Numerical Methods in the Hydrological Sciences

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Preface

This electronic book provides students, instructors and professionals in the hydrological sciences the tools they need to pursue study and research using numerical methods. It will enable them to write programs to solve fairly complex problems, to explore and understand the current literature in which numerical methods are used, and to have the confidence to delve into texts on numerical methods to extend their knowledge to solve new problems.

The book combines an introduction to a suite of useful numerical methods with examples that illustrate their application to a range of hydrological problems. We have adopted *MATLAB*®[1] as the computational framework for the notes because it provides the reader with a set of mathematical and graphical tools that makes it possible to gain solid experience in writing code while avoiding the details of particular programming languages.

There are thirteen weeks in the typical semester and we have provided a chapter for each week. The first chapter presents an introduction to using *MATLAB* for computation, including the basics of command-line operation and a description of how script and function files are used to create code to solve problems. Subsequent chapters cover the solution of nonlinear equations (Chapter 2), numerical differentiation and integration (Chapter 3), numerical solution of ordinary differential equations (Chapters 4 and 5), numerical solution of partial differential equations using finite differences (Chapters 6, 8, and 9), iterative solution of systems of linear equations (Chapter 7), numerical solution of partial differential equations using the finite element method (Chapters 10 and 11), Fourier analysis of time series (Chapter 12), and interpolation of spatial data (Chapter 13). Each chapter uses one or more hydrological examples to illustrate methods as they are developed, and each of the chapter presents a set of homework problems, so readers can test their understanding of how to create code to solve problems.

The book derives from a set of lecture notes for a one-semester graduate course on applications of numerical methods in the hydrological sciences. The notes focus on the basic concepts, algorithms, and skills required for numerical computations and their application. Some background material and mathematical detail are presented in separate boxes; more often readers are directed to texts and papers where details can be found if needed. We recommend that the notes be used in conjunction with a "standard" numerical methods textbook to provide a more complete discussion of the mathematical underpinnings and considerations of the various methods.

The structure and length of the book are well suited to a semester-long course. Instructors can use it as the core of a course or as supplementary material in a broader course on numerical methods. Similarly, hydrology students might use it as a course text or as a supplement to a course with a less hydrological focus. Professionals in hydrology and other fields who want to learn numerical methods (or refresh their knowledge) via a self-study route will find that the notes provide a broad introduction to numerical methods with many examples and problems to illustrate and

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[1] *MATLAB* is a registered trademark of The MathWorks, Inc.

provide experience with such methods. *MATLAB* codes for all example problems are provided; solutions to homework problems at the conclusion of each chapter are available in an appendix to the main text.

We thank the many students who took our course over the past decade for the comments that they made to help us improve the lecture notes. Our students tell us that the notes have proven very useful to them. We hope that they will prove to be as useful to others. Special thanks go to Jeffrey Raffensperger who pioneered the course with us in the 1990s.

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George M. Hornberger is Ernest H. Ern Professor of Environmental Sciences at the University of Virginia, where he has taught since 1970. His research focuses on understanding how hydrological processes affect the transport of dissolved and suspended constituents through catchments and aquifers. Throughout his career, Professor Hornberger has employed numerical analyses in his research and in his teaching. Hornberger received his B.S. and M.S. degrees from Drexel University in 1965 and 1967, respectively. In 1970, he received a Ph.D. in hydrology from Stanford University. He is co-author of two textbooks, including *Numerical Methods in Subsurface Hydrology* (John Wiley & Sons).

Patricia L. Wiberg is Professor of Environmental Sciences at the University of Virginia, where she has taught since 1990. Her research focuses on the mechanics of sediment erosion, transport and deposition, as well as associated evolution of sediment bed properties and morphology. Professor Wiberg's research frequently involves development and application of process-based numerical models and analysis of time series and other data. Wiberg received her B.A. in mathematics from Brown University in 1976 and an M.S. and Ph.D. in oceanography from the University of Washington in 1983 and 1987, respectively. Wiberg and Hornberger, along with Jeffrey Raffensperger and Keith Eshleman, are co-authors of *Elements of Physical Hydrology* (Johns Hopkins University Press).

Introduction

In 1917, L.F. Richardson was attached to a French infantry division on the western front. As World War I raged about him, however, he took upon himself a scientific project of great complexity: to predict the weather over a section of western Europe for a six-hour period given measurements of winds, pressures, temperatures, etc. as a starting point. His numerical computation employed an approximation to the equations describing atmospheric dynamics and was done with the help of a slide rule, the only computational aid available at the time. The computation took six months to complete and was a spectacular failure. (See Hayes, 2001 for a delightful exposition of the story.) Failure or not, Richardson's basic plan was sound, as successful numerical weather prediction proved decades later, with the advent of digital computers.

Computation has been a staple of science, including hydrological science, from the very beginning. *Numerical* methods, whereby solutions to problems are obtained through approximations, are also venerable, with contributions from many famous mathematicians from about the 17th century onward. As was the case for numerical weather prediction, however, the widespread use and refinement of a variety of numerical methods applied to hydrological problems came with the development of digital computers.

Today quite sophisticated software is available commercially that allows hydrologists to solve problems and visualize results on a personal computer. Using such software often does not require much knowledge of how the codes work. However, familiarity with numerical methods (solution techniques and their implementation) can allow a person to modify existing software or to solve new problems that may arise in the course of a professional career. It also provides a solid conceptual basis for evaluating commercial software. Our view is that some experience with numerical methods, even for hydrologists who are "experimentalists" and not "theoreticians", is extremely valuable. For the more theoretically inclined, numerical methods are likely to figure prominently in their approach to solving hydrological problems.

The term "numerical methods" covers a very broad spectrum of topics. Some texts on numerical methods in hydrology focus exclusively on the solution of the partial differential equations governing flow of groundwater (e.g., Wang and Anderson 1982; Bear and Verruijt 1987). Other texts focus on models involving ordinary differential equations (e.g., Walker 1991). Data analysis, especially including analysis of time series and spatial data, is another area where a text on numerical methods in hydrology could be focused (e.g., Middleton 2000). And, of course, texts can "mix and match" (e.g., Carr 1995). We have chosen the latter course and cover in this book a series of topics that we deem of greatest importance given the constraints of covering them in a single semester.

When we first discussed how to structure the course from which these notes derive, we debated how much we could cover given that our students typically had limited programming experience. We finally decided to use *MATLAB* as the computational engine because we were convinced then that we could cover many more applications than we could if we used more standard programming languages while still giving students solid experience in writing code. We outlined a set of topics with which we thought all students should be acquainted.

The result was an ambitious undertaking for a one-semester course. But although students find our course challenging, they generally acknowledge at the completion of the term that they have met with success.

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