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Legislative Bargaining and Partisan Delegation

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For nearly thirty years, one of the most prominent debates in the legislative politics literature has been concerned with the extent to which political parties exert meaningful influence over the lawmaking processes in the United State Congress. Proponents of "strong party" theories of lawmaking suggest that parties control the legislative agenda (e.g., Cox and McCubbins 2005; Diermeier and Vlaicu 2011) and/or distribute favors to their members (as well as twist their arms, figuratively speaking) (e.g., Aldrich and Rohde 2001; Jenkins and Monroe 2012; Minozzi and Volden 2013) to induce them to support the party's policy goals. Alternatively, proponents of "majoritarian" theories of lawmaking note that in the United States Congress (and the U.S. House in particular), nearly all aspects of policymaking and legislative organization are subject to the approval of a simple legislative majority. Hence, it seems implausible that a strong majority party would be empowered with legislative tools that would facilitate outcomes that are inconsistent with the wishes of the legislative majority (Krehbiel 1999). Moreover, given the substantial positive correlation between legislators' policy preferences and their party affiliations in the contemporary Congress, it is very difficult to identify whether legislators might cast votes supporting their parties' positions "*in spite of their disagreement* about the policy in question, or … *because of their agreement* about the policy in question" (Krehbiel 1993, p. 238, emphasis in original).[1]

This often-quoted passage by Krehbiel raises a fundamental question about the prospects for majority party influence in legislatures: Are there conditions under which legislators that belong to the majority party would willingly give up their policymaking autonomy (or some portion thereof) to a party Leader, who then employs that authority to pursue his policy interests – possibly at the expense of some of the other party members' interests? To address this question in a distributive politics context, we consider a simple model in which three legislators, two of whom are partisans, engage in bargaining over particularistic goods, which we formalize as Baron–Ferejohn bargaining over the division of a dollar. As in Calvert and Dietz (2005) and Choate, Weymark, and Wiseman (2017), we assume that a legislator's preferences are defined over the share of the dollar that he receives as well as the share of the dollar that his copartisan receives (if he has a copartisan).

We begin by supposing that a partisan legislator can delegate his proposal rights to his copartisan and characterize the bargaining equilibrium that ensues. We then identify the conditions under which a partisan strictly prefers to delegate his proposal rights to his copartisan rather than retaining them and playing the partisan legislative bargaining game with equal recognition probabilities that is analyzed by Choate, Weymark, and Wiseman (2017). The rules of the legislature determine what the recognition probabilities are if the partisan legislators delegate their proposal rights to a Leader. In order to make our analysis applicable to different institutional arrangements, we only assume that a party Leader's recognition probability is at least one-half. We show that with our model of Baron–Ferejohn bargaining with partisan affiliations, a partisan is willing to endow a party Leader with all of his own proposal-making authority only if (i) the Leader's recognition probability is larger than the probability that one of the partisans is recognized when there is no delegation, (ii) party affiliation is sufficiently strong, and (iii) the legislators are sufficiently impatient. Moreover, when delegation is preferred, it is due to the resulting larger recognition probability for the majority party because its Leader's proposal is the same as what either partisan would propose if they engaged in policymaking without any de facto Leader.

[1]Calvert and Dietz (2005) employ this same quotation in a different context to motivate their analysis. Our model builds on theirs.

The Model

Our model of legislative bargaining with partisanship builds on those of Calvert and Dietz (2005) and Choate, Weymark, and Wiseman (2017). There are three legislators who must decide on a *distribution* $x = (x_1, x_2, x_3)$ of a dollar among themselves, where $x_i \geq 0$ for $i = 1,2,3$ and $\sum_{i=1}^{3} x_i = 1$. The majority party consists of legislators 1 and 2 (the *partisans*). In the period in which agreement on the distribution x is reached, the legislators' utilities are:

$$U^1(x) = x_1 + \alpha x_2, \tag{1}$$

$$U^2(x) = x_2 + \alpha x_1, \tag{2}$$

and

$$U^3(x) = x_3, \tag{3}$$

where $\alpha \in [0,1)$. Thus, in addition to his own share, a partisan cares for his copartisan's share, but with a weight less than 1. The parameter α can be interpreted as the strength of party affiliation. Baron and Ferejohn (1989) suppose that $\alpha = 0$, so each legislator only cares about his own share. Legislators discount future payoffs using a common *discount factor* of $\delta \in [0,1)$. Because $\delta < 1$, the legislators prefer to receive their shares sooner rather than later.[2]

Bargaining is modeled as an infinite-horizon noncooperative game. In each stage of this game, a legislator is recognized to make a proposal for dividing the dollar. The legislature uses a *closed rule*, so that the proposed distribution is voted on without amendment against the status quo. The bargaining ends if a majority votes in favor a proposal, with the dollar distributed accordingly. If a proposal is defeated, after a one period delay, the stage game is repeated. A legislator's *strategy* has two components. In each period, his *proposal strategy* specifies the proposed distribution should he be recognized, whereas his *voting strategy* indicates which distributions he would vote for. As in Baron and Ferejohn (1989), attention is restricted to *stationary strategies* in which the proposal and voting strategies are time invariant. Thus, decisions are not contingent on past history.

The Bargaining Equilibria

We first consider the legislative bargaining game with delegation. Without loss of generality, we suppose that it is legislator 2 who delegates his proposer rights, but not his voting rights, to legislator 1. In the absence of delegation, each legislator is recognized as the proposer with probability $\frac{1}{3}$ and, so, the proposal is made by a partisan with probability $\frac{2}{3}$. The legislative rules may specify a recognition probability for a majority party Leader different from $\frac{2}{3}$ when there is delegation. In order for our analysis to apply to a broad range of legislative arrangements, we suppose that this probability is $\pi \in \left[\frac{1}{2}, 1\right)$ rather than specifying a particular value for π. Requiring π to be at least $\frac{1}{2}$ serves two purposes: (i) it excludes implausible cases where intra-party delegation causes the loss

[2] The conditions that characterize when delegation is preferred also apply when there is no discounting ($\delta = 1$), but the description of the equilibrium with delegation is somewhat more complex than that given in Proposition 1 for $\delta < 1$, so, for simplicity, we assume that there is discounting. In their model of partisan legislative bargaining without delegation, Calvert and Dietz (2005) assume that $\delta = 1$.

of majority proposal power and (ii) it allows us to set aside the more complicated equilibria that can arise in such cases.[3]

Proposition 1 characterizes the stationary subgame perfect equilibrium of the legislative bargaining game with delegation.

Proposition 1. *For any $\delta \in [0,1)$, if legislator 2 delegates his proposal rights to legislator 1 who is recognized as the proposer in any period with probability $\pi \in \left[\frac{1}{2},1\right)$, a set of strategies is a stationary subgame perfect equilibrium if and only if:*

(a) *When legislator 1 is the proposer, he offers* 0 *to his copartisan,* 0 *to the nonpartisan, and proposes for himself to receive* 1.

(b) *When the nonpartisan is the proposer, he offers* 0 *to legislator 1,* $\frac{\delta\pi\alpha}{1-(1-\pi)\delta}$ *to legislator 2, and proposes for himself to receive* $1-\frac{\delta\pi\alpha}{1-(1-\pi)\delta}$.

(c) *For either proposer:*

 (i) *legislator 1 votes for any distribution in which he receives utility at least* $\delta\left[\pi+(1-\pi)\alpha\left(\frac{\delta\pi\alpha}{1-(1-\pi)\delta}\right)\right]$*;*

 (ii) *legislator 2 votes for any distribution in which he receives utility at least* $\frac{\delta\pi\alpha}{1-(1-\pi)\delta}$*;*

 (iii) *the nonpartisan votes for any distribution in which he receives utility at least* $\delta(1-\pi)\left(1-\frac{\delta\pi\alpha}{1-(1-\pi)\delta}\right)$.

Each of the distributions proposed receives the support of the proposer and legislator 2, and so has the support of a majority.

The values in Proposition 1.(c) that specify what each legislator must be offered for his support are their discounted continuation values. In the proof of this proposition, we show that legislator 1 must be offered more for his support than legislator 2 except when $\delta = 0$, in which case they both will accept a zero share. Intuitively, this is the case because the party Leader has proposal power and his copartisan does not. Consequently, when the nonpartisan is recognized, he can keep more for himself by making an offer to legislator 2 rather than to legislator 1, and the amount he offers is the minimum amount needed to ensure that the proposal is accepted, thereby ending the bargaining. In contrast, when the majority party Leader is recognized, he proposes that the entire dollar be allocated to himself. This proposal is supported by his copartisan in spite of him not receiving any of the dollar because of the positive externalities that he obtains from his party Leader acquiring all of the dollar.

To determine whether delegation is beneficial for the two partisans, we compare how they fare in the equilibrium in Proposition 1 and in the equilibrium when there is no delegation and recognition probabilities are equal. Here, we summarize the main features of a partisan's equilibrium strategy in the latter equilibrium and refer the reader to Choate, Weymark, and Wiseman (2017) for a complete characterization of this equilibrium.

[3] See Čopič (2016) for a recent study that analyzes a generic Baron–Ferejohn bargaining game in which players have non-equal recognition probabilities.

The set of possible values of the strength of party affiliation parameter α and the discount factor δ are partitioned into three regions, with the qualitative features of the equilibrium differing between them. There are two α-dependent threshold values of δ, $\underline{\delta}(\alpha) = \frac{6\alpha}{(1+\alpha)(2+\alpha)}$ and $\bar{\delta}(\alpha) = \frac{-3\alpha-\alpha^2+\sqrt{24\alpha+33\alpha^2+6\alpha^3+\alpha^4}}{2(1+\alpha)}$, both of which are increasing in α. Except when $\alpha = 0$, these thresholds differ, with $0 < \underline{\delta}(\alpha) < \bar{\delta}(\alpha) < 1$. For $\delta \in [0, \underline{\delta}(\alpha))$, a partisan proposes to keep all of the dollar for himself when recognized. For $\delta \in \left[\underline{\delta}(\alpha), \bar{\delta}(\alpha)\right)$, a partisan proposer also offers the nonpartisan nothing, but offers his copartisan a positive share. For $\delta \in \left[\bar{\delta}(\alpha), 1\right)$, a partisan proposer offers a positive share to one of the other legislators, with the recipient determined probabilistically. In all three regions, a nonpartisan proposer choses one of the other legislators probabilistically and offers him a share. All proposals receive majority support.

When is Delegation Preferred?

The copartisan of the majority party Leader receives nothing if he delegates his proposal rights and his Leader is recognized. If he does not delegate, then he keeps some or all of the dollar if recognized and, for some parameter values, a positive amount if the other partisan is recognized. In light of these observations, one naturally wonders whether there are any circumstances in which the two partisans benefit from delegation. We show that there exist such circumstances.

In both of the partisan legislative bargaining games being considered, the stage game is the same in every period and the equilibrium is in stationary strategies. Therefore, in both games, any legislator's expected utility is his undiscounted continuation value.[4] In the game without delegation, the two partisans have the same proposal power and, hence, have the same continuation value. As we have seen, legislator 1's continuation value exceeds (resp. is equal to) that of legislator 2 in the delegation game when $\delta > 0$ (resp. $\delta = 0$), so to determine whether both partisans prefer delegation, we only need to determine if legislator 2 does. In Proposition 2, we characterize the parameter restrictions for delegation to be beneficial for the two partisans.

Proposition 2. *The partisan legislators both prefer the expected equilibrium outcome of the partisan legislative bargaining game with delegation to the expected equilibrium outcome of the partisan legislative bargaining game with equal recognition probabilities if and only if*

$$\pi > \frac{2}{3}, \tag{4}$$

$$\alpha > \frac{1}{3\pi - 1}, \tag{5}$$

and

$$\delta < \frac{2[(3\pi-1)\alpha - 1]}{(1+\alpha)[\pi(2+\alpha)-2]}. \tag{6}$$

Thus, there are conditions such that a partisan prefers to delegate his policymaking authority to his copartisan. However, for this to be the case, (i) the probability π that the majority party

[4] As we have noted, the formulas for the continuation values when there is delegation are given in Proposition 1.(c). The functional form for a partisan's continuation value when there is no delegation may be found in the proof of Proposition 2. It depends on whether $\delta \geq \bar{\delta}$ or not.

Leader is recognized must be larger than the probability that someone from the majority party is recognized when there is no delegation, (ii) the strength of party affiliation (α) has to be sufficiently large, and (iii) legislators need to be sufficiently impatient (i.e., δ must be sufficiently small). In particular, in order for a partisan to be willing to delegate his proposal rights, it is necessary that $\alpha > \frac{1}{2}$ and $\delta < \underline{\delta}(\alpha)$. The first inequality follows from (4) and (5). The necessity of the second inequality is established in the proof of Proposition 2.

Thus, for a partisan to prefer to give up his policymaking authority, it must be the case that by delegating, the probability that a partisan is recognized increases. When $\delta < \underline{\delta}(\alpha)$ and legislator 2 keeps his proposal rights, he receives nothing if legislator 1 is recognized, which is what he receives for the same value of δ if he delegates his proposal power to legislator 1 and the latter is recognized. However, if he keeps his proposal rights and is recognized, he will obtain some or all of the dollar. Moreover, delegation is only preferable if it reduces the probability that the nonpartisan is recognized, which makes it less likely that legislator 2 will receive any share of the dollar from a nonpartisan proposal. These observations suggest that legislator 2 would only prefer to have the party Leader propose on his behalf and keep the entire dollar if recognized if the value of party affiliation is sufficiently large so that which of the partisans gets the dollar is less important than that one of them does. But, in order for this to be the case, legislators must be sufficiently impatient. With greater impatience, legislator 2 is willing to give up a larger share of the dollar in order to secure policy agreement in the current period. When δ exceeds the bound in (6), his willingness to sacrifice the share that he could obtain if recognized when there are equal recognition probabilities is not sufficient to make delegation worthwhile.

The bound on α in (5) depends on π and the bound on δ in (6) depends on both α and π. Further insight into a partisan's decision about whether to delegate may be obtained by considering how these bounds vary in response to changes in these parameters.

Proposition 3. *The lower bound on α in* (5) *is decreasing in π and the upper bound on δ in* (6) *is increasing in both α and π.*

The more likely that the party Leader is recognized, the less likely that the nonpartisan is, which makes it less likely that legislator 2 benefits from a nonpartisan proposal. Consequently, he does not need to value his Leader's share as much, or be so willing to sacrifice future benefits, in order to prefer delegation. Nor does he need to be as impatient if he values his Leader's share more. Indeed, because the bound on δ approaches 1 as α and π both approach this value, if the value that a partisan places on his copartisan's share and the probability that the party Leader is recognized are both close to 1, then he prefers to delegate his proposal rights unless he values future benefits almost as much as current ones.

Conclusion

A fundamental question in scholarly debates about the role of parties in legislatures revolves around when one would expect legislators to empower a party Leader to act on their behalf, even if this power could be used in a way that is counter to their direct interests. We have addressed this question using an extension of the well-studied Baron–Ferejohn model of bargaining over particularistic goods in a majoritarian legislature composed of three legislators who have equal proposal and voting power in the absence of delegation, two of whom have partisan ties, as in Calvert and Dietz (2005) and Choate, Weymark, and Wiseman (2017). In this distributive politics context, we have shown that complete delegation of proposal power to a copartisan is sometimes in the interests of the two partisans and have characterized the conditions under which this is the case.

Our analysis contributes to the identification of circumstances in which a *strong* legislative party might emerge in a majoritarian legislature when legislators put aside their own interests in favor of party cohesion. Our findings resonate nicely with Krehbiel's first *Congressional Parties Paradox*: "Parties are said to be strong exactly when, viewed through a simple spatial model, they are superfluous" (Krehbiel 1999, p. 35). In our distributive politics model, by delegating proposal rights to his copartisan, a partisan strengthens his copartisan's power. Yet, the creation of such party authority is only in the interest of both partisans if they already have such strong bonds between themselves that either of them would consent to his copartisan obtaining the entire dollar. Indeed, even if the majority party could acquire all of the proposal rights (i.e., $\pi = 1$), a partisan would not want to delegate proposal authority to a party Leader unless α exceeds $\frac{1}{2}$. If party affiliation is strong, the distribution of a given share of the dollar between the partisans is of secondary importance to the likelihood of one of the partisans being recognized to make a proposal. For this reason, delegation is only in the interest of the partisans if party ties are strong and a partisan proposer can secure all of the dollar, whether or not there is delegation. As a consequence, a party is essentially "superfluous" for the members of a majority party to achieve their ends, unless delegating proposal power to a Leader significantly increases the party's recognition probability.

References

Aldrich, John H. and David Rohde. 2001. "The Logic of Conditional Party Government: Revisiting the Electoral Connection." In *Congress Reconsidered*, edited by Lawrence C. Dodd and Bruce I. Oppenheimer. Washington, DC: CQ Press, Seventh ed., 269–292.

Baron, David P. and John A. Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 83:1181–1206.

Calvert, Randall L. and Nathan Dietz. 2005. "Legislative Coalitions in a Bargaining Model with Externalities." In *Social Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks*, edited by David Austen-Smith and John Duggan. Berlin: Springer, 227–247.

Choate, Thomas, John A. Weymark, and Alan E. Wiseman. 2017. "Partisan Strength and Legislative Bargaining." Working Paper No. 6-2017, Center for the Study of Democratic Institutions, Vanderbilt University.

Čopič, Jernej. 2016. "Asymmetric Legislative Bargaining." Unpublished manuscript, Aix-Marseille School of Economics.

Cox, Gary W. and Mathew D. McCubbins. 2005. *Setting the Agenda: Responsible Party Government in the U. S. House of Representatives*. Cambridge: Cambridge University Press.

Diermeier, Daniel and Razvan Vlaicu. 2011. "Parties, Coalitions, and the Internal Organization of Legislatures." *American Political Science Review* 105:359–380.

Jenkins, Jeffery A. and Nathan W. Monroe. 2012. "Buying Negative Agenda Control in the U.S. House." *American Journal of Political Science* 56:897–912.

Krehbiel, Keith. 1993. "Where's the Party?" *British Journal of Political Science* 23:235–266.

———. 1999. "Paradoxes of Parties in Congress." *Legislative Studies Quarterly* 24:31–64.

Minozzi, William and Craig Volden. 2013. "Who Heeds the Call of Party in Congress?" *Journal of Politics* 75:787–802.

Appendix: Proofs

Proof of Proposition 1. We first establish the necessity part of the proof.

(a) Let p be the probability that legislator 1 seeks only the support of legislator 2 by making him an equilibrium offer of y. Similarly, let q be the probability that legislator 3 seeks the support of legislator 2 by making him an offer. Legislator 2's equilibrium continuation value is

$$V^2 = \left[\pi\left(p((1-\alpha)y+\alpha)+(1-p)\alpha(1-\delta V^3)\right)+(1-\pi)\left(q\delta V^2+(1-q)\alpha\delta V^1\right)\right]. \tag{A.1}$$

In (A.1), $(1-\alpha)y+\alpha$ is legislator 2's utility when he receives y. This utility may be greater than the minimum amount δV^2 needed to obtain legislator 2's support if the nonnegativity constraint on y binds.

Because $\delta V^3 \geq 0$, $\alpha \in [0,1)$, and $y \geq 0$,

$$-\alpha\delta V^3 \leq (1-\alpha)y.$$

Adding α to both sides of this inequality, it follows that

$$\alpha(1-\delta V^3) \leq (1-\alpha)y+\alpha. \tag{A.2}$$

Using (A.2) in (A.1), we obtain

$$V^2 \leq \left[\pi\left((1-\alpha)y+\alpha\right)+(1-\pi)\left(q\delta V^2+(1-q)\alpha\delta V^1\right)\right]. \tag{A.3}$$

Next, we show that either $q=1$ or

$$\alpha\delta V^1 \leq \delta V^2. \tag{A.4}$$

Suppose that $q<1$. Then, legislator 3 weakly prefers to make the minimum offer δV^1 needed to obtain legislator 1's support than to make the minimum offer δV^2 needed to obtain legislator 2's support. That is,

$$1-\delta V^1 \geq 1-\delta V^2$$

or, equivalently,

$$\delta V^1 \leq \delta V^2.$$

Multiplying the left hand side of this inequality by α, we obtain (A.4).

Therefore, either by using (A.4) in (A.3) or by setting $q=1$ in the latter inequality, we have

$$V^2 \leq \left[\pi\left((1-\alpha)y+\alpha\right)+(1-\pi)\delta V^2\right].$$

Solving this inequality for V^2 gives the bound

$$V^2 \leq \frac{\pi\left((1-\alpha)y+\alpha\right)}{1-(1-\pi)\delta}. \tag{A.5}$$

We now show that $y = 0$. On the contrary, suppose that $y > 0$. It then follows that y is chosen so that legislator 2's utility is equal to his discounted continuation value. That is,

$$\delta V^2 = (1-\alpha)y + \alpha. \tag{A.6}$$

Because $\alpha \in [0,1)$ and $y > 0$, the right hand side of (A.6) is positive. If $\delta = 0$, we have a contradiction and, hence, $y = 0$. If $\delta \neq 0$, (A.6) implies that $V^2 > 0$. In this case, replacing $(1-\alpha)y + \alpha$ with δV^2 on the right hand side of (A.5) and dividing both sides of the resulting inequality by V^2, we obtain

$$1 \leq \frac{\delta\pi}{1-(1-\pi)\delta},$$

which holds if and only if $\delta \geq 1$. This is a contradiction because δ is assumed to be less than 1. Therefore, we also have $y = 0$ if $\delta \neq 0$.

When $y = 0$, (A.5) implies that

$$\pi\alpha \geq [1-\delta+\pi\delta]V^2 \geq \pi\delta V^2$$

and, hence, that

$$\alpha \geq \delta V^2,$$

which shows that legislator 2's utility is no smaller than his discounted continuation value when the nonnegativity constraint on his share offer binds.

If $\delta V^3 > 0$, legislator 1's utility is 1 if he offers legislator 2 nothing, whereas it is $1 - \delta V^3 < 1$ if he offers legislator 3 the minimum δV^3 needed to get his support. Hence, legislator 1 is strictly better off seeking the support of legislator 2, and so sets $p = 1$. Thus, when $\delta V^3 > 0$, legislator 1 keeps the whole dollar for himself and offers nothing to the other legislators.

If $\delta V^3 = 0$, legislator 1 can obtain the support of legislator 3 by offering him nothing, so in this case as well, legislator 1 keeps all of the dollar, as was to be shown.

(b) If $\delta = 0$, legislator 3 does not need to offer either of the other legislators any share of the dollar to obtain their support. Hence, when $\delta = 0$, legislator 3 keeps the dollar.

Now suppose that $\delta > 0$. Given that $y = 0$, the continuation values for legislators 1 and 2 are respectively

$$V^1 = \pi + (1-\pi)\left(q\alpha\delta V^2 + (1-q)\delta V^1\right) \tag{A.7}$$

and

$$V^2 = \pi\alpha + (1-\pi)\left(q\delta V^2 + (1-q)\alpha\delta V^1\right). \tag{A.8}$$

We show that $V^1 > V^2$. On the contrary, suppose that $V^1 \leq V^2$. If, in fact, $V^1 < V^2$, then it must be that $q = 0$ because legislator 3 can obtain the support of legislator 1 by offering him δV^1 which is less than the amount δV^2 needed to obtain legislator 2's support. Setting $q = 0$ in (A.7) and (A.8), it then follows that

$$V^1 = \pi + (1-\pi)\delta V^1 > \pi\alpha + (1-\pi)\alpha\delta V^1 = \alpha V^1 = V^2$$

because $\alpha < 1$, which contradicts the assumption that $V^1 < V^2$. Hence, it must be the case that $V^1 \geq V^2$.

It remains to show that $V^1 \neq V^2$. On the contrary, suppose that $V^1 = V^2$. Substituting V^1 for V^2 in (A.7) and (A.8), adding the resulting equations, and solving for V^1, we obtain

$$V^1 = \frac{\pi(1+\alpha)}{2-(1-\pi)(1+\alpha)\delta}. \tag{A.9}$$

Equating the right hand sides of (A.7) and (A.8) with V^1 substituted for V^2, we obtain

$$\pi + (1-\pi)\left(q\alpha\delta V^1 + (1-q)\delta V^1\right) = \pi\alpha + (1-\pi)\left(q\delta V^1 + (1-q)\alpha\delta V^1\right)$$

or, equivalently,

$$\pi(1-\alpha) = (1-\pi)(1-\alpha)(2q-1)\delta V^1.$$

The left hand side of the latter equation is positive, so $\delta V^1 \neq 0$. Solving this equation for q, we obtain

$$q = \frac{1}{2}\left[\frac{\pi}{(1-\pi)\delta V^1} + 1\right].$$

Substituting the value of V^1 from (A.9) into this equation and simplifying the resulting right hand side, we find that

$$q = \frac{1}{(1-\pi)\delta(1+\alpha)} - \frac{1}{2} + \frac{1}{2}.$$

Because $\pi \geq \frac{1}{2}$ and both α and δ are less than 1, it then follows that

$$q \geq \frac{2}{\delta(1+\alpha)} > 1,$$

which is not possible. Thus, $V^1 \neq V^2$ and, therefore, $V^1 > V^2$.

Hence, if $\delta > 0$ and $V^1 > V^2$, we have $\delta V^2 < \delta V^1$. Thus, legislator 3 can obtain the support of legislator 2 by offering him less than is needed to obtain legislator 1's support. Therefore, it must be the case that $q = 1$ when $\delta \neq 0$.

From Part (a), we know that if legislator 1 is the proposer, he offers legislator 2 $y = 0$ with probability $p = 1$, so legislator 2's utility is α. Because $q = 1$, it then follows from (A.1) that

$$V^2 = \pi\alpha + (1-\pi)\delta V^2. \tag{A.10}$$

Solving (A.10) for V^2, we obtain

$$V^2 = \frac{\pi\alpha}{1-(1-\pi)\delta}. \tag{A.11}$$

Because $q = 1$, legislator 3 offers legislator 1 nothing, legislator 2 the amount

$$\delta V^2 = \frac{\delta\pi\alpha}{1-(1-\pi)\delta}, \tag{A.12}$$

and keeps the rest the dollar for himself. Note that

$$\delta V^2 \leq \alpha \tag{A.13}$$

because $\delta\pi\alpha \leq \alpha(1-\delta)+\delta\alpha\pi$. Moreover, $\delta V^2 = 0$ when $\delta = 0$, which is the amount that legislator 3 offers legislator 2 when $\delta = 0$.

(c) Reasoning as in the derivation of (A.10), when $\delta > 0$, the continuation values for legislators 1 and 3 are respectively

$$V^1 = \pi + (1-\pi)\alpha\delta V^2 \tag{A.14}$$

and

$$V^3 = (1-\pi)(1-\delta V^2). \tag{A.15}$$

Using (A.11) to eliminate V^2 from (A.14) and (A.15), we obtain

$$V^1 = \pi + (1-\pi)\alpha\left(\frac{\delta\pi\alpha}{1-(1-\pi)\delta}\right)$$

and

$$V^3 = (1-\pi)\left(1-\frac{\pi\delta\alpha}{1-(1-\pi)\delta}\right).$$

Hence,

$$\delta V^1 = \delta\left[\pi + (1-\pi)\alpha\left(\frac{\delta\pi\alpha}{1-(1-\pi)\delta}\right)\right] \tag{A.16}$$

and

$$\delta V^3 = \delta\left[(1-\pi)\left(1-\frac{\pi\delta\alpha}{1-(1-\pi)\delta}\right)\right]. \tag{A.17}$$

The values in (A.16), (A.12), and (A.17) are the minimum utilities needed to secure the support of legislator 1, 2, and 3, respectively, for all values of $\delta \in [0,1)$, not just for $\delta \in (0,1)$.

It remains to confirm that each of the proposals receives the support of its proposer and at least one of the other legislators. When legislator 1 is the proposer, he receives utility 1, which is the maximum possible utility for him, so votes in favor of his proposal. Legislator 2 receives utility α, which by (A.13) is at least as large as his continuation utility δV^2, and so he also votes in favor of this proposal. When legislator 3 is the proposer, he receives utility

$$1-\delta V^2 > \delta(1-\pi)\left(1-\delta V^2\right) = \delta V^3,$$

and so votes in favor of his proposal. Legislator 2 receives utility equal to his continuation value, and so he also votes in favor of this proposal.

This completes the necessity part of the proof.

For the sufficiency part of the proof, we need to show that the strategies described in the proposition are a stationary subgame perfect equilibrium. In other words, we need to show that no legislator wants to deviate unilaterally from these strategies. To do this, we must show that (i) no legislator in his role as a proposer wants to modify the share offered to one of the other legislators in order to receive his support, (ii) no legislator in his role as a proposer wants to modify the probabilities with which he makes offers to the other legislators, and (iii) no legislator wants to deviate from his voting strategy. All of these claims have already been established in demonstrating necessity. Legislator 1 offers legislator 2 nothing and legislator 3 offers him the smallest amount that he will accept, which guarantees each of these proposers a higher payoff than an offer to the other legislator when $\delta > 0$ and the same payoff when $\delta = 0$, so (i) and (ii) hold. The last part of the proof of necessity establishes (iii). □

Proof of Proposition 2. As noted in the discussion preceding the statement of Proposition 2, (i) in both of the the partisan legislative bargaining games being considered, any legislator's expected utility is his undiscounted continuation value and (ii) in the game without delegation, the two partisans have the same continuation value. In the proof of Proposition 1, we show than legislator 1's continuation value exceeds that of legislator 2 in the delegation game when $\delta > 0$. By Proposition 1.(c), their continuation values are both 0 when $\delta = 0$. Therefore, we only need to determine when legislator 2 prefers to delegate. In the game without delegation, the functional form of the expression for this legislator's continuation value depends on whether $\delta \geq \bar{\delta}$, so there are two cases to consider.

Case 1. For $\delta \in [\bar{\delta}(\alpha), 1)$, using the continuation values for legislator 2 in Proposition 1.(a) in Choate, Weymark, and Wiseman (2017) and in Proposition 1.(c) here, legislator 2 prefers delegation if and only if

$$\frac{\pi\alpha}{1-(1-\pi)\delta} > \frac{(1+\alpha)(\alpha+\delta)}{\delta(3+\alpha)}. \tag{A.18}$$

We now prove that this inequality never holds when $\delta \in [\bar{\delta}(\alpha), 1)$. Simple algebra shows that

$$\begin{aligned}
\pi\alpha\delta(3+\alpha) &> (1+\alpha)(\alpha+\delta)[1-\delta+\pi\delta] \\
\leftrightarrow 2\pi\alpha\delta &> (1+\alpha)(\alpha+\delta)(1-\delta)+\pi\delta^2+\pi\alpha\delta^2 \\
\rightarrow 2\pi\alpha\delta &> (1+\alpha)(\alpha+\delta)(1-\delta)+2\pi\alpha\delta^2 \\
\leftrightarrow 0 &> (\alpha+\delta+\alpha^2+\alpha\delta)(1-\delta)+2\pi\alpha\delta(\delta-1) \\
\leftrightarrow 0 &> (\alpha+\delta+\alpha^2+\alpha\delta-2\pi\alpha\delta)(1-\delta) \\
\rightarrow 0 &> (\alpha+\delta+\alpha^2+\alpha\delta-\alpha\delta-\alpha)(1-\delta) \\
\leftrightarrow 0 &> (\delta+\alpha^2)(1-\delta).
\end{aligned}$$

The right hand side of the last inequality is positive, so we have a contradiction.

Case 2. For $\delta \in [0, \bar{\delta}(\alpha))$, using the continuation values for legislator 2 in Proposition 1.(e) in Choate, Weymark, and Wiseman (2017) and in Proposition 1.(c) here, legislator 2 prefers delegation if and only if

$$\frac{\pi\alpha}{1-(1-\pi)\delta} > \frac{2(1+\alpha)}{6-\delta-\alpha\delta}. \tag{A.19}$$

We first show that this inequality holds if and only if (6) is satisfied and then determine the restrictions on α and π needed to ensure that $\alpha \in [0,1)$, $\delta \in [0, \bar{\delta}(\alpha))$, and $\pi \in \left[\frac{1}{2}, 1\right)$.

The inequality in (A.19) holds if and only if

$$\begin{aligned}
\pi\alpha(6-\delta-\alpha\delta) &> 2(1+\alpha)(1-\delta+\pi\delta) \\
\leftrightarrow 6\pi\alpha-\pi\alpha\delta-\pi\alpha^2\delta &> 2(1+\alpha)+2(1+\alpha)\delta(\pi-1) \\
\leftrightarrow 2[(3\pi-1)\alpha-1] &> (1+\alpha)[\pi(2+\alpha)-2]\delta
\end{aligned} \tag{A.20}$$

Next, we show that (A.20) is equivalent to (6). We do this by showing that the term that multiplies δ on the right hand side of (A.20) is positive. In order to do this, we first show that the left hand side of (A.20) is less than this term. We have

$$
\begin{aligned}
6\pi\alpha - 2(1+\alpha) &< (1+\alpha)[\pi(2+\alpha)-2] \qquad \text{(A.21)}\\
\leftrightarrow 3\pi\alpha &< 2\pi + \alpha^2\pi\\
\leftrightarrow 0 &< (\alpha-2)(\alpha-1) \qquad \text{(A.22)}
\end{aligned}
$$

The right hand side of (A.22) is positive, so (A.21) holds. Hence, if the right hand side of (A.21) is nonpositive, then the left hand side of (A.21) is negative, in which case we must have $\delta > 1$ in order to satisfy (A.20), which is impossible. Therefore, the right hand side of (A.21) is positive and so (A.20) holds if and only if the bound on δ in (6) is satisfied.

The discount factor δ must be nonnegative. This is only the case if the numerator on the right hand side of (6) is positive because, as we have seen, the denominator is positive. This numerator is positive if and only if the bound on α in (5) is satisfied. By (5), $\alpha < 1$ if and only $\frac{1}{3\pi-1} < 1$, which is equivalent to the restriction on π in (4).

It remains to show that when (4), (5), and (6) hold that $\delta < \bar{\delta}(\alpha)$. We show that, in fact, $\delta < \underline{\delta}(\alpha)$; that is, that

$$
\delta < \frac{6\alpha}{(1+\alpha)(2+\alpha)}. \qquad \text{(A.23)}
$$

Using the bound for δ in (6) and recalling that the denominator in this bound is positive, (A.23) holds if and only if

$$
\begin{aligned}
\frac{2[(3\pi-1)\alpha-1]}{(1+\alpha)[\pi(2+\alpha)-2]} &< \frac{6\alpha}{(1+\alpha)(2+\alpha)}\\
\leftrightarrow \frac{3\alpha\pi-\alpha-1}{3\alpha} &< \frac{\pi(2+\alpha)-2}{2+\alpha}\\
\leftrightarrow \pi - \frac{\alpha+1}{3\alpha} &< \pi - \frac{2}{2+\alpha}\\
\leftrightarrow \frac{\alpha+1}{3\alpha} &> \frac{2}{2+\alpha}\\
\leftrightarrow \alpha^2 - 3\alpha + 2 &> 0\\
\leftrightarrow (1-\alpha)(2-\alpha) &> 0.
\end{aligned}
$$

The last inequality holds because $\alpha < 1$. Hence, (A.23) is satisfied. □

Proof of Proposition 3. That the lower bound on α in (5) is decreasing in π follows immediately from its functional form. That the upper bound on δ in (6) is increasing in both α and π was verified using *Mathematica*. □