A Dynamic Tree-Based Item Response Model

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Abstract

- This study presents a dynamic tree-based item response (IRTree) model as a novel extension of the autoregressive generalized linear mixed effect model (dynamic GLMM), capable of modeling differentiated processes indicated by intensive polytomous time series eye-tracking data.
- The dynamic IRTree model is a general modeling framework which can model change processes (trend and autocorrelation) and which allows for the decomposition of data into various sources of heterogeneity.
- An experimental study that employed the visual world eye-tracking technique was used to illustrate the dynamic IRTree model.
- The results of a simulation study showed that parameter recovery of the model was satisfactory and that ignoring trend and autoregressive effects resulted in biased estimates and standard errors of experimental condition effects in the same conditions found in the empirical study.

Study Motivation & Purpose

- Literature reviews on existing IRTree models and time series models lead to the conclusion that there is a disconnect between the available analytic methods and a common data structure in studies of real-time cognitive processes using the visual world eye tracking technique.
- This disconnect can be resolved by combining the IRTree model with the time series model.
- The goal of the novel modeling framework was to allow for:
  - differential processing depending on the response option (based on the tree feature of the model),
  - heterogeneity of the processes (based on the IRT feature of the model), and
  - change processes (trend and autoregressive parameters) as in the time series models and in the dynamic GLMM.
- The novelty of the dynamic IRTree model lies in the combination of three features: the tree feature, the IRT feature, and the dynamic feature. All three are important to answer substantive research questions regarding cognitive processes underlying data from a linguistically-inspired eye tracking study.

Empirical Study: Intensive Polytomous Time Series Data from Eye Trackers

- The data set comes from a study previously published as Ryskin, Benjamin, Tullis, and Brown-Schmidt (2015).
- 152 native English-speaking participants from the University of Illinois at Urbana-Champaign.
- Eye tracking was conducted with Eyelink 1000 eye-trackers.
- Participants took turns instructing each other to click on objects on the computer screen (e.g., "Click on the small elephant."):

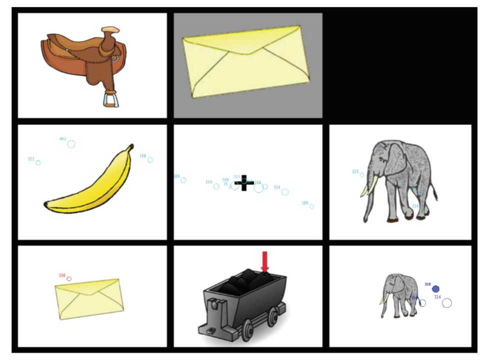

Figure 1: An example of visual stimulus from the perspective of one participant.

- In addition to the target object (e.g., a small elephant), there was a competitor object of a different size (e.g., a large elephant) and unrelated objects (e.g., a banana).
- Each person participated in 288 trials under three experimental conditions (Two Contrasts-Shared; Two Contrasts-Privileged; One Contrast).
- The xy-positions of participants' eye-fixations were recorded in 10 millisecond intervals. Each of the trials yielded eye-fixation data for 112 equally-spaced time points.
- Each trial featured an "item," which was the object that the participant had to click on (e.g., duck, frog, elephant). There were a total of 96 unique items in the dataset.
- Data had a multilevel structure, with time series eye-fixation data (level 1) nested in 288 trials (level 2) cross-classified by person and items (level 3).

Data Description: Change Processes

- Autocorrelations did not converge to zero, even at large values of time lag, indicating that there are trend and autocorrelations. In addition, there was a large dropoff in partial autocorrelations for time lags > 1, suggesting that it was necessary to model just first-order autocorrelations.
- To examine trend over time, we also plotted empirical logits for trials across persons over time, which resulted in a positive and (approximately) linear trend over time. Fitted lines over time were similar between the linear function and Kernal-weighted local polynomial smoothing function, and only small deviations from the linear trend were observed.

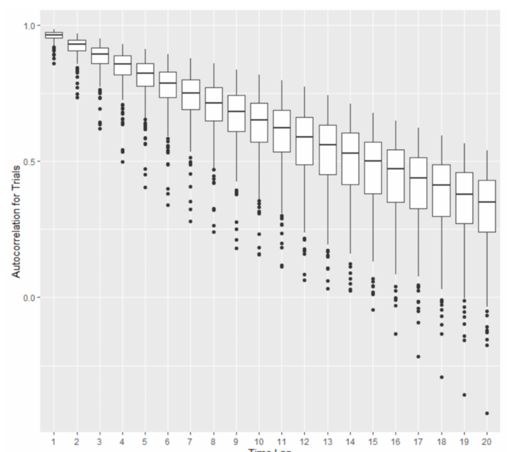

Figure 2: Autocorrelation

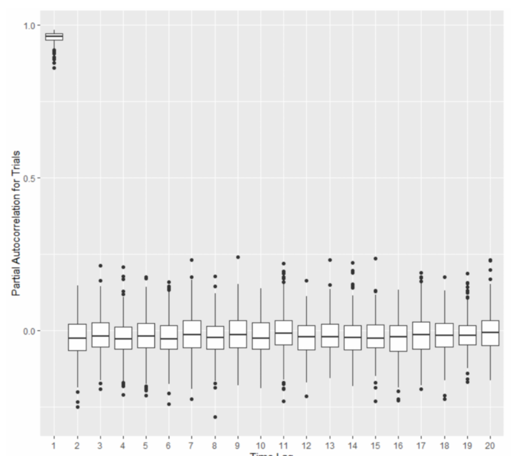

Figure 3: Partial autocorrelation

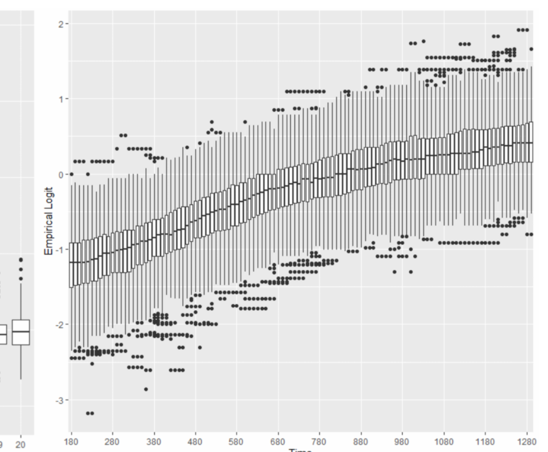

Figure 4: Trend

Model Specification

- To specify a dynamic IRTree model, original polytomous responses y were recoded into binary data y^* nested within two nodes:

Response	*node*	y	y^*
target	1	1	1
competitor	1	2	1
unrelated objects	1	3	0
target	2	1	1
competitor	2	2	0
unrelated objects	2	3	NA

- This structure resulted in missing data when $node = 2$ and $y = 3$, which causes problems when using past observations in modeling.

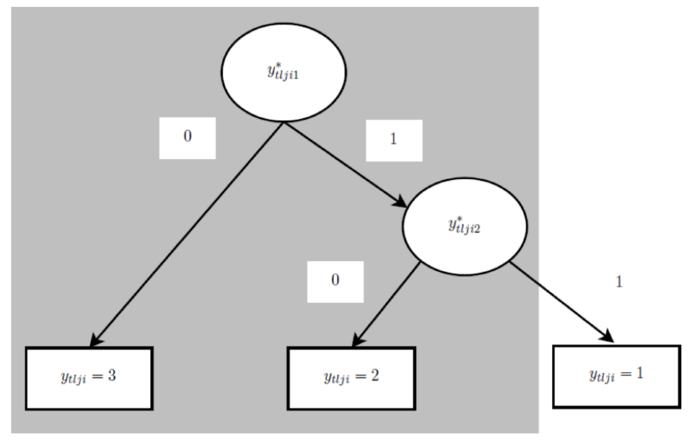

Figure 5: Tree diagram for binary processes (two branches at each node in the tree) within a three-category paradigm.

- Two dynamic IRTree models were used to reconcile this missing data, one using two lag covariates indicating whether the person looked at the target or competitor at the previous time point ($x_{T(t-1)ljir}$ and $x_{C(t-1)ljir}$), and the other using the last preceding observation ($y^*_{(t-1)ljir}$).
- In addition, the three experimental conditions were recoded using Helmert-coded contrasts:

Condition	*contrast*	*privileged*
One Contrast	-1	0
Two Contrasts-Privileged	0.5	0.5
Two Contrasts-Shared	0.5	-0.5

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- The model with lag covariates ($x_{T(t-1)ljir}$) and $x_{C(t-1)ljir}$), called Model 1, is written as follows:

$$\begin{aligned}&\text{logit}[P(y^*_{tljir}|x_{T(t-1)ljir}, x_{C(t-1)ljir}, time_{tljir}, \mathbf{x}, \delta_{ljir}, \lambda_{1jr}, \theta_{jr}, \lambda_{2ir}, \beta_{ir})] =\\ &[x'_{T(t-1)ljir}\lambda_{Tr} + x'_{C(t-1)ljir}\lambda_{Cr} + time'_{tljir}\zeta_r + \mathbf{x}'\gamma_r]\\ &+ \delta_{ljir} + [x'_{T(t-1)ljir}\lambda_{T1jr} + x'_{C(t-1)ljir}\lambda_{C1jr} + \theta_{jr}]\\ &+ [x'_{T(t-1)ljir}\lambda_{T2ir} + x'_{C(t-1)ljir}\lambda_{C2ir} + \beta_{ir}],\end{aligned}$$

where t is an index for time, l is an index for trial, j is an index for person, i is an index for item, and r is an index for node.
- The model with lag covariate $y^*_{(t-1)ljir}$, called Model 2, is written as follows:

$$\begin{aligned}&\text{logit}[P(y^*_{tljir}|y^*_{(t-1)ljir}, time_{tljir}, x, \delta_{ljir}, \lambda_{1jr}, \theta_{jr}, \lambda_{2ir}, \beta_{ir})] =\\ &[y^{*'}_{(t-1)ljir}\lambda_r + time'_{tljir}\zeta_r + x'\gamma_r] + \delta_{ljir} + [y^{*'}_{(t-1)ljir}\lambda_{1jr} + \theta_{jr}] + [y^{*'}_{(t-1)ljir}\lambda_{2ir} + \beta_{ir}]\end{aligned}$$

- Model 1 and Model 2 above each explain y^*_{tljir} using the following fixed and random effects:
- Fixed effects
  - Intercept γ_{0r}
  - Lag fixed effects λ_{Tr} and λ_{Cr} (Model 1 only)
  - Lag fixed effect λ_r (Model 2 only)
  - First experimental condition (*privileged*) γ_{1r}
  - Second experimental condition (*contrast*) γ_{2r}
  - Trend fixed effect ζ_r
- Random effects
  - Trial random effect δ_{ljir}
  - Person random effects λ_{Tr}, λ_{Cr}, and θ_{jr} (Model 1 only)
  - Person random effects λ_r and θ_{jr} (Model 2 only)
  - Item random effects λ_{Tr}, λ_{Cr}, and β_{ir} (Model 1 only)
  - Item random effects λ_r and β_{ir} (Model 2 only)

Parameter Estimation

- The marginal likelihood for Model 1 is written as:

$$\prod_{r=1}^{R}\prod_{j=1}^{J}\prod_{i=1}^{I}\int_{\zeta_{1jr}}\int_{\zeta_{2ir}}\left[\prod_{t=1}^{T_{lji}-1}\prod_{l=1}^{L}\left\{\int_{\delta_{ljir}} P(y^*_{tljir}|y^*_{(t-1)ljir}, time_{tlji},, \delta_{ljir}, \lambda_{1jr}, \theta_{jr}, \lambda_{2ir}, \beta_{ir})g_1(\delta_{ljir})d\delta_{ljir}\right\}\right]d\zeta_{1jr}d\zeta_{2ir}$$

$$\cdot\prod_{r=1}^{R}\prod_{j=1}^{J}\int_{\zeta_{1jr}} g_2(\zeta_{1jr})d\zeta_{1jr}\cdot\prod_{r=1}^{R}\prod_{i=1}^{I}\int_{\zeta_{2ir}} g_3(\zeta_{2ir})d\zeta_{2ir},$$

where $\zeta_{1jr} = [\theta_{j1}, \theta_{j2}, \lambda_{1j1}, \lambda_{1j2}]'$ for random person effects, $\zeta_{2ir} = [\beta_{i1}, \beta_{i2}, \lambda_{2i1}, \lambda_{2i2}]'$ for random item effects, and $g_1(.)$, $g_2(.)$ and $g_3(.)$ are multivariate normal density functions.
- Parameter estimation was implemented using the `glmer` function in `lme4` version 1.1.15 (Bates et al., 2018) `R` package (R Core Team, 2017) for Laplace approximation and using `rStan` (Stan Development Team, 2018) for Bayesian analysis. Estimates from Laplace approximation and Bayesian analysis were comparable.

Empirical Study Results

- The tree approach makes a distinction between processing occurring at the two nodes:
  - Node 1 is for the processing of lexico-semantic information in the intiial words in the phrase (e.g., "the small e-...").
  - Node 2 refers to the resolution of ambiguity between the target and competitor (e.g., the small and large elephants).
- Model-data fit results and correlations between the two random person intercepts confirmed that each node comes with its own dimension.
- Experimental condition effects differed between nodes as well. For example, the *Contrast* effect was positive (0.050) at Node 1, indicating that the lexico-sematic processing of the unfolding expression activates the target and competitor more in the *Two-Contrasts* conditions than in the *One-Contrast* condition. However, the *Contrast* effect was negative (-0.386) at Node 2.
- The trend effect differed depending on the node. The trend was clearly steeper for the ambiguity resolution process (0.031 at Node 2) than for lexico-semantic processing (0.006 at Node 1).