Kahn, Herman. 2017. *On Escalation: Metaphors and Scenarios*. Routledge.

Katz, Yaakov. 2010. "And they struck them with blindness." *The Jerusalem Post* .
URL: https://www.jpost.com/Magazine/Features/And-they-struck-them-with-blindness

Kinzer, Stephen. 2007. *Overthrow: America's Century of Regime Change from Hawaii to Iraq*. Macmillan.

Kreps, Sarah E. and Matthew Fuhrmann. 2011. "Attacking the Atom: Does Bombing Nuclear Facilities Affect Proliferation?" *Journal of Strategic Studies* 34(2):161–187.

Kydd, Andrew H. and Roseanne W. McManus. 2017. "Threats and Assurances in Crisis Bargaining." *Journal of Conflict Resolution* 61(2):325–348.

Lanoszka, Alexander. 2016. "Russian Hybrid Warfare and Extended Deterrence in Eastern Europe." *International Affairs* 92(1):175–195.

Leventoğlu, Bahar and Ahmer Tarar. 2008. "Does Private Information Lead to Delay or War in Crisis Bargaining?" *International Studies Quarterly* 52(3):533–553.

Levy, Jack S. 2011. "Preventive War: Concept and Propositions." *International Interactions* 37(1):87–96.

Lin-Greenberg, Erik. 2022*a*. "Evaluating Escalation: Conceptualizing Escalation in an Era of Emerging Military Technologies." *Journal of Politics* .

Lin-Greenberg, Erik. 2022*b*. "Wargame of drones: remotely piloted aircraft and crisis escalation." *Journal of Conflict Resolution* 66(10):1737–1765.

Liu, Linqun. 2021. "Domestic Constraints in Crisis Bargaining." Typescript, University of Chicago.
URL: https://sites.google.com/site/liqunliu90/home

Lyall, Jason. 2013. "Airpower and Coercion in Counterinsurgency Wars: Evidence from Afghanistan." Typescript, Yale University.
URL: http://www.jasonlyall.com/wp-content/uploads/2013/10/AFGHAN_AIRSTRIKES.pdf

Mazarr, Michael J. 2015. "Struggle in the Gray Zone and World Order." *War on the Rocks* .
URL: https://warontherocks.com/2015/12/struggle-in-the-gray-zone-and-world-order/

McCormack, Daniel and Henry Pascoe. 2017. "Sanctions and Preventive War." *Journal of Conflict Resolution* 61(8):1711–1739.

Mearsheimer, John J et al. 2001. *The tragedy of great power politics*. WW Norton & Company.

Meirowitz, Adam. 2006. "Designing Institutions to Aggregate Preferences and Information." *Quarterly Journal of Political Science* 1(4):373–392.

Meirowitz, Adam and Anne E. Sartori. 2008. "Strategic Uncertainty as a Cause of War." *Quarterly Journal of Political Science* 3(4):327–352.

Meirowitz, Adam, Massimo Morelli, Kristopher W. Ramsay and Francesco Squintani. 2019. "Dispute Resolution Institutions and Strategic Militarization." *Journal of Political Economy* 127(1):378–418.

Milgrom, Paul and Ilya Segal. 2002. "Envelope Theorems for Arbitrary Choice Sets." *Econometrica* 70(2):583–601.

Morrow, James D. 1989. "Capabilities, Uncertainty, and Resolve: A Limited Information Model of Crisis Bargaining." *American Journal of Political Science* 33(4):941–972.

Morrow, James D. and Jessica S. Sun. 2020. "15 Models of Interstate Conflict." *The SAGE Handbook of Research Methods in Political Science and International Relations* .

Morrow, James D and Kevin L Cope. 2021. "The limits of information revelation in multilateral negotiations: A theory of treatymaking." *Journal of Theoretical Politics* 33(4):399–429.

Myerson, Roger B. 1979. "Incentive Compatibility and the Bargaining Problem." *Econometrica* 47(1):61–73.

Nagl, John A, James F Amos, Sarah Sewall, David H Petraeus et al. 2008. *The US Army/Marine Corps Counterinsurgency Field Manual*. Number 3-24 University of Chicago Press.

Opall-Rome, Barbara. 2018. "Declassified: How an Israeli operation derailed Syria's nuclear weapons drive." *Defense News* .
URL: https://www.defensenews.com/global/mideast-africa/2018/03/20/just-declassified-how-an-israeli-operation-derailed-syrias-nuclear-weapons-drive

Owen, Dan. 2021. "Time for a Fully Integrated Dual-Fleet Force Model." *War on the Rocks* .
URL: https://warontherocks.com/2021/03/time-for-a-fully-integrated-dual-fleet-force-m

Paine, Jack, Scott A Tyson, Dirk Berg-Schlosser, Bertrand Badie and Leonardo Morlino. 2020. "Uses and abuses of formal models in political science.".

Pape, Robert A. 1996. *Bombing to Win: Air Power and Coercion in War*. Cornell University Press.

Paul, James A. 2003. Oil Companies in Iraq: A Century of Rivalry and War. In *Global Policy Forum*. pp. 25–26.

Pietrucha, Mike and Jeremy Renken. 2019. "Uplifted: The Case for Small Tactical Airlift." *War on the Rocks* .
URL: https://warontherocks.com/2019/05/uplifted-the-case-for-small-tactical-airlift/

Post, Abigail. 2019. "Flying to Fail: Costly Signals and Air Power in Crisis Bargaining." *Journal of Conflict Resolution* 63(4):869–895.

Powell, Robert. 1999. *In the Shadow of Power: States and Strategies in International Politics*. Princeton University Press.

Powell, Robert. 2006. "War as a Commitment Problem." *International Organization* 60(1):169–203.

Powell, Robert. 2015. "Nuclear Brinkmanship, Limited War, and Military Power." *International Organization* 69(3):589–626.

Qiu, Xiaoyan. 2022*a*. "Rebel strategies and the prospects for peace." *American journal of political science* 66(1):140–155.

Qiu, Xiaoyan. 2022*b*. "State Support for Rebels and Interstate Bargaining." *American Journal of Political Science* .

Reiter, Dan. 2005. "Preventive Attacks Against Nuclear Programs and the "Success" at Osiraq." *Nonproliferation Review* 12(2):355–371.

Ritter, Emily Hencken. 2014. "Policy Disputes, Political Survival, and the Onset and Severity of State Repression." *Journal of Conflict Resolution* 58(1):143–168.

Schelling, Thomas C. 1980. *The strategy of conflict*. Harvard university press.

Schram, Peter. 2019. "Managing Insurgency." *Journal of Conflict Resolution* 63(10):2319–2353.

Schram, Peter. 2021. "Hassling: How States Prevent a Preventive War." *American Journal of Political Science* 65(2):294–308.

Schram, Peter. 2022. "When Capabilities Backfire: How Improved Hassling Capabilities Produce Worse Outcomes." *The Journal of Politics* 84(4).

Schultz, Kenneth A. 1999. "Do Democratic Institutions Constrain or Inform? Contrasting Two Institutional Perspectives on Democracy and War." *International Organization* 53(2):233–266.

Schultz, Kenneth A. 2010. "The Enforcement Problem in Coercive Bargaining: Interstate Conflict over Rebel Support in Civil Wars." *International Organization* 64(2):281–312.

Shachtman, Noah. 2011. "The Secret History of Iraq's Invisible War." *Wired, April* 11.

Shapiro, Jacob N. 2013. *The Terrorist's Dilemma: Managing Violent Covert Organizations*. Princeton University Press.

Shapiro, Jacob N and Nils B Weidmann. 2015. "Is the phone mightier than the sword? Cellphones and insurgent violence in Iraq." *International Organization* 69(2):247–274.

Slantchev, Branislav L. 2011. *Military threats: the costs of coercion and the price of peace*. Cambridge University Press.

Spaniel, William. 2019. *Bargaining over the Bomb: The Successes and Failures of Nuclear Negotiations*. Cambridge University Press.

Spaniel, William. 2020. "Power Transfers, Military Uncertainty, and War." *Journal of Theoretical Politics* 32(4):538–556.

Spaniel, William and Iris Malone. 2019. "The Uncertainty Trade-off: Reexamining Opportunity Costs and War." *International Studies Quarterly* 63(4):1025–1034.

Stokes, Doug. 2007. "Blood for Oil? Global Capital, Counter-Insurgency and the Dual Logic of American Energy Security." *Review of International Studies* 33(2):245–264.

Tarar, Ahmer. 2016. "A Strategic Logic of the Military Fait Accompli." *International Studies Quarterly* 60(4):742–752.

Tarar, Ahmer. 2021. "Risk Preferences, Uncertainty, and War." *International Interactions* 48(2):1–25.

Waltz, Kenneth N. 1979. *Theory of international politics*. Waveland Press.

Online Appendix

Contents

A Proofs

A.1 Proof of Lemma 1

This result, as well as Lemma 3 below, depends on the following auxiliary result on the expected utility from mimicking various types in the absence of hassling.

Lemma A.1. *If $h = 0$ and $\theta' < \theta''$,*

$$\Phi_D(\theta'' \,|\, \theta') \leq \Phi_D(\theta' \,|\, \theta') \leq \Phi_D(\theta' \,|\, \theta'') \leq \Phi_D(\theta'' \,|\, \theta'').$$

Proof. The first and third inequalities follow from (IC). The second follows because W_D is increasing and $h = 0$:

$$\begin{aligned}
\Phi_D(\theta' \,|\, \theta') &= \pi(\theta')[V_D(\theta') - K_D(0, \theta')] + (1 - \pi(\theta'))W_D(\theta') \\
&= \pi(\theta')V_D(\theta') + (1 - \pi(\theta'))W_D(\theta') \\
&\leq \pi(\theta')V_D(\theta') + (1 - \pi(\theta'))W_D(\theta'') \\
&= \pi(\theta')[V_D(\theta') - K_D(0, \theta'')] + (1 - \pi(\theta'))W_D(\theta'') \\
&= \Phi_D(\theta'' \,|\, \theta').
\end{aligned}$$

□

Lemma 1. *If $h = 0$ and $\theta' < \theta''$, then $\pi(\theta') \geq \pi(\theta'')$.*

Proof. Lemma A.1 implies

$$\Phi_D(\theta'' \,|\, \theta'') - \Phi_D(\theta'' \,|\, \theta') \geq \Phi_D(\theta' \,|\, \theta'') - \Phi_D(\theta' \,|\, \theta'),$$

which is equivalent to

$$(1 - \pi(\theta''))[W_D(\theta'') - W_D(\theta')] \geq (1 - \pi(\theta'))[W_D(\theta'') - W_D(\theta')].$$

As $W_D(\theta'') > W_D(\theta')$, this in turn implies $\pi(\theta') \geq \pi(\theta'')$. □

A.2 Proof of Proposition 1

Proposition 1. *Assume θ degrades hassling effectiveness. If $\theta' < \theta''$, then $\pi(\theta') \geq \pi(\theta'')$.*

Proof. For a proof by contradiction, suppose $\theta' < \theta''$ and $\pi(\theta') < \pi(\theta'')$. This implies $\pi(\theta') = 0$ and $\pi(\theta'') = 1$. (VA) implies

$$V_D(\theta'') - K_D(h(\theta''), \theta'') \geq W_D(\theta'').$$

(IC), combined with the assumption that θ' degrades hassling effectiveness, implies

$$W_D(\theta') \geq V_D(\theta'') - K_D(h(\theta''), \theta') \geq V_D(\theta'') - K_D(h(\theta''), \theta'').$$

Combining these inequalities gives $W_D(\theta') \geq W_D(\theta'')$, a contradiction. $\square$

A.3 Proof of Proposition 2

Proposition 2. *Assume θ improves hassling effectiveness, and let $\theta' < \theta''$. If (WURI) holds, then $\pi(\theta') \geq \pi(\theta'')$. If (SURI) holds, then $\pi(\theta') \leq \pi(\theta'')$.*

Proof. We will prove the claims by contraposition. Let $\theta' < \theta''$, and suppose $\pi(\theta') < \pi(\theta'')$ (i.e., $\pi(\theta') = 0$ and $\pi(\theta'') = 1$). We want to prove that this implies (WURI) does not hold. Note that (VA) implies

$$V_D(\theta'') - K_D(h(\theta''), \theta'') \geq W_D(\theta''),$$

while (IC) implies

$$W_D(\theta') \geq V_D(\theta'') - K_D(h(\theta''), \theta').$$

Combining these gives

$$W_D(\theta') + K_D(h(\theta''), \theta') \geq V_D(\theta'') \geq W_D(\theta'') + K_D(h(\theta''), \theta''),$$

which in turn implies

$$W_D(\theta'') - W_D(\theta') \leq K_D(h(\theta''), \theta') - K_D(h(\theta''), \theta').$$

Because $\pi(\theta'') = 1$, this means (WURI) cannot hold, establishing the first claim of the lemma. An analogous argument establishes that (SURI) cannot hold if $\pi(\theta') > \pi(\theta'')$ (i.e., $\pi(\theta') = 1$ and $\pi(\theta'') = 0$). $\square$

A.4 Proof of Lemma 2

Lemma 2. *Assume θ improves hassling effectiveness, (DD) holds, and $\max \mathcal{H} = \bar{h} < \infty$. If $W_D(\theta'') - W_D(\theta') > K_D(\bar{h}, \theta') - K_D(\bar{h}, \theta'')$ for all $\theta', \theta'' \in \Theta$ such that $\theta' < \theta''$, then (WURI) holds.*

Proof. For all $h < \bar{h}$ and $\theta' < \theta''$, (DD) implies

$$K_D(\bar{h}, \theta'') - K_D(h, \theta'') < K_D(\bar{h}, \theta') - K_D(h, \theta'),$$

which is equivalent to

$$K_D(\bar{h}, \theta') - K_D(\bar{h}, \theta'') > K_D(h, \theta') - K_D(h, \theta'').$$

Therefore, under the hypothesis of the lemma, we have

$$W_D(\theta'') - W_D(\theta') > K_D(\bar{h}, \theta') - K_D(\bar{h}, \theta'') \geq K_D(h, \theta') - K_D(h, \theta'')$$

for all $h \in \mathcal{H}$, which implies (WURI). □

A.5 Proof of Lemma 3

Lemma 3. *If $h = 0$ and $\theta' < \theta''$, then $U_D(\theta') \leq U_D(\theta'')$.*

Proof. Immediate from Lemma A.1. □

A.6 Proof of Proposition 3

Proposition 3. *Assume θ improves hassling effectiveness. If $\theta' < \theta''$, then $U_D(\theta') \leq U_D(\theta'')$. The inequality is strict if $\pi(\theta') = 0$ or $h(\theta') > 0$.*

Proof. When θ improves hassling effectiveness, (IC) implies

$$\begin{aligned} U_D(\theta'') &= (1 - \pi(\theta''))W_D(\theta'') + \pi(\theta'')[V_D(\theta'') - K_D(h(\theta''), \theta'')] \\ &\geq (1 - \pi(\theta'))W_D(\theta'') + \pi(\theta')[V_D(\theta') - K_D(h(\theta'), \theta'')] \\ &\geq (1 - \pi(\theta'))W_D(\theta') + \pi(\theta')[V_D(\theta') - K_D(h(\theta'), \theta')] \\ &= U_D(\theta'). \end{aligned}$$ □

If $\pi(\theta') = 0$, then the second inequality above is strict. The same is true if $\pi(\theta') = 1$ and $h(\theta') > 0$.

A.7 Proof of Proposition 4

The proof depends on a more general property of hassling effectiveness and equilibrium utility:

Lemma A.2. *If $\pi(\theta') = \pi(\theta'') = 1$ and θ' has greater hassling effectiveness than θ'', then $U_D(\theta') \geq U_D(\theta'')$ (strictly if $h(\theta'') > 0$).*

Proof. By (IC), we have

$$U_D(\theta') \geq \underbrace{V_D(\theta'') - K_D(h(\theta''), \theta')}_{\Phi_D(\theta'' \mid \theta')} \geq V_D(\theta'') - K_D(h(\theta''), \theta'') = U_D(\theta''). \tag{A.1}$$

The second inequality of Equation A.1 is strict if $h(\theta'') > 0$. □

Proposition 4. *Assume θ degrades hassling effectiveness. There exists $\hat{\theta}$ such that $\pi(\theta) = 1$ for all $\theta < \hat{\theta}$ and $\pi(\theta) = 0$ for all $\theta > \hat{\theta}$. If $\theta' < \theta'' < \hat{\theta}$, then $U_D(\theta') \geq U_D(\theta'')$ (strictly if $h(\theta'') > 0$). If $\hat{\theta} < \theta' < \theta''$, then $U_D(\theta') < U_D(\theta'')$.*

Proof. The claim about $\hat{\theta}$ follows from Proposition 1. The next claim then follows from Lemma A.2. The final claim follows because W_D is strictly increasing in θ. □

A.8 Proof of Proposition 5

Proposition 5. *If $\pi(\theta) = \pi(\theta') = 1$ and $h(\theta) \leq h(\theta')$, then $V_D(\theta) \leq V_D(\theta')$. Furthermore, if $h(\theta) < h(\theta')$, then $V_D(\theta) < V_D(\theta')$.*

Proof. (IC) implies

$$V_D(\theta') - K_D(h(\theta'), \theta') \geq V_D(\theta) - K_D(h(\theta), \theta'),$$

which is equivalent to

$$V_D(\theta') - V_D(\theta) \geq K_D(h(\theta'), \theta') - K_D(h(\theta), \theta').$$

If $h(\theta) \leq h(\theta')$, then the RHS is non-negative, and the first claim follows. If $h(\theta) < h(\theta')$, then the RHS is strictly positive, and the second claim follows. □

A.9 Proof of Proposition 6

We first state a helpful lemma.

Lemma A.3. *Assume (BV) holds. For all $\theta, \theta' \in \Theta$, Φ_D is differentiable with respect to θ, and*

$$\frac{\partial \Phi_D(\theta' \mid \theta)}{\partial \theta} = (1 - \pi(\theta')) \frac{dW_D(\theta)}{d\theta} - \pi(\theta') \frac{\partial K_D(h(\theta'), \theta)}{\partial \theta}.$$

Proof. The existence of $\partial\Phi_D/\partial\theta$ follows from (BV). The expression in the lemma then follows immediately from the definition of Φ_D. □

We then rely on standard mechanism design arguments to establish the proposition.

Proposition 6. *Assume* (BV) *holds. For all* $\theta_0 \in \Theta$,

$$U_D(\theta_0) = U_D(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_0} (1-\pi(\theta))\frac{dW_D(\theta)}{d\theta}\, d\theta - \int_{\underline{\theta}}^{\theta_0} \pi(\theta)\, \frac{\partial K_D(h,\theta)}{\partial\theta}\bigg|_{h=h(\theta)} d\theta.$$

Proof. (IC) implies $U_D(\theta) = \sup_{\theta'\in\Theta} \Phi_D(\theta' \mid \theta)$ for all $\theta \in \Theta$. Therefore, by Milgrom and Segal (2002, Theorem 1),

$$\frac{dU_D(\theta)}{d\theta} = \frac{\partial\Phi_D(\theta' \mid \theta)}{\partial\theta}\bigg|_{\theta'=\theta}$$

at each point where U_D is differentiable. Furthermore, (BV) implies Φ_D is Lipschitz continuous in θ. The claim then follows from Lemma A.3 and Milgrom and Segal (2002, Corollary 1). □

We now state and prove a corollary with a stronger characterization for the case where θ degrades hassling effectiveness.

Corollary A.1. *Assume* θ *degrades hassling effectiveness and* (BV) *holds. There exists* $\hat{\theta} \in \Theta$ *such that* $\pi(\theta) = 1$ *for all* $\theta < \hat{\theta}$ *and* $\pi(\theta) = 0$ *for all* $\theta > \hat{\theta}$. *If* $\underline{\theta} < \hat{\theta} < \bar{\theta}$, *then for all* $\theta_0 < \hat{\theta}$,

$$V_D(\theta_0) = W_D(\hat{\theta}) + K_D(h(\theta_0),\theta_0) + \int_{\theta_0}^{\hat{\theta}} \frac{\partial K_D(h,\theta)}{\partial\theta}\bigg|_{h=h(\theta)} d\theta. \tag{A.2}$$

Proof. Proposition 1 implies the existence of the cutpoint $\hat{\theta}$. We then have $U_D(\theta) = W_D(\theta)$ for all $\theta > \hat{\theta}$. (BV) implies that W_D is continuous and Proposition 6 implies that U_D is continuous, so if $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ we have $U_D(\hat{\theta}) = W_D(\hat{\theta})$. For $\theta_0 < \hat{\theta}$, Proposition 6 then gives

$$\begin{aligned} V_D(\theta_0) &= U_D(\theta_0) + K_D(h(\theta_0),\theta_0) \\ &= U_D(\hat{\theta}) + K_D(h(\theta_0),\theta_0) + \Big[U_D(\theta_0) - U_D(\hat{\theta})\Big] \\ &= W_D(\hat{\theta}) + K_D(h(\theta_0),\theta_0) + \int_{\theta_0}^{\hat{\theta}} \frac{\partial K_D(h,\theta)}{\partial\theta}\bigg|_{h=h(\theta)} d\theta. \end{aligned}$$ □

This result is useful for two reasons. First, it shows that we need relatively little information about the equilibrium to pin down settlement values in flexible-response crisis bargaining games when θ degrades hassling capability. As long as we know the lowest type that goes to war and the amount of hassling exerted by each lower type, we can derive the exact settlement level in equilibrium. Importantly, if two bargaining games result in the same cutpoint type and the same amount of hassling below the cutpoint, they will also result in the exact same terms of settlement for each type of D, even if the bargaining processes themselves are quite dissimilar. Second, as we show in the next section, the necessary condition provided by Corollary A.1 turns out to be sufficient. Specifically, for any non-decreasing hassling plan $h(\theta)$, if we allocate settlement values according to the given formula for V_D, the resulting mechanism is incentive compatible and satisfies voluntary agreements.

B Additional Analysis

B.1 Extent of Hassling

We have shown that greater hassling effectiveness is associated with greater utility in case the crisis ends in a settlement (Proposition 3 and Proposition 4). We have also shown that the only way to improve one's payoff from a settlement is to choose greater levels of hassling (Proposition 5). Intuitively, then, it would appear to follow that more effective types hassle more in equilibrium. This intuition only holds in general when the cost of hassling satisfies the decreasing differences condition.

Proposition B.1. *Assume (DD) holds. If $\pi(\theta) = \pi(\theta') = 1$ and θ' has greater hassling effectiveness than θ, then $h(\theta) \leq h(\theta')$.*

Proof. (IC) implies:

$$V_D(\theta) - K_D(h(\theta), \theta) \geq V_D(\theta') - K_D(h(\theta'), \theta),$$
$$V_D(\theta') - K_D(h(\theta'), \theta') \geq V_D(\theta) - K_D(h(\theta), \theta').$$

A rearrangement of terms gives

$$K_D(h(\theta'), \theta') - K_D(h(\theta), \theta') \leq V_D(\theta') - V_D(\theta) \leq K_D(h(\theta'), \theta) - K_D(h(\theta), \theta).$$

(DD) therefore implies $h(\theta) \leq h(\theta')$. □

Because we can only say that more effective types hassle *weakly* more, this result leaves open two possibilities about why exactly more effective types have better equilibrium payoffs. One is that they hassle the same amount and receive the same settlement, so the higher payoff comes solely from the lower cost of hassling. The other possibility is that they hassle more and get better terms of settlement. Only if decreasing differences holds can we rule out a third possibility—that the more effective types hassle slightly less but at much lower cost, for a net increase in payoff despite the decrease in terms of settlement.

Combined with our earlier results on settlement values, Proposition B.1 implies that the settlement value V_D increases with hassling effectiveness, as long as the decreasing differences condition holds. Can we say anything more specific about the relationship between private type and the equilibrium choice of hassling in a broad class of flexible-response crisis bargaining games? If private strength is associated with lower hassling effectiveness, the answer turns out to be no: virtually any weakly decreasing hassling plan (subject to some continuity restrictions) can be sustained as the equilibrium of some bargaining game.

Proposition B.2. *Assume θ degrades hassling effectiveness and (BV) and (DD) hold. Let h be any non-increasing and absolutely continuous function from $[\underline{\theta}, \bar{\theta}]$ into $\mathcal{H}$. Take any $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$, let $\pi(\theta) = \mathbf{1}\{\theta \leq \hat{\theta}\}$, and let $V_D(\theta)$ be defined by Equation A.2. The direct mechanism (h, π, V_D) satisfies (IC) and (VA).*

Proof. As a preliminary, note that because K_D is Lipschitz and h is absolutely continuous, $K_D(h(\theta), \theta)$ is absolutely continuous when viewed as a function of θ (Cobzaş, Miculescu and Nicolae 2019, Corollary 3.3.9). Consequently, V_D is absolutely continuous and thus differentiable almost everywhere on $[\underline{\theta}, \hat{\theta})$.

Now take any $\theta, \theta' \in \Theta$. If $\theta' < \hat{\theta}$, then

$$\begin{aligned} \Phi_D(\theta' \mid \theta) &= V_D(\theta') - K_D(h(\theta'), \theta) \\ &= W_D(\hat{\theta}) + K_D(h(\theta'), \theta') - K_D(h(\theta'), \theta) + \int_{\theta'}^{\hat{\theta}} \left. \frac{\partial K_D(h, \theta'')}{\partial d\theta''} \right|_{h = h(\theta'')} d\theta''. \end{aligned}$$

Therefore, for almost all $\theta' < \hat{\theta}$, we have

$$\begin{aligned}\frac{\partial \Phi_D(\theta' \mid \theta)}{\partial \theta'} &= \frac{\partial K_D(h(\theta'), \theta')}{\partial h}\frac{dh(\theta')}{d\theta'} + \frac{\partial K_D(h(\theta'), \theta')}{\partial \theta'} \\ &\quad - \frac{\partial K_D(h(\theta'), \theta)}{\partial h}\frac{dh(\theta')}{\theta'} - \frac{\partial K_D(h(\theta'), \theta')}{\partial \theta'} \\ &= \underbrace{\frac{dh(\theta')}{\theta'}}_{\leq 0}\left[\frac{\partial K_D(h(\theta'), \theta')}{\partial h} - \frac{\partial K_D(h(\theta'), \theta)}{\partial h}\right].\end{aligned}$$

Because θ degrades hassling effectiveness, (DD) implies that the term in brackets is non-negative if $\theta \leq \theta'$ and non-positive if $\theta \geq \theta'$. Next, notice that

$$\begin{aligned}&\lim_{\theta' \to \hat{\theta}^-} \Phi_D(\theta' \mid \theta) \\ &\quad = \lim_{\theta' \to \hat{\theta}^-}\left[W_D(\hat{\theta}) + K_D(h(\theta'), \theta') - K_D(h(\theta'), \theta) + \int_{\theta'}^{\hat{\theta}} \frac{\partial K_D(h, \theta'')}{\partial d\theta''}\bigg|_{h=h(\theta'')} d\theta''\right] \\ &\quad = W_D(\hat{\theta}) + K_D(h(\hat{\theta}), \hat{\theta}) - K_D(h(\hat{\theta}), \theta).\end{aligned}$$

Therefore, if $\theta \leq \hat{\theta}$, then

$$\lim_{\theta' \to \hat{\theta}^-} \Phi_D(\theta' \mid \theta) \geq W_D(\hat{\theta}) \geq W_D(\theta) = \lim_{\theta' \to \hat{\theta}^+} \Phi_D(\theta' \mid \theta).$$

Conversely, if $\theta \geq \hat{\theta}$, then

$$\lim_{\theta' \to \hat{\theta}^-} \Phi_D(\theta' \mid \theta) \leq W_D(\hat{\theta}) \leq W_D(\theta) = \lim_{\theta' \to \hat{\theta}^+} \Phi_D(\theta' \mid \theta).$$

Finally, we have $\Phi_D(\theta' \mid \theta) = W_D(\theta)$ for all $\theta' > \hat{\theta}$. Altogether, these findings imply $\Phi_D(\theta' \mid \theta)$ is non-decreasing in θ' if $\theta' \in [\underline{\theta}, \theta]$ and non-increasing in θ' if $\theta' \in [\theta, \bar{\theta}]$. Therefore, (IC) holds. These results also imply $U_D(\theta) \geq \lim_{\theta \to \hat{\theta}^-} \Phi_D(\theta' \mid \theta) = W_D(\theta)$ for all $\theta \leq \hat{\theta}$, so (VA) also holds. □

This result demonstrates that incentive compatibility and voluntary agreements alone place no restrictions on the pattern of hassling across types beyond the fact that more effective types cannot engage in less of it. Consequently, the specifics of the relationship between effectiveness and the degree of hassling are model-dependent. For example, we cannot determine from the primitives alone whether all types hassle to the same degree, or whether there is some separation in levels of hassling. This will depend on how bargaining takes place and the precise effects of hassling choices on non-war payoffs.

B.2 Possibility of Peace

In the context of flexible-response crisis bargaining, we must be explicit about what peace entails. At a minimum, as in ordinary crisis bargaining games, the game must end with a negotiated settlement for all types of D. Furthermore, because transgressions and hassling may be interpreted as forms of low-level conflict, we will focus on equilibria in which C chooses $t = 0$ and each type of D chooses $h = 0$. Mirroring the terminology of Fey and Ramsay (2011), we will say an equilibrium is always peaceful if it meets these conditions.

In our baseline flexible-response context, the sufficient condition for peace is virtually the same as in ordinary crisis bargaining models. In particular, it must be possible to divide the pie so as to simultaneously satisfy both C (assuming C's knowledge of D's type is limited to the prior distribution) and the strongest type of D. In what follows, let $\hat{W}_C = \mathbb{E}[W_C(\theta)] = \int_\Theta W_C(\theta)\, dF(\theta)$, C's prior expectation of its own war payoff.

Proposition B.3. *If $\hat{W}_C + W_D(\bar{\theta}) \leq 1$, then there is a flexible-response crisis bargaining game form with voluntary agreements that has an always peaceful equilibrium.*

Proof. We prove the result by construction. Consider the following extensive form game:

1. C chooses $t \in \mathcal{T}$.
2. D chooses $h \in \mathcal{H}$.
3. C and D simultaneously choose $b_C \in \{0, 1\}$ and $b_D \in \{0, 1\}$.

War occurs if either player chooses $b_i = 1$:

$$\pi^g(b_C, b_D) = \mathbf{1}\{b_C + b_D > 0\}.$$

Baseline payoffs are divided according to the war payoff for the strongest type of D, with each player paying a penalty $Z > 0$ if the other chooses a non-zero flexible response:

$$V_C^g(t, h, b_C, b_D) = 1 - W_D(\bar{\theta}) - Z \cdot \mathbf{1}\{h > 0\},$$
$$V_D^g(t, h, b_C, b_D) = W_D(\bar{\theta}) - Z \cdot \mathbf{1}\{t > 0\}.$$

Assume Z is arbitrarily large—specifically, $Z > \max\left\{1 - W_D(\bar{\theta}) - \hat{W}_C, W_D(\bar{\theta}) - W_D(\underline{\theta})\right\}$. Note that this game has voluntary agreements, as each player can guarantee war by choosing $b_i = 1$.

We claim that the following strategy profile constitutes an equilibrium of this game:

1. C chooses $t = 0$.
2. Following all choices of t, D chooses $h = 0$.
   - C's beliefs about D's type remain at the prior following all (t, h).
3. C chooses $b_C = 1$ if and only if $h > 0$. D chooses $b_D = 1$ if and only if $t > 0$.

Because Z was chosen to be arbitrarily large, the choices of bargaining strategy when $h > 0$ or $t > 0$ are clearly best responses. Now consider the case when $h = t = 0$. For C, deviating to $b_C = 1$ would result in an expected payoff of $\hat{W}_C \leq 1 - W_D(\bar{\theta})$, which is unprofitable. For any type of D, deviating to $b_D = 1$ would result in a payoff of $W_D(\theta) \leq W_D(\bar{\theta})$, which is unprofitable. The bargaining strategies therefore comprise a Bayesian Nash equilibrium. Moving up the game tree, a deviation by C to $t > 0$ or by D to $h > 0$ would result in war, which we have just shown is worse than the payoffs from the proposed strategies. Finally, note that C's beliefs are updated in accordance with Bayes' rule whenever possible. Therefore, the proposed strategy profile is a perfect Bayesian equilibrium. □

The condition of Proposition B.3 is least likely to hold when the distribution of D's type is right-skewed. In this case, C's expected war payoff will be relatively high, since D's type is likely to be low. It will thus be impossible to satisfy the strongest type of D while giving C at least its expected war payoff. Because strong types of D are rare in this setting, the equilibrium chance of conflict will be low, but not zero.

If C's war payoff is independent of D's type (i.e., D's type only affects its cost of war, not its probability of victory), then the condition of Proposition B.3 always holds. A distribution of the pie following the probability of war will be acceptable both to C and to all types of D. The following result is a direct analogue of Proposition 2 in Fey and Ramsay (2011).

Corollary B.2. *If $W_C(\theta) = p - c_C$ and $W_D(\theta) = 1 - p - c_D(\theta)$, where $c_D : \Theta \to \mathbb{R}_+$, then there is a flexible-response crisis bargaining game form with voluntary agreements that has an always peaceful equilibrium.*

Proof. The result follows from Proposition B.3, as $\hat{W}_C + W_D(\bar{\theta}) = 1 - c_C - c_D(\bar{\theta}) < 1$. □

Because we have assumed transgressions and hassling only affect payoffs from negotiations, our conditions for always peaceful equilibria do not depend on the costs of these options. If the war payoffs were also functions of t and h, then the players' reservation values for conflict would depend on their marginal effects and costs. Therefore, in order for flexible responses to materially affect the prospects for peace, responses must shape payoffs in war as well as peace.[29]

B.3 Two-Dimensional Type

In the main framework, we treat D's war payoff and hassling costs as both being functions of a single type parameter, θ. We now relax this assumption, allowing separate types to control D's war payoff and hassling costs. In contrast with the main framework, this allows for situations where two types of D might have identical war payoffs but different hassling costs. Our goal is to show that our qualitative findings on the relationship between private types and the probability of war still hold up in this more realistic environment.

In the extended framework, let D's type be a pair (θ, ξ). As in the main framework, $\theta \in \Theta \subseteq \mathbb{R}$ determines D's war costs. However, D's hassling costs are now a function of $\xi \in \Xi \subseteq \mathbb{R}$. Formally, the cost term for D in case of settlement is now $K_D(h, \xi)$, which strictly increases in both parameters.[30] In terms of our main framework analysis, lower values of ξ correspond to greater hassling effectiveness, and so on. We now write D's utility in a given game form g as

$$u_D^g(t, h, b_C, b_D \mid \theta, \xi) = (1 - \pi^g(b_C, b_D))W_D(\theta) + \pi^g(b_C, b_D)\left[V_D^g(t, h, b_C, b_D) - K_D(h, \xi)\right].$$

As a regularity condition to simplify the statement of Proposition B.5, we assume K_D is continuous in ξ.

We assume the type pair (θ, ξ) is drawn from a joint probability distribution with support on $\mathcal{S} \subseteq \Theta \times \Xi$. We do not impose any statistical relationship between them: θ and ξ may be positively correlated, negatively correlated, or uncorrelated.

The components of a direct mechanism—h, π, and V_D—have the same substantive interpretation as before, but are now functions of the type pair (θ, ξ). D's utility from reporting (θ', ξ') when its true type is (θ, ξ) is now

$$\Phi_D(\theta', \xi' \mid \theta, \xi) = (1 - \pi(\theta', \xi'))W_D(\theta) + \pi(\theta', \xi')\left[V_D(\theta', \xi') - K_D(h(\theta', \xi'), \xi)\right]. \tag{B.3}$$

Mirroring the main analysis, we define $U_D(\theta, \xi) = \Phi_D(\theta, \xi \mid \theta, \xi)$. The incentive compatibility

[29] A similar mechanism drives results in Liu (2021).

[30] The exception, as in the main framework, being when $h = 0$: we assume $K_D(0, \xi) = 0$ for all $\xi \in \Xi$.

and voluntary agreements conditions for a direct mechanism have the same substantive interpretation as before, but we restate them here for the two-dimensional setting. Incentive compatibility is

$$U_D(\theta, \xi) \geq \Phi_D(\theta', \xi' \mid \theta, \xi) \qquad \text{for all } (\theta, \xi), (\theta', \xi') \in \mathcal{S}. \tag{IC$'$}$$

Voluntary agreements is

$$\pi(\theta, \xi) \left[V_D(\theta, \xi) - K_D(h(\theta, \xi), \xi)\right] \geq \pi(\theta, \xi) W_D(\theta) \qquad \text{for all } (\theta, \xi) \in \mathcal{S}. \tag{VA$'$}$$

We continue to restrict attention to direct mechanisms in which $\pi \in \{0, 1\}$.

B.3.1 Increased War Payoff, Degraded Hassling Effectiveness

Proposition 1 in the main analysis shows that if hassling effectiveness decreases with war payoffs, then higher types are more likely to go to war rather than settling. We establish an analogous result for the two-dimensional setting, showing that the chance of war increases with D's war payoff as long as D's hassling effectiveness remains the same or decreases.

Proposition B.4. *Assume $(\theta', \xi'), (\theta'', \xi'') \in \mathcal{S}$. If $\theta'' > \theta'$ and $\xi'' \geq \xi'$, then $\pi(\theta'', \xi'') \leq \pi(\theta', \xi')$.*

Proof. For a proof by contradiction, suppose $\theta'' \geq \theta'$, $\xi'' \geq \xi$, $\pi(\theta', \xi') = 0$, and $\pi(\theta'', \xi'') = 1$. (IC$'$) implies

$$V_D(\theta'', \xi'') - K_D(h(\theta'', \xi''), \theta'') \geq W_D(\theta''),$$

as well as

$$W_D(\theta') \geq V_D(\theta'', \xi'') - K_D(h(\theta'', \xi''), \xi').$$

Combining these, along with the assumption that $\xi'' \geq \xi'$, gives us

$$\begin{aligned} W_D(\theta') &\geq V_D(\theta'', \xi'') - K_D(h(\theta'', \xi''), \xi') \\ &\geq V_D(\theta'', \xi'') - K_D(h(\theta'', \xi''), \xi'') \\ &\geq W_D(\theta''). \end{aligned}$$

But this in turn implies $\theta' \geq \theta''$, a contradiction. □

B.3.2 Increased War Payoff and Hassling Effectiveness

Proposition 2 in the main analysis shows that if hassling effectiveness increases with war payoffs, then the relationship between type and the probability of war depends on auxiliary conditions. Specifically, if the war payoff increases quickly enough relative to the decrease in the cost of hassling—a condition we call WURI—then war is more likely for higher types. In the opposite case, when the SURI condition holds because the hassling cost declines more quickly than the war payoff increases, we have the opposite relationship.

We now prove an analogous result for the two-dimensional setting. We consider the effects of increasing D's war payoff *and* hassling effectiveness on the equilibrium probability of war. If the change in hassling effectiveness is small enough, then the change cannot shift the equilibrium outcome from war to settlement.

Proposition B.5. *Assume $\pi(\theta', \xi') = 0$, and consider any $(\theta'', \xi'') \in \mathcal{S}$ such that $\theta'' > \theta'$. There exists $\hat{\xi} > \xi'$ such that if $\xi'' < \hat{\xi}$, then $\pi(\theta'', \xi'') = 0$.*

Proof. For a proof by contraposition, suppose $\pi(\theta'', \xi'') = 1$. Then (IC′) implies

$$W_D(\theta') \geq V_D(\theta'', \xi'') - K_D(h(\theta'', \xi''), \xi'),$$

as well as

$$V_D(\theta'', \xi'') - K_D(h(\theta'', \xi''), \xi'') \geq W_D(\theta'').$$

Combining these yields

$$K_D(h(\theta'', \xi''), \xi') - K_D(h(\theta'', \xi''), \xi'') \geq W_D(\theta'') - W_D(\theta').$$

The RHS of the above expression is strictly positive. Meanwhile, the LHS approaches 0 as $\xi'' \to \xi'$. Therefore, there is a cutpoint $\hat{\xi} > \xi'$ such that $\xi'' > \hat{\xi}$. □

B.3.3 Equilibrium Payoffs

Our next result generalizes our results on the relationship between private type and equilibrium payoffs (Proposition 3 and Proposition 4) for the setting in which war payoffs and hassling costs are separate type components. We find that D's equilibrium utility is weakly increasing in the private war payoff and weakly decreasing in the cost of hassling. Additionally, the marginal decrease in payoff with the hassling cost is strict for any type that settles and hassles along the path of play.

Proposition B.6.

(a) *For all ξ, if $\theta'' > \theta'$, then $U_D(\theta'', \xi) \geq U_D(\theta', \xi)$. The inequality is strict if $\pi(\theta', \xi) = 0$.*

(b) *For all θ, if $\xi'' > \xi'$, then $U_D(\theta, \xi'') \leq U_D(\theta, \xi')$. The inequality is strict if $\pi(\theta, \xi'') = 1$ and $h(\theta, \xi'') > 0$.*

Proof. Claim (a). Fix any ξ, and assume $\theta'' > \theta'$. If $\pi(\theta', \xi) = 1$, then (IC′) implies

$$U_D(\theta'', \xi) \geq V_D(\theta', \xi) - K_D(h(\theta', \xi), \xi) = U_D(\theta', \xi).$$

Otherwise, if $\pi(\theta', \xi) = 0$, then (IC′) implies

$$U_D(\theta'', \xi) \geq W_D(\theta'') > W_D(\theta') = U_D(\theta', \xi).$$

Claim (b). Fix any θ, and assume $\xi'' > \xi'$. If $\pi(\theta, \xi'') = 0$, then (IC′) implies

$$U_D(\theta, \xi') \geq W_D(\theta) = U_D(\theta, \xi'').$$

Otherwise, if $\pi(\theta, \xi'') = 1$, then (IC′) implies

$$\begin{aligned} U_D(\theta, \xi') &\geq V_D(\theta, \xi'') - K_D(h(\theta, \xi''), \xi') \\ &\geq V_D(\theta, \xi'') - K_D(h(\theta, \xi''), \xi'') \\ &= U_D(\theta, \xi''). \end{aligned}$$

If $h(\theta, \xi'') > 0$, then the second inequality in the above expression is strict. □

B.3.4 Binary Hassling Decision

We can obtain further characterization by assuming the choice of whether to hassle is dichotomous. Formally, assume the space of feasible hassling decisions is $\mathcal{H} = \{0, 1\}$. Without loss of generality, we may now allow ξ to represent the cost of $h = 1$, so that $K_D(h, \xi) = \xi h$. We also let $W_D(\theta) = \theta$ without loss of generality. Equation B.3 now reduces to

$$\Phi_D(\theta', \xi' \,|\, \theta, \xi) = (1 - \pi(\theta', \xi'))\theta + \pi(\theta', \xi') \left[V_D(\theta', \xi') - \xi h(\theta', \xi')\right].$$

As a helpful preliminary, we establish that an important implication of Proposition 5 still holds in the two-dimensional environment: if two types both settle in equilibrium and both have the same hassling choice, then they yield the same settlement value.

Lemma B.4. *For each $h \in \{0, 1\}$, there exists $V_h \in \mathbb{R}$ such that if $\pi(\theta, \xi) = 1$ and $h(\theta, \xi) = h$, then $V_D(\theta, \xi) = V_h$.*

Proof. Suppose $\pi(\theta', \xi') = \pi(\theta'', \xi'') = 1$ and $h(\theta', \xi') = h(\theta'', \xi'') = h$. (IC′) for (θ', ξ') implies

$$V_D(\theta', \xi') - \xi' h \geq V_D(\theta'', \xi'') - \xi' h,$$

and thus $V_D(\theta', \xi') \geq V_D(\theta'', \xi'')$. By the same token, (IC′) for (θ'', ξ'') implies $V_D(\theta'', \xi'') \geq V_D(\theta', \xi')$. Therefore, $V_D(\theta', \xi') = V_D(\theta'', \xi'')$. □

To avoid trivialities in the remaining results, we will restrict attention to direct mechanisms in which all three outcomes—war, settlement with no hassling, and settlement with hassling—are realized by at least one type.

Among the types that settle in equilibrium, the choice of whether to hassle is determined entirely by one's cost of hassling. Naturally, the cost of hassling must be no greater than the marginal increase in the settlement due to hassling.

Lemma B.5. *Let V_0 and V_1 be defined as in Lemma B.4.*

- *If $\pi(\theta, \xi) = 1$ and $h(\theta, \xi) = 0$, then $\xi \geq V_1 - V_0$.*
- *If $\pi(\theta, \xi) = 1$ and $h(\theta, \xi) = 1$, then $\xi \leq V_1 - V_0$.*

Proof. To prove the first claim, suppose $\pi(\theta', \xi') = 1$ and $h(\theta', \xi') = 0$. (IC′), combined with Lemma B.4, implies $V_0 \geq V_1 - \xi'$, which proves the result. The proof of the second claim is analogous. □

Each type's expected utility from settling is thus the upper envelope of the constant V_0 and the linear function $V_1 - \xi$. Whether a type settles or fights depends on the comparison between its war payoff and this upper envelope.

Lemma B.6. *Let V_0 and V_1 be defined as in Lemma B.4.*

- *If $\pi(\theta, \xi) = 0$, then $\theta \geq \max\{V_0, V_1 - \xi\}$.*

- *If* $\pi(\theta, \xi) = 1$, *then* $\theta \leq \max\{V_0, V_1 - \xi\}$.

Proof. Immediate from Lemma B.4 and the incentive compatibility condition. □

These results allow us to fully characterize the set of incentive compatible direct mechanisms in the binary hassling case.

Proposition B.7. *Assume* $\mathcal{S} \subseteq [\underline{\theta}, \overline{\theta}] \times [\underline{\xi}, \overline{\xi}]$. *A direct mechanism is incentive compatible if and only if there exist* $V_0 \in [\underline{\theta}, \overline{\theta}]$ *and* $V_1 \in [V_0 + \underline{\xi}, V_0 + \overline{\xi}]$ *such that:*

(a) If $\pi(\theta, \xi) = 1$, *then* $V_D(\theta, \xi) = V_{h(\theta,\xi)}$.

(b) If $\pi(\theta, \xi) = 1$ *and* $\xi < V_1 - V_0$, *then* $h(\theta, \xi) = 1$.

(c) If $\pi(\theta, \xi) = 1$ *and* $\xi > V_1 - V_0$, *then* $h(\theta, \xi) = 0$.

(d) If $\theta > \max\{V_0, V_1 - \xi\}$, *then* $\pi(\theta, \xi) = 0$.

(e) If $\theta < \max\{V_0, V_1 - \xi\}$, *then* $\pi(\theta, \xi) = 1$.

Proof. Necessity. Suppose the direct mechanism satisfies (IC′). Lemma B.4 implies the existence of V_0 and V_1, and (IC′) implies $V_0 \in [\underline{\theta}, \overline{\theta}]$ and $V_1 \in [V_0 + \underline{\xi}, V_0 + \overline{\xi}]$.[31] Claim (a) then follows from Lemma B.4, claims (b) and (c) follow from Lemma B.5; and claims (d) and (e) follow from Lemma B.6.

Sufficiency. Suppose the direct mechanism satisfies the conditions of the proposition. Property (a) implies that each type's payoff from settlement without hassling is V_0, and each type's payoff from settlement with hassling is $V_1 - \xi$.

Now take any type (θ', ξ') that goes to war: $\pi(\theta', \xi') = 0$. Property (e) implies $\theta' \geq \max\{V_0, V_1 - \xi'\}$, so there is no incentive for (θ', ξ') to deviate to settlement with or without hassling.

Next take any type (θ', ξ') that settles and does not hassling: $\pi(\theta', \xi') = 1$ and $h(\theta', \xi') = 0$. Property (b) implies $V_0 \geq V_1 - \xi'$, so there is no incentive for this type to deviate to settlement with hassling. Property (d) in turn implies $V_0 \geq \theta'$, so there is also no incentive to deviate to war.

[31] A violation of these boundary conditions would imply that one or more outcomes are not reached by any type along the path of play, violating our restriction to mechanisms where all three possibilities occur. For example, $V_0 < \underline{\theta}$ would imply that every type strictly prefers war over settlement with no hassling, so there would be no type for which $\pi(\theta, \xi) = 1$ and $h(\theta, \xi) = 0$.

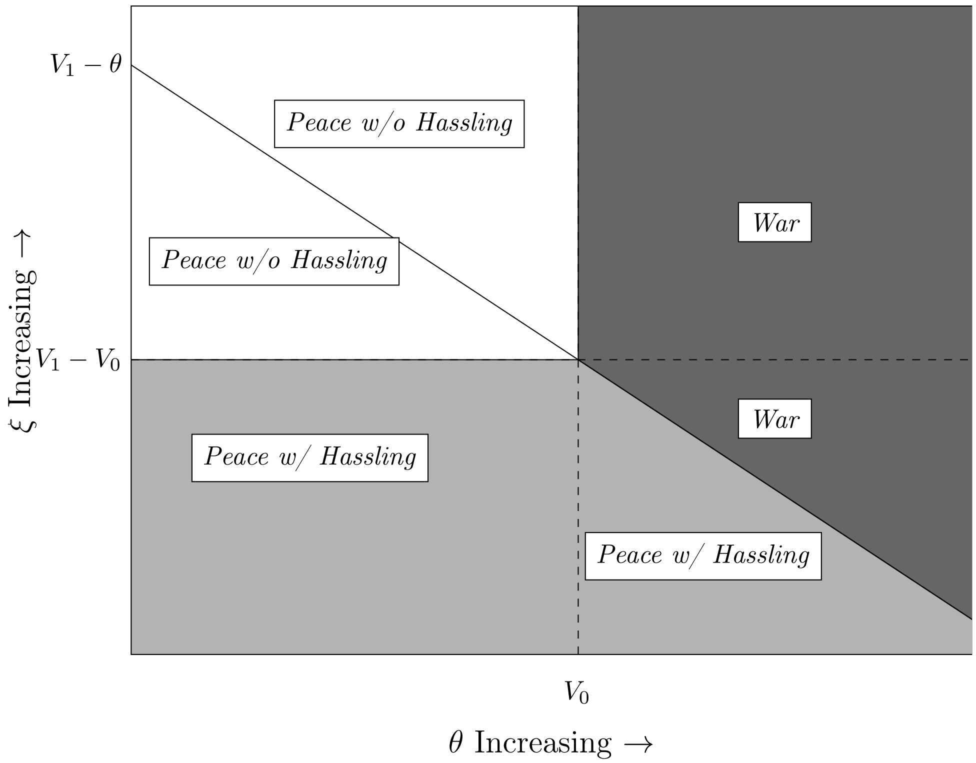

Figure 5: Equilibrium outcomes in the framework with multidimensional types and a binary hassling decision.

Finally, take any type (θ', ξ') that settles and hassles: $\pi(\theta', \xi') = 1$ and $h(\theta', \xi') = 1$. Properties (c) and (d) then imply $V_1 - \xi' \geq \max\{V_0, \theta'\}$, so there is no incentive for this type to deviate to settlement without hassling or to war. □

Figure 5 illustrates the shape of an incentive compatible direct mechanism in the binary hassling case. Naturally, war occurs when D's war payoff and hassling costs are both high (top-right of the figure). If D's war payoff is low (left side), then the equilibrium ends in settlement, with D hassling if its costs of doing so are low (bottom-left) and not hassling otherwise (top-left). The most interesting scenario is when D's war payoff is high and its hassling costs are low (bottom-right). In this case there may be war or settlement with hassling, depending on the exact magnitude of war payoffs compared to hassling costs.

We also see our general results on the two-dimensional framework, Proposition B.4 and Proposition B.5, reflected in the figure. Consider any point in the figure where the equilibrium outcome is war. Per Proposition B.4, any increase in D's war payoff accompanied by no change or an increase in D's hassling costs (moving right or upper right) will still have war as the outcome. Meanwhile, per Proposition B.5, the effect of a simultaneous improvement in D's war payoff and reduction in its hassling costs (moving down and to the right) is conditional. If the reduction in hassling costs is greater than the increase in war payoff, then the type with the higher war payoff may settle instead of going to war.