Vacuum Pair Production/Annihilation and Cardiac String Dynamics

John P. Wikswo

Living State Physics Group

Departments of Physics and Astronomy, Molecular Physiology & Biophysics, and Biomedical Engineering
Vanderbilt Institute for Integrative Biosystems Research and Education
Vanderbilt University

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- Jim Weiss
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Courtesy of Peter Hunter, Auckland

textured-heart-spin.mpg
Where are the heart strings, and who is pulling them?

- The normal heart has none
- The presence of one string is serious
- The presence of several for a very few minutes is fatal

flavio_rabbit_vf.avi

Courtesy of Flavio Fenton, Hofstra
Outline

- The heart is a …
- Cardiac fibrillation
- Spiral waves in the heart
  - Two dimensions – Spiral waves
  - Three dimensions – Scroll waves
- Phase plane analysis
- Singularity identification
  - Simple reentry
  - Fibrillation
- Singularity interactions
  - Attraction vs repulsion versus oscillation
  - Annihilation
  - Creation
- What is needed?
  - Interaction potential
  - String creation operator
The Heart is a...

- Self-assembling,
- Biochemically powered,
- Electrically activated,
- Electrically non-linear,
- Pressure- and volume-regulated,
- Two-stage,
- Tandem,
- Mechanical pump
- With a mean time-to-failure of approximately two billion cycles.
The heart is... electrically activated...

From: The Ciba Collection of Medical Illustrations: Heart, F. H. Netter, 1978

Courtesy of Peter Hunter, Auckland
The heart is an ...

- Electrically activated,
- Mechanical pump
The Normal Heart Beat

Courtesy of Rick Gray and CRML, U. Alabama Birmingham
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Courtesy of Flavio Fenton
The heart is an electrically activated mechanical pump...

with a mean time-to-failure of approximately two billion cycles....

Normal  Tachycardia  Fibrillation  Defibrillation
Induction of Fibrillation

Courtesy of Rick Gray and CRML, U. Alabama Birmingham
Termination of Fibrillation

Courtesy of Rick Gray and CRML, U. Alabama Birmingham
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Leon Glass and *Physics Today*
Spiral and Scroll Waves in Nature

- A generic property of excitable media
- Have been shown to occur in
  - Circulating waves of bioelectric activity in cardiac and retinal tissue
  - Autocatalytic chemical reactions, such as Belousov-Zhabotinsky reaction (BZ)
  - cAMP waves in slime mold *Dictyostelium discoideum*
  - Intracellular calcium release in oocytes
  - Oxidation of CO on crystal surfaces in ultrahigh vacuum conditions
- Cardiac fibrillation involves multiple scroll waves in 3-D
Cardiac fibrillation occurs at the spatial scale of the entire heart, and involves multiple, interacting spiral and/or scroll waves!

Physics Today and Leon Glass, Montreal
Transmural versus intramural scroll waves in reentrant arrhythmias and fibrillation

- Transmural waves can exist in 2-D (thin) or 3-D (thick)
- Intramural waves require ~1 cm wall thickness

Courtesy of Arkady Pertsov, Syracuse
Transition from Normal Rhythm to Ventricular Tachycardia to Ventricular Fibrillation

Single spiral wave = Tachycardia
Multiple spiral waves = Fibrillation = SCD

Movies courtesy of Flavio Fenton

Initiation of Spiral Wave Reentry

S1-S2 crossed-field stimulation
A “Simple” Spiral Wave

The nature of the spiral is set by the non-linear properties of the excitable medium

– Linear core
– Epicycloidal meander
– Circular core

Courtesy of Flavio Fenton
Nonlinear Properties Determine the Trajectories

- Six Phenotypes
  - Circular
  - Epicycloidal
  - Cycloid
  - Hypercycloidal
  - Hypermeander
  - Linear core

- Winfree, Krinsky, Barkley, Efimov, Jalife, Pertsov, Gray, Roth, Fenton, Garfinkel, Chen …

Courtesy of Flavio Fenton
Non-linear dynamics of **reentry**, fibrillation, and defibrillation

- **Reentry** -- Self-sustained excitation due to propagating activation wave fronts in the heart that continue to re-excite different regions of tissue rather than terminating after a single excitation.

- **Anatomical reentry** -- activation wave fronts that travel in one direction around an anatomical obstacle.

- **Functional reentry** -- activation circulate around a dynamical phase singularity.
Spiral Wave and Figure-of-Eight Reentry

- **Spiral Wave:**
  - S1 vert line
  - S2 horiz line

- **Figure-of-Eight**
  - S1 vert line
  - S2 point
Spiral Wave, Figure-of-Eight, and Quatrefoil Reentry

- **Spiral Wave (A)**
  - S1 vertical line
  - S2 horizontal line
  - One singularity (plus boundary)

- **Figure-of-Eight (B)**
  - S1 vertical line
  - S2 point
  - Two singularities

- **Quatrefoil (C & D)**
  - Anisotropic cable
  - S1 point
  - S2 point
  - Cathodal (C) or anodal (D) have opposite rotations
  - Four singularities
Optical Imaging of Quatrefoil Reentry


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Transform into Phase Space

• The problem: a given voltage can either be rising or falling

• The solution: represent the cardiac action potential in terms of “phase” in the cardiac cycle:
  – 0, 1, 2, 3 …
  – 1%, 2%, 3%, 3%, 5%, …
  – 0°, 5°, 10°, 15°, 20°, 25°, …

• One definition of phase (of many):

\[
\phi(x, y, t) = \tan^{-1} \left[ \frac{V_m(x, y, t)}{dV_m(x, y, t)/dt} \right]
\]

Pictures by Mark Bray
From Voltage to Phase Space

Four singularities of indeterminate phase, i.e., points surrounded by all colors.

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Mark Bray
Why look for strings?

• Movies of the surface potentials are complicated
• It is not clear how much of the information is needed

• Model based upon
• Movies by Mark Bray

fhnplus_scroll_wave_breakup_surface_fps60.avi
Wavefronts are Better

- The wavefronts are better
- Require description of the dynamics of the entire system
Strings Alone May Be Best

• Surface singularities are simpler
• Filaments (strings) are the best
• Do they interact in a manner that can allow us to ignore the rest of the problem?
• HOW DO WE FIND THEM??

Mark Bray
Local Phase and the Wave Vector $\vec{k}$

- The spatial gradient of the phase $\phi$ is the wave vector $\vec{k}$

$$\vec{k} = -\nabla \phi(x, y)$$

**Topological Charge $\vec{k}$**

$$n_t \equiv \frac{1}{2\pi} \oint_c \nabla \phi \cdot d\ell$$

$$n_t \equiv \frac{-1}{2\pi} \oint_c \vec{k} \cdot d\ell$$
Phase and Topological Charge

- **Curl** $k$ is proportional to the topological charge!

$$\hat{z} \cdot [\nabla \times \vec{k}(\vec{x})] = \frac{\partial k_y}{\partial x} - \frac{\partial k_x}{\partial y} = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \oint_c \vec{k}(\vec{r}) \cdot d\ell$$

- It can be shown that the differential curl evaluates as exactly zero, except at the singularity, where it is undefined.

- At the singularity, the line integral around the singularity must be used directly to find the topological charge.

“Use of Topological Charge to Determine Filament Location in a Numerical Model of Scroll Wave Activity,” M.-A. Bray and J.P. Wikswo, Jr., *IEEE Trans BME*, in press
Phase Singularities in Cardiac Reentry

Phase (\(\phi\)) plot

\[ \text{Curl } k = \text{Curl } (\nabla \phi) \]

The phase singularities can be identified by computing the curl of the gradient of the phase distribution.

Mark Bray
Topological Charge

\[ n_t \equiv \frac{1}{2\pi} \oint_c \nabla \phi \cdot \mathbf{d}\ell \]

\[ n_t \equiv \frac{1}{2\pi} \oint_c \mathbf{k} \cdot \mathbf{d}\ell \]

- Topological charge \( n_t \) is zero about any closed path that does not encircle a phase singularity.
- \( n_t \) is +1 or -1 for a path that encircles a singularity with a single arm.
- Topological charge is conserved, i.e., singularities are created and destroyed in pairs.
Singularity Motion During Spiral Wave Breakup

Voltage

Curl of Phase

Courtesy of Rick Gray

BRDR.avi
Filaments in Three Dimensions

What looks like a figure-of-eight reentrant wave from the surface...

...is actually a 3-D scroll wave in the underlying myocardium with a filament connecting the two singularities

- Filaments are the 3-D analogue of the 2-D phase singularity

Mark Bray
Topological charge

- **Curl** $k$ may be approximated by
  1) a differential operator, or
  2) as a discretized contour interval that is in fact a convolution operation of an image with two Nabla windows

\[
\left( \nabla \times \vec{k} \right) \cdot \hat{z} \propto \nabla_x \otimes k_y + \nabla_y \otimes k_x,
\]

\[
\nabla_x = \begin{bmatrix}
+1 & +1 & +1 \\
0 & 0 & 0 \\
-1 & -1 & -1
\end{bmatrix} \quad \nabla_y = \begin{bmatrix}
-1 & 0 & +1 \\
-1 & 0 & +1
\end{bmatrix}
\]

Filaments in Three Dimensions

• Filaments are the 3-D analogue of the 2-D phase singularity

Mark Bray

bz_scroll_ring_surface.avi
bz_scroll_ring_filament_plus_wavefront.avi
Because curl is a three-dimensional vector operator, this convolution approach can be extended readily to 3-D in order to visualize scroll wave filaments.

Mark Bray
String Dynamics

- Strings with positive line tension shrink (Paniflov, Rudenko and Krinsky, Biophysics, 31: 926 (1986))

bz_scroll_ring_(filament).avi
bz_scroll_wave_stable_surface_fps60.avi
bz_scroll_wave_stable_filament_plus_wavefront_fps60.avi
bz_scroll_wave_stable_filament_fps60.avi
String Dynamics


• If they touch a surface, a pair of singularities is produced

• Topological charge is conserved

Movie Courtesy of Flavio Fenton
A Little Negative Line Tension

fhnplus_scroll_ring_k8_4panel.avi

Mark Bray
A Lot of Negative Line Tension

fhnplus_scroll_ring_k40_filament_plus_wavefront.avi
fhnplus_scroll_ring_k40_filament.avi
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Courtesy of Mark Bray
Quatrefoil Reentry

- Follows repeated stimuli applied at a single site
- Has been used to demonstrate the importance of unequal bidomain anisotropies in cardiac electrodynamics
- Provides a reproducible, controlled system for study of the interactions of phase singularities and their accompanying filaments
Quatrefoil Reentry

- We replicate the experimentally observed quatrefoil reentry configuration using a simulated pair of adjacent circular filaments (scroll rings) oriented along their symmetry axes with varying initial radii and separation distances.

Mark Bray
Reaction-Diffusion System

- We use a two-variable model of the Belousov-Zhabotinsky (BZ) reaction using the Field-Koros-Noyes formulation

\[
\frac{dv}{d\tau} = \frac{1}{\epsilon} \left[ v(1-v) - \left( 2q\alpha \frac{w}{1-w} + \beta \right) \frac{v - \mu}{v + \mu} \right] + \nabla^2 v
\]

\[
\frac{dw}{d\tau} = x - \alpha \frac{w}{1-w} + \delta \nabla^2 w
\]

where \( v \) is the bromous acid concentration, \( w \) is the relative ferroin concentration, and \( \delta = D_w/D_v \) (\( \delta = 1 \) in this case)

- For \( \delta = 1 \), \( \frac{d(R^2)}{dt} = -2D \)

- With this BZ formulation, a single ring shrinks with a relative absence of translational drift; permits us to observe interaction without large single ring dynamics
Methodology

- Modeled 3-D system using an axisymmetric cylindrical coordinate system $(z, \rho, \theta)$, such that all results are independent of angle $\theta \rightarrow$ Need only to examine 2-D $(z, \rho)$ plane
- Started rings at initial separation $(Z_0)$ and initial radius $(R_0)$ and examined life-time $(T_L)$ and motion in $(z, \rho)$ plane
- Simulated cathode and anode break with appropriate initial conditions
Initial Conditions

Experimental

Cathode break

Anode break

Numerical

Numerical – Wave Fronts

\[Z_0\]

\[R_0\]
Simulated Singularity Interactions

- Start with a pair of vortex rings of fixed diameter and positive line tension
- Measure decay time as a function of separation and initial size

BZ: Anode break

BZ: Cathode break

Mark Bray
Cathodal Break


1: Enhanced decay, attraction, and mutual annihilation per Elphick and Meron, Physica D, 53: 385 (1991)

M Bray and J. Wikswo, in preparation
Cathode break movie

$R_0 = 128, Z_0 = 52$

$t = 195$

$t = 600$

$t = 990$

$t = 1410$

$t = 1800$

$t = 2025$

Filament annihilation by collision

$R_0 = 128, Z_0 = 60$

$t = 225$

$t = 600$

$t = 990$

$t = 1410$

$t = 1800$

$t = 3840$

Self-annihilation by shrinkage
Anodal Break


1: Enhanced decay, attraction, and mutual annihilation per Elphick and Meron, Physica D, 53: 385 (1991)

2: Extended lifetime


Mark Bray

Mutual annihilation by collision

Self-annihilation by shrinkage
Anode break movie

$R_0 = 128, \ Z_0 = 52$

$R_0 = 128, \ Z_0 = 60$

Filament annihilation by collision

Delayed self-annihilation by shrinkage
Anodal Break Trajectories

Mark Bray
Initial Velocity = Force
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Courtesy of Jim Weiss
String Creation and Annihilation: Positive Line Tension with Fiber Rotation

Vacuum loop creation/annihilation  Loop pinch-off  Vacuum loop creation and coupling

Movies Courtesy of Flavio Fenton

• Wave break occurs when the leading edge of a wave runs into the tail of a preceding wave
• Wavebreaks create filaments which create reentrant activation
Future Questions

• For both cases, what parameters determine attractive versus repulsive behavior? Parameter gradients?

• Can a kinematic relationship be derived for the scroll ring interactions?
  – Is the effective mass constant or not, since it is a dissipative system?
  – Can the ring interaction be described by a point-to-point potential, and if so, are there obvious centers of action?

• In a field model, how do you introduce string creation from the vacuum?
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- **Rick Gray**
- Peter Hunter
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- Mark Lin
- Neils Otani
- Arkardy Pertsov
- Nathalie Virag
- Jim Weiss
- And many others

Courtesy of Peter Hunter, Auckland

fhnplus_scroll_wave_break_4panel.avi

textured-heart-spin.mpg
The End