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A Theory of Competitive Partisan Lawmaking

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ABSTRACT

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A Theory of Competitive Partisan Lawmaking*

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Abstract

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Although workhorse models of legislative choice are starkly different from one-another in their assumptions about majority party advantages, they are essentially identical in their denial of rights and resources to the minority party. This paper provides a theoretical account of lawmaking in which the minority party receives *some* procedural rights or resources, albeit fewer than those of the majority party. We find important exceptions to the general thrust of extant theories and argue that a more nuanced appreciation for how competitive parties' procedural and resource tools may allow future work to better integrate legislative politics with interest group and electoral politics.

Our theory is situated between two extreme tendencies in current theories of lawmaking. The first extreme is composed of theories that are lopsidedly partisan. These models postulate that legislators in the majority party—and only the majority party—collude to form a "procedural cartel" (e.g., Cox and McCubbins 1993, 2002, 2005). The primary mechanisms in the exercise of concentrated power are agenda-setting and favor-trading. Although different authors conceive of agenda setting powers differently, the basic thrust of this work involves the assignment of gatekeeping and/or closed rules to the majority party or its leader. Theories that treat the majority party with such deference—and the minority party with such insouciance—are aptly labeled *monopartisan,* because, at best, their proponents give only conditional lip service to the minority party.

> Provided that the majority party has . . . more powers and resources to employ than the minority party, then legislation should reveal this fact. In particular, the greater the degree of satisfaction of the condition of conditional party government, the farther policy outcomes should be skewed from the center of the whole Congress toward the center of opinion in the majority party (Aldrich and Rohde 2000, p.34).[1]

In other words, minority party legislators may try to imitate the majority party by endowing their leaders with resources to influence legislative policymaking. In all likelihood, however, the minority party's natural supply of allocable resources and its ability to generate them endogenously are dwarfed by the resources of the majority party. Policies, therefore, are predicted to diverge not only from the preferences of

[1] See also Rohde (1991), Sinclair (1995), and Smith (2007).

most minority party legislators but also from the most-preferred position of the House's median voter. Consistent with the cartel theory, then, the minority party is neither seen nor heard, and the majority party is the big winner.

The complementary extreme consists of theories that are radically nonpartisan. These models postulate that legislators are individualistic and, as such, behave in accordance with their primitive preferences irrespective of their party affiliations. Neither of the parties plays a formal-analytical role, and, indeed, their omission is at times proudly conspicuous. Weingast and Marshall, for instance, emphasize the party exclusion by stating as an assumption that "*parties place no constraints on the behavior of individual representatives*" (1988, p.137, italics in original). Mayhew, likewise, minces no words: "The fact is that no theoretical treatment of the United States Congress that posits parties as analytic units will go very far" (1974, p.27). More recently, the pivotal politics theory, too, is brashly nonpartisan (Krehbiel 1996, 1998; Brady and Volden 1998).

> The point is not that majority party organizations and their deployment of resources are inconsequential. Rather, it is to suggest that competing party organizations bidding for pivotal voters roughly counterbalance one another, so final outcomes are not much different from what a simpler but completely specified nonpartisan theory predicts (Krehbiel 1998, p.171).

In other words, as long as both parties are endowed with rights and resources in approximate parity, the parties' counteractive influence may result in lawmaking outcomes in the neighborhood of the chamber median.

Although most theories of lawmaking fit comfortably within these two categories, portrayals of parties in most empirical research are nearly always situated between the extremes of monopartisanship and nonpartisanship. A distinguishing feature of this middle ground is that the otherwise silent minority party is given *some* voice. Examples include empirical studies of bipartisanship as a form of cooperation and measured by cosponsorship activity and roll-call voting behavior (e.g., Harbridge 2010, 2011; Clark 2013) and bipartisanship in the form of acquiescence to the executive in the making of foreign policy (e.g., Kendall 1984-85, McCormick and Wittkopf 1990, Meernik 1993, and Nelson 1987). These works do not directly address relative majority- and minority-party influence, however, so the net consequence of two-party competition remains uncertain.

A few works address the issue of minority party influence more directly, however. In a path-breaking article, Jones (1968) builds on a notion of "counteractive" party influence and presents a typology for minority party influence, suggesting conditions under which the minority party is likely to be influential. Krehbiel and Wiseman (2005) advance a concept of "legislative bipartisanship" that is compatible with Jones's counteractive minority party and suggest that the minority party is influential commensurate with its relative electoral strength vis-à-vis the majority party.[2] Binder (1996) presents similar arguments in her exploration of minority parliamentary rights,[3] while Lebo, McGlynn and Koger (2007) introduce a theory of "strategic party government" wherein the majority and minority parties are assumed to choose a level of party cohesion that has consequences for policy outcomes, which presumably serve their electoral fortunes.

While these perspectives all acknowledge some role for the minority party in lawmaking and thereby provide needed balance to the literature, they also share a shortcoming: none offers an explicit theory of majority- and minority party strategic interaction in lawmaking. As a result, the connections between exogenous variables of interest (e.g., committee seats, parliamentary rights, transferrable resources) and the endogenous variables (individual behavior and collective choices) are murky. This theoretical gap, in turn, inhibits our understanding of the roles of *both parties* in legislatures, and makes it impossible to resolve potentially contradictory claims regarding the policy impact of the minority party in competitive partisan legislatures.

To begin to fill the gap between radical nonpartisanship and lopsided monopartisanship, this paper introduces a framework for assessing theoretical possibilities of minority party influence in a partisan legislature.[4] In seeking a theoretical *juste-milieu*, we hope not only to acquire a deeper understanding of nonpartisan and monopartisan theories but also to gain new insights from two new models that—in two distinct and independent ways—reserve for the minority party a figurative seat at the lawmaking table.

The analysis begins by revisiting and building on Snyder's (1991) seminal model of vote-buying.[5] We consider a closed-rule legislature with an endogenous proposal put

[2]Dixit, Grossman, and Gul (2000) develop a more rigorous theory of partisan competition over common resources, which is similar to the argument advanced by Krehbiel and Wiseman (2005).

[3]Schickler (2000) engages a similar question to Binder (1996) by studying the determinants of rules changes that seem to (ex ante) benefit the majority party, but he fails to provide a clear mapping between choice of rules and policy outcomes. Rather, it is assumed that rules that benefit the majority or minority parties lead to outcomes that are favored by the majority or minority party, respectively.

[4]Other recent work that can be interpreted as studying minority party influence—though not as a main objective–includes Den Hartog and Monroe (2011), Diermeier and Vlaicu (2011) and Volden and Bergman (2006).

[5]We use the terms vote-buying, favor-trading, side-payments, resource transfers, and bribes synonymously. Although some might associate vote-buying with illegal transactions this is not our primary conception. Logrolls, implicit promises of support, and a general expectation of good will on future issues in exchange for help today are all manifestations of what we call transfers.

forth by a majority party leader who can allocate side-payments to crucial voters who otherwise prefer the status quo to the bill. This first model provides a preliminary insight into the relative strategic benefits of restrictive procedures versus side-payments in a monopartisan legislature. It also lays the foundation for our original contributions: two new models of *competitive partisanship* in which the two parties' leaders have conflicting interests and hence compete for the votes of moderates. The first model is a simple procedural variation on the monopartisan model. It gives the minority party leader one and only one strategic tool to exploit—the ability to counter the majority party's bill with a single amendment—while leaving intact the majority party leader's monopoly supply of transferrable resources. The change from the closed rule to a modified-closed rule, which we call *agenda-based competition*, has a significant, counteractive, moderating effect on the baseline monopartisan equilibrium. The second model is also incremental with respect to monopartisanship. Instead of affording the minority agenda rights, however, the model of *resource-based competition* allows both parties—rather than just the majority party—to engage in vote-buying. When parties are evenly matched in regards to resources, gridlock is very likely to ensue, because the costs that the majority party must incur to secure policy change (by ensuring that the minority party cannot succeed in counteractive vote-buying) are not worth the policy gains it would experience. Moreover, the ability for even a highly resource-handicapped minority party to make side-payments acutely constrains the ability of the majority to pull the outcome away from the legislative median, as happens in the monopartisan model, and thereby gives the minority party a big policy bang for its small resource buck.

Although our models are motivated by parties within legislatures, the analytic results are also amenable to broader, extra-legislative interpretations concerning resources, agenda-rights, and the relative powers of competitive parties. For example, our finding that the minority party benefits more than the majority party from an increment in resources also has implications for interest groups' partisan strategies, party competition in the electorate, and revealed preferences for any type of reform concerning money and parties in legislative politics.

1 Assumptions

We confine our attention to a one-dimensional policy space in which legislators' ideal points on a continuum form a uniform distribution over the interval $[-\frac{1}{2}, \frac{1}{2}]$. Therefore, the legislature's median voter m has ideal point $x_m = 0$. Legislators' preferences are defined over the policies over which they vote and the side-payments that they may receive from party leaders. Specifically, the utility of voter with ideal point x from voting for policy p and receiving a side-payment or resource transfer t is defined as:

$$U(p,t) = -\left(x - p\right)^2 + t$$

where p is a generic policy in $\Re^1$ (either a bill b, an amendment a, or an exogenous status quo q), and $t \geq 0$ is the transfer or side-payment that a leader offers this legislator in exchange for a vote. When it is needed to avoid ambiguity, we denote the transfer to a specific legislator with ideal point x by $t(x)$. Legislators are position-taking oriented insofar as they receive promised payments for the act of voting a specified way—not for the realization of the collective choice. This is standard in vote-buying models (Snyder 1991, Groseclose 1996, Snyder and Groseclose 1996).[6]

In each of three models, at least one party leader also has policy preferences. We assume the majority party leader R has an ideal point $x_R > 0$ on the right side of the policy space, and the minority party leader L has an ideal point $x_L < 0$ on the left. Leaders differ from other legislators in two respects. First, their preferences are outcome-based rather than action-based; that is, leaders' payoffs are a function of what the legislature as a whole chooses—not on leaders' voting actions.[7] Second, leaders may have at their disposal a non-negative endowment of resources that they can distribute to rank-and-file legislators in exchange for their votes.

More formally, in each of the three models at least one party leader selects a transfer schedule, which is a mapping from the policy space into the non-negative real numbers. We let $t^j(\cdot)$ denote a schedule of this form, and the value of the transfer by leader j to a legislator with ideal point x is then denoted $t^j(x)$.[8] We denote the total cost to the leader of party j of a schedule t^j by $T^j = \int_{-\frac{1}{2}}^{\frac{1}{2}} t^j(x)dx$.

Groseclose and Snyder (1996) model vote-buying competition by assuming that the second-moving vote-buyer (the minority party in our model) has a fixed pool of resources and is willing to expend all of these resources to block *any* policy movements away from his ideal point. In contrast, they assume that the first-moving vote-buyer (the majority party in our model) does value its resources, thereby capturing the trade-off between spending money and gaining policy benefits. We depart from this asymmetric modelling

[6]Extensions to these canonical vote-buying models have since been advanced by Console-Battila and Shepsle (2009), Dal Bo (2007), and Snyder and Ting (2005).

[7]Leaders may be assumed to vote or not to vote, as long as the median voter is defined accordingly.

[8]When the meaning is clear we suppress the argument (x) and the superscript j.

strategy and instead assume that both the first-mover (majority party) and the second-mover (minority party) value resources and therefore incur costs from their transfer.

More specifically, in variants of the model in which a party has resources at its disposal to allocate to legislators, we assume that the party has an infinite (sufficiently large) resource endowment, denoted E^j for party j, and that preferences are quasi-linear in policy and total transfers, T.[9] Formally, the leader of party j has preferences represented by the utility function over the final policy p and transfer schedule $t^j(\cdot)$:

$$U_j(p, T^j) = -(x_j - p)^2 - T^j,$$

where (recall) T^j is the total amount of transfers authorized by party leader j. Note that we assume that rank-and-file legislators' preferences are not influenced by their party affiliations, per se. Indeed, we treat all rank-and-file legislators identically in regard to party labels and focus instead on their preference heterogeneity. For convenience, we assume that all legislators place the same per unit value on transfers as one-another and regardless of the source.[10]

Given these assumptions, the blueprint for analysis is straightforward. We are interested in two independent facets of potential counteractive minority party influence. While assessing the analytic possibilities, we wish to specify conditions under which minority party influence arises endogenously. In other words, to what extent is minority party influence a property of equilibrium play in a well-specified game? We investigate two such games that reflect different dimensions of party competition. *Agenda-based* competition is defined in terms of whether rights to propose policies are shared by party leaders or are monopolized by the majority party leader.[11] *Resource-based* competition is defined in terms of whether both parties have endowments available for disbursements as side-payments, or whether endowments, also, are monopolized by the majority party. This simple three-model scheme allows for transparent comparisons of the two different forms of minority-majority interaction with a fixed, monopartisan, baseline model.

[9]By assuming that the resource endowment is infinite we focus on cases in which policy issues are sufficiently contentious that both parties can bring meaningful resources to bear on the vote-buying competition. As an extension we also consider a scenario in which the the minority party has a finite resource endowment and is constrained in the manner that Groseclose and Snyder (1996) analyze, although this is not our preferred formulation.

[10]This is a strong assumption that we revisit in the Discussion. For an informal treatment of party-specific valuations of resource transfers, see Lawrence, Maltzman and Smith (2006).

[11]Although a presenting a comprehensive catalogue is beyond the scope of this paper, we think of agenda access as being a proper subset of *procedural rights*. The latter, larger set includes, but is not limited to, recognition, speech-making, bill-introduction, and co-sponsorship rights—all of which the minority party is granted in most two-party legislatures (e.g., see Clark 2013, Jackman 2013).

2 A monopartisan legislature

First we summarize the baseline case in which the majority party monopolizes both procedural rights and transferrable resources. This game is a close analytic approximation of Cox and McCubbins's (2005) verbal discussion of a "procedural cartel,"[12] and is analytically identical to Snyder's one-sided vote-buying model with an endogenous proposal (1991, Proposition 2). An empirical manifestation of the procedure is the U.S. House of Representatives' closed rule, that is, a single up or down vote on a proposal that was generated by a centralized majority party leadership.[13]

The formal *monopartisan game* has three stages:

1. The majority party leader R proposes a bill b and offers a schedule of transfers $t(\cdot)$ giving resources $t(x)$ to legislator x.[14]

2. Legislators with ideal point, x, cast their votes $v(x)$ for or against the bill b implicitly comparing b to an exogenous status quo q and taking into account transfers $t(x)$.

3. The winning policy $p \in \{b, q\}$ is realized, transfers t occur, and players receive payoffs.

Our three propositions each summarize equilibrium behavior in one of the models. Formal statements and proofs are relegated to an appendix so that the main body can supply more heuristics, intuition, and interpretations.

Proposition 1 *In the unique*[15] *subgame perfect Nash equilibrium to the monopartisan game, behavior depends on the location of the status quo as follows:*

[12]In *Setting the Agenda*, the formal model is an *open* rule with gatekeeping, identical to that in Denzau and Mackay (1983) but relabeled the "cartel agenda model" and interpreted as "negative agenda power." Much of the verbal discussion of agenda setting, however, is more compatible with the closed rule of Romer and Rosenthal (1978), also known as the "setter model."

[13]More specifically, the model approximates the House's closed rule when the rule also denies the minority party its once-traditional right to offer one motion to recommit. In recent years, this additional contraction of bipartisan procedural rights is increasingly common (Wolfensburger 2003; see also Krehbiel and Meirowitz 2002 and Roberts 2005).

[14]We henceforth conserve on notation by suppressing the superscript R in the transfer term $t(\cdot)$ unless the minority party leader L, too, can make side-payments (Proposition 3).

[15]Uniqueness is subject to the qualification that, when q is exactly equal to $-1+\sqrt{1+2x_R}$, behavior may be as in parts b or c of the proposition. Throughout we will ignore this inconsequential form of multiplicity.

(a) Unconstrained, costless agenda-setting. *For extreme status quo points ($q < -x_R$ and/or $q > x_R$), the majority party leader proposes a bill at her ideal point, offers no transfers to legislators, and* $b^* = x_R$ is the outcome.

(b) Gridlock. *For status quo points* $q \in [-1 + \sqrt{1 + 2x_R}, x_R]$*, the majority party leader proposes a bill equal to the status quo, offers no transfers, and* $b^* = q$ *is the outcome.*

(c) Constrained, costly agenda-setting. *For status quo points* $q \in [-x_R, -1+\sqrt{1 + 2x_R}]$, *the majority party leader proposes* $b^* = \frac{2\sqrt{4+2q+q^2+6x_R}-4-q}{3}$ *and offers transfers* $t^*(x) = (x - \frac{2\sqrt{4+2q+q^2+6x_R}-4-q}{3})^2 - (x - q)^2$ *to all legislators with ideal points* $x \in [0, \frac{b^*+q}{2}]$ *to make them indifferent between voting for* b^* *and the status quo* q*. The bill,* b^* *passes with a minimum-majority.*

Proof. *See Appendix.*

The main strategic tension for the majority leader is that she always wants to pull policy toward her ideal point, but the median voter and other voters in his proximity oppose such changes and require compensation.[16] Any such incremental shifts beyond those obtainable in the classic setter model are, therefore, costly for the majority leader. Given that the majority leader R has monopoly access to the agenda, and that the median voter has an ideal point normalized at $x_m = 0$, the leader can always guarantee an outcome of at least q (or the reflection of q about 0) without expending any transferrable resources. This is because the median voter is pivotal, and so any policy proposal that makes him indifferent to the status quo passes with the support of the median and all voters to his right. The focal issue is whether the leader can obtain a more right-leaning policy than q (or $-q$) via resource transfers. The proposition states that she sometimes can and specifies exactly when, why, and at what cost in side-payments.

Cases (a) and (b) exhibit behavior that duplicates that observed in the canonical setter model without side-payments (Romer and Rosenthal 1978). For the most extreme status quo points ($q < -x_R$ and $q > x_R$), the majority party leader optimizes by proposing a bill b equal to her ideal point and offering no transfers. That proposal passes because the median strictly prefers it to the status quo. For more intermediate status quo points (those between $-1 + \sqrt{1 + 2x_R}$ and x_R), however, the agenda setter optimizes by (trivially) proposing a bill equal to the status quo. Although vote-buying to get a better bill is possible, it is not optimal for the agenda setter, because any rightward

[16]We use the female gender form for leaders and male for the median voter.

movement of the policy beyond the status quo costs the leader more in side-payments than the gain in utility she would obtain from an only slightly more desirable policy. Because the endogenous price of buying policy gains is too high, gridlock prevails.

Case (c), which comprises the remaining status quo points, is the more interesting part of the proposition, because here behavior deviates from that in the canonical setter model. An optimizing agenda setter, in effect, goes through the following thought process. She contemplates an array of possible bills ($b > q$ for $q > 0$, $b > -q$ otherwise), their associated vote-determining cutpoints $(b+q)/2$, and the cost of making the cheapest series of transfers that will just secure passage of that bill. She then selects the utility-maximizing b^* and T^* combination. The proposition reveals that the majority leader R's optimal strategy is to propose a bill strictly greater than that in the setter model and to make side-payments to a narrow band of pivotal voters whose ideal points span 0 (the median) up to and including the midpoint (cutpoint) between the bill and status quo, $x = \frac{b^*+q}{2}$. Figure 1 provides a specific example.

[Figure 1 here.]

Example 1

Designate as majority party agenda setter the legislator with ideal point $x^R = \frac{1}{4}$. For this specific illustration, assume also that the status quo is slightly left of center, $q = -\frac{1}{10}$.

An opportunistic majority party leader observes that the status quo is not stable. For instance, a bill of $b = 0$ would pass with no side-payments (as would other bills up to $+\frac{1}{10}$). How might she obtain the best balance between moving policy to the right and expending minimal resources subject to the constraints that her proposed bill attracts a simple majority of yes votes? She knows, of course, that typically there is a trade-off between desirable policy shifts and conservation of her resources, so she does not necessarily want to move policy all the way to her ideal point. The equilibrium involves finding the cheapest way to buy any particular policy and then finding, among those policies, the one policy for which the marginal gain from policy utility equals the marginal cost of the cheapest vote-buying strategy that attracts a simple majority of votes.

The leader's first step is transparent. She can obtain .2 units of policy gain: a change from $q = -\frac{1}{10}$ to $b = \frac{1}{10}$ for free, according to canonical agenda setting logic. But can she do better, and, if so, how much better and at what cost? That is, at what point $b^* > -q = \frac{1}{10}$ do the net benefits of the transfers-for-votes cease to be positive? The proposition answers that the farthest to the right that the majority leader is willing to propose a bill is $b^* \approx 0.236$, which involves making transfer payments as a function of ideal points x of approximately $t(x) = .00458 - 0.672x$ to all legislators located at or between the median voter ($x_m = 0$) and $x = \frac{b^*+q}{2} \approx .0681$. This leads to a total payment $T^*$of approximately .00156. Any additional rightward shift of the bill would require paying a greater number of legislators *and* paying greater amounts to those who are already receiving payments. These incremental expenses are not worth the small policy benefit. ■

Figure 1 also provides a convenient summary of the similarities and differences between agenda-setting with and without resource transfers—or, in our nomenclature, the *setter model* and the *monopartisan model,* respectively. For extreme values of the status quo and values in the interior neighborhood of the setter's ideal point (cases (a) and (b) in Proposition 1), the two theories exhibit identical behavior. In contrast, for a wide band of moderate status quo policies, the majority party exhibits net policy gains from its monopoly on resources combined with its agenda monopoly.

It bears emphasis, however, that these are net *policy* gains and, as such, tell only part of the story. The complete story must add that such outcomes come at a price that, in Example 1, is increasing in q throughout most of the interval in Figure 1 in which vote-buying occurs.

3 Agenda-based competition

What are the consequences of giving the minority party more voice? Specifically, what happens if the minority party leader, L, is given the right to craft and offer a single counterproposal—call it an amendment, denoted a—to the majority party leader's bill, b? Such an arrangement approximates a modified-closed rule in the U.S. House of Representatives or, likewise, the motion to recommit with instructions in many legislatures, including the U.S. House. Our formulation of agenda-based competition relaxes only the proposal monopoly in the monopartisan baseline model; the majority party's resource monopoly remains intact. Formally, then, the stages of the *agenda-based competitive-partisan game* are:

1. The majority party leader proposes a bill b and offers a schedule of transfers $t(\cdot)$ to legislators.

2. The minority party leader proposes an amendment a to the bill.

3. Legislators vote first on whether to amend the bill (i.e., whether a or b faces the status quo q in the final vote), and second on whether to pass the (possibly-amended) bill or to accept the status quo.

4. The winning policy $p \in \{a, b, q\}$ is realized, transfers t occur, and players receive payoffs.

We interpret the transfer schedule $t(x)$ as a pledge of resource transfers to legislator x for voting in support of b over a and b over q. Proposition 2 characterizes the equilibrium.

Proposition 2 *In the essentially unique*[17] *subgame perfect Nash equilibrium to the agenda-based competitive partisan game, for any status quo q:*

(a) *The majority party leader proposes a bill* $b^* = -1 + \sqrt{1 + 2x_R}$ and offers positive transfers $t^*(x) = (x + 1 - \sqrt{1 + 2x_R})^2$, *to all legislators with ideal points* $x \in [0, -1 + \sqrt{1 + 2x_R}]$ *such that each such voter is indifferent between her bill-and-transfer pair and a hypothetical policy located at her ideal point x.*

(b) *An amendment, a^*, is made but does not pass given R's transfer schedule; therefore, many amendments are best responses on the equilibrium path.*

(c) *The amendment a^* fails, the bill b^*passes, and the sum of transfers is* $T^* = \frac{1}{3}(-1 + \sqrt{1 + 2x_R})^3$.

Proof. *See Appendix.*

The core intuition in the proposition is evident in the special case of the game *without* transfers. When the minority party leader L has the right to offer an amendment, the majority party leader R, as first mover, cannot simply optimize with respect to the exogenous status quo but must also anticipate and optimize with respect to the forthcoming minority party amendment. This feature of the game gives rise to an implicitly dynamic form of counteractive convergence. Specifically, if the majority leader were to attempt to extract the same-sized rightward policy shift that she successfully obtains in the closed-rule agenda-setting model, the minority leader, as second mover, could counteract with an amendment that is slightly closer (on the left) to the median

[17]For portions of the parameter space no proposal by the minority party will pass, and, therefore, a large range of different amendments are consistent with equilibrium play. All such equilibria yield the same outcomes and payoffs.

voter than is the majority leader's contemplated bill (on the right). Anticipating this, the majority leader will moderate her power-grabbing ambitions. But then the minority party leader will undercut the majority leader again. This reasoning can be iterated creating a figurative race to the center, the limit of which is a median-voter outcome.

Now consider the competitive-partisan game in which the majority can make side-payments. Proposition 2 reveals how the median gravitational pull of the simpler model is asymmetrically attenuated by the majority party's monopoly over resources. Sizing up the game ex ante, the majority party leader R sees that, in the absence of side-payments, the minority party leader L can achieve a median-voter outcome as in the race-to-the-center special case. Therefore, to get anything better than that, R must compensate moderate voters for any hypothetical bill to the right of the median. In other words R must compensate some voters in a manner that protects its bill b from any possible amendment a that the minority may be willing to put on the agenda. Consider first an incremental majority party power grab, $b = x_m + \varepsilon$. Such a strategy is intuitive, because it makes the majority party leader and most of her party members better off. It is relatively inexpensive, because only the median voter and those directly to his right require compensation—and only a small amount of compensation at that. So long as the majority agenda setter pays all legislators located between (and including) $x_m = 0$ and ε so that they each weakly prefer the bill to a hypothetical amendment located at their ideal points, such a strategy is impervious to counteraction by the minority party, because the minority party has no resources with which to compete. Therefore, R can in fact obtain a non-median, majority party-leaning outcome.

This cautious power grab by the majority leader is not optimal, however, unless and until her marginal cost of side-payments catches up with her marginal benefit from the rightward policy shift. R therefore considers bills farther and farther to the right until the cost and benefit margins are equal. En route to this optimally placed bill, however, total transfer costs rise quickly, because not only is the size of the compensation-demanding coalition increasing in b, but so, too, are the per capita costs. The schedule of equilibrium transfers $t(x)$ has the property that each side-payment recipient with ideal point x receives a utility equal to what he would get if policy were at his ideal point and there were no transfers. Viewed this way, it is clear that when the minority party can propose an amendment, a sizeable share of what would otherwise be the majority leader's rents instead goes to the pivotal recipients of transfers.

The equilibrium bears an important similarity with, and an important difference from, the optimal "bribe function" in Snyder's (1991) model. The similarity is that the greatest side-payment goes to the median voter, because she is most harmed by the optimal rightward shift of $b^*$and must therefore be compensated most for her vote. Moving right, then, as the pivotal block of legislators become increasingly hospitable towards the bill, they require less and less compensation. In other words, as in Snyder's model, individuals' side-payments are monotonically decreasing in distance from the median (moving towards the vote buyer).

The difference between this equilibrium and Snyder's is somewhat subtler, yet more significant. In Snyder's model, bribes are required only up to the cutpoint midway between the status quo and the optimal bill. With the addition in our model of the possibility of a counteractive proposal, however, the majority leader must compensate voters beyond the cutpoint, and all the way up to the right-most voter whose ideal point equals the optimal bill. Otherwise, any such member can be poached by a minority-party amendment that lies slightly to the left of his ideal point. Power grabs by the majority party are therefore much more expensive in the presence of the mere possibility of a counteractive proposal by the minority party.

Example 2

Figure 2 revisits the same parametric setup as Example 1. For this case, equilibrium transfers are $t(x)^* \approx (x - 0.2247)^2$ to all legislators who have ideal points $x \in [0, 0.2247]$.

[Figure 2 about here]

To better understand the intuition underlying minority party influence in the equilibrium, consider in greater detail what happens *out* of equilibrium. Suppose the majority party leader miscalculates and, say, offers a legislator located at .1 a transfer 0.015 units instead of the slightly greater equilibrium value of $(.1 + 1 - \sqrt{1 + .5})^2 \approx 0.0155$. This presents an opportunity for minority party exploitation. The resourceless minority leader L cannot outbid her adversary R with side payments, but she can acquire a more favorable policy than $b^* \approx 0.2247$, which is the outcome on the equilibrium path. All L must do to fare better is to poach the underpaid legislator at .1 by proposing an amendment a that is the legislator's utility-equivalent to the proposed bill plus her promised side-payment (i.e., the policy that generates $-(.1 - b)^2 + .015$). Thus the minority can find amendments that induce an interval of legislators to vote against b. Such amendments are necessarily to the left of the stiffed legislator's ideal point $x = 0.1$ (in this case, $a^* \approx 0.0763$), and it is crafted to defeat R's erroneously devised (b, t) pair by a minimal majority.

More generally, the optimal amendment strategy of the minority party leader is to look for such an error and exploit the left-most instance in the manner described for legislator with $x = .1$. If no such error occurs (which it won't, in equilibrium), L proposes any amendment, including, possibly, $a^* = 0$, which, in equilibrium, is inconsequential because b^* always wins. ■

Figure 2 also clarifies the bigger picture by comparing, for all q, equilibrium outcomes of the monopartisan and the agenda-based competition models. The figure speaks directly to the question posed above about the value to the minority party of a simple one-and-done amendment right. The minority party is always at least as well off with a proposal right than without it, and the shading in the figure represents this erosion of majority party influence relative to the monopartisan model. Moreover, in contrast with Figure 1, this difference represents total *utility* gains to the minority party; there are no background side-payment costs, because the minority leader may not buy votes in this game.

The majority party leader, meanwhile, incurs a significant reduction in overall utility relative to the monopartisan game. Although she is able figuratively to buy a constant policy shift away from the median voter, the constant cost of doing so is sufficiently great that, for all q, her transfer costs exceed her policy benefits and, thus, she is worse off under agenda-competition in spite of her resource monopoly.

A final characteristic, which differentiates agenda-competition from each of the other models we analyze, is that the minority party's minimal access to the agenda essentially renders the status quo q irrelevant, therefore, gridlock never occurs except at the sole point at which q by happenstance exactly equals b^*.

4 Resource-based competition

Narratives of majority party leadership in the U.S. Congress regularly describe not only the concomitant interplay of majority leaders' side-payments and shaping legislation but also their keen attention to how the minority party will respond to majority initiatives.[18] Minority party responses may come in the form of alternative proposals, as addressed above, or in the form of competing side-payments, which we undertake next. To do this, we revert back to the monopartisan model (Proposition 1) to use as a frame of reference and then introduce and solve a variation of a "two-sided vote-buying model"

[18] A good recent example and detailed narrative is Kaiser's (2013) account of Senate action on the Dodd-Frank Wall Street Reform and Consumer Protection Act.

(Snyder and Groseclose 1996). A substantively unique feature in our approach is to consider a game in which the bill is endogenous, i.e., a calculated action by the majority party agenda setter. Formally, the stages of the *resource-based competitive-partisanship game* are:

1. The majority party leader R proposes a bill, b, and offers a schedule of transfers $t^R(\cdot)$ to legislators.

2. The minority party leader L offers a schedule of transfers $t^L(\cdot)$ to legislators.

3. Legislators cast their votes for or against the bill (implicitly versus the status quo).

4. The winning policy $p \in \{b, q\}$ is implemented, transfers T occur, and players receive payoffs.

Snyder and Groseclose show that counteractive vote-buying strategies come in diverse forms, and it should not be surprising that endogenizing bill formation further complicates matters significantly. Due to the technical and somewhat tedious nature of the analysis, most of the derivation of results is in the Appendix.

In a sentence, the behavior implicit in the equilibrium can be summarized as anticipatory and counteractive lowest-price coalition-building. To see the logic, we can break it down further by referencing players and their incentives. As optimizing agenda setter, the majority party leader R must do the following when forming her proposal b and transfer schedule $t^R(x)$. First, for all possible bills b that R prefers to the status quo q, she must anticipate how much the minority leader L is willing to spend to preclude rightward movements in policy. In the Appendix we parameterize this willingness-to-spend value as $W^L(b, q)$. While doing this, R assumes L will respond to R's strategy with the lowest-cost minimum-winning coalition that maintains the status quo q. Player R then uses her knowledge of $W^L(b, q)$ as a constraint when optimizing in her choice of the bill and her transfer schedule. In equilibrium, the minority leader never engages in counteractive vote-buying, even though she is able to, because the majority leader is always in one of two positions. Either it does not pay for her to move the bill farther to the right than the status quo, much like what happens in the monopartisan game (Proposition 1). In this case, $b^* = q$, so L gets q without needing side-payments. Or else, it does pay for R to move the policy, in which case she does so in the least-cost way: with a bill, $b^* > q$ plus a transfer schedule that can be blocked only by a counteractive transfer schedule by L that would cost L exactly its maximum willingness to spend,

$W(b, q)$. In other words, this condition is effectively crafted to price the minority party leader out of the vote-buying market.

A salient feature of equilibrium behavior in the resource-competition game is the preponderance of gridlock, particularly in conditions that seem, *a priori*, to be empirically most plausible. To help provide intuition and precision for this broad claim, it is helpful to elaborate on a diverse sample of substantively interesting combinations of the parameter space. Six such cases are summarized in Proposition 3.

Proposition 3 *In the subgame perfect Nash equilibrium to the resource-based competitive partisan game, the following conditions for gridlock and policy change hold:*

(a) If $q > x_R$, then $b^* = x_R$ and $T_R^* = T_L^* = 0$.

(b) If $q = x_R$, then $b^* = q = x_R$ and gridlock occurs.

(c) If $q = x_L$, then policy always changes via majority party resource transfers.

(d) If $q = 0$, then policy changes only if $x_R > -2x_L$.

(e) If $x_L = -x_R \leq \frac{1}{4}$ policy changes only if $q > x^R$ or $q < q^*$ where q^* solves $2(x_R - q) = (2\sqrt{q + x_R} + q)^2$.

(f) If $x_L = -x_R > \frac{1}{4}$ policy changes only if $q > x^R$ or $q < q^*$ where $q^* = min\{q', q''\}$ where q' solves $2(x_R - q) = (2\sqrt{q + x_R} + q)^2$ and q'' solves $x_R = \frac{1}{4} - 2q + q^2$.

Proof. *See Appendix.*

Case (a) conforms with the standard setter model. It is analytically familiar and behaviorally straightforward but almost surely empirically rare. If the status quo is more extreme than the majority party monopoly agenda setter, then R proposes her ideal point. The minority party prefers this change to the status quo; hence, there is no threat of her engaging in counteractive vote-buying. As a result, no resource transfers are required, and the majority party leader wins big for free, with a new policy being located at her ideal point.

Cases (b) and (c) establish rough bounds on the range of status quo points that we regard as relatively empirically plausible. At the upper boundary is case (b) (also the lower limit of case (a)) where $q = x_R$. When existing policy is already at the majority party agenda leader's ideal point R, by definition, she cannot be made better off, so she simply proposes $b^* = q = R$, and gridlock results. At the lower boundary is case (c) where $q = x_L$. This case represents a scenario in which, for instance, a powerful left-of-center *former* majority party was evidently legislatively successful in enacting policies, yet was electorally unsuccessful in retaining seats. Consequently, the Left Party now finds itself in the minority, enjoying—for the time being, at least—a favorable, inherited status quo. Case (c) of the proposition affirms that this situation is unstable. Specifically, the majority party leader can always craft a bill b and a nonzero transfer schedule $t^R(x)$, forming a supermajority coalition to move policy to the right. In other words, gridlock *never* occurs for sufficiently left-of-center status quo points. Yet, for the majority to break gridlock and figuratively correct the disequilibrium she *always* uses side payments. Depending on the parameter values of leaders' ideal points, total side payments can be substantial.

Case (d) represents perhaps the most plausible scenario or, at least, the most neutral starting point. The status quo policy is located at the legislative median, thereby favoring neither the majority nor the minority party. Gridlock occurs here, too, unless an extreme condition is met, namely, that the majority party leader has an ideal point that is at least twice as extreme than as the minority party leader's. While conceptually imaginable, such a situation seems empirically improbable.[19] Plausibility conjectures aside,the behavioral logic of this asymmetry condition bears emphasis. Consider various values of the minority party leader's ideal point, beginning at zero and moving to the left by an increment of δ. When status quo policies are centrist, as in this example, the proposition implies that any leftward shift by the minority party leader by an amount δ must be matched by a rightward shift of the majority party leader by an amount greater than 2δ for vote buying to be net-beneficial to the majority. But this possibility ceases to exist for all $\delta > \frac{1}{4}$, because at that point $-2x_L > \frac{1}{2}$, i.e.,the majority party leader will have exceeded the outer boundary of her party and the maximum of our ideal point space. Therefore, simply having a minority party leader with an ideal point $x_L < -\frac{1}{4}$ (i.e., in the lower quadrant of the policy space) unilaterally destroys the necessary condition for effective majority-party vote buying. Consequently, gridlock reigns yet again.

Finally, cases (e) and (f) fix leaders' ideal points at symmetric, (e) moderate and (f) extreme locations on opposite sides of the median voter, as in the standard setup in the theoretical literature on parties in legislatures. The key finding in this symmetric case is that gridlock occurs for a region of status quo points that starts left of the median (less than 0) and ends at x_R. The lower bound of this gridlock interval moves farther

[19]Seemingly most likely candidates in the last half-century are Speaker Newt Gingrich and House Minority Leader Richard Gephardt. But was Gingrich *twice* as conservative as Gephardt was liberal, relative to the Republican-leaning median House member?

from the median as the majority leader becomes more extreme. The gridlock interval expands for two reasons as the party leaders become more polarized. First, the interval of status quo points that fall between the median and the majority leader expands. Second, also enlarged is the set of status quo points that could easily be moved by the majority leader in model 1; these are now too costly to move given the minority party leader's willingness and ability to pay for gridlock.[20]

Example 3

For $x_L = -\frac{1}{4}$ and $x_R = \frac{1}{4}$, we solve for the conditions under which gridlock occurs as a function of q, which, in turn, allows solving for the equilibrium bill proposal b^* as a function of q. If the status quo lies at, or to the left of, a critical point q^* such that gridlock does not occur, then the final policy b solves a first order condition equating the marginal gain to R from policy movements, $2(\frac{1}{4} - b)$, with the marginal cost of the transfer T'. (The marginal cost is defined in Claim 1 in the Appendix.) Figure 3 plots the equilibrium policy outcome as a function of q. Three substantively different intervals are evident: a left-of-center interval in which gridlock is broken at a cost to the majority party, a large central-to-right gridlock interval, and an unlikely right interval in which policy springs back to the majority leader's ideal point. ■

[Figure 3 here]

Combining these various features of equilibrium in the resource-competition game, we extract three additional implications and interpretations.

First, competitive bidding by two party leaders (Proposition 3) relative to vote-buying by only the majority party (Proposition 1) has a dramatic impact, both on policy outcomes and on players' payoffs. The policy impact is illustrated graphically in Figure 3 by the lightly hatched space between black (thick) and blue (medium-thick) lines. If anything, this difference understates the minority party's benefits from its entry into the vote-buying market, because the figure shows only the *policy* losses to the majority. At the same time, and indeed whenever gridlock is broken, the majority party pays handsomely for the modest changes it effects. We say "modest" because, as Figure 3 illustrates, when status quo points lie on the minority side of the spectrum, equilibrium outcomes stay on the minority side of the spectrum, unless they are very extreme. This is a much different result than other simple-majority theories of lawmaking.

[20] For example, in the case of $x_R = \frac{1}{4}$ the critical gridlock cutoff is approximately $q^* = -.065$. When $x_R = \frac{1}{2}$ the critical value gridlock value is approximately $q^* - .118$.

Some nonobvious empirical implications are closely related. Inasmuch as the gridlock interval for the equilibrium is a proper superset of all points between the chamber median and majority party leader, the model shares the implication of Cox and McCubbins's "cartel agenda model" that the majority party is never "rolled" (defeated when a majority of the majority opposes the bill). Data will therefore not be generated for those situations under either theory—but, for much different reasons. In the cartel agenda model, the reason is that majority-party gatekeeping keeps such bills off of what otherwise would be an open-rule agenda that, in turn, would result in an undesirable outcome at the chamber medium. In the resource-competition model, the majority party's procedural deck is more favorably stacked by the closed-rule, whose advantage to the majority party agenda setter was illustrated above. Yet, even so, in the presence of minority-party resources, the majority leader finds it not to be cost-effective to build a winning coalition for any $b > q$ after q reaches a critical point q^* that lies on the minority side of the chamber median. The models are not fully observationally equivalent, however. In fact, for all $q < 0$, they are different in substantively interesting ways. Below q^* the cartel model predicts full convergence to the chamber median, while the resource-competition model predicts only partial convergence. Above q^* the cartel model again predicts full convergence, while the resource-competition model now predicts gridlock.[21] In both conditions, the extra margin of minority party influence in the resource-competition model is directly traceable to its ability to make counter-offers.

Finally, it follows broadly from this analysis and discussion that, as the resource-competition model plays out, the minority party somehow gets a bigger bang for its buck than the majority party does. One way to illustrate this fact is to ask how much it costs the majority to overcome an off-the-equilibrium path transfer of T^L. Although in equilibrium the answer depends on the parameters in complicated ways, one qualitative feature is robust. The majority must expend more than T^L to overcome T^L. Thus, even though this model (like Snyder and Groseclose's counterpart) predicts that only the majority party expends resources, it is a mistake to conclude that the majority party is the greater beneficiary from resources. On the contrary, the minority party is actually more positively affected by its ability to bring resources to the table. This surprising finding is likely to have additional interesting implications if a model such as ours were to be imbedded in models in which interest groups are connected with party leaders, or in electoral models in which campaign finance limits are evaluated. We conjecture,

[21]Given this expanded gridlock interval, the pattern of realized outcomes in the House would be expected to look much like those in the Senate, when it operates under a binding filibuster constraint a la the pivotal politics model.

for example, that it can be shown analytically that the resource-disadvantaged minority party might *object* to rationing of resources (broadly construed) more than the resource-advantaged majority party, even though majority leaders are observed to do much more with their resources than minority leaders.

At first glance, it might seem that these insights and implications are due to the assumption that both parties have infinite resources. We investigate this conjecture—and, implicitly, the robustness of our finding—in a final extension.

4.1 Extension: Minority Party with Minimal Resources

Thus far, we have considered scenarios in which both parties have sufficient resources to allocate as much as they wish in their attempts to sway voters. While these assumptions about resources are the norm in the theoretical literature, scholars with an affinity for applied monopartisan theory question whether real-world minority parties have sufficient resources to mount successfully the kinds of effective counteractive influence illustrated above (e.g., Aldrich and Rohde, 2000). Building on the analysis of Proposition 3, we can easily accommodate instances in which the majority party has a large—indeed huge—resource advantage over the minority. Two kinds of substantive questions guide our analysis. First, are more resources always better than fewer for the minority party? More technically, is the relationship between minority party resources and the ability to constrain the majority party's policy gains (and/or to make such gains more expensive) monotonically increasing? Are these relationships smooth, or are there threshold effects and discontinuities? Second, what are the implications of a cap on minority party resources for minority party influence?

To address the first question, we now assume that the minority party's resource endowment, E is finite and sufficiently small that, in the resource-competition game solved above, the minority party does not have enough resources to block the equilibrium proposal and transfer schedule in the unconstrained equilibrium in Proposition 3. If the minority party cannot afford to spend the resources that make it indifferent between retaining the status quo q or letting b pass (equivalently, if $W^L(b, q) > E$.), then the minority party's indifference condition is no longer a relevant constraint that the majority leader faces when constructing her optimal bill and bribe. Rather, the majority leader might pass b by spending less, or it might prefer to pass a different bill altogether while spending a different amount.

The added structure of a cap on minority party resources allows for a more direct characterization of the marginal effect of the amount the minority leader is able to credibly threaten to spend (E) on the amount that the majority leader must spend (T^*) for a given bill if she is to call the minority leader's bluff effectively. In the Appendix we include an analog to Proposition 3. Here, we limit our analysis to the familiar example in which $q = 0$, $x_L = -\frac{1}{4}$, and $x_R = \frac{1}{4}$. The first step is to show how the equilibrium bill b^* varies as a function of the minority resource endowment E. For instance, at what critical size E' does the minority's endowment take effect and begin to provide the minority with the kind of tangible benefits that are illustrated in the unconstrained-budget case?

Recall that in the absence of the constraint E, this preference configuration induces gridlock. Unknown is whether gridlock occurs even with very small E, i.e., a very stringent resource constraint. Suppose, for example, that E were the legislative resource equivalent of a penny. Under these circumstances it seems certain that the resource-advantaged majority party would engage in at least some vote-buying to alter the status quo. Indeed, more generally, and drawing on the implicit characterization in the Appendix, one of two cases must occur. When E is sufficiently large, gridlock of the sort illustrated in Proposition 3 occurs. When E is sufficiently small, the majority party selects a $b^* > q$ while making side-payments to a non-flooded coalition.[22] Specifically, for a non-flooded coalition to be formed it must be true that:

$$E \leq \frac{b}{2} - b^2 + \frac{b^3}{2} \tag{1}$$

When the equilibrium involves a non-flooded coalition (i.e. when the above is satisfied) the equilibrium bill, b satisfied a first order condition that characterizes optimality for R. This condition takes the form:

$$\frac{3b^2}{4} + 2b + \frac{4E^{-\frac{1}{2}}b^{-\frac{1}{2}}}{3(2)^{-\frac{1}{2}}} - \frac{1}{2} = 0 \tag{2}$$

Notice that, here, the exogenous resource level E enters into the implicit characterization of b. To see what happens when the minority is very constrained, i.e. E is very small, we proceed as follows. For E close to 0, the feasible solutions to (2) are close to solutions of the following polynomial:

$$\frac{3b^2}{4} + 2b - \frac{1}{2} = 0 \tag{3}$$

[22]The term is Snyder and Groseclose's. A coalition is said to be non-flooded if not every voter in the winning coalition receives a transfer and flooded if, loosely, some votes are free to the leader.

In our example, the solution is $b^* \approx .23$. Substituting this value into the right hand side of inequality (1) which is the necessary condition for a non-flooded coalition to be formed (and thus (1) is a necessary condition for (2) to be the appropriate first order condition for an equilibrium bill), we obtain $\frac{.23}{2} - .23^2 + \frac{.23^3}{2} \approx .07$. This condition is satisfied for values of E close to 0. Accordingly, for low enough values of E the equilibrium involves a non-flooded coalition, and the solution is characterized by equation (2). Using the implicit function theorem we can verify that this solution b^* is a decreasing function of E. In other words, starting from a very tight constraint (E very small), as the minority party's resources increase, the policy enacted by the majority decreases, meaning that it moves away from the majority party leader's ideal point and towards the minority leader's ideal point. Moreover, this impact of minority party resources is felt immediately with arbitrarily small E and is smooth and monotonic as the minority party becomes better and better endowed.

Further analysis reveals that there is no minority resource endowment, E, that is large enough, yet binding, such that a flooded coalition is formed by the majority party leader. Hence, we see that for the special case in which the minority and majority parties' ideal points are located symmetrically opposite the median at $-\frac{1}{4}$ and $\frac{1}{4}$, respectively, and the status quo is located at zero, one of two outcomes occurs. If the minority party has a large resource endowment, the equilibrium is as described in Example 3. That is, gridlock ensues, and, therefore, the majority party receives no benefit whatsoever from its resource advantage even when paired with monopoly agenda control. Alternatively, if the endowment is relatively small, the majority party proposes a bill that is somewhat to the right of the median voter's ideal point, a non-flooded coalition is formed, and the equilibrium bill, b, is a decreasing function of the minority party's endowment, E.

Overall, this extension is consistent with two conclusions. First, approximate parity in resources—although a commonsensical premise—is *not* a necessary condition for minority party influence. The minority party needs neither one-for-one resources nor even proportional resources relative to the majority party, because even a more poorly endowed, tightly-budget-constrained minority party puts a significant damper on majority party policy gains. Specifically, when the status quo favors the minority party ($q < 0$), its mere threat of counteractive resource provision keeps new policies from gravitating to the chamber median voter, as would happen, for example, in a perfectly resource-free, party-cartel, or open-rule legislature. Complementarily, when the status quo favors the majority party ($q < 0$), even a small pool of minority party resources is sufficient for precluding a large majority party gain via vote-buying. In both of these cases, the mere existence of a minority party with resources makes the majority party's modest policy gains come at a high price. Second, these observations and the comparative statics within them appear to be smooth and monotonic. Consistent with intuition, the greater are the minority party's resources, the more influential it will be as measured by special outcomes. Somewhat more surprisingly, this feature takes effect even with minuscule resources. No critical mass is required for minority party influence.

5 Discussion

Empirical tests of these findings exceed the scope of our current analysis, however, several of our findings can be constructively connected to existing empirical claims. For example, the quoted excerpt from Aldrich and Rohde in the Introduction alludes to a hypothesis of interest to scholars in the cottage industry of polarization studies.[23] The authors suggest that as the condition for "conditional party government" (Rohde 1991) is increasingly met, outcomes will become more extreme and consistent with majority party preferences. In contrast to this claim, the results of Proposition 3, suggest that, as the parties become more polarized, gridlock becomes ever more pervasive, and policies figuratively get stuck in the vast interior between party extremes. Moreover, as illustrated by our numerical example, for those cases in which policy change is possible, increasing the distance between the minority leader's ideal point and the status quo yields equilibrium policies that are relatively closer to the minority party's than the majority party's interests. Similarly, although Krehbiel (1998) suggests that party pressures may roughly cancel out one-another, yielding outcomes in the neighborhood of the chamber median, the analysis here suggests that the size of that so-called neighborhood may be more like that of a small state. Outcomes at any given time can be quite far from the chamber median and, indeed, proximal to *either* of the two party leaders, depending on historically inherited status quo policies.

Although the rights-resources framework is motivated by research mostly on congressional politics and US government, it can nearly as easily be applied and interpreted in parliamentary, non-presidential, or multi-party settings. Granted, modeling a comparable three-or-more-party vote-buying game would be much more complicated, but this is not really necessary for illustrating what we take to be our primary analytical insight. For the passage of more moderate policies, it is sufficient to give only a *small* endowment

[23] See, for example, Bartels (2008), Fiorina, Abrams and Pope (2006), Hetherington (2009), and McCarty, Poole, and Rosenthal (2006).

of resources—or only *one* proposal—to only *one* party on the non-majority-party side of the policy spectrum.

Among the assumptions more vulnerable to criticism is the strictly sequential nature of our notion of competitive partisanship. In the case of resource transfers, such a structure is indeed an analytic convenience, but one that is reasonably well-defended elsewhere (see Groseclose 1996, for example).[24] Multi-round sequential bidding or more literal market-like mechanisms for vote-trading are nevertheless other possibilities that could, in principle, be embedded into the framework.[25] In the case of amendment activity, it is not difficult to reverse the order of offering proposals, as in, for instance Weingast's (1989) "fighting fire with fire" argument.[26] This alteration does not seem to be very informative or defensible, however. For one thing, the analytical consequences are not much different if the minority moves first and the majority moves last, because this model, too, has the median-gravitational, race-to-the-center property discussed in Section 3. For another thing, designating the majority-party as last mover in agenda-setting seems not to comport with the procedural facts in the legislative body in which we are most interested at this stage of the research agenda.[27]

Another unresolved issue is how the results change under competitive vote-buying with party-specific pricing rather than with purely preference-based compensation. It should be noted, however, that it is not abundantly clear what assumptions should be made in this regard. On first pass, it seems intuitive to think that votes of majority party moderates are cheaper to the majority party leader than are votes of minority party moderates, and vice versa from the minority party leader's perspective. On

[24]If we deviated from a sequential offers assumption, the model would effectively become a version of the well-studied "Colonel Blotto" game, for which pure strategy equilibria do not exist. While mixed strategy equilibria can be characterized in these types of games (i.e., Barelli, Govindan, and Wilson 2012), extracting the empirical implications from such equilibria characterizations are difficult for various substantive applications (and the study of legislative politics, in particular).

[25]See, for example, Baron (2006), and Dekel, Jackson and Wolinsky (2009).

[26]The modeling in the article is two-dimensional and thus seemingly more general than our framework. However the paper does not include derivation and proofs of equilibira, so generality is in question. Propositions are stated and proved for a similar model in Krehbiel and Meirowitz (2002) that also disputes analytically some of Aldrich and Rohde's "conditional party government" claims.

[27]Alluded to earlier are two possibly noteworthy qualifications. First, the argument that the House's motion to recommit—while it exists—is rarely used (e.g., Roberts 2005) does not hold water in the present context, because in both of our models of competitive partisanship, the minority party does not have to use its designated weapon for it to be influential in equilibrium. Indeed, the minority party's amendment is not accepted in the agenda-competition game and it expends no resources in the resource-competiton game. A more serious qualification is that, although the US House's motion to recommit is, by long-standing precedent, a right reserved for the minority party leader or his designee, as we noted above (footnote 13) it can be and sometime is taken away (Wolfensburger 2003). Doing this, however, requires a special order and is subject to a majority vote.

second pass, if, as is commonly assumed, the majority party wants to remain in the majority, then its leaders are uniquely sensitive to the electoral fates of its legislators from swing districts which tend to elect moderates. Moderates, therefore, may be high on leaders' lists for getting a pass on voting for the party's position, in which case a leader's crossing the aisle while shopping for votes may be cost effective. Ultimately, which is the better assumption seems to be an empirical question.[28]

A more promising next step is to begin to endogenize some of the exogenous components in our framework as a way of analyzing competitive-partisanship from the perspective of organizational design or institutional choice. Why, how, and from whom do party leaders acquire procedural rights and transferable resources? Are the processes of determining the delegation of procedural authority actually *partisan* processes, or do they simply appear to be partisan after the fact because delegators are well-sorted into preference types? Are procedures such as the House's motion to recommit durable institutions because they protect the minority *party* or because they protect minority *viewpoints* or preferences, which are highly correlated with partisanship on the issues of the day? These are challenging questions, but questions that are likely to be addressed more effectively by considering theoretical possibilities in conjunction with empirical analysis.[29] The framework of competitive partisanship provides a parsimonious and potentially useful way of conceptualizing ongoing research on legislative parties, preferences, behavior, and institutions.

[28]A similar possibility is to explore how our results change if side-payments offered by the majority party were generally more valuable to all legislators than those offered by the minority. Here, too, there is no guarantee that the intuitive expectation will be fulfilled. Analytically, this may simply be another way of saying the majority party leader has more resources than her minority counterpart—not resources that, per unit, are more valuable. The present framework already allows for this possibility, and, indeed, our examples adopted it as an assumption.

[29]For elements of, or findings bearing on, institutional choice similar to what we have in mind, see Anzia and Cohn (2013), Dewan and Spirling (2011), Diermeier and Vlaicu (2011), Jenkins and Monroe (2012).

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6 Appendix

6.1 Monpartisanship model

Proof of Proposition 1

The proof is essentially the same as that of Snyder (1991)'s proof of Proposition 2. Using standard arguments the following characterization obtains.

Definte $\bar{q} = -1 + \sqrt{1 + 2x_R}$.

For $q \geq \bar{q}$, the bill is $b^* = \frac{2\sqrt{4+2q+q^2+6x_R}-4-q}{3}$, . The transfers are $t_i = (x_i - \frac{2\sqrt{4+2q+q^2+6x_R}-4-q}{3})^2 - (x_i - q)^2$ to all legislators with ideal points $x_i \in [0, \frac{b^*+q}{2}]$, total transfers is $T = \frac{4}{27}(\sqrt{4 + 2q + q^2 + 6x_R}(8 - q^2 + q + 3x_R) - 16 - 6q - q^3 - 18x_R)$.

In all cases, voting is side-payment sincere: that is, $\forall i \in N$, $v_i^* =$ Yes if $(x_i - b)^2 + t_i \geq -(x_i - q)^2$ and No otherwise.

6.2 Agenda Competition model

Proof of Proposition 2

The equilibrium of the Agenda Competition game is derived via backwards induction. In stage 4, a voter with ideal point x will vote for a new policy over the status quo if one of the following inequalities hold:

$$-(x - b)^2 + t(x) \geq -(x - q)^2 \tag{4}$$

$$-(x - a)^2 \geq -(x - q)^2 \tag{5}$$

with expression (4) being the relevant constraint if b is chosen by the legislature over a in stage 3, and (5) being the relevant constraint otherwise.

In stage 3, then, given that both b and a will beat q if they reach a vote against q, a voter with ideal x will vote for the majority party bill, b, over the minority party amendment, a, if the following holds:

$$-(x - b)^2 + t(x) \geq -(x - a)^2. \tag{6}$$

If either policy, b or a would fail against q then the appropriate substitution is needed in

(6). Working back to stage 2, the minority leader L would have already observed b and $t(\cdot)$ as proposed by the majority leader R, and must now choose the optimal amendment a^* that maximizes its utility subject to the constraint that it can beat the majority party leader's bill (taking into account the observed transfers from R to selected legislators), and can also beat the status quo.

The minority party leader's problem (if she is going to successfully pass an amendment $a < b$) can be represented as the following:

$$\begin{aligned}
&\max_a -(x_L - a)^2 \text{ such that:}\\
&-(x-a)^2 > -(x-b)^2 + t(x)\text{for a set of voters of measure no less than } \tfrac{1}{2},\\
&-(0-a)^2 \geq -(0-q)^2.
\end{aligned}$$

The first constraint captures the requirement that a majority will support a over b despite the transfers offered by the majority with the observation that in equilibrium voters will need to resolve indifference in favor of b, and the second constraint captures the requirement that the minority amendment will pass over the status quo (here using the fact that the median is representative on the binary choice between a and q). Note that the first constraint may not be possible to satisfy. If this set is empty and b will pass over q than every proposal by the minority is optimal (and the final policy will be b). If b will not pass over q than the minority's optimum is to propose the policy closest to her ideal that beats q–which is well-defined and mirrors the case of a monopolist from the previous section.

It is useful to define the function $\hat{x}(x, t(x)) \equiv x + \sqrt{x^2 - 2xb + b^2 - t(x)}$. This is the policy that yields a voter with ideal point x the same utility (without a transfer) as the policy b and the transfer $t(x)$. For a voter obtaining no transfer this is just the reflection point.

Finally, moving back to stage 1, the majority party leader will either select the most constraining bill and no transfers, $b = 0$ or it will choose b^* and $t(\cdot)$ to maximize his/her utility, subject to the constraint that the minority cannot find an amendment that beats b and moves closer to the minority's ideal. This requires that for every possible a that the minority would prefer to b^* there is at most one value of x for which $t(x) > 0$ and $-(x-a)^2 \geq -(x-b)^2 + t(x)$. This condition captures the requirement that no amendment can attract the support of a non null set of voters that the majority is spending resources on. Were this condition false than either after this amendment b would still pass and the majority would be wasting resources, or following this amendment b would fail and the majority missed the chance to obtain a policy better than q. Hence if the majority is changing policy to b the majority party leader will chose b and T to maximize:

$$U(b) = -(x_R - b)^2 - T(b)$$
$$\text{where} \quad T(b) = \int_0^b (x-b)^2 dx.$$

Hence, $U(b) = -(x_R - b)^2 - \frac{1}{3}b^3$, and $U'(b) = 2x_R - 2b - b^2$, which implies that $b^* = -1 + \sqrt{1+2x_R}$, and the majority party leader will pay out transfers $t(x) = (x + 1 - \sqrt{1+2x_R})^2$, to all legislators $x \in [0, -1 + \sqrt{1+2x_R}]$. It is straightforward to demonstrate that b^* beats $q \quad \forall q \in \Re$.

It remains only to check that the majority prefers to play this strategy and obtain the value $U(b^*)$ as opposed to selecting $b = 0$ which will result in a final policy of 0. Substituting the above value of b^* we obtain the relevant condition

$$-(x_R - 1 - \sqrt{1+2x_R})^2 - \frac{1}{3}(-1 + \sqrt{1+2x_R})^3 \geq -(x_R)^2$$

which is true on the policy space with strict inequality for $x_R \neq 0$ and equality for $x_R = 0$

Drawing on this analysis, we characterize the unique subgame perfect Nash equilibrium to the agenda-based competition as:

$$b^* = -1 + \sqrt{1+2x_R} \quad \text{and} \quad t(x) = \begin{cases} (x + 1 - \sqrt{1+2x_R})^2, & \forall\, x \in [0, -1 + \sqrt{1+2x_R}] \\ 0, & \text{otherwise} \end{cases},$$

Total payments $T^* = \frac{1}{3}(-1 + \sqrt{1+2x_R})^3$, $a^*(b, T)$ as described above and voting strategies, v^* that are sincere. The equilibrium outcome is then $x^* = b^*$.

6.3 Resource Competition model

Our presentation of the analysis of the resource competition model is less complete and thus the strucure of this section of the Appendix differs from the previous two sections. We begin by laying out the details of how the game is analyzed, then present two intermediate results and end by deriving and stating a generalization of Proposition 3. The generalization is stated here as Proposition 3a and appears at the botttom of this section. This section concludes with a formal statement and proof of the equilibrium characterization in Extension 4.1

We must first partially characterize the equilibrium. The conditions can be obtained from the following observations. First, for any fixed bill b that R might propose, it is necessary to characterize precisely the transfer schedule for R that exactly exhausts L's willingness to spend to defeat the proposed bill. The insights about the structure of these solutions are then used to determine conditions on optimal proposals b^* by R. A strategy by R can be reformulated slightly as a per-voter transfer schedule $t^R(\cdot)$ (as before), and a supermajority margin s, where the overall size of the bill-supporting coalition is the fraction $\frac{1}{2} + s$. In equilibrium, the transfers are chosen so as to equalize the utility obtained by each voter who receives a positive side-payment. The strategies can be characterized by the parameter s (i.e., the size of the supermajority margin) and then s is a choice variable by R, who selects b and s to maximize her utility.

We begin by taking b and q as fixed, with $b > q$. The most that L is willing to spend to block b is given by the utility difference (to L) from the two policies, $-(x_L - b)^2 + (x_L - q)^2$. This policy utility differential is denoted W^L. Therefore, optimality (and thus, equilibrium) requires that, under the best response that L can play, bribed voters are indifferent between voting for b or q, and that enough of them resolve this indifference in favor of b. Thus in equilibrium it must cost *exactly* W^L to build a coalition of voters that is sufficiently large to block b, such that each voter in the coalition is indifferent between voting for b or q. Accordingly, if R is going to make transfers to pass b then she would select the least costly transfer schedule that could be blocked by W^L.[30]

To construct such a schedule we begin by defining $v(x; b, q) = -(b - x)^2 + (x - q)^2$, which is the difference in policy utility to a voter with ideal point x from voting for b over q. This difference can be simplified to:

$$v(x; b, q) = 2(b - q)y - (b^2 - q^2). \tag{7}$$

We let $v^{-1}(t; b, q) = \frac{t}{2(b-q)} + \frac{(b+q)}{2}$ denote the inverse of the difference in utility to voter y. This quantity is the ideal point of the voter who obtains utility difference t from the choice between b and q. When the meaning is clear we suppress the (b, q) arguments. We may write the willingness to pay for a bill b over the status quo q for the majority

[30] The analysis hinges on determing how indifference must be resolved. It is not possible to support equilibria in which L decides to block b if it costs her exactly W^L to do this. Given this type of profile, R would want to deviate and spend slightly more. Reasoning about how voters/legislators resolve indifference also involves these considerations, a standard feature of equilibrium analysis in many bargaining problems.

party leader as:

$$W^R(b,q) = 2(b-q)x_R - (b^2 - q^2). \tag{8}$$

Likewise, we can characterize the willingness to pay for the minority party leader to block a bill, b, given the status quo q, as:

$$W^L(b,q) = -2(b-q)x_L - (b^2 - q^2). \tag{9}$$

For fixed W^L, b, and q, the problem of selecting an optimal transfer schedule (for party leader R) is solved in Snyder and Groseclose. Correcting a minor typo in their proposition and employing our notation, we obtain the relevant characterization of vote buying strategies. More specifically, the optimal supermajority, s, is obtained by minimizing R's total expenditure,

$$T_R(b,s) = \int_{-s}^{\min\{v^{-1}(\frac{W^L}{s}),\frac{1}{2}\}} (\frac{W^L}{s} - v(y))dy. \tag{10}$$

With quadratic preferences over policy, the function $v(\cdot)$ is linear, and thus this problem reduces to one of minimizing the area of simple geometric shapes, subject to the constraint that the minority is driven to indifference (and in equilibrium it won't engage in vote-buying). Snyder and Groseclose employ the labels of *flooded* and *non-flooded* coalitions to distinguish between cases in which only a subset of the legislators supporting the majority-favored bill are actually bribed (nonflooded) and those in which all legislators in the majority's coalition are bribed (flooded). Employing our notation, a nonflooded coalition occurs when $v^{-1}(\frac{W^L}{s}) \leq \frac{1}{2}$, while a flooded coalition ensues when $v^{-1}(\frac{W^L}{s}) > \frac{1}{2}$.

As a function of exogenous parameters and the proposal, b Snyder and Groseclose's result yields the subgame perfect transfer costs for the majority party leader. We restate their result here and add the partial derivitives of the transfer with respect to the policy b as subsequent analysis will build on these characterizations.

Fact 1 (Subgame perfect transfers): For fixed b and W^L the the the following transfer schedules correspond to the solution of Snyder and Groseclose's model:

(1) When $q < 0$, $-2x_L \leq \frac{(b+q)^2}{4} - (b+q)$, $T = 0$;

(2) (*Nonflooded Coalition*)

When $q < 0$, $\frac{(b+q)^2}{4} - (b+q) < -2x_L < \frac{(b+q)^2}{2} - (b+q)$,

$$T = -2(b-q)x_L + (b^2 - q^2) - \frac{b-q}{4}(b+q)^2; \tag{11}$$

$$\frac{dT}{db} = -2x_L + 2b - \frac{3b^2 + 2bq - q^2}{4} \tag{12}$$

(3) (*Nonflooded Coalition*)
When $\frac{(b+q)^2}{2} - (b+q) \leq -2x_L \leq \frac{[1-(b+q)]^2}{2} - (b+q)$, [31] $s = \min\{\sqrt{\frac{b+q}{2} - x_L}, \frac{1}{2}\}$ [32]

$$T = [v^{-1}(\frac{W^L}{s}) + s][\frac{W^L}{s} + (b^2 - q^2)] - (b-q)[v^{-1}(\frac{W^L}{s})^2 - s^2] \tag{13}$$

$$\frac{dT}{db} = 2[v^{-1}(\frac{W^L}{s}) + s][\frac{b - x_L}{s} + b] - [v^{-1}(\frac{W^L}{s})^2 - s^2] \tag{14}$$

where $v^{-1}(\frac{W^L}{s}) = \frac{\frac{b+q}{2} - x_L}{s} + \frac{b+q}{2}$, $\frac{W^L}{s} = 2(b-q)\frac{\frac{b+q}{2} - x_L}{s}$
(4) *(Flooded and Nonuniversalistic Coalition)*
When $\frac{[1-(b+q)]^2}{2} - (b+q) < -2x_L < \frac{1}{2} - \frac{b+q}{2}$, s is uniquely and well defined by $4s^3 + 2(b+q)s^2 = b + q - 2x_L$

$$T = [\frac{W^L}{s} + (b^2 - q^2)](\frac{1}{2} + s) - (b-q)(\frac{1}{4} - s^2) \tag{15}$$

$$\frac{dT}{db} = 2(\frac{b - x_L}{s} + b)(s + \frac{1}{2}) - [\frac{1}{4} - s^2] \tag{16}$$

(5) *(Flooded and Universalistic Coalition)*
When $-2x_L \geq \frac{1}{2} - \frac{b+q}{2}$, $s = \frac{1}{2}$

$$T = 3(b^2 - q^2) - 4(b-q)x_L \tag{17}$$

$$\frac{dT}{db} = 2(3b - 2x_L) \tag{18}$$

To characterize necessary and sufficient condtions for gridlock we conduct a local analysis for small changes from q. In particular we use the partial derivatives, $\frac{dT}{db}$ in each of the cases and the derivative of policy utility, $2(b - x_R)$ evalutated at $b = q$ to characterize conditions under which R prefers to make a small move from q. This

[31] When $q \geq 0$, $\frac{(b+q)^2}{2} - (b+q) \leq -2x_L$ is automatically satisfied since $x \geq b \geq q \geq x_L$.
[32] When $q \geq 0$, we only have *nonuniversalistic* coalition, i.e. $s < \frac{1}{2}$.

argument is appropriate because, although the transfer functions, T are not concave we show in the next lemma (proof appears in supplementary appendix) that the total utility is single-peaked in b in each case.

Lemma 1 (Concavity of R's objective): As a function of b, the utility to R from making subgame perfect transfers, $V(b)$ is concave

Given the concavity of $V(W^L)$ the following result is obtained by using the characterization in Fact 1 to determine when $V'(b) > 0$ at $b = q$. If this inequality holds then gridlock does not occur. If it fails then gridlock occurs.

Proposition 3A (Resource Competition and Gridlock): The parameters (x_L, q, x_R) induces gridlock in equilibrium (i.e. there is no policy change or vote-buying) if and only if the exogenous parameters (x_L, q, x_R) are in one of the sets:

$A_1 = \{(x_L, q, x_R) : q < 0, q^2 - 2q < -2x_L < 2q^2 - 2q, 2(x_R - q) \leq -2x_L - (q^2 - 2q)\}$

$A_2 = \{(x_L, q, x_R) : 2q^2 - 2q \leq -2x_L < \frac{(1-2q)^2}{2} - 2q, 2(x_R - q) \leq (2\sqrt{q - x_L} + q)^2\}$

$A_3 = \{(x_L, q, x_R) : \frac{(1-2q)^2}{2} - 2q \leq -2x_L < \frac{1}{2} - q, 2(x_R - q) \leq (\frac{q - x_L}{s} + q)(s + \frac{1}{2}) - (\frac{1}{4} - s^2),$ where s is uniquely and well defined by $4s^3 + 2(b+q)s^2 = b + q - 2x_L \}$

$A_4 = \{(x_L, q, x) : -2x_L > \frac{1}{2} - q, x_R \leq 4q - 2x_L\}$

The statement of Proposition 3A is much more general than that of the result, Proposition 3 in the paper. The former is a characeterization of necessary and sufficient conditions for gridlock. The latter, Proposition 3 in the body is a corrollary listing several substantively interesting conditions for gridlock. Proposition 3 is obtained by substitution of the stated parametric assumptions into the definitions of the sets in Proposition 3A.

6.4 Comments on Extension 4.1: Resource Competition with a finite minority party resource endowment

In the Resource Competition extension in which the minority has the finite resource endowment E^L we focus on the case of $q = 0$ and $x_L = -\frac{1}{4}, x_R = \frac{1}{4}$. Under the assumption that E is binding, a full characterization of the equilibrium follows:

Proposition 4 (Resource Competition with a binding endowment): Assume that E^L is binding and that gridlock does not occur. Then (i) the optimal bill induces a non-flooded coalition and satisfies the first order condition.

$$2(x_R - b) = \frac{1}{4}(3b - q)(b + q + 4\sqrt{\frac{E^L}{2(b-q)}}) \tag{19}$$

(ii) Moreover this occurs if at the value of b solving the above first order condition the following is true:

$$E^L \leq \frac{b-q}{2}(1 - (b+q))^2 \tag{20}$$

7 Supplementary Appendix

Proof of Lemma 1

Proof. As $V(b)$ is the difference of the quadratic policy loss (concave in b) and the transfer, it is sufficient to show that the transfer is convex in b.

Case 1 is trivial as the transfer is identically 0 on this part of the parameter space and thus its second derivitive is 0.

Case 2: Differentiating (9) we obtain $T'' = \frac{4-3b-q}{2}$. Since both b and q are contained in the policy space $[0,1]$, the numerator is at least 0 and thus the term is non-negative.

Case 3: As s can take on one of two possible expressions we handle this case in two parts.

(3.1) If $s = \sqrt{\frac{b+q}{2} - x_L}$, we have $v^{-1}(\frac{W^L}{s}) = s + \frac{b+q}{2}$. Hence,

$$\frac{dT}{db} = [v^{-1}(\frac{W^L}{s}) + s]\{2[\frac{b-x_L}{s} + b] - [v^{-1}(\frac{W^L}{s}) - s]\} \tag{21}$$

$$= (2s + \frac{b+q}{2})(2\frac{b-x_L}{s} + 2b - \frac{b+q}{2}) \tag{22}$$

In case 3 the parameters satisfy $\frac{(b+q)^2}{2} - (b+q) \leq -2x_L$. So $2s \geq |b+q|$. As a result $2s + \frac{b+q}{2}$ is always non-negative. We also know that $2s + \frac{b+q}{2} = 2\sqrt{\frac{b+q}{2} - x_L} + \frac{b+q}{2}$ is strictly increasing in b. Therefore, in order to show $\frac{dT}{db}$ is strictly increasing in b, we now only need to show that $2(\frac{b-x_L}{s}) + 2b - \frac{b+q}{2}$ is strictly increasing in b, and the value is non-negative when $b = q$. The value of $2(\frac{b-x_L}{s}) + 2b - \frac{b+q}{2}$ when $b = q$ is $2\sqrt{q - x_L} \geq 0$. In the following, we show that $\frac{b-x_L}{s}$ is a strictly increasing function, so that $2(\frac{b-x_L}{s}) + 2b - \frac{b+q}{2}$ is strictly increasing in b:

$$\frac{b-x_L}{s} = \frac{b-x_L}{\sqrt{\frac{b+q}{2} - x_L}} \tag{23}$$

$$= \sqrt{2}\frac{b-(2x_L-q)+(x_L-q)}{\sqrt{b-(2x_L-q)}} \tag{24}$$

$$= \sqrt{2}(\sqrt{b-(2x_L-q)} - (q-x_L)\frac{1}{\sqrt{b-(2x_L-q)}} \tag{25}$$

is strictly increasing in b. As a result $\frac{dT}{db}$ is strictly increasing in b.

(3.2) If $s = \frac{1}{2}$, we have $v^{-1}(\frac{W^L}{s}) = b + q - 2x_L + \frac{b+q}{2} = \frac{3}{2}(b+q) - 2x_L$. Hence,

$$\frac{dT}{db} = [v^{-1}(\frac{W^L}{s}) + \frac{1}{2}]\{2[2(b - x_L) + b] - [v^{-1}(\frac{W^L}{s}) - \frac{1}{2}]\} \tag{26}$$

$$= (\frac{3}{2}(b+q) - 2x_L + \frac{1}{2})[4(b - x_L) + 2b - (\frac{3}{2}(b+q) - 2x_L) + \frac{1}{2}] \tag{27}$$

$$= (\frac{3}{2}(b+q) - 2x_L + \frac{1}{2})[6b - 2x_L - \frac{3}{2}(b+q) + \frac{1}{2}] \tag{28}$$

$$= (\frac{3}{2}(b+q) - 2x_L + \frac{1}{2})(\frac{9}{2}b - 2x_L - \frac{3}{2}q + \frac{1}{2}) \tag{29}$$

Because both $\frac{3}{2}(b+q) - 2x_L + \frac{1}{2}$ and $\frac{9}{2}b - 2x_L - \frac{3}{2}q + \frac{1}{2}$ are strictly increasing in b and non-negative when $b \geq q \geq x_L$, $\frac{dT}{db}$ is strictly increasing.

Case 4: For this case

$$\frac{[1-(b+q)]^2}{2} - (b+q) < -2x_L < \frac{1}{2} - \frac{b+q}{2}. \tag{30}$$

By Proposition 3, we know that s is uniquely and well defined by

$$4s^3 + 2(b+q)s^2 = b + q - 2x_L \tag{31}$$

and

$$\frac{dT}{db} = 2(\frac{b - x_L}{s} + b)(s + \frac{1}{2}) - [\frac{1}{4} - s^2]. \tag{32}$$

Step: 4.1: We first establish that: $2s + (b+q) \geq 0$.

If $b + q \geq 0$, the result follows as s is non-negaitve.

If $b + q < 0$, the shape of the function $4s^3 + 2(b+q)s^2$ (as a function of s) has the following properties. It is strictly decreasing on $[0, -\frac{1}{3}(b+q)]$, and strictly decreasing on $[-\frac{1}{3}(b+q), \frac{1}{2}]$, and has two zero points: 0 and $-\frac{1}{2}(b+q)$. Thus $4s^3 + 2(b+q)s^2$ is negative on $[0, -\frac{1}{2}(b+q)]$, so that the solution to equation (*31*), s^*, always satisfies the condition that $s \geq -\frac{1}{2}(b+q)$, i.e. $2s + (b+q) \geq 0$.

Step 4.2: If we take derivatives with respect to b in equation (*31*), we have

$$(12s^2 + 4(b+q)s)\frac{ds}{db} + 2s^2 = 1. \tag{33}$$

Hence,

$$\frac{ds}{db} = \frac{1-2s^2}{(12s^2+4(b+q)s)} \tag{34}$$

$$= \frac{1-2s^2}{4s^2+4s(2s+(b+q))} > 0 \tag{35}$$

Step 4.3: Expanding $\frac{dt}{db}$ yields

$$\frac{dT}{db} = 2(b-x_L) + \frac{b-x_L}{s} + 2bs + b - \frac{1}{4} + s^2 \tag{36}$$

In order to show that this function is strictly increasing, we only need to show that $\frac{b-x_L}{s}$ and $2bs+s^2$ are strictly increasing.

According to equation (*31*),

$$\frac{b-x_L}{s} = 4s^2 + 2(b+q)s - \frac{(q-x_L)}{s} \tag{37}$$

$$= 2s^2 + 2[s^2+(b+q)s] - \frac{(q-x_L)}{s} \tag{38}$$

Because $[s^2+(b+q)s]' = 2s\frac{ds}{db} + s + (b+q)\frac{ds}{db} = \frac{ds}{db}(2s+(b+q)) + s \geq 0$, $[s^2+(b+q)s]$ is strictly increasing. Hence, $\frac{b-x_L}{s}$ is strictly increasing.

Because $(2bs+s^2)' = 2s + 2b\frac{ds}{db} + 2s\frac{ds}{db} = 2s + (2b+2s)\frac{ds}{db} \geq 2s + (2b-(b+q))\frac{ds}{db} = 2s + (b-q)\frac{ds}{db} \geq 0$, $2bs+s^2$ is strictly increasing. Hence, $\frac{dT}{db}$ is strictly increasing.

Case 5: *When* $-2x_L \geq \frac{1}{2} - \frac{b+q}{2}$ by Proposition 3, we know that

$$\frac{dT}{db} = 2(3b-2x_L). \tag{39}$$

Hence, $T(b)$ is convex. ■

Proof of Proposition 4

Proof. Drawing on the logic behind Fact 1, a full characterization of the various types of proposals and coalitions that could be formed when E^L is exogenous is obtained by substituting E^L for the function $W^L(b,q)$ (defined in equation 6 of the appendix) in the appropriate objective functions. A full characterization includes the non-flooded case appearing in Propostion 4 as well as the adding the following additional cases:

(2) If the optimal bill is such that a flooded and nonuniversalistic coalition is built, then the first order condition is

$$2(x_R - b) = (s^*)^2 - \frac{1}{4} + b(2s^* + 1) \tag{40}$$

where s is determined by $\quad 4(b-q)s^3 = -2(b^2 - q^2)s^2 + E$

A flooded and nonuniversalistic coalition is optimal if at the value of b given by the above system, the following conditions are true:

$$\frac{b-q}{2}(1-(b+q))^2 \leq E^L \leq \frac{b-q}{2}(1+(b+q)) \tag{41}$$

(3) If the optimal bill is such that a flooded and universalistic coalition is built, then first order condition is

$$b = \frac{x_R}{2}. \tag{42}$$

As our statement of Proposition 4 indicates that only a non-flooded coaltion can be formed in an equilibrium under the assumption that E^L binds (as well as our parametric assumptions) we now show that there is no minority resource endowment, E^L, that is large enough, yet binding, such that a flooded coalition is formed by the majority party leader. To see this formally, it is sufficient to check whether there are values of E^L for which the solution $b(E^L)$ found from (2) in the paper is such that (1) (in the paper) is not satisfied yet E^L is sufficiently small that it constrains the minority's choices. This last condition is pinned down by the formula for $W^L(b, q)$ in equation (6). At $q = 0, x_L = -\frac{1}{4}$ we obtain, $W^L(b, 0) = \frac{b}{2} - b^2$. So iff $E^L < W^L(b, 0)$ the constraint is binding. So from (38) in the Appendix and this identity we see that in order for a non-flooded bill to be suboptimal, and in order for the resource constraint to bind in the minority's behavior we need: $E^L > \frac{b}{2}(1-b)^2$ and $E^L < \frac{b}{2} - b^2$. As the former inequality simplifies to $E^L > \frac{b}{2} - b^2 + \frac{b^3}{2}$ we cannot satisfy both conditions. Thus for these values of the ideal points, the minority constraint, E^L can only be binding if it induces a non-flooded coalition to be built. ■

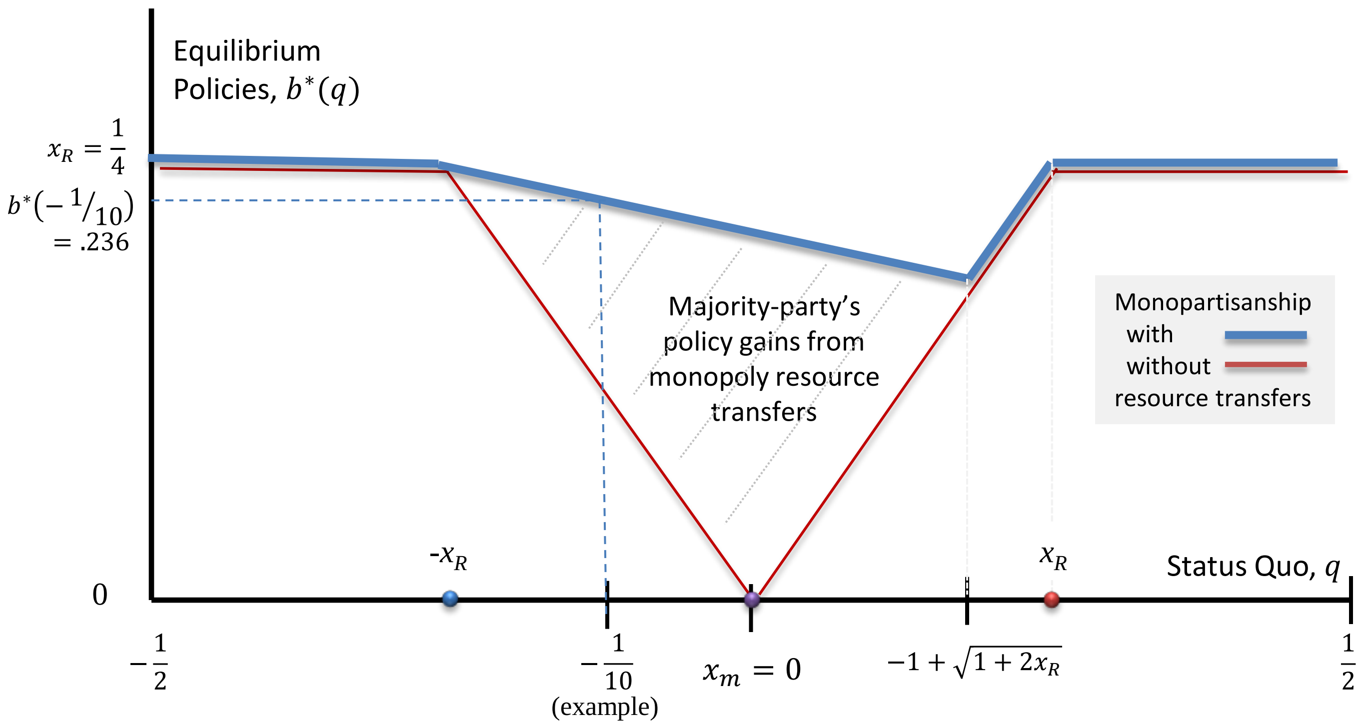

Figure 1 and Example 1. Monopartisanship with and without resource transfers

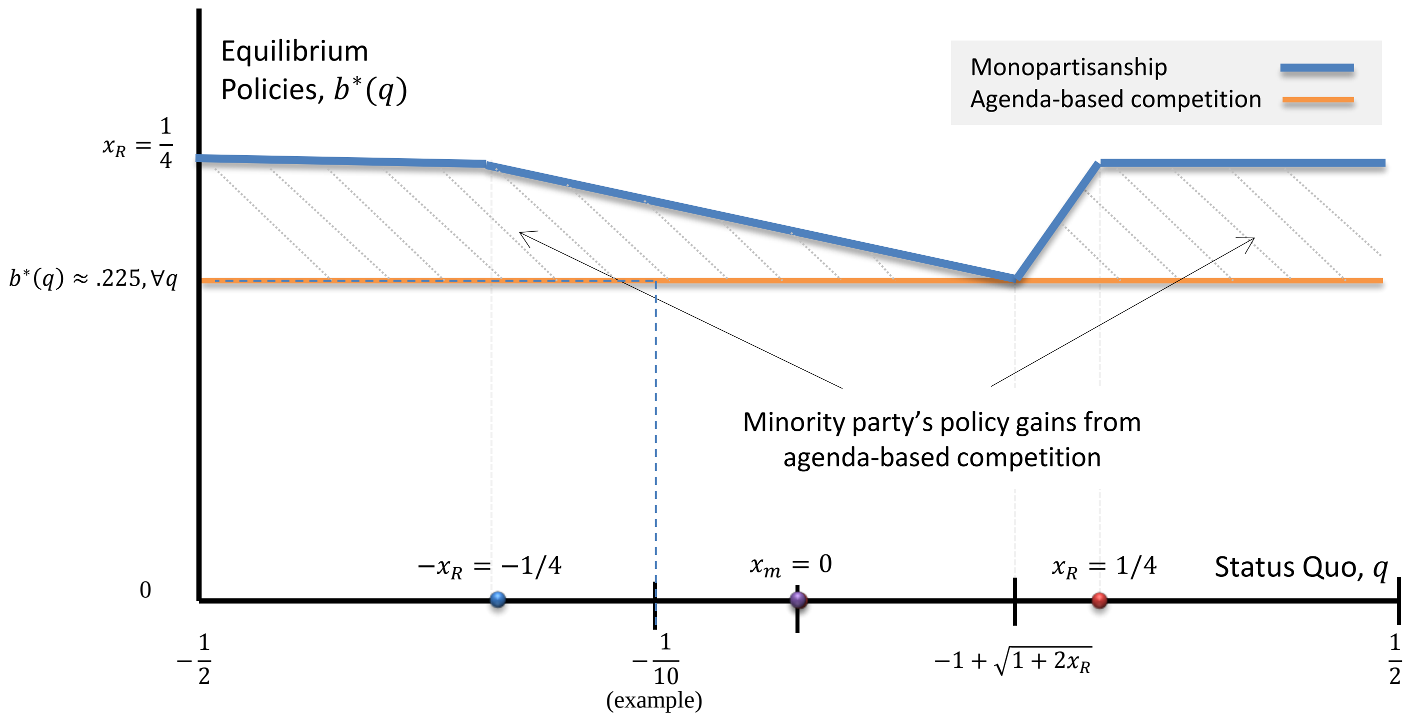

Figure 2 and Example 2. Agenda-based competition with a majority-party resource monopoly

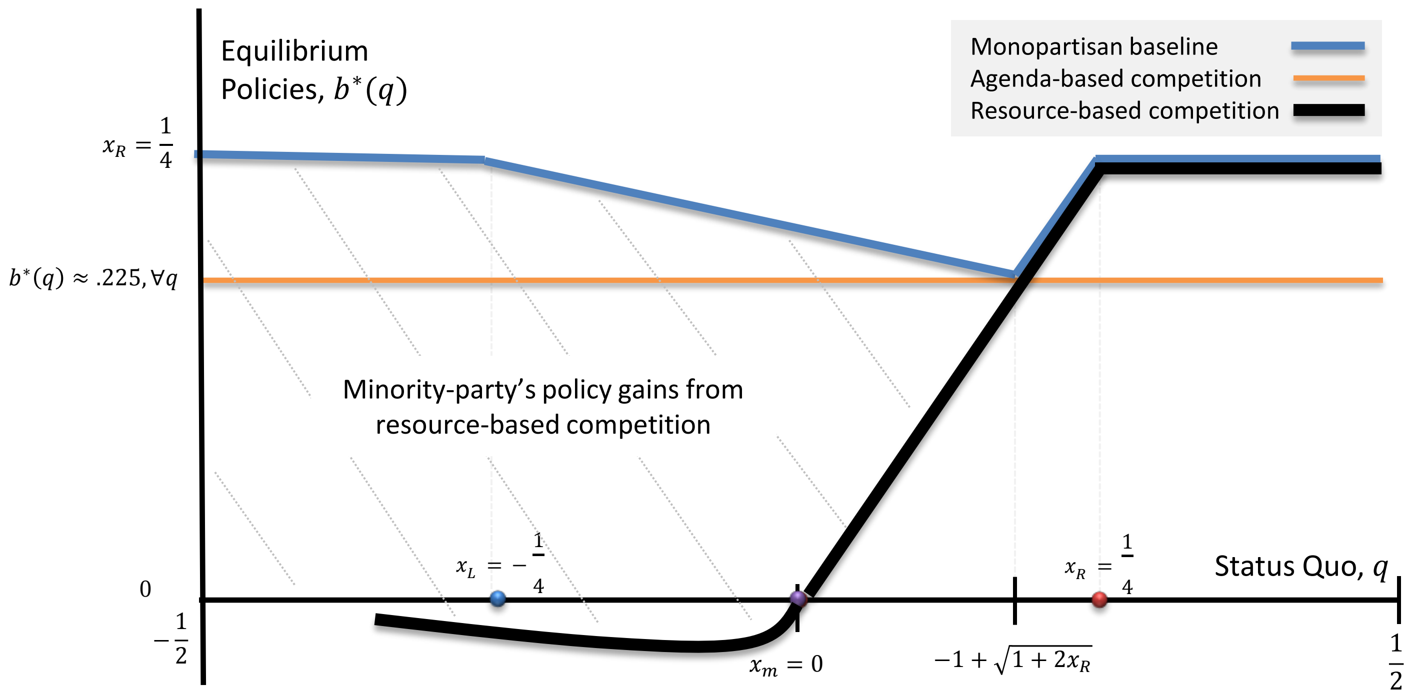

Figure 3 and Example 3. Resource-based competition with a majority-party agenda monopoly, and summary of the three competitive-partisanship games