

# RE-ELECTION THROUGH DIVISION

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ABSTRACT. We provide a positive analysis of effort allocation by a politician facing reelection when voters are uncertain about the politician’s preferences on a divisive issue. We then use this framework to derive normative conclusions on the desirability of transparency, term limits, and independence of executive power. There is a pervasive incentive to “posture” by overproviding effort to pursue the divisive policy, even if all voters would strictly prefer to have a consensus policy implemented. As such, the desire of politicians to convince voters that their preferences are aligned with the majority can lead them to choose strictly pareto dominated effort allocations in the first period. When politicians care a lot about reelection, transparency of politicians’ effort choices is valuable in systems with weak executive institutions, whereas, when executive institutions are strong, transparency is harmful. Conversely, when politicians are impatient or there are term limits that reduce the value of remaining in office, transparency can be beneficial when executive institutions are strong, but can be harmful when they are weak.

*Keywords:* Posturing, Transparency, Effort Allocation, Strength of Executive Institutions.

*JEL Classification numbers:* D72, D78, D82.

## 1. INTRODUCTION

In any principal-agent relationship – voter-politician or owner-manager – the key dimensions of the contract (implicit or explicit) are the *scope* of the agent’s delegation, the structure of *contingent rewards*, and the amount of *monitoring* of actions. These three dimensions, of course, take different forms depending on the application. In the important evaluation of a policy

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*Date:* September 21, 2011.

We would like to thank Justin Fox, Johannes Hörner, Navin Kartik, Andrea Prat, Larry Samuelson, Ken Shotts, Francesco Squintani, and Mike Ting for helpful comments.

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maker's incentives, the scope of the agent's possibilities depends for example on the independence of executive power; the contingent rewards of office holding can for example be affected by the number of terms of potential reelection; and monitoring of a politician's action is typically referred to as an issue of transparency. Is a transparency reform always "good" or does it depend on the degree of independence of the executive and on whether there are term limits? Similarly, does the desirability of term limits or any other alteration of the reward structure depend on the level of transparency and independence of executive power? These are the type of institutional design questions we are interested in, for which we believe that a new theoretical framework is necessary.

In any democracy that gives policy makers the possibility of re-election, incumbent politicians make policy choices considering the current relative importance of the potential choices as well as their impact on re-election prospects, with weights that obviously vary across politicians' preferences and across institutional settings. Sometimes the difficult choices are between different policies on the same issue (e.g. fiscal discipline choices versus fiscal stimulations for the economy) and in some other contexts the most difficult choices are about what issues to focus on the most during the term in office (e.g. attempting reforms of judicial, financial, or education systems versus management of the economy without attempting reforms). Campaign advisors may play an important role as well, and sometimes the choice of actions taken and the choice of which actions to advertize the most or campaign about, do not align with our perception of importance ranking. Gay marriage, gun control, and other divisive issues like NPR or PBS funding have for example reached a level of importance in the political debate in the U.S. that could be viewed as excessive with respect to other large problems concerning our economic, financial, and social welfare system crises.<sup>1</sup>

The first thing to understand is what drives an incumbent politician's choice about the allocation of effort across the various policy issues, allowing issues to differ in terms of importance as well as in terms of how divisive they are. Common value issues have typically an importance that

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<sup>1</sup>See Fiorina et al. (2006, 202): "Most citizens want a secure country, a healthy economy, safe neighborhoods, good schools, affordable health care, and good roads, parks, and other infrastructure. These issues do get discussed, of course, but a disproportionate amount of attention goes to issues like abortion, gun control, the Pledge of Allegiance, medical marijuana, and other narrow issues that simply do not motivate the great majority of Americans."

varies over time and states of Nature (like economic reforms or national security issues), while the positions on divisive issues are less sensitive to the business cycle or exogenous events (like the positions on abortion or gay marriage). Thus it makes sense to think that in some periods the common value issues could be of dominant importance, while in some other periods the absence of pressing common value reform or action needs can bring up the relevance of decisions on divisive issues. What we show is that *even when* there exist very important common value issues that everybody agrees should be solved first, incumbent politicians tend to *overprovide* effort on divisive issues, in order to signal their type. The uncertainty that voters have about the preferences of politicians on divisive issues, coupled with possibility that there will be disagreement about which issue should be solved in a future period, is what causes politicians to “posture” by focusing effort on divisive issues, rather than common value ones, in order to signal that they hold the majority preferences on that issue.<sup>2</sup> This incentive to posture still exists even if all voters agree that the common value issue is more important and would receive a higher payoff if that issue was dealt with; hence, posturing may involve first period effort allocations which are strictly pareto dominated.<sup>3</sup> The cost of these posturing incentives vary with the efficiency or independence of executive institutions: when the overprovision of effort on one issue translates into reduced effort on important issues the cost of posturing incentives is obviously higher than when it is feasible to do both.

In the first part of the paper we will assume that voters (the principal) can observe the effort allocation choices by the incumbent politician (the agent). Using standard refinements from the signalling literature, we show that when a politician is impatient or is not excessively office-motivated, then the unique equilibrium is a separating one, in which majority type politicians

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<sup>2</sup>We refer to focusing effort on actions with the maximal electoral benefit, rather than the greatest policy benefit, as posturing (e.g. Fox 2007). Closely related is the literature on pandering (e.g. Canes-Wrone et al. 2001, Maskin and Tirole 2004) which examines the incentives for politicians to take actions which voters think are in their interest, possibly at the expense of actions which actually are, in order to signal competence or congruence with the voters. In our setting voters may understand that the politician’s action does not maximize their first period welfare, but may re-elect her anyhow if it signals she is more likely to share the voters’ preferences.

<sup>3</sup>As has been noted in the previous literature (e.g. Fearon 1999) there can often be a friction between incentivizing politicians to implement desirable policies (sanctioning) and choosing candidates who will implement desirable policies in the future (selection). In our model, because voters cannot commit to a re-election rule ex-ante, the selection motive causes politicians to choose suboptimal policies in the first period.

focus primarily on the divisive issue, whereas politicians whose position on the divisive issue is minoritarian focus on the common value issue and thereby give up re-election. On the other hand, for sufficiently high patience or re-election interests, the only equilibrium is a pooling equilibrium in which both types posture by focusing on the divisive issue. The parameter space in which the equilibrium is posturing is larger the more the electorate is polarized on the divisive issue. If anything, in the debate about whether polarization originates in society or in politics, our model suggests that more divisive policies are determined by elite's incentives, but such elite's incentives to divide are increasing in the existing polarization in public opinion.

In the second part of the paper we ask what happens when voters cannot observe politicians' allocation of effort choices, and can only observe the actions' consequences. In some cases it is more difficult than in others (for the nature of the issues or for the level of information development of the country or because of explicit regime choices) to observe actual effort or focus, and only results are observable, perhaps even with delay. This is a transparency issue that has recently attracted some attention (see e.g. Prat 2005). We show that transparency can be harmful when politicians are constrained in their amount of decision power and have low rewards from reelection (or term limits) and in the opposite scenario in which executive institutions are powerful and rewards from office are very high (for example because there are no term limits or there is even tenure in office). Only when there is a "mismatch" between strength of executive power and rewards from being reelected, a transparency enhancement is unambiguously to be advocated. The intuition behind the finding that when executive institutions or the politician's talent are high transparency is harmful when the politician is strongly office motivated, is that in that case if the politician has a minoritarian position on the divisive issue she would posture on the divisive issue much more often than when effort signalling is not possible. This case of strong executive institutions and office motivated politicians is particularly interesting because in this particular case transparency is harmful *both* in terms of discipline and in terms of sorting, whereas for the other combinations of parameters there is always a trade off.

As known at least from Holmstrom (1979), welfare is increasing in transparency when complete contingent contracts can be written. However, in our setting it is clearly realistic to assume that politicians cannot commit on a sequence of actions in the long term, and this gives rise to the intuitive possibility that transparency may not be desirable, as in other career concern models (see e.g. Holmstrom 1999 and Dewatrippont 1999). In career concern models such as those, the

typical result is that transparency is bad for discipline but good for sorting. Our results point out that the effects of transparency on discipline and sorting depend crucially on preference and institutional parameters. The closest paper to ours for the issue of transparency is certainly Prat (2005), who also considers the potentially negative effects of transparency of actions in a model of career concerns. Like in Prat (2005), in our setting transparency of actions can be bad both for discipline and sorting, but only when politicians are strongly motivated by re-election purposes and in presence of strong executive institutions. The main reason why transparency of actions is bad in Prat (2005) is the incentive it gives to "conformism", whereas in our setting the main reason is in the greater incentives that transparency of actions may give the incumbent politician to focus on divisive issues rather than common value issues in order to signal their type. This "posturing" by elected officials differs from conformism and is equally important.<sup>4</sup>

The paper is organized as follows: in section 2 we present the model; section 3 contains the full equilibrium characterization when effort allocation across issues is observable; section 4 will instead display the equilibrium outcomes when the politicians' actions are not observable, distinguishing between high and low executive power and high and low rewards from election. In section 5 we will offer some concluding remarks and more connections with the literature and opportunities for future research.

## 2. MODEL

We consider a two-period model in which a politician chooses a policy on behalf of the public. In each period there are two dimensions  $A$  and  $B$  and the politician has to decide how to allocate effort between passing bills on the two issues. That is, the politician allocates effort  $w^A \in [0, 1]$  to issue  $A$  and  $w^B \in [0, 1]$  to issue  $B$ , and faces the constraint  $w^A + w^B \leq W$ , where  $W \in (0, 2)$ . We normalize the status quo policy to be 0 in each dimension, and assume that if effort  $w^A$  is exerted on issue  $A$  then the policy will be  $p^A = 1$  with probability  $w^A$  and 0 with probability  $1 - w^A$ . Similarly devoting effort  $w^B$  to issue  $B$  results in policy  $p^B = 1$  with probability  $w^B$  and  $p^B = 0$  with probability  $1 - w^B$ . We interpret the parameter  $W$  as representing the ability

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<sup>4</sup>For interesting connections between conformism and posturing/pandering, see Che, Dessein and Kartik (2011). See Maskin and Tirole (2004) for a systematic treatment of pandering and see Morelli and Van Weelden (2011) for results on how the incentives to pander to public opinion by an incumbent politician relate to the divisiveness of issues and to the informational advantage of politicians.

of the politician to pursue her agenda: when  $W$  is small the politician knows that no matter which policy she pursues it is unlikely to take effect; when  $W \approx 2$ , she is able to get both policies implemented with high probability if she so chooses; for intermediate values of  $W$  the politician faces a tradeoff where she can implement one policy but will find it difficult to get everything she wants implemented. Thus  $W$  will also be interpretable as being related to the efficiency of decision making institutions and/or to the independence of executive power.

In each period,  $t \in \{0, 1\}$  the stage game utility of voter  $i$  is

$$-\gamma(\theta_t - p_t^A)^2 - (1 - \gamma)(x_i^B - p_t^B)^2 + v_t^j,$$

where  $p^A$  and  $p^B$  are the policies implemented and  $v^j$  is the quality or “valence” of incumbent  $j$ . We assume that the voters hold common values over dimension  $A$  but heterogenous preferences in dimension  $B$ .<sup>5</sup> So we assume that  $\theta_t \in \{0, 1\}$  reflects whether all voters prefer policy  $p^A = 1$  or  $p^A = 0$ . Assume that the probability that  $\theta_t = 1$  is  $q \in (0, 1)$ . Conversely, the voters may be type  $x^B = 1$  or  $x^B = 0$  reflecting whether their preferred policy in dimension  $B$  is 0 or 1. We focus on the case in which the majority of voters prefer policy 1 in dimension  $B$  so the fraction of voters with  $x_i^B = 1$  is  $m \in (\frac{1}{2}, 1)$ . We assume that  $\gamma \in (\frac{1}{2}, 1)$  so that all voters care more about the issue  $A$  than issue  $B$ . The assumption that  $\gamma \in (\frac{1}{2}, 1)$  is not necessary for our results to hold, but corresponds to the case where all players prefer  $A$  to be done first, and so biases against politicians choosing  $B$ .

The valence term says that voters, in addition to caring about which policy is implemented, receive some additional payoff from having a politician who is high quality or who they like personally. We assume that the distribution of valence among politicians is symmetric, mean 0 with continuous density on  $[-\epsilon, \epsilon]$ , where  $\epsilon > 0$ . The candidate’s valence is unknown to both the candidate and voters at time 0, but is revealed to everyone when the candidate is in office. As

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<sup>5</sup>The assumption that we have common values over  $A$ , and so voters cannot update their beliefs about the politician’s preferences over  $A$ , will simplify the analysis and play a key role in the equilibrium selection argument we present. The key feature necessary for our results, however, is that the voters update less about the politician’s preference on the  $A$  dimension than on the  $B$  dimension. Even if the  $A$  dimension were not pure common values, when there is also uncertainty about the true state of the world, it is always the case that there is less opportunity for updating on the issue in which there is a broad consensus than there is on an issues where voters are more divided. As such, there is greater incentive for pandering or posturing over the divisive issue. See Morelli and Van Weelden (2011) for details.

the candidate does not know their valence when making a decision, and the voters will learn the candidate's valence regardless of her action, the addition of the valence component will serve only to ensure that voters are (generically) not indifferent between re-electing the candidate and not, and to ensure that the probability of re-election will vary continuously with the voters' beliefs about the politician's type. We focus on the case where  $\epsilon$  is small.

We assume that the distribution of politician preferences is the same as the distribution of voter preferences. However the politician also receives benefit  $\phi$  from being in office.<sup>6</sup> So the stage game utility of politician  $j$  if  $p_t^A$  and  $p_t^B$  is implemented is

$$\phi - \gamma(\theta_t - p_t^A)^2 - (1 - \gamma)(x_j^B - p_t^B)^2,$$

if they are in office, and

$$-\gamma(\theta_t - p_t^A)^2 - (1 - \gamma)(x_j^B - p_t^B)^2,$$

if not.

The assumption that the distribution of politician types is the same as the distribution of voters is consistent with a citizen-candidate (e.g. Besley and Coate 1997, Osborne and Slivinski 1996) view of electoral representation – as politicians are citizens as well they must be found among the population of voters. Note, however, that in order for our results to go through, we only require that the more divisive issue is the one with the greater heterogeneity in politician preferences – and hence also the issue on which voters have the greatest ability to update about the politician's type.

Voters form beliefs about the type of the politician. As there are only two types we can define

$$\mu(w^A, w^B) = Pr(x_j^B = 1 | w^A, w^B)$$

That is,  $\mu$  is the probability the politician is the majority type given allocation  $w^A$  and  $w^B$ .

The game is twice repeated with discount factor  $\delta \in (0, 1)$ . The timing of the game is as follows.

- (1) In period 0 a politician is randomly selected to be in office for that period.

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<sup>6</sup> $\phi$  could include monetary and non monetary rewards from being elected, or could also represent the reduced form of the continuation value of remaining in office, which is for example very sensitive to whether there are term limits (and hence after the second period for example the political payoffs are over) or one could be reelected forever.

- (2)  $\theta_0$  is realized and publicly observed.
- (3) The politician decides how to allocate effort ( $w^A$  and  $w^B$ ). Two subcases:
  - (a) The voters observe the effort decision – transparency case;
  - (b) Voters do not observe policy effort decisions – no transparency case.
- (4) The incumbent's valence  $v_j$  is realized and publicly observed.
- (5) The policy is determined for period 0 with all players receiving their utilities for period 0.
- (6) Voters observe outcomes and update beliefs on incumbent, then vote whether to re-elect the politician or not. If the politician is not re-elected a random replacement is drawn.
- (7)  $\theta_1$  is realized, and the politician decides how to allocate effort in period  $t = 1$ .
- (8) The policy is realized with all players receiving their payoff for period 1.

### 3. EQUILIBRIUM WITH TRANSPARENCY

We look for Perfect Bayesian Equilibria, restricting attention to those in which all voters always hold the same beliefs about the politician's type. We begin by first solving for how the politicians behave in period  $t = 1$ . As  $\gamma > 1/2$ , all politicians, as well as all voters, care more about issue  $A$  than issue  $B$ . Hence, in the second period, the politician will first focus on addressing issue  $A$ , if any change is desired on that issue ( $\theta_1 = 1$ ). If the politician is in the majority then he also prefers to act on issue  $B$  and will exert any left over effort after securing the preferred policy in dimension  $A$  on policy  $B$ . The minority type will never exert effort on policy  $B$ . We then have the following lemma.

**Lemma 1. *Politician Action in the Second Period***

*In period  $t = 1$ ,*

- (1) *a politician of the majority type will choose  $w^A = \min\{W, 1\}$  and  $w^B = 1 - w^A$  when  $\theta_1 = 1$ , and  $w^B = \min\{W, 1\}$  and  $w^A = 0$  when  $\theta_1 = 0$ .*
- (2) *a politician of the minority type will choose  $w^A = \min\{W, 1\}$  when  $\theta_1 = 1$  and  $w^A = 0$  when  $\theta_1 = 0$ , and  $w^B = 0$  for either  $\theta_1 \in \{0, 1\}$ .*

Now that we have determined the behavior of the politician in the second period, we can consider the decision faced by the voters. Voters who are of the majority type will support the incumbent if they believe she is sufficiently more likely to be of the majority type, relative to a



random replacement; similarly, voters of the minority type will support the incumbent if they believe that she is sufficiently less likely to be the majority type than a random replacement. How high a probability voters must place on the candidate being their desired type to support them depends on the politician's valence. We assume that all voters vote for the candidate they prefer, and that the politician is re-elected if and only if they receive at least half the votes. Note that this means that the politician will be re-elected if and only if the voters of the majority type support them being re-elected.

We now look for the parameters under which Separating and Pooling Equilibria exist. When  $\epsilon$  is small, in a separating equilibrium, only the majority type will ever be re-elected, and they will be re-elected with certainty. Note also that, in any pooling equilibrium, the politician will be re-elected if and only if  $v^j \geq 0$ , which we have assumed will happen with probability  $1/2$ .

**Lemma 2. *Voter Behavior***

- (1) *There exists  $\bar{\epsilon} > 0$  such that, for all  $\epsilon \in [0, \bar{\epsilon})$ , if the voters know the politician is the majority type with certainty,  $\mu(w^A, w^B) = 1$ , she is re-elected with probability 1. If the voters know the politician is the minority type with certainty,  $\mu(w^A, w^B) = 0$ , she is re-elected with probability 0.*
- (2) *For any  $\epsilon$ , if the voters know the politician is the majority type with probability  $m$ ,  $\mu(w^A, w^B) = m$ , then she is re-elected with probability  $1/2$ .*

Now that we have determined the voter behavior, given the beliefs induced by the politician's action, we now turn to analyzing the first period behavior of the politician. As this is a signaling game it will admit many equilibria, especially when the candidates receive a large intrinsic value from holding office ( $\phi$  is high). While it is possible to support many different first period actions by assigning unnatural off-path beliefs, applying Criterion D1 from Cho and Kreps (1987) will greatly reduce the number of equilibria. As a majority type has a greater willingness than a minority type to take action  $B$ , criterion D1 will require that voters believe that a politician who exerted greater effort on  $B$  to be of the majority type.

As this is not a standard sender-receiver game we must first define how this condition applies in our setting. Note also that, while Cho and Kreps (1987) define these refinements in terms of Sequential Equilibrium, because there are a continuum of potential actions, we analyze the game using Perfect Bayesian Equilibrium. For our purposes, the only relevant restriction on off-path

beliefs from working with Sequential Equilibrium rather than Perfect Bayesian is that all voters hold the same beliefs at all information sets, and we restrict attention to equilibria which have that property.

In order to facilitate the definition, we first define

$$u^*(x_j^B, \theta_0)$$

to be the expected utility of a type  $x_j^B$  politician from playing her equilibrium strategy in a given Perfect Bayesian Equilibrium, given that the state is  $\theta_0$ . Further we define

$$u(w^A, w^B, \theta_0, \mu | x_j^B)$$

to be the expected utility, given the equilibrium strategies of the other players, of a type  $x_j^B$  politician from choosing allocation  $(w^A, w^B)$  at time 0 when the state is  $\theta_0$  if the belief the voters form about her type from choosing that allocation is  $\mu$ .

**Definition 1. Criterion D1 (Cho and Kreps 1987)**

A Perfect Bayesian Equilibrium satisfies criterion D1 if,

- (1) at all information sets all voters hold the same beliefs,  $\mu$ , about the politician's type.
- (2) if for some off-path allocation  $(w^A, w^B)$ , and  $x^B \in \{0, 1\}$ ,

$$\{\mu \in [0, 1] : |u(w^A, w^B, \theta_0, \mu | x^B) > u^*(x^B, \theta_0)\},$$

is a proper superset of

$$\{\mu \in [0, 1] : |u(w^A, w^B, \theta_0, \mu | 1 - x^B) \geq u^*(1 - x^B, \theta_0)\},$$

then

$$\mu(x^B | w^A, w^B) = 1.$$

Criterion D1 simply says that, if the voters see an out of equilibrium allocation chosen, they should believe it was chosen by the type of politician who would have an incentive to deviate for the least restrictive set of beliefs. As the majority type receives positive utility from implementing  $B$ , while the minority type receives a negative payoff from doing so, the majority type has a greater incentive to take action  $B$  than the minority type does. There is one caveat to this however. As most of the politicians are in the majority (by definition), the minority type is less likely to be replaced by another candidate of the same type if they are defeated, and so they

value re-election more. In order to rule out the possibility that minority types would have a greater incentive to spend a small amount of time on  $B$  than the majority types would, we make the following assumption.

**Assumption 1.**

$$m \leq \frac{1}{2\gamma}.$$

This assumption ensures that the majority type has greater incentive to choose  $B$  than the minority type for every value of  $\phi$ .<sup>7</sup>

We now consider how applying criterion D1, coupled with Assumption ??, allows us to make precise predictions from this model. Consider first the case in which  $\delta$  and/or  $\phi$  are low, so the politicians are much more concerned with the policy implemented in the current period than they are about securing re-election. Consequently, in equilibrium, both types must focus the bulk of their energies on ensuring that  $A$  is implemented. Notice, however, that the equilibrium must involve the majority type separating themselves by placing strictly positive effort on  $B$ : as the majority type has a strictly greater incentive to choose  $B$  than the minority type, by placing slightly more weight on  $B$  the majority type can reveal themselves to be the majority type and guarantee certain re-election. Hence, for  $\phi$  low, we will have a separating equilibrium with minority types focusing on  $A$  and majority types exerting just enough effort on  $B$  to reveal themselves to be the majority type. The requirement D1 on off-path beliefs guarantees that this separating equilibrium will be the one with the minimum effort diverted from  $A$  to  $B$ .

Now consider the case where  $\delta$  and  $\phi$  are high, and so the primary concern of politicians is to secure re-election. In this setting, though the majority type still has an incentive to try to separate by putting additional emphasis on issue  $B$ , the minority type is no longer willing to forsake re-election in order to be able to focus her efforts on her preferred policy. As the minority type would always have an incentive to mimic the majority type, and the majority type would always have an incentive to try to separate by attaching more effort to  $B$ , the only possible

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<sup>7</sup>If assumption 1 is violated, i.e. under very high values of  $m$  or very high values of  $\gamma$ , there could be values of  $\phi$  small enough such that the minority type, knowing that their replacement is extremely unlikely to share their policy preferences, might be more willing to choose a small level of  $w^B$  than the majority type would. As this is only an issue when  $\phi$  is very small, we could replace Assumption 1 with a condition that  $\phi$  is non-trivial.

equilibrium would involve all politicians putting maximal effort  $w^B = \min\{W, 1\}$  on issue  $B$  in the initial period.

Finally note that, in a separating equilibrium, emphasizing issue  $B$  results in re-election with certainty. In contrast, in a pooling equilibrium, emphasizing issue  $B$  results in re-election with probability  $1/2$ . For intermediate levels of office-motivation then it would not be possible to have a D1 equilibrium that is either separating (since the minority type would have an incentive to deviate and mimic the majority type) or pooling (since the minority type would prefer to lose election than be re-elected with probability  $1/2$  by exerting effort on the issue they don't like). For this range of parameters the equilibrium will be partial-pooling, with all majority types emphasizing issue  $B$ , and minority types randomizing between focusing all their effort on  $A$  and losing election, and focusing on  $B$  and being re-elected with probability between  $1/2$  and  $1$ .

As the above discussion suggests, we have the following characterization of the equilibria with off-path beliefs which satisfy D1.

**Proposition 1. *Characterization of Equilibrium with D1 Beliefs***

*For all  $\epsilon \in (0, \bar{\epsilon})$ , there exists a Perfect Bayesian Equilibrium which satisfies criterion D1. Further, there exist  $\bar{\phi}$  and  $\phi^*$  with  $0 < \bar{\phi} < \phi^*$  such that, in any equilibrium, when  $\theta_0 = 1$ ,*

- (1) *if  $\phi \in (0, \bar{\phi})$  the majority type chooses  $w^B = w_*(\phi) > 0$  and  $w^A = W - w^B$  and the minority type randomizes with a possibly degenerate probability between  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  and  $w^B = w_*(\phi)$ ,  $w^A = W - w^B$ . If the minority type chooses allocation  $w^B = w_*(\phi)$ ,  $w^A = W - w^B$  with positive probability, it must be that the probability of randomization is small enough that*

$$Pr(\text{Re-elected} | w^B = w_*(\phi), w^A = W - w^B) = 1.$$

- (2) *if  $\phi \in (\bar{\phi}, \phi^*)$  the majority type chooses  $w^B = \min\{W, 1\}$ ,  $w^A = W - w^B$  and the minority type randomizes with a non-degenerate probability between  $w^B = \min\{W, 1\}$ ,  $w^A = W - w^B$  and  $w^A = \min\{W, 1\}$ ,  $w^B = 0$ , so that the probability of re-election is*

$$Pr(\text{Re-elected} | w^B = \min\{W, 1\}, w^A = W - w^B) \in (\frac{1}{2}, 1).$$

- (3) *if  $\phi > \phi^*$  all politicians choose  $w^B = \min\{W, 1\}$  and  $w^A = W - w^B$ .*

So we have that, when the system induces sufficiently low office holding rewards coming from reelection, we get a separating equilibrium in which the majority type chooses  $w^B > 0$  and allocates the rest of her effort to issue  $A$ , whereas the minority type allocates all effort to  $A$ . With high enough office holding rewards, on the other hand, all politicians focus first on issue  $B$  in order to maximize their probability of being re-elected. Finally, for an intermediate level of office holding rewards, the minority type randomizes between posturing and revealing themselves to be in the minority by pursuing issue  $A$ . How much should the office holding rewards be in order to be in each range is determined by the parameters of the model, and we consider how these ranges are determined in the next section.

**3.1. Comparative Statics.** Let us consider how  $\bar{\phi}$  and  $\phi^*$  vary with the parameters. We have the following proposition.

**Proposition 2. *Range of Existence for a Separating Equilibrium***

- (1)  $\bar{\phi}$  and  $\phi^*$  are strictly increasing in  $W$  for  $W \in (0, 1)$  and strictly decreasing for  $W \in (1, 2)$ .
- (2)  $\bar{\phi}$  and  $\phi^*$  are strictly decreasing in  $\delta$ .
- (3)  $\bar{\phi}$  and  $\phi^*$  are strictly decreasing in  $m$ .

Before discussing what each part of this result means, recall the importance of  $\bar{\phi}$  and  $\phi^*$  in the D1 equilibrium we characterized in Proposition ???. It is only possible to support separating equilibria when  $\phi \leq \bar{\phi}$ , and only possible to support an equilibrium that does not involve everyone pooling on  $B$  when  $\phi < \phi^*$ . Hence  $\bar{\phi}$  and  $\phi^*$  are indices of how likely it would be (in a world of randomized parameter values) to live in an equilibrium without pervasive posturing incentives.

The first part of the proposition says that  $\bar{\phi}$  and  $\phi^*$  are non-monotonic in  $W$ . If  $W$  is small, it will be difficult to support a separating equilibrium. As politicians know that the effort allocation they choose is unlikely to have an effect on the resulting policy, they have a greater incentive to choose the allocation most likely to get them re-elected – in this case, that means pooling on  $B$ .<sup>8</sup> As  $W$  increases, effort is more likely to lead to the policy being implemented, so the incentive for the politician to allocate effort to her preferred policy increases. However, as  $W$  gets larger than 1, further increases in  $W$  will make it more difficult to support a separating equilibrium.

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<sup>8</sup>Fox and Stephenson (2011) present a model where judicial review, by insulating politicians from their policy choices, can increase posturing.

This is because, when  $W$  is large, politicians are capable of getting both the  $A$  and the  $B$  policy implemented with a high probability. As the greatest cost of effort on the  $B$  policy, when  $W \leq 1$ , is that it comes directly at the expense of effort which could be allocated to the  $A$  policy, the costs of choosing the  $B$  policy are lower when  $W$  is large. As the policy choice for the politician is most stark when  $W = 1$ , this is when it is possible to support a separating equilibrium for the widest range of parameters.

The second part of the proposition says that  $\bar{\phi}$  and  $\phi^*$  are both decreasing in  $\delta$ : as posturing involves choosing sub-optimal policies in the current period in order to secure re-election, the more patient politicians are, and hence the more they care about the future, the lower the benefit from holding office necessary to induce posturing.

The third part of the proposition, that  $\bar{\phi}$  and  $\phi^*$  are increasing in  $m$ , shows that it becomes more difficult to support a separating equilibrium the smaller the minority. When  $m$  is large, minority type politicians have a greater incentive to hold office, because a replacement is unlikely to share their policy preferences. Therefore, in equilibrium, we get more posturing.

#### 4. EQUILIBRIUM WITH UNOBSERVABLE EFFORT CHOICES

We now consider the incentives when the effort allocation is not transparent. That is, we assume that the voters can observe only the outcome but not  $w^A$  or  $w^B$ . As the incentive for the politician to take each action depends on the beliefs the voters form after observing each outcome, the beliefs at off-path information sets can play a key role even here in determining the politician's incentives. Further, since, in our model, a non-status-quo policy can never result if a politician does not exert positive effort on the issue, off-path information sets are produced by many politician strategies – if the politician's equilibrium strategy involves  $w^j = 0$  for some  $j \in \{A, B\}$  then observing  $p^j = 1$  is off the equilibrium path. So, in order to make precise predictions, we want to apply the logic of criterion D1 to pin down off-path beliefs and select among equilibria. With non-transparency about the politician's effort, this game is not a sender-receiver game, and so we must adapt the notion of D1 from Cho and Kreps (1987) to an environment in which the action taken by the politician is not observed. To do so, we first define

$$M(x^B, w^A, w^B) = \{\mu \in [0, 1] : u(w^A, w^B, \theta_0, \mu | x^B) > u^*(x^B, \theta_0)\}.$$

That is,  $M$  determines the (possibly empty) set of beliefs for which a politician of type  $x^B$  would have a strict incentive to deviate to  $(w^A, w^B)$  instead of their equilibrium strategy. We then say that the beliefs at off path information set  $(p^A, p^B)$  are consistent with D1 if, whenever there is one type that would be willing to choose *every* effort allocation which would result in  $(p^A, p^B)$  being observed with positive probability for a wider range of beliefs than the other type, the voters must believe the politician is of that type if they ever observe  $(p^A, p^B)$ . This leads to the definition of criterion D1 in a non-transparent environment.

**Definition 2. Criterion D1 with Non-Transparency**

*In the Non-Transparent Action game a Perfect Bayesian Equilibrium satisfies criterion D1 if,*

- (1) *at all information sets  $(p^A, p^B)$  all voters hold the same beliefs,  $\mu$ , about the politician's type.*
- (2) *whenever there exists an off-path policy outcome  $(p^A, p^B)$  and type  $x^B \in \{0, 1\}$  such that, for all feasible  $(w^A, w^B)$  effort allocations for which information set  $(p^A, p^B)$  is reached with positive probability,  $M(x^B, w^A, w^B)$  is a superset of  $M(1 - x^B, w^A, w^B)$ , and if  $M(x^B, w^A, w^B) \neq \emptyset$ , a proper superset, then*

$$\mu(x^B | p^A, p^B) = 1.$$

In the analysis we now separate the  $W < 1$  case from the  $W > 1$  case as the implications of non-transparency differ substantially between the two cases.

4.1. **The  $W \leq 1$  case.** Focusing on the minority type, i.e., on an incumbent with  $x^B = 0$ , The first result of this section is that if the value of office and the discount factor are sufficiently small, the absence of transparency leads to the best possible outcome in terms of first period welfare.

**Proposition 3.** *For any  $W \leq 1$  and for any combination of  $\delta, \gamma, q, m$  such that*

$$(1) \quad \delta < \frac{2\gamma - 1}{(1 - q)(1 - \gamma)(1 - m)W},$$

*there exists a bound  $\phi_1^{BA} > 0$  such that  $\forall \phi \leq \phi_1^{BA}$  the unique D1 equilibrium when effort allocation cannot be observed is a pooling one with no posturing, with both types choosing to focus on A, hence yielding a higher first period welfare than under transparency.*

This result that for low enough  $\phi$  there is a benefit of not having transparency because posturing incentives are eliminated altogether is interesting, but of course the benefit of this is only in terms of first period welfare (discipline), while the separating equilibrium prevailing with transparency has higher expected second period welfare (due to sorting), at least for the majority type of voter. As the sanctioning and sorting effects of transparency go in opposite directions, the overall effect on the welfare of the majority type voters, and for the voters as a whole, is ambiguous.

Let us now examine the opposite case, when  $\phi$  is sufficiently high. We have the following proposition.

**Proposition 4.** *For any  $W \leq 1$ , if  $\phi > \phi_0^{A0} \equiv \frac{2\gamma}{\delta} - (1-q)(1-\gamma)mW$ , in the unique D1 equilibrium the majority type chooses  $w^B = W$  and the minority type mixes between putting effort on B and making no effort whatsoever. In this case an institutional reform yielding transparency would be good for first period welfare.*

For high values of  $\phi$  the lack of transparency creates even further welfare loss in the first period beyond that already determined by posturing incentives. A way to interpret propositions ?? and ?? together is that in the presence of weak executive institutions (low  $W$ ), for example with many checks and balances that reduce the probability of accomplishing multiple policy reforms, introducing transparency will induce better first period behavior when politicians are very patient and/or very office motivated, whereas it can induce even greater distortions in first period behavior when politicians are impatient and/or mainly policy motivated.

If  $\phi \in (\phi_1^{BA}, \phi_0^{AB})$ , then a separating equilibrium exists in which the majority (resp. minority) type puts the whole effort on B (resp. A). Finally, for  $\phi \in (\phi_0^{AB}, \phi_0^{A0})$ , there are multiple equilibria with minority types mixing. Because of multiple equilibria in this region it would be difficult to make welfare comparisons with the transparency case, hence in the above propositions we have focused exclusively on the two regions, high and low, where the comparison is clearest.

As we will see, the conclusion that a transparency reform would be particularly desirable in the absence of term limits but potentially harmful in the presence of term limits is a conclusion that only applies in contexts where executive institutions are weak ( $W < 1$ ), while in the presence of strong executive institutions the desirability of transparency reforms could actually correlate with term limits in the opposite way.



4.2. **The  $W \geq 1$  case.** As with the  $W < 1$  case, we focus on high and low values of  $\phi$ . Recall that, with transparency, if  $\phi$  is low we have a separating equilibrium in which the minority type chooses  $w^A = 1$  and  $w^B = 0$ , and the majority type chooses  $w^B \in (W - 1, 1)$ ,  $w^A = W - w^B$ .

**Proposition 5.** *For any  $W > 1$ , if  $\delta < \min\left\{\frac{m(1+qW-2q)}{W-1}, \frac{2\gamma-1}{(3-W)[(1-m)(1-\gamma)(1+(W-2)q)]}\right\}$ , and effort allocations are not observable, there exists  $\phi' > 0$  such that the unique D1 equilibrium for  $\phi \leq \phi'$  involves all minority candidates choosing  $w^A = 1, w^B = 0$  and all majority candidates choosing  $w^A = 1, w^B = W - 1$  in state  $\theta_0 = 1$ . This is the same first period behavior as with transparent actions,, but, as selection is worse, it provides lower second period utility to majority voters.*

We now turn to the high office motivation case. In the transparency case, if the voters observe any effort allocation other than that chosen by the majority type, they will know with certainty that the politician deviated, and so is the minority type. So, in order to support a posturing equilibrium we need only to check that the politician does not have an incentive to deviate to her most preferred policy and lose re-election. Hence we can support a posturing equilibrium if and only if

$$\gamma(2 - W) + (1 - \gamma) \leq \frac{1}{2}\delta[\phi + m(1 - \gamma)[(1 - q) + q(W - 1)]],$$

or equivalently

$$\phi \geq \phi^* = \frac{2(1 - \gamma(W - 1))}{\delta} - m(1 - \gamma)[1 + q(W - 2)].$$

As in the transparency case, the minority type has the option of deviating to her preferred first-period policy  $w^A = 1, w^B = 0$  and being re-elected with probability 0: she will prefer this to pooling if and only if  $\phi < \phi^*$ . Note, however, that she could also deviate to choose  $A$  with higher probability putting the rest of the effort on  $B$ , and get re-elected with probability greater than 0, so there is an additional deviation we need to check to verify that a posturing equilibrium can be supported. She will not have an incentive to shift weight from  $B$  if and only if

$$(2 - W) \leq \frac{2 - W}{2}\delta[\phi + m(1 - \gamma)[(1 - q) + q(W - 1)]]$$

or equivalently

$$\phi \geq \phi_{NA}^* \equiv \frac{2}{\delta} - m(1 - \gamma)[1 + q(W - 2)]$$

Notice that  $\phi_{NA}^* > \phi^*$ , so it is more difficult to support a posturing equilibrium when the action is non-transparent. We then get the following proposition.

**Proposition 6.** *For  $\phi \geq \phi_{NA}^*$  the unique equilibrium involves all politicians choosing  $w^B = 1, w^A = W - 1$ . If  $\phi$  is in the non-empty interval  $(\phi^*, \phi_{NA}^*)$ , in equilibrium, the minority type must choose  $w^B < 1, w^A = W - w^B$  with positive probability.*

Note that, on the range  $\phi \in (\phi^*, \phi_{NA}^*)$  the welfare comparison is unambiguous: the minority type places more effort on  $A$ , which gives higher payoff to everyone in the first period. Further, because  $A$  is more likely to occur when the politician is the minority type, the voters learn about the politician's type, so the politician is more likely to be retained if she is the majority type. Hence, as in Prat (2005) non-transparency over actions is beneficial in this range, both in terms of the first period action, and in terms of selection for the future. When  $\phi \in (\phi^*, \phi^{NA})$ , as non-transparency makes posturing less effective, in equilibrium the minority types do not posture as much as they did with transparency. This breaks the equilibrium in which all politicians pool with maximal posturing, leading to more efficient policy choices by politicians, and more learning by voters.

Note that this does not mean that we cannot have a posturing equilibrium when the action is non-transparent and  $W > 1$ . When  $\phi > \phi_{NA}^*$  the benefits from holding office are great enough that even with non-transparency the equilibrium must involve maximal posturing by all politicians. As, regardless of the transparency regime, voters cannot update about the politicians in a pooling equilibrium, when  $\phi > \phi_{NA}^*$  the equilibrium outcome is identical with or without transparency of the effort choices.

## 5. CONCLUSIONS

We have considered the incentives of politicians to “posture” by focusing their efforts on issues which, though perhaps not the most important issues from the voters' perspective, present the greatest opportunity for politicians to signal their preferences to voters. We have shown that this incentive can lead politicians to spend their time pursuing policies which are not only harmful to the minority, but also an inefficient use of time from the majority's perspective. Further, we have shown that greater transparency about how politicians allocate their time, while often beneficial, can increase posturing, and so decrease the first period welfare of the voters as well as the selection of politicians who share the objectives of the majority of the electorate.

There has been much written on how American politics has become polarized (e.g Fiorina et al. 2006, Abramowitz 2010), and how a disproportionate amount of time (Fiorina 2006) and media attention (Prat and Stromberg 2010) is devoted to divisive issues at the expense of issues which it is believed voters consider more important. While our model cannot be directly applied to study polarization and partisanship – there are no parties in this model, only a clearly defined majority that all politicians need to win over – one could imagine extending this framework to an environment with parties in which different politicians are predominantly concerned with signaling to members of their own party – for example if they first needed to win a primary in order to stand for office. In such an environment if Republicans posture in order to show that they are committed to not raising taxes by refusing to accept even mild tax increases, while Democrats posture to show they are committed to maintaining government spending on social issues, it may not be possible for politicians to reach an agreement on a common value goal (e.g dealing with the debt crisis, maintaining the nation’s credit rating) even if all involved agree that is an important priority.<sup>9</sup> A full analysis of posturing in the face of parties, primaries, or special interest groups is left to future work.

Further, though the choice of institutions depends on factors that are obviously outside our model, the above results suggest that if we wanted to analyze an institutional design problem in which multiple institutional reforms are compared, the desirability of each institutional reform cannot be evaluated in “isolation” from the others. In particular, consider the binary choice between imposing term limits (say with only one reelection allowed) versus a status quo of no term limits. Even though this question requires a fully dynamic framework for proper analysis, which is in our research agenda, one can already see what our results could be by interpreting a low (resp. high)  $\phi$  as a reduced form consequence of having (resp. not having) term limits.<sup>10</sup>

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<sup>9</sup>Appearing on *Meet the Press* soon after Standard and Poor downgraded the U.S. debt rating, former chairman of the Council of Economic Advisors Austan Goolsbee wondered if “for the sake of the economy, ‘Can’t we wait on the things that we’re going to yell at each other about and start on the things that we agree on?’” (Goolsbee 2011) Our analysis provides an explanation for why it is often difficult to get politicians to come together to address the “things we agree on.”

<sup>10</sup>See Smart and Sturm (2011) for an analysis of the effect of term limit on the incentives to pander in a model based on Maskin and Tirole (2004).

Focusing on the desirability of transparency reforms, one message of our paper is that transparency reforms are more likely to be desirable when either executive power is weak (low  $W$ ) and there are no term limits (high  $\phi$ ), or when the opposite is true, i.e., when executive power is strong and there are term limits, whereas in the remaining two combinations of the institutional system (in a stylized world of binary high-low choices on all dimensions) a transparency reform can lead to negative outcomes.

Similar caveats on institutional reform evaluations could be generated by our framework or extensions of it also for the other two dimensions, i.e., for the desirability of term limits or for the desirability of executive efficiency. Term limits, by inducing a low  $\phi$ , are likely to generate positive voter welfare effects when executive institutions are weak (low  $W$ ) and political actions are non transparent, and in the opposite mix where executive power is strong and political actions are transparent, but the introduction of term limits could be detrimental in the other two cases. These considerations suggest to us that our framework could be very useful for future normative institutional analysis.

Finally, the results of this paper can be fruitfully applied to organization theory: the scope of a manager's action space, the reward structure, and the degree of monitoring of her actions are three important components of the optimal contract the principal wants to set up, and our framework suggests how these three instruments should be changed as a function of the share-holders' heterogeneity of preferences, the relative importance of the issues the manager has to decide on, and the uncertainty about how such relative importance may change in the future. Since the parameters of the model would need to have a slightly different interpretation (for example high versus low  $\phi$  could be generated by internal career steps in the hierarchy or tenure prospects or by some other source of payoff incentives that may differ from the reelection prospects we talk about), a full blown analysis of our model for organization design is left for future research.

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## 7. APPENDIX A: EQUILIBRIUM CHARACTERIZATION

**7.1. Separating Equilibria.** We now consider the parameters under which it is possible to support a fully separating equilibrium. As the politician has a continuum of actions, and the effort levels are observable, we could specify a large number of equilibria which exhibit some degree of separation. We focus on equilibria in which, when  $\theta_0 = 1$ , the majority type, who benefits from effort spent on both issues, always chooses  $w^A + w^B = W$ .

**Definition 3.** *A Separating Equilibrium is one in which at time  $t = 0$ :*

- (1) *in state  $\theta_0 = 1$  the majority type chooses  $w^A \in [0, 1]$ ,  $w^B \in [0, 1]$  such that  $w^A + w^B = W$ ;*
- (2) *in state  $\theta_0 = 1$  the minority type chooses  $w^A = \min\{W, 1\}$  and  $w^B = 0$ .*

In a fully separating equilibrium, if the minority type chooses an allocation other than the allocation chosen by the majority type, her type will be revealed. Hence the minority type will choose the allocation they most prefer from a policy perspective, as specified in the above definition.

The upper bound on patience such that a separating equilibrium exists is

$$(2) \quad \bar{\delta} = \begin{cases} \frac{W}{\phi + (1-\gamma)(1-q)mW} & \text{if } W \leq 1, \\ \frac{1 - (W-1)\gamma}{\phi + m(1-\gamma)(1+(W-2)q)} & \text{if } W > 1. \end{cases}$$

The interval  $[w_*(\delta), w^*(\delta)]$  defines the range of  $w^B$  that the majority type must be choosing in a separating equilibrium when  $\delta \in (0, \bar{\delta}]$ .

These bounds are

$$(3) \quad w_*(\delta) = \begin{cases} \delta(\phi + (1 - \gamma)(1 - q)mW) & \text{if } W \leq 1, \\ \min\{(W - 1)\gamma + \delta(\phi + m(1 - \gamma)(1 - (2 - W)q)), W - 1\} & \text{if } W > 1. \end{cases}$$

and

$$(4) \quad w^*(\delta) = \begin{cases} \min\{\delta \frac{\phi + (1 - \gamma)(1 - q)(1 - m)W}{(2\gamma - 1)}, W\} & \text{if } W \leq 1, \\ \min\{(W - 1)(2\gamma - 1) + \delta \frac{\phi + (1 - m)(1 - \gamma)(1 + q(2 - W))}{2\gamma - 1}, 1\} & \text{if } W > 1. \end{cases}$$

We now prove that there exists a separating equilibrium if and only if politicians are not too patient.

**Lemma 3. Existence of a Separating Equilibrium**

*Under Assumption 1 there exists a separating equilibrium if and only if  $\delta \in (0, \min\{\bar{\delta}, 1\}]$ . For each  $\delta \in (0, \min\{\bar{\delta}, 1\}]$  we have a separating equilibrium if and only if the minority type chooses  $w^A = \min\{W, 1\}$ , and  $w^B = 0$  and the majority type chooses  $w^B \in [w_*(\delta), w^*(\delta)]$  and  $w^A = W - w^B$ .*

**7.2. Pooling Equilibria.** We now consider for which parameters there would exist a pooling equilibrium. Note that the majority type never prefers to choose  $w^A + w^B < W$  when  $\theta = 1$ , though for appropriately chosen beliefs this could be supported in equilibrium. We look for equilibria in which the majority type always sets  $w^A + w^B = W$  and the minority type chooses the same allocation as the majority type.

**Definition 4.** *A pooling equilibrium is one in which, when  $\theta_0 = 1$ , both the majority and minority type choose the same allocation  $w^A$  and  $w^B$  with  $w^A + w^B = W$ .*

Note that in a pooling equilibrium the politician will be re-elected with probability 1/2. We now consider, for each  $W$  the possibility of a pooling equilibrium with  $w^B = w' \in [0, \min\{W, 1\})$ . Now define

$$(5) \quad \underline{\delta} = \begin{cases} 0 & \text{if } W \leq 1, \\ \frac{2(W-1)(1-\gamma)}{\phi + m(1-\gamma)(1+(W-2)q)} & \text{if } W > 1. \end{cases}$$

and

$$(6) \quad w^B(\delta) = \begin{cases} \min\{\frac{\delta}{2}[\phi + (1-q)m(1-\gamma)W], W\} & \text{if } W \leq 1, \\ \min\{(W-1)\gamma + \frac{\delta}{2}[\phi + m(1-\gamma)(1+(W-2)q)], 1\} & \text{if } W > 1. \end{cases}$$

When  $W > 1$ , since pooling would involve the minority type choosing  $w^B > 0$ , some patience will be required to support a pooling equilibrium. The bound  $w^B(\delta)$  determines how high  $w^B$  can be supported in a pooling equilibrium. Note that if  $\delta$  is small we can only support a pooling equilibrium with  $w^B$  small. However, when  $\delta$  is larger it is possible to support a much wider range of pooling equilibria. Define

$$(7) \quad \delta^* \equiv \begin{cases} \frac{2W}{\phi + (1-\gamma)(1-q)mW} & \text{if } W \leq 1, \\ \frac{2(1-(W-1)\gamma)}{\phi + m(1-\gamma)(1+(W-2)q)} & \text{if } W > 1. \end{cases}$$

This determines the minimum level of patience for which we can support a pooling equilibrium with any  $w^B \in [\max\{0, W-1\}, \min\{W, 1\}]$ . We have the following proposition.

**Lemma 4. *Existence of a Pooling Equilibrium***

*For all  $\delta \geq \underline{\delta}$  there exists pooling equilibria. When  $\delta \geq \delta^*$  it is possible to support any  $w^B \in [\max\{0, W-1\}, \min\{W, 1\}]$  in a pooling equilibrium. When  $\delta \in (\underline{\delta}, \delta^*)$  these equilibria can involve any  $w^B$  with*

$$w^B \in [\max\{0, W-1\}, w^B(\delta)].$$

*and  $w^A = W - w^B$ .*

**7.3. Partial-pooling Equilibria.** We now look for equilibria which are neither fully separating nor fully pooling. In such an equilibrium we must then have at least one type randomizing. As allocating effort to  $B$  instead of  $A$  is more costly for the minority type than the majority it will never be an equilibrium for both types to randomize over the same two allocations, so in a partial pooling equilibrium there will be only one allocation taken by both types with positive probability. Below we characterize the equilibria which are neither pooling or separating.

**Lemma 5. *Partial-Pooling Equilibria***

*In any Perfect Bayesian Equilibrium in which in state  $\theta_0 = 1$  the majority type always chooses  $w^A + w^B = W$ , if the equilibrium is neither Separating nor Pooling, at least one type must be randomizing with a non-degenerate probability. Further, each type must randomize over at most*



two allocations, and there will be exactly one allocation which both types choose with positive probability. The equilibrium must take one of the following forms:

- (1) The minority type randomizes between allocations  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  and  $w^B = w'$ ,  $w^A = W - w'$ , where  $w' \leq w_*(\delta)$ . The majority type always chooses  $w^B = w'$ ,  $w^A = W - w'$ . An equilibrium of this form can be supported if and only if  $\delta \in (\frac{1}{2}\underline{\delta}, \delta^*)$ .
- (2) The minority type always chooses some allocation  $w^B = w'$ ,  $w^A = W - w'$  and the majority type randomizes between  $w^B = w'$ ,  $w^A = W - w'$  and  $w^B = w''$ ,  $w^A = W - w''$  where  $w'' > w'$ .
- (3) The minority type randomizes between  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  and  $w^B = w'$ ,  $w^A = W - w'$ , where  $w' \geq w_*(\delta)$ . The majority type randomizes between  $w^B = w'$ ,  $w^A = W - w'$  and  $w^B = w''$ ,  $w^A = W - w''$  where  $w'' > w'$ .

We have now fully characterized the possible equilibria, given that the majority type always chooses an allocation with  $w^A + w^B = W$ . This characterization is used, along with refinements on off-path beliefs, to make more precise predictions.

## 8. APPENDIX B: PROOFS

We first prove the results for the characterization of equilibrium with transparency. We then apply the refinement D1 on the off-path equilibrium beliefs and show how this produces a unique equilibrium prediction.

**8.1. Proof of Characterization Results.** *Proof of Lemma 1* Immediate.  $\square$

*Proof of Lemma 2* First note that, in a pooling equilibrium, the incumbent is the majority type with probability  $m$  and the minority type with probability  $1 - m$ , which is the same distribution of politician types as a potential replacement. Hence, all voters will vote for the candidate if and only if the candidates' valence  $v_j$  is at least the average valence of the population of candidates,  $E[v] = 0$ . By symmetry this happens with probability  $1/2$ , so the probability of being re-elected in a pooling equilibrium is  $1/2$  for all  $\epsilon$ .

Now we consider a separating equilibrium. We consider the payoffs to the majority type for two cases:  $W \leq 1$  and  $W > 1$ . If the candidate is a majority type with valence  $v_j$  then the

expected payoff is

$$-q[(1 - W)\gamma + (1 - \gamma)] - (1 - q)(1 - \gamma)(1 - W) + v_j,$$

if  $W \leq 1$  and

$$-q(1 - \gamma)(2 - W) + v_j,$$

if  $W > 1$ . Similarly, if the candidate is a minority type the expected payoff is

$$-q(1 - W) - (1 - \gamma) + v_j,$$

if  $W < 1$  and

$$-(1 - \gamma) + v_j$$

when  $W > 1$ . Combining these equations payoffs with the fact that a random replacement will have an expected valence of 0 we have that the expected payoff from a random replacement is

$$-q(1 - W)\gamma - (1 - \gamma)[qm + (1 - q)(1 - W)m + (1 - m)]$$

when  $W < 1$  and

$$-(1 - \gamma)[q(2 - W) + (1 - m)(q(2 - W) + (1 - q))]$$

when  $W > 1$ . We can then conclude that the majority voter will vote to re-elect a majority incumbent if and only if

$$v_j \geq \begin{cases} -(1 - \gamma)(1 - m)(1 - q)W & \text{if } W \leq 1, \\ -(1 - \gamma)(1 - m)(q(2 - W) + (1 - q)) & \text{if } W > 1, \end{cases}$$

and will vote to re-elect the a minority incumbent if and only if

$$v_j \geq \begin{cases} (1 - \gamma)m(1 - q)W & \text{if } W \leq 1, \\ (1 - \gamma)m(q(2 - W) + (1 - q)) & \text{if } W > 1, \end{cases}$$

Defining

$$\bar{\epsilon} = \begin{cases} (1 - \gamma)(1 - m)(1 - q)W & \text{if } W \leq 1, \\ (1 - \gamma)(1 - m)(q(2 - W) + (1 - q)) & \text{if } W > 1, \end{cases}$$

we have that, for all  $\epsilon \in [0, \bar{\epsilon})$  the majority of voters will vote to re-elect the majority type and reject the minority type with certainty. A majority type will then always be re-elected, and a minority type never will be re-elected, in a separating equilibrium.  $\square$

*Proof of Lemma 3* We divide this into two cases:  $W \leq 1$  and  $W > 1$ . Consider first the case in which  $W \leq 1$ . We start by looking for parameters for which the politician is re-elected if and only if  $w^B = w' \in (0, W]$ . In order for the minority type to be willing to take action  $A$  and lose re-election it must be that the payoff from allocating effort to  $A$  in the first period,

$$\phi - (1 - W)\gamma + \delta[-q(1 - W)\gamma - (1 - q)m(1 - \gamma)W],$$

is at least as high as the utility from allocating  $w'$  to  $B$  and being re-elected,

$$\phi - \gamma(1 - W + w') - w'(1 - \gamma) + \delta[\phi - q(1 - W)\gamma].$$

Hence, in order for the minority type to have an incentive to choose  $w^A = W$  in the first period we must have,

$$w' \geq \delta[(1 - q)m(1 - \gamma)W + \phi].$$

That is, that the disutility in policy today from choosing  $w^B = w'$  instead of  $w^B = 0$ , which is  $w'$ , must be at least as large as discounted the disutility of losing office tomorrow,  $\phi$ , as well as the policy loss from having a random replacement. This policy loss is equal to  $m$ , the probability they will not be of the same type, multiplied by  $1 - q$ , the probability there will be disagreement in the preferred policy, multiplied by  $W$ , the probability the policy the politician pursues will be implemented, multiplied by  $(1 - \gamma)$ , the disutility from having  $w^B = W$  in the second period for a minority type.

For the majority type to have an incentive to choose  $w^B = w'$  instead of  $w^B = 0$  it must be that the policy loss from implementing  $B$  instead of  $A$  is not enough to make the politician lose office. That is we must have that the utility from focusing on  $B$ ,

$$\phi - \gamma(1 + w' - W) - (1 - \gamma)(1 - w') + \delta[\phi - q\gamma(1 - W) - q(1 - \gamma) - (1 - q)(1 - \gamma)(1 - W)]$$

is at least as large as from exerting all effort on  $A$ ,

$$\phi - \gamma(1 - W) - (1 - \gamma) + \delta[-q\gamma(1 - W) - q(1 - \gamma) - (1 - q)(1 - \gamma)(1 - W)m - (1 - q)(1 - \gamma)(1 - m)]$$

That is, the majority type has an incentive to implement  $B$  with probability  $w'$  if and only if

$$(2\gamma - 1)w' \leq \delta[(1 - q)(1 - m)(1 - \gamma)W + \phi].$$

So we have an equilibrium for

$$\delta \in \left[ \frac{(2\gamma - 1)w'}{\phi + (1 - \gamma)(1 - q)(1 - m)W}, \frac{w'}{\phi + (1 - \gamma)(1 - q)mW} \right].$$

Next note that, for any  $w' \in (0, W)$  this interval is non-empty. To see this, note that

$$\frac{(2\gamma - 1)w'}{\phi + (1 - \gamma)(1 - q)(1 - m)W} < \frac{w'}{\phi + (1 - \gamma)(1 - q)mW},$$

if and only if

$$(2\gamma - 1)\phi + (2\gamma - 1)(1 - \gamma)(1 - q)mW < \phi + (1 - \gamma)(1 - q)(1 - m)W.$$

This equation can then be simplified to

$$(1 - q)W[2\gamma m - 1] < 2\phi.$$

We we have that when

$$\phi > \max\left\{\frac{1 - q}{2}(2\gamma M - 1)W, 0\right\},$$

this always produces a non-empty interval for each  $w'$ . Next note, that as  $w'$  varies from 0 to  $W$  every  $\delta \in (0, \frac{W}{\phi + (1 - \gamma)(1 - q)mW}]$  is in one of the non-empty intervals. Hence for appropriately chosen  $w'$  – that is, if the fraction of time spent by the majority type is appropriately chosen to maintain incentives – for any  $\delta \leq \frac{W}{\phi + (1 - \gamma)(1 - q)mW}$  we can support a separating equilibrium. Further, we can re-write the condition

$$\delta \in \left[ \frac{(2\gamma - 1)w'}{\phi + (1 - \gamma)(1 - q)(1 - m)W}, \frac{w'}{\phi + (1 - \gamma)(1 - q)mW} \right]$$

as

$$w' \in \left[ \delta(\phi + (1 - \gamma)(1 - q)mW), \delta \frac{\phi + (1 - \gamma)(1 - q)(1 - m)W}{(2\gamma - 1)} \right].$$

This gives us an upper and lower bound on how the effort is allocated by the majority type when  $\theta_0 = 1$  in a separating equilibrium.

Now we consider the case in which  $W > 1$ , so  $\min\{W, 1\} = 1$ . Now, in state  $\theta_0 = 1$  the utility to the minority type to choosing  $w^A = 1, w^B = 0$ , her most preferred policy is

$$\phi + \delta[-q(W - 1)(1 - \gamma)m - (1 - q)m(1 - \gamma)],$$

and the utility from  $w^B = w' \in [W - 1, 1]$  and  $w^A = W - w'$  is

$$\phi - (1 + w' - W)\gamma - (1 - \gamma)w' + \delta[\phi]$$

Hence, we have that the minority type is optimizing if and only if

$$w' - (W - 1)\gamma \geq \delta[\phi + m(1 - \gamma)(1 + (W - 2)q)]$$

Now consider the majority type. By choosing  $w^B = w'$ ,  $w^A = W - w'$  her utility is

$$\phi - \gamma(1 + w' - W) + (1 - w')(1 - \gamma) + \delta[\phi - q(1 - \gamma)(2 - W)].$$

By choosing the myopically optimal allocation –  $w^A = 1$ ,  $w^B = W - 1$  – her utility is

$$\phi - (1 - \gamma)(2 - W) + \delta[-qm(1 - \gamma)(2 - W) - (1 - m)(1 - \gamma)].$$

Hence, in order to have the majority type willing to focus on  $B$  we must have

$$(1 + w' - W)(2\gamma - 1) \leq \delta[\phi + (1 - m)(1 - \gamma)(1 + q(W - 2))].$$

Hence both types are optimizing and he have an equilibrium if and only if

$$(8) \quad \delta \in \left[ \frac{(1 + w' - W)(2\gamma - 1)}{\phi + (1 - m)(1 - \gamma)(1 + q(W - 2))}, \frac{w' - (W - 1)\gamma}{\phi + m(1 - \gamma)(1 + (W - 2)q)} \right].$$

We now show that this interval is non-empty. To prove this it is sufficient to show that

$$\frac{(1 + w' - W)(2\gamma - 1)}{\phi + (1 - m)(1 - \gamma)(1 + q(W - 2))} < \frac{w' - (W - 1)\gamma}{\phi + m(1 - \gamma)(1 + (W - 2)q)},$$

which is equivalent to

$$\frac{(2\gamma - 1)}{\phi + (1 - m)(1 - \gamma)(1 + q(W - 2))} < \frac{1}{\phi + m(1 - \gamma)(1 + (W - 2)q)}.$$

Note that this equation holds if and only if

$$2(1 - \gamma)\phi > (1 - \gamma)(1 + q(W - 2))[(2\gamma - 1)m - (1 - m)],$$

which holds whenever

$$\phi > \frac{1 + q(W - 2)}{2}[2\gamma - 1],$$

or  $\phi > \bar{\phi}$ .

Finally, note when  $w' = W - 1$  this interval contains 0, that both ends of the interval are increasing in  $w$ , and that when  $w' = 1$  the upper bound becomes  $\frac{1 - (W - 1)\gamma}{\phi + m(1 - \gamma)(1 + (W - 2)q)}$ . Hence as  $w'$  varies from  $W - 1$  to 1, we get a separating equilibrium for all  $\delta \in [0, \frac{w' - (W - 1)\gamma}{\phi + m(1 - \gamma)(1 + (W - 2)q)}]$ .

To conclude, note that

$$\delta \in \left[ \frac{(1 + w' - W)(2\gamma - 1)}{\phi + (1 - m)(1 - \gamma)(1 + q(W - 2))}, \frac{w' - (W - 1)\gamma}{\phi + m(1 - \gamma)(1 + (W - 2)q)} \right],$$

if and only if

$$w' \in \left[ (W - 1)\gamma + \delta(\phi + m(1 - \gamma)(1 + q(2 - W))), (W - 1)(2\gamma - 1) + \delta \frac{\phi + (1 - m)(1 - \gamma)(1 + q(2 - W))}{2\gamma - 1} \right].$$

So we have upper and lower bounds on the  $w'$  spent on issue  $B$ .  $\square$

*Proof of Lemma 4* Consider first the case in which  $W \leq 1$ . If  $w^B = w'$  then  $w^A = W - w'$ . Now consider the incentives faced by the minority type. If she follows the above strategy her payoff is

$$\phi - \gamma(1 + w' - W) - (1 - \gamma)w' + \delta \left[ \frac{\phi}{2} - q(1 - W)\gamma - (1 - q)(1 - \gamma)W \frac{m}{2} \right]$$

And by choosing her most preferred allocation and losing re-election  $w^A = W$ ,  $w^B = 0$  her payoff is

$$\phi - \gamma(1 - W) + \delta [-q(1 - W)\gamma - (1 - q)m(1 - \gamma)W]$$

So we have that the minority type is optimizing if and only if

$$w' \leq \frac{\delta}{2} [\phi + (1 - q)m(1 - \gamma)W].$$

Note that when  $w' = 0$ , so all effort is spent on  $A$ , this constraint is satisfied for any  $\delta$ . Now consider the majority type. By following the prescribed strategy her utility is

$$\phi - \gamma(1 + w' - W) - (1 - \gamma)(1 - w') + \delta \left[ \frac{\phi}{2} - q\gamma(1 - W) - q(1 - \gamma) - (1 - q)(1 - \gamma)(1 - W) \frac{1 + m}{2} - (1 - q)(1 - \gamma) \frac{1 - m}{2} \right]$$

and by choosing her most preferred allocation and losing re-election her payoff is

$$\phi - \gamma(1 - W) - (1 - \gamma) + \delta [-q\gamma(1 - W) - q(1 - \gamma) - (1 - q)(1 - \gamma)(1 - W)m - (1 - q)(1 - \gamma)(1 - m)]$$

So the majority type is optimizing if and only if

$$w'(2\gamma - 1) \leq \frac{\delta}{2} [\phi + (1 - q)(1 - m)(1 - \gamma)W].$$

Next we note that if the minority type is optimizing, so is the majority type. To see this note that

$$\frac{\delta}{2} [\phi + (1 - q)m(1 - \gamma)W] \leq \frac{\delta}{2(2\gamma - 1)} [\phi + (1 - q)(1 - m)(1 - \gamma)W],$$

is equivalent to

$$\phi \geq \frac{1-q}{2}[2\gamma m - 1],$$

which holds whenever  $\phi > \bar{\phi}$ .

Hence, we have that it is always possible to support a pooling equilibrium with when  $W \leq 1$  and that this pooling equilibrium can involve any level of  $w^B \in [0, W]$  no higher than

$$\frac{\delta}{2}[\phi + (1-q)m(1-\gamma)W].$$

Now we consider the case in which  $W > 1$ . In this case they must be choosing  $w^B = w' \in [W-1, 1]$  and so we must have  $w^B > 0$ . We consider first the incentives faced by the minority type. Now, in state  $\theta_0 = 1$  the utility to the minority type to choosing  $w^A = 1, w^B = 0$ , her most preferred policy is

$$\phi + \delta[-q(W-1)(1-\gamma)m - (1-q)m(1-\gamma)],$$

and the utility from  $w^B = w' \in [W-1, 1]$  and  $w^A = W - w'$  is

$$\phi - (1+w'-W)\gamma - (1-\gamma)w' + \delta[\frac{\phi}{2} - q(W-1)(1-\gamma)\frac{m}{2} - (1-q)(1-\gamma)\frac{m}{2}].$$

Hence, we have that the minority type is optimizing if and only if

$$w' - (W-1)\gamma \leq \frac{\delta}{2}[\phi + m(1-\gamma)(1+(W-2)q)].$$

Note that this is most easily satisfied when  $w' = (W-1)$ , in which case it reduces to

$$(W-1)(1-\gamma) \leq \frac{\delta}{2}[\phi + m(1-\gamma)(1+(W-2)q)].$$

Note further that, when this holds, the minority type has an incentive to follow the specified strategy if and only if

$$w' \leq (W-1)\gamma + \frac{\delta}{2}[\phi + m(1-\gamma)(1+(W-2)q)].$$

Now we consider the majority type. If she allocates effort as specified then her payoff is

$$\phi - \gamma(1+w'-W) - (1-\gamma)(1-w') + \delta[\frac{\phi}{2} - q\frac{1+m}{2}(1-\gamma)(2-W) - \frac{1-m}{2}(1-\gamma)],$$

and, if she allocates  $w^A = 1$  in the first period, her payoff is

$$\phi - (1-\gamma)(2-W) + \delta[-qm(1-\gamma)(2-W) - (1-m)(1-\gamma)].$$

So we have that the majority type is optimizing if and only if

$$(2\gamma - 1)(1 + w' - W) \leq \frac{\delta}{2}[\phi + (1 - m)(1 - \gamma)(1 + q(2 - W))].$$

Finally, we conclude that this is satisfied whenever

$$w' - (W - 1)\gamma \leq \frac{\delta}{2}[\phi + m(1 - \gamma)(1 + (W - 2)q)].$$

To see this, note that because  $\phi > \bar{\phi}$ ,

$$\frac{w' - (W - 1)}{\phi + m(1 - \gamma)(1 + (W - 2)q)} > \frac{(2\gamma - 1)(1 + w' - W)}{\phi + (1 - m)(1 - \gamma)(1 + q(2 - W))}.$$

and so

$$\frac{w' - (W - 1)\gamma}{\phi + m(1 - \gamma)(1 + (W - 2)q)} > \frac{(2\gamma - 1)(1 + w' - W)}{\phi + (1 - m)(1 - \gamma)(1 + q(2 - W))}.$$

We can then conclude that there exists an equilibrium whenever

$$(W - 1)(1 - \gamma) \leq \frac{\delta}{2}[\phi + m(1 - \gamma)(1 + (W - 2)q)].$$

□

*Proof of Lemma 5* We begin by showing that in any equilibrium which is not Pooling or Separating at least one type must be randomizing, there must be one and only one allocation which is chosen with positive probability, and both types randomize over, at most, two allocations. First note that at least one type must randomize: if both were following pure strategies they would either choose the same allocation (pooling) or different (separating). So now we show that we cannot have both types choosing the same two different allocations each with positive probability. We do this by contradiction. Suppose there are two allocations  $w^B = w', w^A = W - w'$  and  $w^B = w'', w^A = W - w''$ . Suppose WLOG that  $w' > w''$ , and let  $p'$  and  $p''$  be the probability of being re-elected after choosing allocations  $w^B = w', w^A = W - w'$  and  $w^B = w'', w^A = W - w''$  respectively. There are two cases to consider:  $W \leq 1$  and  $W > 1$ .

Consider first the case in which  $W \leq 1$ . In order for the minority type to be indifferent between  $w^B = w', w^A = W - w'$  and  $w^B = w'', w^A = W - w''$  it must be that

$$w' - w'' = \delta(p' - p'')[\phi + (1 - \gamma)m(1 - q)W],$$

and for the majority type to be willing to randomize it must be that

$$(w' - w'')(2\gamma - 1) = \delta(p' - p'')[\phi + (1 - \gamma)(1 - m)(1 - q)W].$$



Equation the expressions for  $\frac{\delta(p'-p'')}{w'-w''}$  this requires that

$$\frac{1}{\phi + (1 - \gamma)m(1 - q)W} = \frac{2\gamma - 1}{\phi + (1 - \gamma)(1 - m)(1 - q)W},$$

or equivalently that

$$\phi = \frac{1 - q}{2}[2\gamma m - 1],$$

which cannot hold when  $\phi > \bar{\phi}$ .

Now consider the case in which  $W > 1$ . In order for the minority type to be indifferent between  $w^B = w', w^A = W - w'$  and  $w^B = w'', w^A = W - w''$  it must be that

$$w' - w'' = \delta(p' - p'')[\phi + (1 - \gamma)m(1 + q(W - 2))],$$

and for the majority type to be willing to randomize it must be that

$$(w' - w'')(2\gamma - 1) = \delta(p' - p'')[\phi + (1 - \gamma)(1 - m)(1 + q(W - 2))].$$

Equation the expressions for  $\frac{\delta(p'-p'')}{w'-w''}$  this requires that

$$\frac{1}{\phi + (1 - \gamma)m(1 + q(W - 2))} = \frac{2\gamma - 1}{\phi + (1 - \gamma)(1 - m)(1 + q(W - 2))},$$

or equivalently that

$$\phi = \frac{1 + q(W - 2)}{2}[2\gamma m - 1],$$

which cannot hold when  $\phi > \bar{\phi}$ . We can then conclude that there cannot exist two actions which both types choose with positive probability for either the case where  $W \leq 1$  or the case where  $W > 1$ .

We have now established that we cannot have both types randomize over the same two actions. Hence there is no more than one action which both types choose with positive probability. We now note that it is not an equilibrium in which one type chooses two different actions which are not chosen by the other type. To see this, note that if only one type were to choose a certain allocation the voters would infer her type with certainty. For a minority type this would mean she would be re-elected with probability 0. Note however, that if the minority type knew she was going to be re-elected with probability 0 she would have a strict preference to choose allocation  $(\min\{W, 1\}, 0)$ , and so could not be randomizing. Now suppose the majority type were randomizing over two allocations which the minority type never chose. Note that at both those allocations the majority type the voters would infer that she were the majority type with

certainty and she would be re-elected with probability 1. Note also that we have assumed that, in equilibrium, the majority type always chooses  $w^A + w^B = W$ . As the majority type has a strict preference for effort spent on  $A$ , she cannot be indifferent between the allocations chosen in this period, and so cannot be randomizing.

Finally note that we must have one action that both types take with positive probability – if not, by the above argument, we would have a separating equilibrium. We can then conclude that any equilibrium which is neither pooling or separating must involve at least one type randomizing, that there is exactly one allocation which both types choose with positive probability, and that no type randomizes over more than two allocations.

We now look for the possible equilibria. As at least one type must be randomizing, and neither type can be randomizing over more than two alternatives there are three possibilities: (1) minority type randomizes, majority type plays a pure strategy; (2) minority type plays a pure strategy while the majority randomizes; (3) both types are randomizing. We consider each of these three cases in turn.

Consider case (1). If minority is randomizing and the majority is playing a pure strategy, then when the minority chooses the allocation the majority type never chooses, she will reveal herself to be the minority type. Hence, we have established that she must choose  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  with positive probability. Now consider the other allocation she randomizes over. By Proposition 1, we can support a separating equilibrium with any  $w^B = w' \geq w_*(\delta)$ . As the utility of the minority type is decreasing in  $w^B$  for any this means that for any  $w^B > w_*$  the minority type has a strict preference to implement  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  to implementing  $w^B$ . Next note that the as the probability that a politician who chose  $w^B = w' \leq w_*$  is the majority type is higher than for the population as a whole the probability of being re-elected after choosing  $w^B$  is  $p \in (1/2, 1]$ .

Next note that, if  $\delta < \frac{1}{2}$ , the minority type strictly prefers implementing  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  to being re-elected with probability 1 after choosing  $w^A = 1$ ,  $w^B = W - 1$ , so we cannot support an equilibrium with randomization. Further, when  $\delta \geq \delta^*$ , for any  $p \in (1/2, 1]$  the minority type would strictly prefer to implement  $w^B = \min\{W, 1\}$ ,  $w^A = W - w^B$  and be re-elected with probability  $p$  to implementing  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  and not being re-elected. Hence we cannot have a equilibrium with randomization unless  $\delta \in (\frac{1}{2}\delta, \delta^*)$ . Finally note that in order to have the minority type willing to randomize it must be that the probability of re-election for

$w^B = w' \in (\max\{W - 1, 0\}, w_*(\delta))$  satisfies

$$w' = \delta p[\phi + (1 - q)(1 - \gamma)mW]$$

if  $W \leq 1$  and

$$w' - (W - 1)\gamma = \delta p[\phi + (1 - \gamma)m(1 + (W - 2)q)]$$

so we must have

$$p = \begin{cases} \frac{w'}{\delta[\phi + (1 - q)(1 - \gamma)mW]} & \text{if } W \leq 1, \\ \frac{w' - (W - 1)\gamma}{\delta[\phi + (1 + (W - 2)q)(1 - \gamma)m]} & \text{if } W > 1, \end{cases}$$

Hence the minority type must randomize in such a way as to support this probability. We now consider case (2).

We conclude by considering case (3).  $\square$

**8.2. Proof of Uniqueness Results.** We first show that, in any equilibrium satisfying criterion D1, the majority type must always choose  $w^A + w^B = W$ .

**Lemma 6.** *There cannot exist a Perfect Bayesian Equilibrium satisfying criterion D1 in which the majority type chooses any allocation  $w^A, w^B$  satisfying  $w^A + w^B < W$  with positive probability in state  $\theta_0 = 1$ .*

*Proof.* Suppose there exists an allocation  $(w_*^A, w_*^B)$  chosen with positive probability on the equilibrium path, in state  $\theta_0 = 1$ , by the majority type, and suppose  $w_*^A + w_*^B < W$ . Let  $p^* \in [0, 1]$  be the probability with which the politician is re-elected after choosing  $w_*^A, w_*^B$ . Now consider a different allocation  $(w', w'')$  with  $w' \geq w_*^A$ ,  $w'' \geq w_*^B$ , and at least one of the inequalities strict. Now define  $u^x(w^A, w^B, p)$  to be the utilities to the politicians of each type,  $x \in \{0, 1\}$ , from implementing a given policy  $(w^A, w^B)$  in state  $\theta_0 = 1$ , if the probability of re-election after choosing policy  $(w^A, w^B)$  is  $p$ . Now define

$$p_1 = \inf\{p' : u^1(w', w'', p') > u^1(w_*^A, w_*^B, p^*)\}$$

and

$$p_2 = \min\{p' : u^0(w', w'', p') \geq u^0(w_*^A, w_*^B, p^*)\}$$

Then  $p_1$  defines the probability of re-election for which the majority type would have a strict incentive to choose  $(w', w'')$  if  $p > p_1$ . Similarly  $p_2$  defines the minimum probability of re-election for which the minority type would have a weak incentive to choose  $(w', w'')$ .

We now show that, in any equilibrium, we must have  $p_1 < \min\{1, p_2\}$ . First, note that  $u^1(w', w'', p') > u^1(w_*^A, w_*^B, p^*)$  if and only if

$$(1 - \gamma)(w' - w_*^A) + \gamma(w'' - w_*^B) > \delta(p^* - p')[\phi + (1 - q)(1 - m)(1 - \gamma)W],$$

when  $W \leq 1$ , and if and only if

$$(1 - \gamma)(w' - w_*^A) + \gamma(w'' - w_*^B) > \delta(p^* - p')[\phi + (1 - m)(1 - \gamma)(1 + q(W - 2))],$$

when  $W > 1$ . Conversely,  $u^0(w', w'', p') > u^0(w_*^A, w_*^B, p^*)$  if and only if

$$(1 - \gamma)(w' - w_*^A) + \gamma(w_*^B - w') > \delta(p^* - p')[\phi + (1 - q)m(1 - \gamma)W],$$

when  $W \leq 1$ , and if and only if

$$(1 - \gamma)(w' - w_*^A) + \gamma(w_*^B - w') > \delta(p^* - p')[\phi + m(1 - \gamma)(1 + q(W - 2))],$$

when  $W > 1$ . Now since  $w' \geq w_*^A$ ,  $w'' \geq w_*^B$ , with at least one inequality strict, we can see immediately that  $(1 - \gamma)(w' - w_*^A) + \gamma(w'' - w_*^B) > 0$ , so we must have  $p_1 < p^* \leq 1$ . Similarly, because

$$\phi + (1 - q)m(1 - \gamma)W > \phi + (1 - q)(1 - m)(1 - \gamma)W,$$

$$\phi + (1 + q(W - 2))m(1 - \gamma)W > \phi + (1 + q(W - 2))(1 - m)(1 - \gamma)W,$$

and

$$(1 - \gamma)(w' - w_*^A) + \gamma(w'' - w_*^B) \geq (1 - \gamma)(w' - w_*^A) + \gamma(w_*^B - w'),$$

we must have  $p_1 < p_2$ . So we can conclude that  $p_1 < \min\{p_2, 1\}$ .

Finally, given that  $p_1 < \min\{p_2, 1\}$ , note that we cannot have an equilibrium, satisfying criterion D1, in which the majority type ever chooses  $(w_*^A, w_*^B)$ . To see this, note that  $(w', w'')$  cannot be on path: As  $p_1 < \min\{p_2, 1\}$ , if the majority type ever chooses  $(w_*^A, w_*^B)$  over  $(w', w'')$  the minority type must strictly prefer  $(w_*^A, w_*^B)$  over  $(w', w'')$  and so the minority type can never choose  $(w', w'')$ . As the voters would then assign beliefs that the politician is the majority type with certainty, she would be re-elected with probability 1, and, as  $p_1 < 1$ , the politician would have a strict incentive to choose  $(w', w'')$  over  $(w_*^A, w_*^B)$ . Further,  $(w', w'')$  cannot be off the equilibrium path – if it were, by criterion D1 the voters must believe the candidate is the majority type with probability 1. As the probability of re-election would then be 1, the majority type would have a strict incentive to deviate to  $(w', w'')$ .

So we can conclude that in any PBE with criterion D1 the majority type must choose  $w^A + w^B = W$  in state  $\theta_0 = 1$ .  $\square$

Next we show that, as choosing  $B$  instead of  $A$  is less costly for the majority type than the minority type, a deviation to exerting less effort on  $B$  is beneficial for a larger set of beliefs for the minority type than the majority type.

**Lemma 7.** *Consider an allocation  $w^B > 0$  and  $w^A = W - w^B$ , and suppose the probability of being re-elected after that allocation is  $p \in (0, 1]$ . Then, for any allocation  $w'$  and  $w'' < w^B$ , then one of the following must hold:*

- (1) *neither type would ever be willing to choose allocation  $(w', w'')$  for any beliefs.*
- (2) *both types would be willing to choose allocation  $(w', w'')$  for all beliefs.*
- (3) *the set of beliefs for which the minority type strictly prefers  $(w', w'')$  to  $(W - w^B, w^B)$  is a proper superset of those for which the majority type would weakly prefer  $(w', w'')$  to  $(W - w^B, w^B)$ .*

*Proof.* Consider an allocation  $w^B > 0$  and  $w^A = W - w^B$  and another allocation  $w', w''$  where  $w'' < w^B$ . We must show that, the set of beliefs for which the minority would prefer  $(w', w'')$  is either a proper superset of the beliefs for which the majority type would weakly prefer  $(w', w'')$ . Let  $p \in (0, 1]$  be the probability of being re-elected by implementing  $w^B > 0, w^A = W - w^B$ .

We prove this separately for the case in which  $W \leq 1$  and when  $W > 1$ . Consider first the case in which  $W \leq 1$ . Then the minority type would have a strict incentive to preference to implement  $(w', w'')$  if and only if the re-election probability  $p' \in [0, 1]$  is such that

$$(w' + w^B - W)\gamma + (w^B - w'')(1 - \gamma) > \delta(p - p')[\phi + (1 - \gamma)(1 - q)mW],$$

or equivalently

$$p' - p > p_0 \equiv \frac{-(w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma)}{\delta[\phi + (1 - \gamma)(1 - q)mW]}.$$

Now consider the majority type. She will have a weak incentive to prefer  $(w', w'')$  if and only if the re-election probability  $p' \in [0, 1]$  is such that

$$(w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma) \geq \delta(p - p')[\phi + (1 - \gamma)(1 - q)(1 - m)W],$$

or equivalently

$$p' - p \geq p_1 \equiv \frac{-(w' + w^B - W)\gamma + (w^B - w'')(1 - \gamma)}{\delta[\phi + (1 - \gamma)(1 - q)(1 - m)W]}.$$

We now show that  $p_0 < p_1$ . To see this, note that we can write

$$\begin{aligned} p_0 &= \frac{(W - w'' - w') - (w^B - w'')\gamma - (w^B - w'')(1 - \gamma)}{\delta[\phi + (1 - \gamma)(1 - q)mW]} \\ &= \frac{W - w'' - w'}{\delta[\phi + (1 - \gamma)(1 - q)mW]} - \frac{w^B - w''}{\delta[\phi + (1 - \gamma)(1 - q)mW]}. \end{aligned}$$

and

$$\begin{aligned} p_1 &= \frac{(W - w'' - w') - (w^B - w'')\gamma + (w^B - w'')(1 - \gamma)}{\delta[\phi + (1 - \gamma)(1 - q)(1 - m)W]} \\ &= \frac{W - w'' - w'}{\delta[\phi + (1 - \gamma)(1 - q)(1 - m)W]} - \frac{(w^B - w'')(2\gamma - 1)}{\delta[\phi + (1 - \gamma)(1 - q)(1 - m)W]}. \end{aligned}$$

So, since

$$\frac{W - w'' - w'}{\delta[\phi + (1 - \gamma)(1 - q)mW]} \leq \frac{W - w'' - w'}{\delta[\phi + (1 - \gamma)(1 - q)(1 - m)W]}$$

and  $w^B > w''$  it is sufficient to show that

$$\frac{1}{\phi + (1 - \gamma)(1 - q)mW} > \frac{(2\gamma - 1)}{\phi + (1 - \gamma)(1 - q)(1 - m)W}.$$

Cross multiplying, this holds whenever

$$\phi > \frac{1 - q}{2}(2\gamma m - 1)W,$$

which is guaranteed by the assumption that  $\phi > \bar{\phi}$ . As we have now established that  $p_0 < p_1$  we can then conclude that either the set of beliefs which give the minority type a strict preference for  $(w', w'')$  are a proper subset of those which give the minority type a weak incentive – or that  $(w', w'')$  is preferred for either all beliefs, or for no beliefs, the voters could hold.

Now consider the case in which  $W > 1$ . Then the minority type would have a strict incentive to preference for  $(w', w'')$  if and only if the re-election probability  $p' \in [0, 1]$  is such that

$$(w' + w^B - W)\gamma + (w^B - w'')(1 - \gamma) > \delta(p - p')[\phi + m(1 - \gamma)(1 + (W - 2)q)],$$

or equivalently

$$p' - p > p_0 \equiv \frac{-(w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma)}{\delta[\phi + m(1 - \gamma)(1 + (W - 2)q)]}.$$

Now consider the majority type. She will have a weak incentive to prefer  $(w', w'')$  if and only if the re-election probability  $p' \in [0, 1]$  is such that

$$(w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma) \geq \delta(p - p')[\phi + (1 - m)(1 - \gamma)(1 + (W - 2)q)],$$

or equivalently

$$p' - p \geq p_1 \equiv \frac{-(w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma)}{\delta[\phi + (1 - m)(1 - \gamma)(1 + (W - 2)q)]}.$$

We now show that  $p_0 < p_1$ , as we did for the case  $W \leq 1$ . To see that this holds, note that

$$p_0 = \frac{W - w'' - w'}{\delta[\phi + m(1 - \gamma)(1 + (W - 2)q)]} - \frac{w^B - w''}{\delta[\phi + m(1 - \gamma)(1 + (W - 2)q)]}$$

and

$$p_1 = \frac{W - w'' - w'}{\delta[\phi + (1 - m)(1 - \gamma)(1 + (W - 2)q)]} - \frac{(w^B - w'')(2\gamma - 1)}{\delta[\phi + (1 - m)(1 - \gamma)(1 + (W - 2)q)]}.$$

As

$$\frac{W - w'' - w'}{\delta[\phi + m(1 - \gamma)(1 + (W - 2)q)]} \leq \frac{W - w'' - w'}{\delta[\phi + (1 - m)(1 - \gamma)(1 + (W - 2)q)]}$$

and  $w^B > w''$  it is sufficient to show

$$\frac{2\gamma - 1}{\phi + (1 - m)(1 - \gamma)(1 + (W - 2)q)} < \frac{1}{\phi + m(1 - \gamma)(1 + (W - 2)q)}.$$

Simplifying, this holds if

$$\phi > \frac{1 + (W - 2)q}{2}(2\gamma m - 1),$$

or equivalently whenever  $\phi > \bar{\phi}$ . As we have  $p_0 < p_1$  we can then conclude that either the set of beliefs which give the minority type a strict preference for  $(w', w'')$  are a proper subset of those which give the minority type a weak incentive – or that  $(w', w'')$  is preferred for either all beliefs, or for no beliefs, the voters could hold.  $\square$

The above lemma will be useful for determining the beliefs after observing that the politician chose a level of  $w^B$  lower than what is on the equilibrium path. Combining the above lemma with criterion D1, except for deviations which no politicians would make regardless of the induced beliefs, the voters would have to believe such a deviation was made by the minority type.

We now consider the beliefs after observing the politician allocate more weight to  $B$  than expected. As the majority politician would pay a smaller policy cost than a minority politician for increasing  $w^B$ , they would be willing to make such an action for a less-restricted set of beliefs.

The following lemma then shows that, if the highest equilibrium level of effort is such that the politician is not re-elected with certainty, then in any equilibrium consistent with D1, in order for the majority type not to have an incentive to deviate, it must be that the majority type is allocating maximal effort to  $B$ .

**Lemma 8.** *In any equilibrium satisfying criterion D1 in which the majority type is re-elected with probability  $p < 1$ ,  $w^B = \min\{W, 1\}$  and  $w^A = W - w^B$  when  $\theta_0 = 1$ .*

*Proof.* We show, by contradiction, that there cannot exist a Perfect Bayesian Equilibrium satisfying criterion D1 in which the majority type ever chooses an allocation  $w^B < \min\{W, 1\}$  and is re-elected with probability less than 1 after taking that action. We prove this result by considering the case  $W \leq 1$  and  $W > 1$ .

Suppose the majority type chooses allocation  $w^B < \min\{W, 1\}$ ,  $w^A = W - w^B$  with positive probability, and suppose the probability of re-election after choosing that action is  $p \in [0, 1)$ . Now consider the allocation  $(W - w', w')$ , with  $w' > w^B$ . Note that, by Proposition 3, if  $(W - w', w')$  is on the equilibrium path, the voters must believe the politician is the majority type with certainty, and so the probability of re-election is 1.

Consider the case in which  $W \leq 1$ . The majority type would have a strict incentive to deviate to  $(W - w', w')$  if and only if the probability of re-election,  $p'$  is such that

$$(w' - w^B)(2\gamma - 1) < \delta(p' - p)[\phi + (1 - q)(1 - \gamma)(1 - m)W],$$

or if

$$p' - p > p_1 \equiv \frac{(w' - w^B)(2\gamma - 1)}{\delta[\phi + (1 - q)(1 - \gamma)(1 - m)W]}.$$

Similarly, the minority type has a weak incentive to deviate if and only if

$$w' - w^B < \delta(p' - p)[\phi + (1 - q)(1 - \gamma)mW],$$

which is equivalent to

$$p' - p \geq p_0 \equiv \frac{w' - w^B}{\delta[\phi + (1 - q)(1 - \gamma)mW]}.$$

Now note that  $p_1 < p_0$ . This follows because

$$\frac{(w' - w^B)(2\gamma - 1)}{\delta[\phi + (1 - q)(1 - \gamma)(1 - m)W]} < \frac{w' - w^B}{\delta[\phi + (1 - q)(1 - \gamma)mW]},$$



if and only if

$$\phi > \frac{1-q}{2}(2\gamma m - 1),$$

which holds whenever  $\phi > \bar{\phi}$ . Finally, note that, as  $w' \rightarrow w^B$ ,  $p_1 \rightarrow 0$ . Hence there exist  $w' > w^B$  such that the majority type would have a strict incentive to choose  $(W - w', w')$  even if they would be re-elected with probability less than 1. Note that, by Proposition 3, since the minority type can never be choosing  $w'$ , if the majority type chooses it with positive probability in equilibrium, the voters must assign probability 1 to them being the majority type, giving the majority politician a strict incentive not to choose  $w^B$ , breaking the purported equilibrium. Further, the set of beliefs under which the majority type would have a strict incentive to deviate are a proper superset of those for which the minority type would have a weak incentive to deviate. Hence, by criterion D1, if  $(W - w', w')$  the voters must assign probability 1 to any politician who chose  $(W - w', w')$  being the majority type, and such a politician would be re-elected with certainty. We have verified, however, the the majority type would then have a strict incentive to deviate. We can therefore conclude that there cannot exist a Perfect Bayesian Equilibrium in which the majority type chooses an allocation with  $w^B < \min\{W, 1\}$  and gets re-elected with probability less than 1 when  $W \leq 1$ .

We now turn to the case where  $W > 1$ . The majority type would have a strict incentive to deviate to  $(W - w', w')$  if and only if the probability of re-election,  $p'$  is such that

$$(w' - w^B)(2\gamma - 1) < \delta(p' - p)[\phi + (1 - \gamma)(1 - m)(1 + (W - 2)q)],$$

or if

$$p' - p > p_1 \equiv \frac{(w' - w^B)(2\gamma - 1)}{\delta[\phi + (1 - \gamma)(1 - m)(1 + (W - 2)q)]}.$$

Similarly, the minority type has a weak incentive to deviate if and only if

$$w' - w^B \leq \delta(p' - p)[\phi + m(1 - \gamma)(1 + (W - 2)q)],$$

which is equivalent to

$$p' - p \geq p_0 \equiv \frac{w' - w^B}{\delta[\phi + m(1 - \gamma)(1 + (W - 2)q)]}.$$

Now note that  $p_1 < p_0$ . This follows because

$$\frac{(w' - w^B)(2\gamma - 1)}{\delta[\phi + (1 - \gamma)(1 - m)(1 + (W - 2)q)]} < \frac{w' - w^B}{\delta[\phi + (1 - \gamma)m(1 + (W - 2)q)]}$$

if and only if

$$\frac{2\gamma - 1}{\phi + (1 - \gamma)(1 - m)(1 + (W - 2)q)} < \frac{1}{\phi + (1 - \gamma)m(1 + (W - 2)q)}$$

if and only if

$$\phi > \frac{1 + (W - 2)q}{2} [(2\gamma - 1)m - (1 - m)] = \frac{1 + (W - 2)q}{2} [2\gamma m - 1],$$

which holds whenever  $\phi > \bar{\phi}$ . Finally, note that, as  $w' \rightarrow w^B$ ,  $p_1 \rightarrow 0$ . Hence there exist  $w' > w^B$  such that the majority type would have a strict incentive to choose  $(W - w', w')$  even if they would be re-elected with probability less than 1. Note that, by Proposition 3, since the minority type can never be choosing  $w'$ , if the majority type chooses it with positive probability in equilibrium, the voters must assign probability 1 to them being the majority type, giving the majority politician a strict incentive not to choose  $w^B$ , breaking the purported equilibrium. Further, the set of beliefs under which the majority type would have a strict incentive to deviate are a proper superset of those for which the minority type would have a weak incentive to deviate. Hence, by criterion D1, if  $(W - w', w')$  the voters must assign probability 1 to any politician who chose  $(W - w', w')$  being the majority type, and such a politician would be re-elected with certainty. We have verified, however, that the majority type would then have a strict incentive to deviate. We can therefore conclude that there cannot exist a Perfect Bayesian Equilibrium in which the majority type chooses an allocation with  $w^B < \min\{W, 1\}$  and gets re-elected with probability less than 1 when  $W > 1$ .  $\square$

With these two lemmas we can now determine when a separating, pooling, and partial-pooling equilibrium consistent with D1 will exist. We begin by considering separating equilibria, and show that only the minimally separating equilibrium is consistent with criterion D1 on off-path beliefs.

**Lemma 9.** *If*

$$\delta \leq \bar{\delta} \equiv \begin{cases} \frac{W}{\phi + (1 - \gamma)(1 - q)mW} & \text{if } W \leq 1, \\ \frac{(1 - (W - 1)\gamma)}{\phi + m(1 - \gamma)(1 + (W - 2)q)} & \text{if } W > 1. \end{cases}$$

there exists a unique *Separating Equilibrium* which satisfies the *Intuitive Criterion*. It involves the minority choosing  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  and the majority type choosing

$$w^B = w_*(\delta) = \begin{cases} \delta(\phi + (1 - \gamma)(1 - q)mW) & \text{if } W \leq 1, \\ (W - 1)\gamma + \delta(\phi + m(1 - \gamma)(1 + (W - 2)q)) & \text{if } W > 1. \end{cases}$$

and  $w^A = W - w^B$ . The equilibrium behavior can be supported by beliefs which satisfy criterion D1.

*Proof.* We begin by showing that the equilibrium described can be supported by off-path beliefs which satisfy criterion D1. As D1 is a more restrictive requirement on the off-path beliefs than the Intuitive Criterion this would then show that the equilibrium satisfies the Intuitive Criterion as well. First note that, since the politician is re-elected with certainty after allocating  $w^B = w_*(\delta)$ ,  $w^A = W - w_*(\delta)$ , and all politicians strictly prefer to implement  $w^B = w_*(\delta)$ ,  $w^A = W - w_*(\delta)$  to any allocation with  $w^B > w_*(\delta)$ , so all allocations with  $w^B > w_*$  are equilibrium dominated. The beliefs after such allocations are then irrelevant for no politician would choose such an allocation for any beliefs. Now consider the beliefs after  $w^B = w' \in (0, w_*(\delta))$ . Note that by construction the minority type is indifferent between  $w^B = 0$  and  $w^B = w_*(\delta)$  in the initial period. Hence, by Lemma 4, the set of beliefs for which the majority type would have a weak incentive to deviate to  $w^B = w'$  are a proper subset of those for which the minority type would have a strict incentive to deviate. As we have established in Proposition that it is possible to support the specified strategies as an equilibrium with appropriate beliefs, and the beliefs determined by D1 are the most punitive possible for a deviation, we have that the above strategies constitute a Perfect Bayesian Equilibrium which satisfies criterion D1.

Having now established that this equilibrium satisfies criterion D1 on off-path beliefs, we now turn to showing that no other separating equilibrium satisfies the Intuitive Criterion, and so doesn't satisfy D1.

Consider a Perfect Bayesian Equilibrium in which  $w^B > w_*(\delta)$ . Now consider the effort allocation  $w' \in (w_*(\delta), w^B)$ ,  $w^A = W - w'$ . We show that such an allocation is equilibrium dominated for the minority type, but not the majority type. Consider first the minority type. Since we can support a separating equilibrium with  $w^B = w_*$  and  $w_A = W - w_*$ , the minority candidate cannot strictly prefer to  $w^B = w_*$ ,  $w_A = W - w_*$ , even if she knows that she would then be re-elected in the next period. Further, as the politician strictly prefers the allocation

$w^B = w_*$ ,  $w_A = W - w_*$  to  $w^B = w'$ ,  $w_A = W - w'$ , she would then have a strict incentive not to choose  $w^B = w'$ ,  $w_A = W - w'$  for any voter beliefs. So the allocation  $w^B = w'$ ,  $w_A = W - w'$  is equilibrium dominated for the minority type. Now consider the majority type. Note first that the politician prefers allocation  $w', w_A = W - w'$  to  $w^B, W - w^B$  at time 0, so if the beliefs were such that she would be re-elected with probability 1 by choosing  $w', w_A = W - w'$  she would have an incentive to choose that allocation. Therefore,  $w', w_A = W - w'$  is equilibrium dominated for the minority type, but not the majority type, and so, by the Intuitive Criterion, the voters must believe that any candidate who took that action was the majority type with certainty.

Finally note that, since the Intuitive Criterion guarantees that the voters must believe a politician who chose allocation  $w' \in (w_*(\delta), w^B)$ ,  $w^A = W - w'$  was the majority type with certainty. Hence the probability of re-election is the same as from choosing  $w^B$  and  $w^A = W - w^B$ . As the politician receives greater utility in the first period by increasing  $w^A$  and decreasing  $w^B$ , she would not be optimizing by choosing  $w^B = w'$ . We can then conclude that it is not possible to support a separating equilibrium with  $w^B > w_*(\delta)$  in which the off-path beliefs satisfy D1.  $\square$

This then says that the only separating equilibrium to satisfy the Intuitive Criterion is the minimally separating one: the lowest weight the majority type could place on  $B$  to separate themselves. We now consider the off-path beliefs supporting a pooling equilibrium.

**Lemma 10.** *There exists a pooling equilibrium which satisfies Criterion D1 on off-path beliefs if and only if*

$$\delta \geq \delta^* \equiv \begin{cases} \frac{2W}{\phi + (1-\gamma)(1-q)mW} & \text{if } W \leq 1, \\ \frac{2(1-(W-1)\gamma)}{\phi + m(1-\gamma)(1+(W-2)q)} & \text{if } W > 1. \end{cases}$$

*In this pooling equilibrium  $w^B = \min\{W, 1\}$  and  $w^A = W - w^B$ .*

*Proof.* By Lemma 8, since in a pooling equilibrium the politician is always re-elected with probability  $1/2$ , we cannot have a pooling equilibrium which satisfies D1 unless in that equilibrium all politicians choose  $w^B = \min\{W, 1\}$  if  $\theta_0 = 1$ . We then first determine the range of parameters for which a pooling equilibrium exists with  $w^B = \min\{W, 1\}$  if  $\theta_0 = 1$ .

Finally we show that when  $\delta < \delta^*$  there cannot exist a pooling equilibrium with  $w^B = \min\{W, 1\}$ . This follows because by Proposition 2 the maximum  $w^B$  which can be supported is

$$\bar{w} = \begin{cases} \min\{\frac{\delta}{2}[\phi + (1-q)m(1-\gamma)W], W\} & \text{if } W \leq 1, \\ \min\{(W-1)\gamma + \frac{\delta}{2}[\phi + m(1-\gamma)(1+(W-2)q)], 1\} & \text{if } W > 1. \end{cases}$$

Further note that when  $W \leq 1$ ,

$$\frac{\delta^*}{2}[\phi + (1-q)m(1-\gamma)W] = \frac{2W}{2(\phi + (1-q)m(1-\gamma)W)} = W.$$

Also, as  $\frac{\delta}{2}[\phi + (1-q)m(1-\gamma)W]$  is increasing in  $\delta$ , so when  $\delta < \delta^*$ ,  $\frac{\delta}{2}[\phi + (1-q)m(1-\gamma)W] < W$ . Similarly, when  $W > 1$ ,

$$\begin{aligned} (W-1)\gamma + \frac{\delta^*}{2}[\phi + m(1-\gamma)(1+(W-2)q)] &= \\ (W-1)\gamma + \frac{(1-(W-1)\gamma)}{\phi + m(1-\gamma)(1+(W-2)q)}[\phi + m(1-\gamma)(1+(W-2)q)] &= 1. \end{aligned}$$

As before, since,  $(W-1)\gamma + \frac{\delta}{2}[\phi + m(1-\gamma)(1+(W-2)q)]$  is increasing in  $\delta$  it is less than 1 when  $\delta < \delta^*$ .

Hence, when  $\delta < \delta^*$ ,  $\bar{w}$  is less than  $\min\{W, 1\}$ . We can therefore conclude that there cannot exist a Perfect Bayesian Equilibrium satisfying D1 when  $\delta < \delta^*$ .

Having now established that there exists a pooling equilibrium with  $w^B = \min\{W, 1\}$  if and only if  $\delta \geq \delta^*$ , we now show that when  $\delta \geq \delta^*$  the beliefs supporting the equilibrium satisfy criterion D1. Now, by Lemma 4, the range of beliefs for which the minority type would have a strict incentive to choose any  $w^B = w' < \min\{W, 1\}$  are a proper superset of those for which the majority type would have a weak incentive to choose that allocation. Hence, in order to be consistent with criterion D1 the voters must believe any  $w^B < \min\{W, 1\}$  was chosen by a minority type, and so the politician would never be re-elected. As these are the most punitive beliefs the voters could hold after a deviation, the beliefs determined by criterion D1 are sufficient to support an equilibrium.

We can then conclude that, when  $\delta \geq \delta^*$ , there exists a unique pooling equilibrium consistent with criterion D1 involves all politicians choosing  $w^B = \min\{W, 1\}$  in state  $\theta_0 = 1$ . Further, when  $\delta < \delta^*$  we cannot have a pooling equilibrium consistent with criterion D1.  $\square$

So we have that when  $\delta < \bar{\delta}$  there exists a unique separating equilibrium but no pooling equilibrium which satisfies criterion D1. When  $\delta > \delta^*$  there exists a unique pooling equilibrium, but no separating equilibrium. Note also that  $\delta^* = 2\bar{\delta} > \bar{\delta}$  so for  $\delta \in (\bar{\delta}, \delta^*)$  neither a separating or pooling equilibrium would exist which is consistent with D1. We now explore the possibility of a semi-separating equilibrium. For this range, there exists a unique semi-separating equilibrium in which the minority-type randomizes so that the politician is re-elected with probability between 1/2 and 1 after choosing the posturing allocation.

**Lemma 11.** *For all  $\epsilon \in (0, \bar{\epsilon})$ , there exists a Semi-Separating Equilibrium in which the majority type chooses  $w^A + w^B = W$  in state  $\theta_0 = 1$ , and which satisfies criterion D1 if and only if  $\delta \in (0, \min\{\delta^*, 1\})$ . In this equilibrium,*

- (1) *if  $\delta \in (\frac{1}{2}\bar{\delta}, \min\{\bar{\delta}, 1\})$  the majority type chooses  $w^B = w_*(\delta)$ ,  $w^A = W - w^B$  and the minority randomizes between  $w^A = \min\{W, 1\}$ ,  $w^B = 0$  and  $w^B = w_*(\delta)$ ,  $w^A = W - w^B$ . The level of randomization is such that*

$$Pr(\text{Re-elected} | w^B = w_*(\delta), w^A = W - w^B) = 1.$$

- (2) *if  $\delta \in (\min\{\bar{\delta}, 1\}, \min\{\delta^*, 1\})$  the majority type chooses  $w^B = \min\{W, 1\}$ ,  $w^A = W - w^B$  and the minority type randomizes with a non-degenerate probability between  $w^B = \min\{W, 1\}$ ,  $w^A = W - w^B$  and  $w^A = \min\{W, 1\}$ ,  $w^B = 0$ . The level of randomization is such that*

$$Pr(\text{Re-elected} | w^B = \min\{W, 1\}, w^A = W - w^B) = \bar{\delta}/\delta.$$

*Proof.* By Lemma 8 we know that the equilibrium must either involve all majority types choosing  $w^B = \min\{W, 1\}$  or have the majority type re-elected with probability 1. Note immediately that this implies that the behavior of the majority type must be of the form in case (1) of Proposition 3, in which the minority type randomizes and the high-type chooses a pure strategy.

We first look for an equilibrium in which  $w^B < \min\{W, 1\}$ . By Lemma 5 we know that we must have the majority type re-elected with probability 1. Further, we have established in Proposition 3 that such an equilibrium can be supported if and only if  $\delta \in (\frac{1}{2}\bar{\delta}, \bar{\delta})$ . For the probability of re-election to be 0 it must be that the minority type chooses  $w^B, W - w^B$  with small enough probability that  $\mu(1|w^B, W - w^B)$  is sufficiently high that the probability of re-election at that

information set is 1. In order for the minority type not to have a strict incentive to choose  $w^B$  then it must be that  $w^B = w_*(\delta)$ .

Now we consider equilibria in which  $w^B = \min\{W, 1\}$ . The majority type would then choose  $w^B = \min\{W, 1\}$ ,  $w^A = W - w^B$ , and the minority type would randomize between  $w^B = \min\{W, 1\}$ ,  $w^A = W - w^B$  and  $w^A = \min\{W, 1\}$ ,  $w^B = 0$ . We need only check that the minority type is indifferent between the two allocations. We now need to determine under which conditions it is possible for the probability of being re-elected,  $p$  makes the minority type indifferent. Note that, as the probability with which the politician is the majority type is at least as high as the probability for the population as a whole, so we need to look for equilibria with  $p \in (\frac{1}{2}, 1]$ . We first show that we must have  $p = \bar{\delta}/\delta$ . First consider the case in which  $W \leq 1$ . Then for the minority type to be indifferent it must be that

$$W = \delta p[\phi + (1 - q)(1 - \gamma)mW],$$

so

$$p = \frac{W}{\delta[\phi + (1 - q)(1 - \gamma)mW]}.$$

Next note that when  $W \leq 1$ ,

$$\bar{\delta} = \frac{W}{\phi + (1 - \gamma)(1 - q)mW},$$

so

$$p = \bar{\delta}/\delta.$$

Similarly, when  $W > 1$  it must be that

$$(1 - \gamma) + \gamma(2 - W) = \delta p[\phi + (1 - \gamma)m(1 + (W - 2)q)],$$

so

$$p = \frac{1 - (W - 1)\gamma}{\delta[\phi + (1 - \gamma)m(1 + (W - 2)q)]}.$$

So, since

$$\bar{\delta} = \frac{1 - (W - 1)\gamma}{\delta[\phi + (1 - \gamma)m(1 + (W - 2)q)]},$$

$$p = \bar{\delta}/\delta.$$

Note finally, that as  $p = \frac{\bar{\delta}}{\delta}$  and we need  $p \in (1/2, 1]$ , this is possible if and only if

$$\delta \in [\bar{\delta}, 2\bar{\delta}) = [\bar{\delta}, \delta^*)$$

□

We have now established that, for  $\delta < \bar{\delta}$  the only equilibrium to satisfy criterion D1 is the minimally separating equilibrium. When  $\delta > \delta^*$  the unique equilibrium to satisfy D1 is the pooling equilibrium. And when  $\delta \in (\bar{\delta}, \delta^*)$  the unique equilibrium is semi-separating. Combining these lemmas then proves Proposition 1.

*Proof of Proposition 1* From Lemmas 9 – 11 we have a sincere equilibrium if and only if  $\delta < \bar{\delta}$ , and posturing equilibrium if and only if  $\delta > \delta^*$ , and a partial-pooling equilibrium if and only if  $\delta \in (\bar{\delta}, \delta^*)$ . Now since

$$\bar{\delta} \equiv \begin{cases} \frac{W}{\phi + (1-\gamma)(1-q)mW} & \text{if } W \leq 1, \\ \frac{1-(W-1)\gamma}{\phi + m(1-\gamma)(1+(W-2)q)} & \text{if } W > 1, \end{cases}$$

and

$$\delta^* \equiv \begin{cases} \frac{2W}{\phi + (1-\gamma)(1-q)mW} & \text{if } W \leq 1, \\ \frac{2(1-(W-1)\gamma)}{\phi + m(1-\gamma)(1+(W-2)q)} & \text{if } W > 1, \end{cases}$$

we have that there exists a sincere equilibrium if and only if

$$\phi < \bar{\phi} \equiv \begin{cases} \frac{W}{\delta} - (1-\gamma)(1-q)mW & \text{if } W \leq 1, \\ \frac{1-(W-1)\gamma}{\delta} - m(1-\gamma)(1+(W-2)q) & \text{if } W > 1, \end{cases}$$

a posturing equilibrium if and only if

$$\phi > \phi^* \equiv \begin{cases} \frac{2W}{\delta} - (1-\gamma)(1-q)mW & \text{if } W \leq 1, \\ \frac{2(1-(W-1)\gamma)}{\delta} - m(1-\gamma)(1+(W-2)q) & \text{if } W > 1, \end{cases}$$

and a partial-pooling equilibrium if and only if  $\phi \in (\bar{\phi}, \phi^*)$ . Finally note that as  $\delta, m, q, \gamma \in (0, 1)$  and  $W < 2$  that this implies that

$$0 < \bar{\phi} < \phi^*.$$

□

*Proof of Proposition 2* Immediate from inspection of

$$\bar{\phi} \equiv \begin{cases} \frac{W}{\delta} - (1-\gamma)(1-q)mW & \text{if } W \leq 1, \\ \frac{1-(W-1)\gamma}{\delta} - m(1-\gamma)(1+(W-2)q) & \text{if } W > 1, \end{cases}$$

and

$$\phi^* \equiv \begin{cases} \frac{2W}{\delta} - (1-\gamma)(1-q)mW & \text{if } W \leq 1, \\ \frac{2(1-(W-1)\gamma)}{\delta} - m(1-\gamma)(1+(W-2)q) & \text{if } W > 1. \end{cases}$$



□

**8.3. Proof of Results with Non-Transparency.** *Proof of Proposition 3:* Focusing first on the minority type, she would never prefer to shift effort from  $A$  to  $B$  (even if  $p^B = 1$  would guarantee reelection) if

$$(9) \quad \phi \leq \phi_0^{AB} \equiv \frac{1}{\delta} - (1-q)(1-\gamma)mW.$$

There is no profitable deviation for a minority type, because even the other option, i.e., doing nothing, is dominated: the condition under which the minority type does not want to switch from doing  $A$  to doing nothing is

$$(10) \quad \phi \leq \phi_0^{A0} \equiv \frac{2\gamma}{\delta} - (1-q)(1-\gamma)mW > \phi_0^{AB}.$$

On the other hand, the condition under which the majority type does not want to shift effort from  $A$  to  $B$  is

$$(11) \quad \phi \leq \phi_1^{BA} \equiv \frac{2\gamma-1}{\delta} - (1-q)(1-\gamma)(1-m)W.$$

For every  $\phi \leq \min\{\phi_0^{AB}, \phi_1^{BA}\}$  it is clear that pooling on  $A$  is the unique equilibrium. For any value of  $\gamma$ ,  $\phi_1^{BA}$  is increasing in  $m$ , whereas  $\phi_0^{AB}$  is decreasing in  $m$ . These bounds are equal when  $m = \frac{1}{2} + \frac{1}{\delta(1-q)W}$ . Thus, if  $\frac{1}{2\gamma} \leq \frac{1}{2} + \frac{1}{\delta(1-q)W}$ , i.e., if

$$(12) \quad \gamma \geq \frac{\delta(1-q)W}{2 + \delta(1-q)W},$$

then  $\phi_1^{BA} < \phi_0^{AB}$  for every set of parameter values that satisfy assumption 1. Given that  $\gamma > 1/2$  by assumption and  $\frac{1}{2} > \frac{\delta(1-q)W}{2 + \delta(1-q)W}$  because  $\delta(1-q)W < 1$ , (??) always holds. Hence  $\phi_1^{BA}$  is the only relevant upper bound on  $\phi$  to sustain the pooling on  $A$  equilibrium, and this is strictly positive when (??) holds. □

*Proof of Proposition 5:*

Let us first see that the minority type would choose the same effort allocation as under transparency. With transparency, the IC constraint of the minority type necessary to support the separating equilibrium is

$$\phi \leq \frac{(1-\gamma)(W-1)}{\delta} - m(1-\gamma)(1+qW-2q),$$

whereas with non-transparency the IC constraint of the minority type is

$$\phi \leq \frac{1 - \gamma}{\delta} - m(1 - \gamma)(1 + qW - 2q),$$

so it is easier to prevent the minority type from mimicking the majority type.

Now consider the behavior of the majority type. Note that the majority type may have incentive to exert more effort on  $B$  (i.e.,  $w^B = 1$ ) because if  $w^B < 1$  then the benefit from reelection is gone with probability  $1 - w^B$ . If the majority type chooses  $w^B = W - 1 + w$  and  $w^A = 1 - w$  then the cost in the first period is  $w(2\gamma - 1)$ . Now, note that by choosing  $w = 0$ , her probability of re-election is  $W - 1$ . whereas she will now be re-elected if and only if it is not the case that  $A$  happens and  $B$  doesn't. As  $A$  happens with probability  $1 - w$  and  $B$  doesn't happen with probability  $2 - W - w$  this is then

$$1 - (1 - w)(2 - W - w).$$

So by choosing  $w^B = W - 1 + w$  the change in probability of re-election is  $w(3 - W - w)$ .

Hence, she has an incentive to choose  $w = 0$  if and only if

$$w(2\gamma - 1) \geq w(3 - W - w)\delta[\phi + (1 - m)(1 - \gamma)(1 + (W - 2)q)]$$

for all  $w \in [0, W - 1]$ . Note that as the left hand side is linear in  $w$ , but the right hand side is concave, this is true if and only if it hold in the limit as  $w$  approaches 0, or equivalently if

$$\delta \leq \frac{2\gamma - 1}{(3 - W)[\phi + (1 - m)(1 - \gamma)(1 + (W - 2)q)]}.$$

So the majority type has no incentive to deviate if

$$\phi \leq \frac{2\gamma - 1}{(3 - W)\delta} - (1 - m)(1 - \gamma)(1 + (W - 2)q).$$

Defining  $\phi' = \min\{\frac{(1-\gamma)(W-1)}{\delta} - m(1-\gamma)(1+qW-2q), \frac{2\gamma-1}{(3-W)\delta} - (1-m)(1-\gamma)(1+(W-2)q)\}$ , and noting that because  $\delta < \min\{\frac{m(1+qW-2q)}{W-1}, \frac{2\gamma-1}{(3-W)[(1-m)(1-\gamma)(1+(W-2)q)]}\}$ ,  $\phi' > 0$ , we are done.

□

*Proof of Proposition 6:* In order to have the minority type indiff between  $B$  and 0 it must be that

$$1 - \gamma = (\pi_B - \pi_0)[\delta(\phi + (1 - q)(1 - \gamma)mW)]$$

where  $\pi_i$  is the prob of being reelected after outcome  $i$ . Assume that after seeing  $p^A = 1$ , the incumbent is not reelected; after seeing nothing, reelected with prob  $\pi_0$ ; after  $B$ , reelected with prob  $\pi_B$ . The above indifference condition therefore implies that

$$\pi_B - \pi_0 = \frac{1 - \gamma}{\delta[\phi + (1 - q)(1 - \gamma)mW]}.$$

Note in order to have an equilibrium we must have that neither type wants to deviate to  $A$ . Recall that the probability of re-election after  $A$  is  $\pi_A = 0$ , so for the minority type we must have

$$\pi_0 - 0 \leq \frac{\gamma}{\delta[\phi + (1 - q)(1 - \gamma)mW]}$$

Note that this implies that  $\pi_B - \pi_0 < \pi_0$ , so we have  $0 < \pi_0 < 1/2 < \pi_B < 1$ . Now if the bound for the minority type holds with equality then

$$\pi_0 = \frac{\gamma}{\delta[\phi + (1 - q)(1 - \gamma)mW]}$$

and so

$$\pi_B = \frac{1}{\delta[\phi + (1 - q)(1 - \gamma)mW]}.$$

Note that this can be satisfied with a  $\pi_0 < 1/2$  only if

$$\delta > \frac{2\gamma}{\phi + (1 - q)(1 - \gamma)mW}.$$

We now show that this equilibrium is unique for  $\delta > \frac{2\gamma}{\phi + (1 - q)(1 - \gamma)mW}$ .  $\square$