Supplementary Materials for

Modeling Multivariate Count Time Series Data with a Vector Poisson Log-Normal Additive Model: Applications to Testing Treatment Effects in Single-Case Designs

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Scatter and Lag Plots among the Three Behaviors

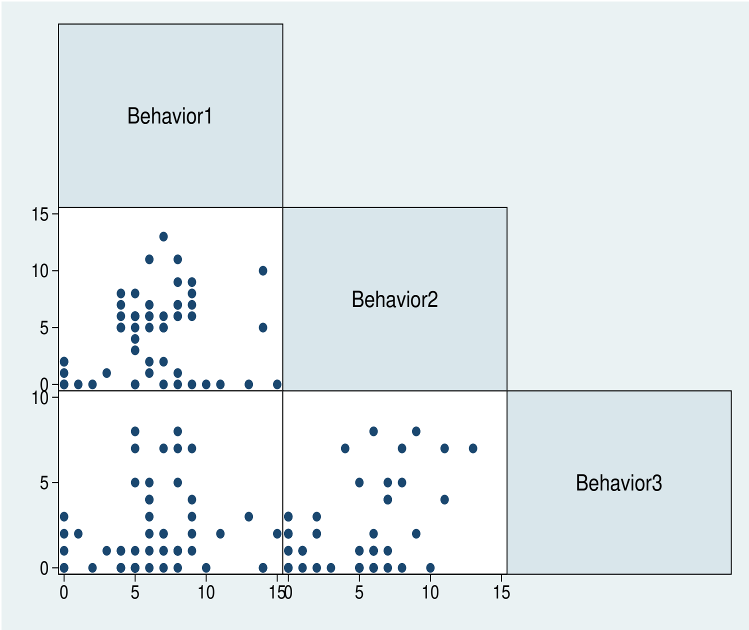

Figure 1. A scatter plot among the three behaviors.

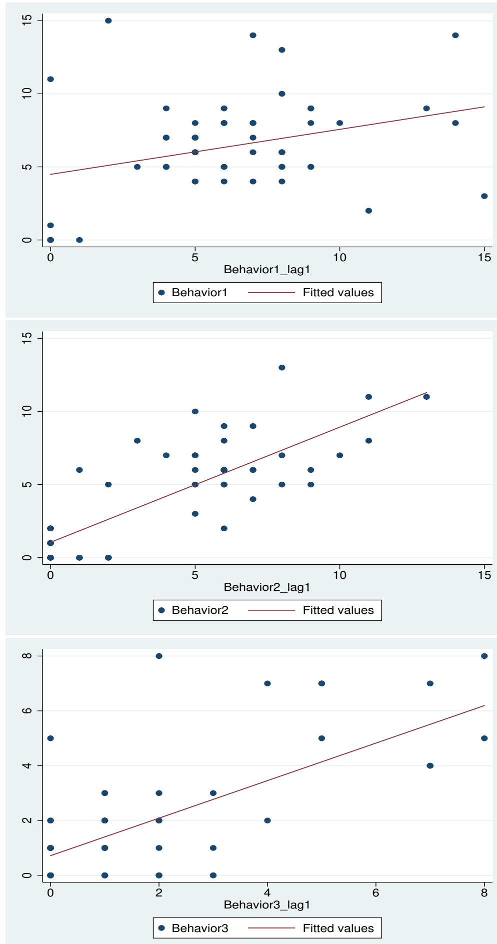

Figure 2. First-order lag plots for Behavior 1 (a teacher's descriptive praise; top panel), Behavior 2 (promoting social interactions; middle panel), and Behavior 3 (redirections; bottom panel).

V-PLN-A Specification of Model4

Model4 for Behavior 1 ($d = 1$) is written as

$$log(\lambda_{1t}) = \beta_{10} + \beta_{11}LC_{1t1} + \beta_{12}SC_{1t1} + \beta_{13}LC_{1t2} + \beta_{14}SC_{1t2}$$

$$+\phi_{11}y_{1(t-1)} + \phi_{12}y_{2(t-1)} + \phi_{13}y_{3(t-1)} + f_1(session_{1t}) + \epsilon_{1t}. \quad (1)$$

Model4 for Behavior 2 ($d = 2$) is written as

$$log(\lambda_{2t}) = \beta_{20} + \beta_{21}LC_{2t1} + \beta_{22}SC_{2t1} + \beta_{23}LC_{2t2} + \beta_{24}SC_{2t2}$$

$$+\phi_{21}y_{1(t-1)} + \phi_{22}y_{2(t-1)} + \phi_{23}y_{3(t-1)} + f_2(session_{2t}) + \epsilon_{2t}. \quad (2)$$

Model4 for Behavior 3 ($d = 3$) is written as

$$log(\lambda_{3t}) = \beta_{30} + \beta_{31}LC_{3t1} + \beta_{32}SC_{3t1} + \beta_{33}LC_{3t2} + \beta_{34}SC_{3t2}$$

$$+\phi_{31}y_{1(t-1)} + \phi_{32}y_{2(t-1)} + \phi_{33}y_{3(t-1)} + f_3(session_{2t}) + \epsilon_{3t}. \quad (3)$$

Parameters are defined as

- β_{10}, β_{20}, and β_{30} are intercepts for Behavior 1, Behavior 2, and Behavior 3, respectively,
- β_{11}, β_{21}, and β_{31} are the first level change for Behavior 1, Behavior 2, and Behavior 3, respectively,
- β_{12}, β_{22}, and β_{32} are the first slope change for Behavior 1, Behavior 2, and Behavior 3, respectively,
- β_{13}, β_{23}, and β_{33} are the second level change for Behavior 1, Behavior 2, and Behavior 3, respectively,
- β_{14}, β_{24}, and β_{34} are the second slope change for Behavior 1, Behavior 2, and Behavior 3, respectively,

- $\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix}$; diagonal terms in Φ are autoregressive coefficients and off-diagonal terms in Φ are cross-regressive coefficients, and

- ϵ_{1t}, ϵ_{2t}, and ϵ_{3t} are random variables for Behavior 1, Behavior 2, and Behavior 3, respectively.

Data Description of OpenBUGS Code for a V-PLN-A Model

Below, X1 in the OpenBUGS code is described. X2 and X3 can be specified in a similar way to X1. X1 is structured as a 53×12 data matrix:

```
X1=structure(.Data =c(
1,0,0,0,0,0,0,0,-0.582433372,-0.47379295,-0.582433372,-0.220298633     ,
1,0,0,0,0,0,2,0,-0.431364031,-0.484016037,-0.546521063,-0.211179622    ,
1,0,0,0,0,1,0,2,-0.282106384,-0.493193635,-0.51105495,-0.202069937     ,
1,0,0,0,0,0,2,2,-0.136472122,-0.500280258,-0.476481225,-0.192978902    ,
1,0,0,0,0,0,0,3,0.00372706,-0.504230418,-0.443246085,-0.183915841      ,
1,1,0,0,0,0,1,1,0.136679471,-0.503998626,-0.411795723,-0.174890081     ,
1,1,0,1,0,11,0,2,0.260573416,-0.498539394,-0.382576334,-0.165910946    ,
1,1,0,2,0,2,0,0,0.373597202,-0.486807235,-0.356034113,-0.15698776      ,
1,1,0,3,0,15,0,2,0.473939138,-0.467756661,-0.332615254,-0.14812985     ,
1,1,0,5,0,3,1,1,0.559787529,-0.440342184,-0.312765953,-0.13934654      ,
1,1,0,6,0,5,0,1,0.629330683,-0.403518316,-0.296932403,-0.130647155     ,
1,1,0,7,0,7,2,2,0.680756906,-0.356239569,-0.2855608,-0.12204102        ,
1,1,0,8,0,7,0,2,0.712254506,-0.297460455,-0.279097337,-0.11353746      ,
1,1,0,9,0,8,1,0,0.72201179,-0.226135487,-0.27798821,-0.1051458         ,
1,1,0,10,0,10,0,0,0.709003453,-0.141903024,-0.28234839,-0.096909526    ,
1,1,0,11,0,8,0,1,0.67534975,-0.047136818,-0.290967956,-0.089008773     ,
1,1,0,12,0,13,0,3,0.623957321,0.055105533,-0.302305765,-0.081657834    ,
1,1,0,13,0,9,0,3,0.55773281,0.16176643,-0.314820672,-0.075071005       ,
1,1,0,14,0,5,0,0,0.47958286,0.269788273,-0.326971532,-0.06946258       ,
1,1,0,15,0,6,1,1,0.392414112,0.376113464,-0.337217203,-0.065046855     ,
1,1,0,16,0,8,6,1,0.299133209,0.477684404,-0.34401654,-0.062038123      ,
1,1,0,17,0,6,2,3,0.202646794,0.571443494,-0.3458284,-0.060650679       ,
1,1,0,18,0,4,5,0,0.105861508,0.654333134,-0.341111637,-0.06109882      ,
1,1,0,19,0,7,5,0,0.011683995,0.723295727,-0.328325108,-0.063596838     ,
1,1,0,20,0,4,5,1,-0.076979104,0.775273672,-0.30592767,-0.06835903      ,
1,1,0,22,0,5,3,0,-0.157221145,0.807209371,-0.272378177,-0.075599689    ,
1,1,0,23,0,4,8,0,-0.226135487,0.816045226,-0.226135487,-0.085533112    ,
1,1,0,24,0,7,5,1,-0.281502473,0.799787017,-0.166345441,-0.098167826    ,
1,1,0,26,0,6,6,2,-0.323850394,0.760694047,-0.094901828,-0.112689297    ,
1,1,0,27,0,5,5,0,-0.354394525,0.702088999,-0.014385422,-0.128077227    ,
1,1,0,28,0,4,7,0,-0.374350143,0.627294556,0.072623002,-0.143311316     ,
1,1,0,29,0,9,6,1,-0.384932525,0.539633402,0.163542669,-0.157371263     ,
1,1,0,31,0,8,6,1,-0.387356946,0.442428219,0.255792804,-0.16923677      ,
1,1,0,32,0,4,6,1,-0.382838683,0.339001691,0.346792633,-0.177887536     ,
1,1,0,33,0,5,6,0,-0.372593012,0.2326765,0.433961381,-0.182303262       ,
1,1,0,34,0,7,6,1,-0.357835209,0.126775329,0.514718273,-0.181463647     ,
1,1,0,35,0,14,5,0,-0.339780552,0.024620862,0.586482535,-0.174348394    ,
1,1,0,37,0,14,10,0,-0.319644315,-0.070464218,0.646673391,-0.159937201  ,
1,1,0,38,0,8,7,1,-0.298641776,-0.155157229,0.692710068,-0.13720977     ,
1,1,0,39,0,5,6,0,-0.27798821,-0.226135487,0.72201179,-0.1051458        ,
1,1,0,40,0,6,6,0,-0.258676294,-0.280848518,0.73267555,-0.063027821     ,
1,1,0,41,0,5,8,5,-0.240808301,-0.319834686,0.725509405,-0.011349685    ,
1,1,0,42,0,7,13,7,-0.224263904,-0.344404562,0.701999182,0.04909193     ,
1,1,0,43,0,8,11,7,-0.208922776,-0.355868719,0.663630706,0.117500344    ,
1,1,0,44,0,6,11,4,-0.194664591,-0.355537727,0.611889802,0.193078876    ,
1,1,0,45,0,9,8,7,-0.181369021,-0.34472216,0.548262295,0.275030848      ,
1,1,0,47,0,9,7,4,-0.168915739,-0.324732589,0.474234011,0.362559578     ,
1,1,0,48,0,9,9,2,-0.157184419,-0.296879586,0.391290774,0.454868388     ,
1,1,0,49,0,5,6,8,-0.146054734,-0.262473723,0.300918412,0.551160596     ,
1,1,1,50,0,8,9,8,-0.135406356,-0.222825572,0.204602747,0.650639524     ,
1,1,1,51,1,6,5,5,-0.125118959,-0.179245705,0.103829607,0.752508492     ,
1,1,1,52,2,8,7,5,-0.115072216,-0.133044694,8.48E-05,0.85597082         ,
1,1,1,53,3,5,4,7,-0.1051458,-0.085533112,-0.1051458,0.960229827
),.Dim = c(53, 12)),
```

There is no observation at the first time point $t = 0$ for the first-order lag outcomes ($\mathbf{y}_{t-1}$). In this study, the lag covariate is treated as a missing variable and its subsequent response variable was not used in MCMC implementation. As a result, there are 53 rows instead of 54 rows for 54 time points. Each column of X1 (specified in Equation 7 in the paper) is defined as:

- Column 1: A covariate of an intercept, a vector of 1
- Column 2: The first level-change dummy coded variable at time t for Behavior 1, LC_{1t1}
- Column 3: The second level-change dummy coded variable at time t for Behavior 1, LC_{1t2}
- Column 4: The first slope-change dummy coded variable at time t for Behavior 1, SC_{1t1}
- Column 5: The second slope-change dummy coded variable at time t for Behavior 1, SC_{1t2}
- Column 6: The first-order lag variable for Behavior 1, $y_{1(t-1)}$
- Column 7: The first-order lag variable for Behavior 2, $y_{2(t-1)}$
- Column 8: The first-order lag variable for Behavior 3, $y_{3(t-1)}$

The last four columns in X1 are CRS basis functions (specified in Equation 6 in the paper):

- Column 9: The first basis function, b_{11}
- Column 10: The second basis function, b_{12}
- Column 11: The third basis function, b_{13}
- Column 12: The fourth basis function, b_{14}

A V-PLN-A Model with Nonlinear Treatment Effects and Nonlinear Trend over Sessions

Below, a V-PLN-A model with nonlinear treatment effects ($f_d(SC_{dt1})$ and $f_d(SC_{dt2})$) and nonlinear trend over sessions ($f_d(session_{dt})$) is specified for the three phases (baseline-treatment-baseline [maintenance]) SCD:

$$\begin{aligned} log(\lambda_{dt}) = \beta_{d0} + \beta_{d1} LC_{dt1} + f_d(SC_{dt1}) + \beta_{d2} LC_{dt2} + f_d(SC_{dt2}) \\ + \phi_{d1} y_{1(t-1)} + \ldots + \phi_{dD} y_{D(t-1)} + f_d(session_{dt}) + \epsilon_{dt}, \end{aligned} \tag{4}$$

where

- LC_{dt1} is the first level-change dummy-coded variable at time t for a behavior d,
- SC_{dt1} is the first slope-change variable at time t for a behavior d; SC_{dt1} is the interaction between a dummy-coded variable for baseline vs. intervention and a sequence number of sessions,
- LC_{dt2} is the second level-change dummy-coded variable at time t for a behavior d,
- SC_{dt2} is the second slope-change variable at time t for a behavior d; SC_{dt2} is the interaction between a dummy-coded variable for intervention vs. maintenance and a sequence number of sessions,
- $y_{d(t-1)}$ is the first-order lag variable,
- β_{d0} is the intercept at the baseline for a behavior d,
- β_{d2} is the second level change (treatment vs. maintenance) for a behavior d,
- ϕ_{dd} is the first-order AR effect for a behavior d; $\phi_{dd'}$ is the first-order CR effect for a behavior d and a behavior $d^{'}$,
- $f_d(SC_{dt1})$ is a smooth function for data-driven first slope change (baseline vs. treatment) for a behavior d,
- $f_d(SC_{dt2})$ is a smooth function for data-driven second slope change (treatment vs. maintenance) for a behavior d,

- $f_d(session_{dt})$ is a smooth function for data-driven trend for a behavior d, and
- ϵ_{dt} is the random variable for a behavior d at time t.