Rhetoric in Legislative Bargaining with Asymmetric Information[1]

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Abstract

We analyze a three-player legislative bargaining game over an ideological and a distributive decision. Legislators are privately informed about their their ideological intensities, i.e. the weight they place on the ideological decision relative to the weight they place on the distributive decision. Communication takes place before a proposal is offered and majority rule voting determines the outcome. We show that it is not possible for all legislators to communicate informatively. In particular, the legislator who is ideologically more distant from the proposer may not communicate informatively, but the closer legislator can communicate whether he would "compromise" or "fight" on ideology. Surprisingly, the proposer may be worse off when bargaining with two legislators (under majority rule) than with one (who has veto power), because competition between the legislators may result in less information conveyed in equilibrium. Despite separable preferences, the proposer is always better off making proposals for the two dimensions together.

JEL classification: C78, D72, D82, D83

1 Introduction

Legislative policy-making typically involves speeches and demands by legislators that may shape the proposals made by the leadership. For example, in the 2010 health care overhaul in the U.S., one version of the Senate bill included $100 million in Medicaid funding for Nebraska as well as restrictions on abortion coverage in exchange for the vote of Nebraska Senator Ben Nelson. As another example, consider the threat in 2009 by seven members of the U.S. Senate Budget Committee to withhold their support for critical legislation to raise the debt ceiling unless a commission to recommend cuts to Medicare and Social Security is approved.[1] Would these senators indeed have let the United States default on its debt, or was their demand just a bluff? More generally, what are the patterns of demands in legislative policy-making? How much information do they convey? Do they influence the nature of the proposed bills? Who gets private benefits and what kind of policies are chosen under the ultimately accepted bills?

To answer these questions, it is necessary to have a legislative bargaining model in which legislators make demands before the proposal of the bills. One approach is to assume that the demands serve as a commitment device, that is, the legislators refuse any offer that does not meet their demands.[2] While this approach offers interesting insights into some of the questions above, it relies on the strong assumption that legislators commit to their demands.[3] In this paper, we offer a different approach that allows legislators to make speeches but to which they are not committed when casting their votes. The premise of our approach is that only individual legislators know which bills they prefer to the status quo. So even if the legislators do not necessarily carry out their threats, their demands may be meaningful rhetoric in conveying private information and dispelling some uncertainty in the bargaining process.

We model rhetoric as cheap-talk messages as in Matthews (1989). In our model (1) three legislators bargain over an ideological and a distributive decision; (2) one of the legislators, called the chair, is in charge of formulating the proposal; (3) each legislator other than the chair is privately informed about his own preferences; (4) communication takes place before a proposal is offered; (5) majority rule voting determines whether the proposal is implemented.

[1] http://thehill.com/homenews/senate/67293-sens-squeeze-speaker-over-commission

[2] This is the approach taken by Morelli (1999) in a complete information framework. He does not explicitly model the proposal-making and the voting stages. As such, the commitment assumption is implicit.

[3] Politicians often make empty threats. See, for example, http://thehill.com/homenews/news/14312-gopsays-it-can-call-reids-bluffs.

Each legislator's position on a unidimensional ideological spectrum is publicly known, but his ideological intensity, i.e. the weight he places on the ideological dimension relative to the distributive dimension is his private information. As such, the chair is unsure how much private benefit she has to offer to a legislator to gain his support for a policy decision, but she can use the messages sent in the communication stage to make inferences about his ideological intensity (i.e. his type). We focus on a class of equilibrium called simple monotone equilibrium in which types who send the same message form an interval, and the proposal does not depend on the message of a legislator if he receives no private benefit. We show that in any simple monotone equilibrium: (1) At most one legislator's messages convey some information about his preferences (Proposition 4, (i)). (2) In particular, if the legislator whose position is closer to the chair's wants to move the policy in the same direction as the chair does, then it is impossible for the other legislator (i.e. the legislator whose position is further away from the chair's) to be informative (Proposition 4, (ii)). (3) Although the closer legislator may be informative, even he can convey only limited information (Proposition 5).

To establish these results, we first show if the type distributions have increasing hazard rates, then in a simple monotone equilibrium, the chair offers positive private benefit to at most one legislator. Suppose one legislator is offered positive private benefit while the other is offered none. Then the legislator who is excluded (i.e., who gets no private benefit) strictly prefers the status quo and will vote against the proposal whereas the legislator who is included (i.e., who gets positive private benefit) becomes pivotal and can guarantee a payoff at least as high as his status quo payoff. Alternatively, suppose no legislator is offered any private benefit. Then the chair's optimal proposal must make the closer legislator just willing to accept. Note that if the closer legislator wants to move policy in the same direction as the chair does, then the chair's optimal proposal must move the policy away from the status quo towards her own ideal. Hence, although the closer legislator is indifferent between this proposal and the status quo, the more distant legislator is made worse off than the status quo. It follows that the more distant legislator would like to maximize his chance to be included in a proposal, thereby undermining the credibility of his rhetoric. As to the closer legislator, it is possible for him to have (at most) two equilibrium messages signaling his ideological intensity. Specifically, he sends the "fight" message when he places a relatively high weight on the ideological dimension and the chair responds with a proposal that involves minimum policy change and gives neither legislator any private benefit since the message indicates that there is no room for making a deal. When he places a relatively low weight on the ideological dimension, he sends the "compromise" message and the chair responds by offering some private benefit in exchange for moving the policy closer to her own ideal. The threshold type of the closer legislator is indifferent between sending the "fight" and the "compromise" messages because either way he gets a payoff equal to his status quo payoff, and a single-crossing property guarantees that other types' incentive constraints are satisfied as well. It is impossible for even the closer legislator to convey more precise information about his ideological intensity. In particular, once the chair believes that the closer legislator places a relatively low weight on ideology and responds by including him in a proposal, the legislator now has an incentive to exaggerate his ideological intensity and demand a better deal from the chair, but this undermines the credibility of his demands. Somewhat ironically, the proposal induced by the "fight" message always passes in equilibrium, but the proposal induced by the "compromise" message may fail to pass in equilibrium.

Surprisingly, bargaining with two legislators rather than one (who can veto a bill) might hurt the chair even though with majority rule, the chair's bargaining position is improved. Under complete information, this improvement in the bargaining position immediately implies that the chair is better off when bargaining with two legislators. Under asymmetric information, however, the number of legislators also affects the amount of information transmission. In particular, increased competition may undermine the legislators' incentives to send the "fight" message, resulting in less information transmitted in equilibrium and this hurts the chair.

Since the players bargain over both an ideological dimension and a distributive dimension, a natural question is whether it is better to bundle the two issues in one bill or negotiate over them separately. In our model bundling always benefits the chair because she can exploit the differences in the other legislators' trade-offs between the two dimensions, and use private benefit as an instrument to make deals on policy changes that she wants to implement. This result, however, depends on the nature of uncertainty regarding preferences. In a related working paper (Chen and Eraslan, 2011), we show that bundling may result in informational loss when ideological positions are private information; in that case, bundling might hurt the chair.

Before turning to the description of our model, we briefly discuss the related literature. Starting with the seminal work of Baron and Ferejohn (1989), legislative bargaining models have become a staple of political economy and have been used in numerous applications. Like our paper, some papers in the literature include an ideological dimension and a distributive dimension (see, for example, Austen-Smith and Banks (1988), Banks and Duggan (2000), Jackson and Moselle (2002), and Diermeier and Merlo (2004)), but all these papers take the form of sequential offers and do not incorporate demands. A smaller strand of literature, notably Morelli (1999), instead models the legislative process as a sequential demand game where the legislators commit to their demands.[4] With the exceptions of Tsai (2009), and Tsai and Yang (2010 a, b), who do not model demands, all of these papers assume complete information.

The literature on cheap talk has largely progressed in parallel to the bargaining literature. Exceptions are Farrell and Gibbons (1989), Matthews (1989), and Matthews and Postlewaite (1989). Of these Matthews (1989) is the most closely related. Our model differs from his by having multiple senders and a distributive dimension in addition to an ideological dimension. Furthermore, in our model, legislators are privately informed about their ideological intensities, whereas in Matthews (1989), the private information is about the ideological position of the sender. Our paper is also related to cheap talk games with multiple senders (see, for example, Gilligan and Krehbiel (1989), Austen-Smith (1993), Krishna and Morgan (2001a, b) Battaglini (2002) and Ambrus and Takahashi (2008)). Our framework differs from these papers because it has voting over the proposal made by the receiver and also incorporates a distributive dimension.

In the next section we describe our model. We first consider the complete information model as a benchmark in Section 3. We then study the bargaining game in which the legislators' ideological intensities are uncertain. In Sections 4, we analyze the simpler game with only one legislator (other than the chair) and then move on to analyze the game with two legislators in Section 5. We discuss extensions and generalizations in Section 6.

2 Model

Three legislators play a three-stage game to collectively decide on an outcome that consists of an ideological component and a distributive component, for example, setting the level of environmental regulation and dividing government spending across districts. Legislator 0 is the proposer (the chair of the legislature) in charge of formulating a proposal.[5] From now on we simply refer to legislator 0 as the chair, and use the term legislator to refer to the other two players.[6] Denote an outcome by $z = (y; x)$ where y is an ideological decision and

[4]See also Vidal-Puga (2004), Montero and Vidal-Puga (2007), and Breitmoser (2009).

[5]We use "her" as the pronoun for the chair and "him" as the pronoun for legislators 1 and 2.

[6]When we use i and j to index the legislators, we omit the quantifiers $= 1, 2$ or $j = 1, 2$. When we refer to both legislator i and legislator j, we implicitly assume $j \neq i$. For legislator $i = 1, 2,$, we let $-i$ denote the

$x = (x_0, x_1, x_2)$ is a distributive decision. The set of feasible ideological decisions is $Y = \mathbb{R}$, and the set of feasible distributions is $X = \{x \in \mathbb{R}^3 : \sum_{i=0}^{2} x_i \leq c, x_1 \geq 0, x_2 \geq 0\}$ where x_i denotes the private benefit of legislator i and $c \geq 0$ is the size of the surplus (or, as it is referred to in the literature, "cake") available for division. We say that proposal $(y; x)$ *includes* legislator i if $x_i > 0$ and *excludes* legislator i if $x_i = 0$. The status quo allocation is denoted by $s = (\tilde{y}; \tilde{x})$ where $\tilde{y} \in Y$ and $\tilde{x} = (0, 0, 0)$.[7]

The payoff of each player $i = 0, 1, 2$ depends on the ideological decision and his/her private benefit. We assume that the players' preferences are separable over the two dimensions. Specifically, player i has a quasi-linear von Neumann-Morgenstern utility function given by

$$u_i(z, \theta_i, \hat{y}_i) = x_i + \theta_i v(y, \hat{y}_i),$$

where $z = (y; x)$ specifies the outcome, $\hat{y}_i$ denotes the ideal policy of player i and $\theta_i \in [0, \infty)$ is the weight that player i places on his/her payoff from the ideological decision relative to the distributive decision. The marginal rate of substitution, $(\partial u_i/\partial y)/(\partial u_i/\partial x_i) = \theta_i(\partial v/\partial y)$, measures player i's preference for ideology relative to private benefit and it depends on both θ_i and $\hat{y}_i$. With fixed $\hat{y}_i$, the (absolute value of the) marginal rate of substitution is increasing in θ_i. For expositional convenience, we call θ_i the parameter of ideological intensity.

Legislator $i = 1, 2$ privately observes the realization of θ_i, called his type, a random variable with probability distribution P_i. The set of possible types of legislator i is $\Theta_i = [\underline{\theta}_i, \overline{\theta}_i] \subset \mathbb{R}_+$, called his type space. Let F_i denote the distribution function of θ_i, i.e., $F_i(t) = P_i(\theta_i \leq t)$. We assume that F_i is continuous and has full support on Θ_i. The legislators' types are independently distributed. Although θ_i is legislator i's private information, its distribution and other aspects of his payoff function, including $\hat{y}_i$, are common knowledge. In the remainder of the paper, $\hat{y}_i$ is fixed and we use $u_i(z, \theta_i)$ to denote legislator i's payoff from outcome z when his type is θ_i.

For simplicity we assume the chair's preference is commonly known. Without loss of generality, assume $\hat{y}_0 < \tilde{y}$, i.e., the chair would like to move the policy to the left of the status quo. To simplify notation, we write $u_0(z) = x_0 + \theta_0 v(y, \hat{y}_0)$ as the chair's payoff from outcome z.

legislator $j = 1, 2$ with $j \neq i$. As is conventional in game theory, when we write (m_i, m_j), we refer to a message profile in which legislator i sends the message m_i and legislator j sends the message m_j. We use analogous notation for other vector of variables involving the two legislators.

[7] The assumption that $\tilde{x} = (0, 0, 0)$, together with the assumption on X, requires that the total surplus for reaching an agreement is non-negative, legislator 1's and legislator 2's status quo cake shares are the same, and the chair's proposal cannot offer cake shares lower than his status quo for either legislator 1 or 2.

We make the following assumptions on v: (1) v is twice differentiable; (2) $v_{11}(y, \hat{y}_i) < 0$ for all $y \in Y$ (which implies that v is concave in y), and $v(\cdot, \hat{y}_i)$ reaches its maximum at $\hat{y}_i$; (3) v satisfies the single-crossing property in $(y, \hat{y}_i)$, i.e., if $v(y', \hat{y}_i) = v(y, \hat{y}_i)$ and $y' > y$, then $(\hat{y}'_i - \hat{y}_i)(v(y', \hat{y}'_i)) - v(y, \hat{y}'_i)) > 0$ for all y, y', $\hat{y}_i \in Y$. This property implies that if legislator i with position $\hat{y}_i$ is indifferent between two policies y' and y where y' is to the right of y, then, any legislator whose position is to the right of $\hat{y}_i$ prefers y' to y and any legislator whose position is to the left of $\hat{y}_i$ prefers y to y'. Note that the familiar quadratic-loss function, $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, satisfies all of these assumptions.

The bargaining game consists of three stages. In the first stage, each legislator i observes his type θ_i and the legislators simultaneously send private messages to the chair. In the second stage, the chair observes the legislators' messages and makes a proposal in $Y \times X$. In the last stage, the players vote on the proposal; the voting rule is majority rule. Without loss of generality we assume that the chair always votes for the proposal. So if at least one of legislators 1 and 2 votes for the proposal, then it passes. Otherwise, the status quo $s = (\tilde{y}; \tilde{x})$ prevails.

The set of allowed messages for legislator i, denoted by M_i, is an abstract, finite set that has more than two elements. The messages have no literal meanings (we discuss their equilibrium meanings later); they are also "cheap talk" since they do not affect the players' payoffs directly. The assumption that M_i is finite rules out the possibility of separating equilibria, but we show that separating equilibria are not possible anyway, i.e. separating equilibria are not possible even if M_i's are infinite.

A strategy for legislator i consists of a message rule in the first stage and an acceptance rule in the third stage. A message rule $\mu_i : \Theta_i \to M_i$ for legislator i specifies the message he sends as a function of his type. An acceptance rule $\gamma_i : Y \times X \times \Theta_i \to \{0, 1\}$ for legislator i specifies how he votes as a function of his type: he votes for a proposal z if $\gamma_i(z, \theta_i) = 1$ and against it if $\gamma_i(z, \theta_i) = 0$.[8] The strategy set for legislator i consists of pairs of measurable functions (μ_i, γ_i) satisfying these properties. The chair's strategy set consists of all proposal rules $\pi : M_1 \times M_2 \to Y \times X$ where $\pi(m_1, m_2)$ is the proposal she offers when receiving (m_1, m_2). We focus on pure strategies and discuss conditions under which it is not restrictive to disallow mixed strategies later.

[8] Technically a legislator's acceptance rule can depend on his message. However, subgame perfection implies that independent of the message he sent, legislator i accepts a proposal if and only if he prefers it to the status quo. As such, we suppress the dependence of γ_i on m_i.

Fix a strategy profile (μ, γ, π). Say that a *message profile* $m = (m_1, m_2)$ *induces* proposal z if $\pi(m) = z$, and a *message* m_i *can induce* proposal z if there exists type θ_j such that $m = (m_i, \mu_j(\theta_j))$ induces proposal z. Proposal z is *elicitable* if some message profile induces it; *proposal* z *can be elicited* by type profile $\theta = (\theta_1, \theta_2)$ if it is induced by a message profile m with $m_i = \mu_i(\theta_i)$; proposal z is *elicited* by type θ_i if it is induced by a message profile m with $m_i = \mu_i(\theta_i)$ and $\{\theta_j : \mu_j(\theta_j) = m_j\}$ is nonempty. Proposal z is *accepted* by legislator i of type θ_i if $\gamma_i(z, \theta_i) = 1$ and *rejected* by legislator i of type θ_i if $\gamma_i(z, \theta_i) = 0$. If a proposal z is induced by m, then, legislator i is *pivotal with respect to* z if $\gamma_j(z, \theta_j) = 0$ for all θ_j such that $\mu_j(\theta_j) = m_j$ and *non-pivotal with respect to* z otherwise.

Equilibrium: In order to define an equilibrium for this game, let $\beta_i(z|m_i)$ denote the probability that legislator i votes to accept proposal z conditional on sending message m_i. Given the strategy (μ_i, γ_i) of legislator i, β_i is derived by Bayes' rule whenever possible.

An equilibrium is a strategy profile (μ, γ, π) such that the following conditions hold for all $i \neq 0$, $\theta_i \in \Theta_i$, $y \in Y, x \in X$ and $m \in M_1 \times M_2$:

$$(E1)\ \ \gamma_i(z, t_i) = \begin{cases} 1 & \text{if } u_i(z, t_i) \geq u_i(s, t_i), \\ 0 & \text{if } u_i(z, t_i) < u_i(s, t_i). \end{cases}$$

(E2) $\pi(m) \in \arg\max_{z' \in Y \times X} u_0(z')\beta(z'|m) + u_0(s)(1 - \beta(z'|m))$, where

$$\beta(z'|m) = 1 - \big(1 - \beta_1(z'|m_1)\big)\big(1 - \beta_2(z'|m_2)\big)$$

is the conditional probability that z' is accepted.

(E3) if $\mu_i(\theta_i) = m_i$, then $m_i \in \arg\max_{m_i'} V_i(m_i', \theta_i)$ where

$$V_i(m_i', \theta_i) = \int_{\Theta_j} \big(\gamma_j(z, \theta_j) u_i\left(\pi\left(m_i', \mu_j(\theta_j)\right), \theta_i\right)$$
$$+ (1 - \gamma_j(z, \theta_j)) \max\{u_i\left(\pi\left(m_i', \mu_j(\theta_j)\right), \theta_i\right), u_i(s, \theta_i)\}\big) dF_j(\theta_j).$$

Condition (E1) is an implication of subgame perfection for the last stage of the game: it requires the legislators to accept proposals that they prefer to the status quo.[9] Condition

[9] We assume a legislator accepts z whenever indifferent between z and s. If $z = s$, the assumption is inconsequential as s would prevail whether or not legislator i accepts it. Otherwise, this assumption is not restrictive.This is because if legislator i does not accept a proposal (not equal to s) when indifferent, then an optimal proposal does not exist for the chair. To see this, note that if the chair has an optimal proposal $(y, x) \neq s$, then at least one legislator i must strictly prefer it to s. But then there exists $\epsilon > 0$ such that either (y, x') with $x_i' = x_i - \epsilon$ or (y', x) with $y' = y - \epsilon$ is another proposal that legislator i strictly prefers to s and makes the chair better off, contradicting the optimality of (y, x).

(E2) requires that equilibrium proposals maximize the payoff of the chair and that her belief is consistent with Bayes' rule. Condition (E3) requires that a legislator elicits only his most preferred proposals among the ones that are possible in equilibrium (in the sense that there is some message that induces it), taking into account the acceptance rule of the other legislator.

For expositional simplicity, from now on we assume that in equilibrium, if $\beta(z|m) = 0$, then $\pi(m) \neq z$; i.e., if a proposal is rejected with probability 1, then the chair does not propose it.[10]

Say that a proposal z is *elicited in the equilibrium* (μ, γ, π) if there exist $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$ such that z is elicited by (θ_1, θ_2); i.e. $z = \pi(\mu_1(\theta_1), \mu_2(\theta_2))$. It is sometimes convenient to classify equilibria by the number of elicited proposals. Define the *size* of an equilibrium to be its number of elicited proposals: $\#\{z \in Y \times X | (\theta_1, \theta_2) \in \Theta_1 \times \Theta_2 \text{ such that } z = \pi(\mu_1(\theta_1), \mu_2(\theta_2))\}$.

For any fixed strategy profile (μ, γ, π), denote by $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2)$ the *outcome* for the type profile (θ_1, θ_2) under (μ, γ, π); i.e., $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = \pi(\mu_1(\theta_1), \mu_2(\theta_2))$ if $\gamma_i(\pi(\mu_1(\theta_1), \mu_2(\theta_2)), \theta_i) = 1$ for at least one of $i = 1, 2$ and $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = s$ otherwise. Say that two equilibria (μ, γ, π) and (μ', γ', π') are *outcome-equivalent* if $\phi^{\mu,\gamma,\pi} = \phi^{\mu',\gamma',\pi'}$.

A *babbling equilibrium* is an equilibrium (μ, γ, π) in which $\mu_i(\theta_i) = \mu_i(\theta_i')$ for all $\theta_i, \theta_i' \in \Theta_i$, $i = 1, 2$, i.e., all types of legislator i send the same message, and $\pi(m) = \pi(m')$ for all $m, m' \in M_1 \times M_2$, i.e., the chair responds to all message profiles with the same proposal. As is standard in cheap-talk models, a babbling equilibrium always exists.

3 Benchmark: complete information

We start by analyzing the benchmark game of complete information, i.e., θ_i is common knowledge. Since there is no private information, the legislators' messages are irrelevant for the chair's belief and her proposal. The modifications of the players' strategies and equilibrium conditions are straightforward and omitted. We next characterize the chair's equilibrium proposal.

If $v(\hat{y}_0, \hat{y}_i) \geq v(\tilde{y}, \hat{y}_i)$ for some legislator i, i.e., if there is a legislator who prefers the chair's ideal policy to the status quo policy, then the chair's problem is trivial: she proposes her ideal and keeps all the private benefit herself. From now on, we assume $v(\hat{y}_0, \hat{y}_i) < v(\tilde{y}, \hat{y}_i)$ for $i = 1, 2$. Note that since $\hat{y}_0 < \tilde{y}$, this implies that $\hat{y}_0 < \hat{y}_i$.

A useful piece of notation is $e(\hat{y}_i) = \min\{y : v(y, \hat{y}_i) = v(\tilde{y}, \hat{y}_i)\}$. Since $v(y, \hat{y}_i)$ is increasing

[10]This is not a restrictive assumption if $c > 0$ because the chair strictly prefers the proposal $(\tilde{y}; c, 0, 0)$ (which is accepted with probability 1) to the status quo, so z is not a best response. If $c = 0$, however, it is possible that z is a best response, but not a unique one (for example, s is another best response).

in y when $y < \hat{y}_i$, under the assumption that $v(\hat{y}_0, \hat{y}_i) < v(\tilde{y}, \hat{y}_i)$, we have $\hat{y}_0 < e(\hat{y}_i) \leq \tilde{y}$, and $e(\hat{y}_i)$ is the policy y that is closest to the chair's ideal that leaves legislator i indifferent between the status quo policy $\tilde{y}$ and y. Note that $e(\hat{y}_i)$ is nondecreasing in $\hat{y}_i$ and in addition, $e(\hat{y}_i) = \tilde{y}$ if $\hat{y}_i \geq \tilde{y}$ and $e(\hat{y}_i) < \hat{y}_i < \tilde{y}$ if $\hat{y}_i < \tilde{y}$.

To start, suppose the chair face only one legislator, legislator 1. Assume that legislator 1 has veto power, i.e., for any proposal to pass, he must vote for it.

Given θ_1, the chair chooses $z^1(\theta_1) = (y^1(\theta_1); x^1(\theta_1))$ to solve

$$\max_{z \in Y \times X} u_0(z) = c - x_1 + \theta_0 v(y, \hat{y}_0)$$

subject to $x_1 + \theta_1 v(y, \hat{y}_1) \geq \theta_1 v(\tilde{y}, \hat{y}_1)$. Since $u_0(z)$ is decreasing in x_1, for x_1^1 to be optimal, it must satisfy $x_1^1 = \theta_1 \left(v(\tilde{y}, \hat{y}_1) - v(y^1, \hat{y}_1)\right)$. To satisfy $x_1^1 \geq 0$, we must have $v(\tilde{y}, \hat{y}_1) \geq v(y^1, \hat{y}_1)$. Thus, substituting for x_1 in the chair's maximization problem, y^1 must be a solution to

$$\max_{y \in Y} c - \theta_1 \left(v(\tilde{y}, \hat{y}_1) - v(y, \hat{y}_1)\right) + \theta_0 v(y, \hat{y}_0)$$

subject to $v(\tilde{y}, \hat{y}_1) \geq v(y, \hat{y}_1)$. Since $v_{11} < 0$, the objective function is strictly concave and hence y^1 is unique. If $\theta_1 v_1(e(\hat{y}_1), \hat{y}_1) + \theta_0 v_1(e(\hat{y}_1), \hat{y}_0) > 0$, then the constraint that $v(\tilde{y}, \hat{y}_1) \geq v(y, \hat{y}_1)$ is binding and we have a corner solution $y^1 = e(\hat{y}_1)$ and $x_1^1 = 0$. Otherwise, there exists a unique $y^1 < e(\hat{y}_1)$ such that $\theta_1 v_1(y^1, \hat{y}_1) + \theta_0 v_1(y^*, \hat{y}_0) = 0$; in this case, $x_1^1 > 0$.

When the chair faces two legislators instead of one, her bargaining position is improved since the voting rule is the majority rule. Let $z^2(\theta_2)$ denote the chair's optimal proposal when facing legislator 2 with ideological intensity θ_2. If $u_0\left(z^i(\theta_i)\right) \geq u_0\left(z^j(\theta_j)\right)$, then $z^i(\theta_i)$ is optimal for the chair when he faces two legislators. Notice that it is possible that the legislator whose ideal policy is further away from the chair's is included in an optimal proposal. This can happen when he puts sufficiently less weight on ideology than the other legislator does. Next, we analyze the model in which the legislators' ideological intensities are private information.

4 One sender

Although our focus is on the legislative bargaining game with three players and majority rule, it is useful to first consider a simpler game in which there is only one legislator (sender) other than the chair. In addition to gaining useful intuition from the analysis, the model is interesting in its own right because it is applicable to situations of bilateral bargaining over two issues.

Letting Γ^S denote the game in which the set of legislators other than the chair is given by S, in the section we consider the case in which $S = \{1\}$.

The modification of the players' strategies and equilibrium conditions in $\Gamma^{\{1\}}$ are straightforward and omitted. To characterize equilibria, we establish the following lemma, which says that between two proposals that offer different transfers: (i) if type θ_1 weakly prefers the proposal that gives him a larger transfer, then any lower type (i.e., any type who places a lower weight on ideology) strictly prefers the proposal that gives him a larger transfer; (ii) if type θ_1 weakly prefers the proposal that gives him a smaller transfer, then any higher type strictly prefers the proposal that gives him a smaller transfer. (Proofs omitted from the text are in Appendix A.)

Lemma 1. *(i) If type θ_1 weakly prefers $z' = (y'; x')$ to $z = (y; x)$ where $x'_1 > x_1$, then any type $\theta'_1 < \theta_1$ strictly prefers z' to z. (ii) If type θ_1 weakly prefers $z'' = (y''; x'')$ to $z = (y; x)$ where $x''_1 < x_1$, then any type $\theta''_1 > \theta_1$ strictly prefers z'' to z.*

A special case of Lemma 1 is worth noting: Suppose type θ_1 is indifferent between the status quo s and $z = (y; x)$ where $x_1 > 0$. If $\theta'_1 < \theta_1$, then type θ'_1 strictly prefers z to s; if $\theta'_1 > \theta_1$, then type θ'_1 strictly prefers s to z. This immediately implies that legislator 1 does not fully reveal his type in equilibrium.[11] To see this, note that in a separating equilibrium, legislator 1 would receive only his status quo payoff as the chair would make a proposal that leaves him just willing to accept. But then type θ_1 would want to mimic a higher type (i.e., exaggerate his ideological intensity) in order to get a better deal from the chair. In fact, we have a much stronger result which says that there exists at most one equilibrium proposal that gives legislator 1 positive private benefit and an equilibrium has at most size two. Before deriving this result and characterizing size-two equilibria, it is useful to first characterize size-one equilibria.

4.1 Size-one equilibria

We focus on babbling equilibrium since any size-one equilibrium is outcome equivalent to a babbling equilibrium. Let z' be the proposal elicited in a babbling equilibrium.

To find z', note that by Lemma 1, if $u_1\left(z, \bar{\theta}_1\right) \geq u_1\left(s, \bar{\theta}_1\right)$, then $u_1\left(z, \theta_1\right) \geq u_1\left(s, \theta_1\right)$ for all $\theta_1 \in \Theta_1$ and so z is always accepted; if $u_1\left(z, \underline{\theta}_1\right) < u_1\left(s, \underline{\theta}_1\right)$, then $u_1\left(z, \theta_1\right) < u_1\left(s, \theta_1\right)$ for all

[11] To be more precise, legislator 1 does not fully reveal his type in equilibrium except in the degenerate case where $z^1\left(\theta_1\right) = \left(e\left(\hat{y}_1\right); c, 0\right)$ for every $\theta_1 \in \Theta_1$. In this degenerate case, even if legislator 1 fully reveals his type, the chair still always makes the same proposal $\left(e\left(\hat{y}_1\right); c, 0\right)$ and we have a size-one equilibrium.

$\theta_1 \in \Theta_1$ and z is always rejected; if $u_1\left(z, \bar{\theta}_1\right) < u_1\left(s, \bar{\theta}_1\right)$ and $u_1\left(z, \underline{\theta}_1\right) \geq u_1\left(s, \underline{\theta}_1\right)$, then there exists $\theta_1 \in \Theta_1$ such that $u_1\left(z, \theta_1\right) = u_1\left(s, \theta_1\right)$ and z is accepted with probability $F_1\left(\theta_1\right)$.

Let $t_1\left(z\right)$ denote the highest type who is willing to accept z if z is accepted with positive probability and set $t_1\left(z\right)$ to $\underline{\theta}_1$ if z is accepted with probability 0. Formally

$$t_1\left(z\right) = \begin{cases} \max\{\theta_1 \in \Theta_1 : u_1\left(z, \theta_1\right) \geq u_1\left(s, \theta_1\right)\} & \text{if } u_1\left(z, \underline{\theta}_1\right) \geq u_1\left(s, \underline{\theta}_1\right), \\ \underline{\theta}_1 & \text{otherwise.} \end{cases}$$

For z' to be the proposal elicited in a babbling equilibrium, it must satisfy

$$z' \in \arg\max_{z \in Y \times X} u_0\left(z\right) F_1\left(t_1\left(z\right)\right) + u_0\left(s\right)\left[1 - F_1\left(t_1\left(z\right)\right)\right].$$

Equivalently, we can formulate the chair's problem as choosing the highest type who is willing to accept her proposal. Let θ_1' be the highest type willing to accept z'. Let $V(\theta_1) = u_0\left(z^1\left(\theta_1\right)\right)$ denote the chair's highest payoff when facing legislator 1 of type θ_1. Then we have

$$\theta_1' \in \arg\max_{\theta_1 \in \Theta_1} V\left(\theta_1\right) F_1\left(\theta_1\right) + u_0\left(s\right)\left(1 - F_1\left(\theta_1\right)\right). \tag{1}$$

If the solution is unique, it is without loss of generality to consider only pure strategies. We close this section by discussing sufficient conditions for uniqueness.

A sufficient condition for θ_1' to be unique is that the objective function is strictly concave. Another sufficient condition for uniqueness is that the objective function is strictly increasing in θ_1. Lemma 7 in Supplementary Appendix shows that in the uniform-quadratic case (i.e., θ_1 is uniformly distributed and $v\left(y, \hat{y}_1\right) = -\left(y - \hat{y}_1\right)^2$), if $y_1 \leq \tilde{y}$, then the objective function is strictly increasing in θ_1 and (1) has a unique solution at $\bar{\theta}_1$; if $y_1 > \tilde{y}$, then (1) may have an interior solution as well as a solution at $\bar{\theta}_1$ but this happens only non-generically.

4.2 Size-two equilibria

The main finding in this subsection is that legislator 1 can credibly convey some information, but only in a limited way. We first show that the number of proposals elicited in an equilibrium is at most two and then characterize size-two equilibria and provide existence conditions.

The following lemma says that there can be at most one proposal elicited in equilibrium that gives legislator 1 strictly positive private benefit.

Lemma 2. *Suppose proposals $z' = (y'; x')$ and $z'' = (y''; x'')$ are elicited in an equilibrium in $\Gamma^{\{1\}}$. If $x_1' > 0$ and $x_1'' > 0$, then $z' = z''$.*

To gain some intuition, suppose there are two equilibrium proposals z' and z'' that give legislator 1 positive private benefits. Then there exists a type θ_1' who elicits z' and is indifferent between z' and the status quo, and another type θ_1'' who elicits z'' and is indifferent between z'' and the status quo. Without of loss of generality assume $\theta_1'' > \theta_1'$. But then by Lemma 1 type θ_1' strictly prefers to elicit z'', a contradiction. So only one equilibrium proposal can have $x_1 > 0$. For such a proposal, it must be true that $y < e(\hat{y}_1)$. When proposing it, the chair makes some transfer to legislator 1 in exchange for moving the policy towards her own ideal.

Now consider a proposal $(y; c, 0)$ that does not give legislator 1 any private benefit. If $e(\hat{y}_1) \leq y \leq \tilde{y}$, then all types accept it; if $y < e(\hat{y}_1)$, no type accepts it. Since $v(y, \hat{y}_0)$ is decreasing in y when $y \geq e(\hat{y}_1)$, we must have $y = e(\hat{y}_1)$.

Hence there are at most two proposals elicited in an equilibrium: one is $(e(\hat{y}_1); c, 0)$ and the other is $(y; c - x_1, x_1)$ with $y < e(\hat{y}_1)$ and $x_1 > 0$. In what follows, let $\breve{z}$ denote the proposal $(e(\hat{y}_1); c, 0)$. Let type θ_1^* be the type indifferent between $(y; c - x_1, x_1)$ and $\breve{z}$. By Lemma 1, if $\theta_1 < \theta_1^*$, then type θ_1 strictly prefers $(y; c - x_1, x_1)$ to $\breve{z}$ and hences elicits $(y; c - x_1, x_1)$. If $\theta_1 > \theta_1^*$, then type θ_1 strictly prefers $\breve{z}$ to $(y; c - x_1, x_1)$. A type $\theta_1 > \theta_1^*$ may elicit $\breve{z}$ and accept it or elicit $(y; c - x_1, x_1)$ and reject it because either way he gets the status quo payoff. To summarize, we have the following result.

Proposition 1. *In $\Gamma^{\{1\}}$: (i) At most two proposals are elicited in any equilibrium. (ii) In a size-two equilibrium, the elicited proposals are $\breve{z}$ and $(y; c - x_1, x_1)$ with $y < e(\hat{y}_1)$ and $x_1 > 0$. There exists a type θ_1^* such that if $\theta_1 < \theta_1^*$, type θ_1 elicits $(y; c - x_1, x_1)$ and accepts it; if $\theta_1 \geq \theta_1^*$, type θ_1 either elicits $(y; c - x_1, x_1)$ and rejects it or elicits $\breve{z}$ and accepts it.*

Proposition 1 says that type above θ_1^* may either elicit $(y; c - x_1, x_1)$ and reject the proposal, or elicit $\breve{z}$ and accept it. Note, however, that if there were any possibility of a "tremble" by legislator 1 at the voting stage, that is, if he might vote for a proposal even though he strictly prefers the status quo to it, then his best message rule is to safely elicit $\breve{z}$ if $\theta_1 > \theta_1^*$. The chair benefits if all $\theta_1 > \theta_1^*$ elicits $\breve{z}$, since the chair prefers the outcome $\breve{z}$ to the status quo.

Suppose the types who elicit the same proposal in equilibrium send the same message,[12] and m_1^1 induces $(y; c - x_1, x_1)$ and m_1^2 induces $\breve{z}$. We can interpret m_1^1 as the "compromise" message and m_1^2 as the "fight" message. Since any type below θ_1^* sends m_1^1, when the chair receives m_1^1, she infers that legislator 1 is likely to have a low ideological intensity, and responds

[12] This loses no generality because any size-two equilibrium is outcome equivalent to such an equilibrium.

with a "compromise" proposal that moves the policy towards her own ideal. Only types above θ_1^* sends m_1^2. When the chair receives m_1^2, she infers that legislator 1 is intensely ideological, and responds with a proposal that involves minimum policy change and no transfer for legislator 1. Note that, multiple size-two equilibria exist with different set of elicited proposals corresponding to different thresholds θ_1^*.

Existence: Recall that $z^1(\theta_1)$ is the chair's optimal proposal when θ_1 is known.

Proposition 2. *A size-two equilibrium exists in* $\Gamma^{\{1\}}$ *if (i)* $z^1\left(\bar{\theta}_1\right) = \breve{z}$ *and (ii)* $z^1\left(\underline{\theta}_1\right) = (y; c - x_1, x_1)$ *for some* $y < e(\hat{y}_1)$ *and* $x_1 > 0$.

The conditions in Proposition 2 require the chair's optimal proposal to be $\breve{z}$ when she is sure that legislator 1 is of the highest type and to be a proposal that has $y < e(\hat{y}_1)$ and $x_1 > 0$ when she is sure that legislator 1 is of the lowest type. Intuitively, under these conditions, there exists a type $\theta_1^* \in (\underline{\theta}_1, \bar{\theta}_1)$ such that $\breve{z}$ is optimal when the chair believes that $\theta_1 \in \left(\theta_1^*, \bar{\theta}_1\right)$ and $(y; c - x_1, x_1)$ is optimal when the chair believes that $\theta_1 \in (\underline{\theta}_1, \theta_1^*)$, which in turn guarantees that a size-two equilibrium exists.

4.3 Comparative statics: equilibria of different sizes

A natural question is whether the players are better off in an equilibrium of higher size. The chair clearly (weakly) prefers a size-two equilibrium to a size-one equilibrium because her decisions are based on better information in a size-two equilibrium. As to legislator 1, consider the following two cases. (i) Suppose $\breve{z}$ is elicited in a size-one equilibrium. Then legislator 1's payoff is the same as his status quo payoff. Since in any size-two equilibrium, the payoff of any type $\theta_1 \geq \theta_1^*$ is the same as his status quo payoff and the payoff of any type $\theta_1 < \theta_1^*$ is strictly higher than his status quo payoff, legislator 1 is better off in a size-two equilibrium. (ii) Suppose $z' \neq \breve{z}$ is elicited in a size-one equilibrium. Whether legislator 1 is better off in a size-two equilibrium depends on the size-two equilibrium under consideration. But it is worth noting that for any size-one equilibrium in which z' is rejected with positive probability, a size-two equilibrium exists in which every type of legislator 1 has the same payoff as that in the size-one equilibrium.[13] In this sense, legislator 1 is again (weakly) better off in a size-two equilibrium.

[13] To construct it, let $\theta_1' < \bar{\theta}_1$ be the type who is just willing to accept z'. Let $\mu_1(\theta_1) = m_1^1$ for $\theta_1 \leq \theta_1'$, $\mu_1(\theta_1) = m_1^2$ for $\theta_1 > \theta_1'$, $\pi\left(m_1^1\right) = z'$, $\pi\left(m_1^2\right) = \breve{z}$ and $\pi(m) \in \{\pi\left(m_1^1\right), \pi\left(m_1^2\right)\}$ for any other $m_1 \in M_1$. In this size-two equilibrium, the payoff for any $\theta_1 < \theta_1'$ is $u_1(z', \theta_1)$ and the payoff for any $\theta_1 \geq \theta_1'$ is $u_1(s, \theta_1)$, the same as in the size-one equilibrium.

5 Two senders

We now analyze $\Gamma^{\{1,2\}}$, the game with two legislators. Without loss of generality, assume that $\hat{y}_1 \leq \hat{y}_2$, which implies that $e(\hat{y}_1) \leq e(\hat{y}_2)$. Since legislator 1's ideal point is closer to the chair's, we call legislator 1 the *closer* legislator and legislator 2 the *more distant* legislator. We focus on a class of equilibria called **monotone equilibria**. An equilibrium (μ, γ, π) is monotone if it satisfies the following property: for any $\theta_i' \leq \theta_i''$ and $i = 1, 2$, if $\mu_i(\theta_i') = \mu_i(\theta_i'')$, then $\mu_i(\theta_i) = \mu_i(\theta_i')$ for any $\theta_i \in [\theta_i', \theta_i'']$. In a monotone equilibrium, the set of types that send the same message is an interval, possibly a singleton.

5.1 Proposals elicited in monotone equilibria

Say that a proposal $(y; x)$ is a *one-transfer proposal* if either $x_1 > 0$ or $x_2 > 0$ but not both, a *two-transfer proposal* if both $x_1 > 0$ and $x_2 > 0$, and a *no-transfer proposal* if $x_1 = 0$ and $x_2 = 0$. The following lemma provides a sufficient condition under which no proposal elicited in a monotone equilibrium is a two-transfer proposal.

Lemma 3. *Suppose in $\Gamma^{\{1,2\}}$, F_i has a differentiable density function f_i for $i = 1, 2$. If $f_i(\theta_i)/(1 - F_i(\theta_i))$ is strictly increasing in θ_i, then any proposal elicited in a monotone equilibrium has $x_i > 0$ for at most one legislator $i \neq 0$.*

Notice that $f_i(\theta_i)/(1 - F_i(\theta_i))$ is the hazard rate. Lemma 3 says that if the prior on θ_i satisfies the increasing hazard rate property, then no proposal elicited in a monotone equilibrium is a two-transfer proposal. Many distribution functions, including uniform, normal, log-normal and beta distributions, have increasing hazard rates. This property is also frequently used in the economics and political science applications.[14]

Too see why Lemma 3 holds, consider the support of the chair's posterior on θ_i. If it is a singleton for at least one of the legislators, say legislator 1, then the chair's posterior on θ_1 is degenerate, which implies that given any proposal, the chair knows whether legislator 1 will accept or reject it. A two-transfer proposal is not optimal because if legislator 1 accepts it, then the chair is strictly better off reducing x_2 and if legislator 1 rejects it, then the chair is strictly better off reducing x_1. If the support of the posterior on θ_i is not a singleton for both $i = 1, 2$, then a two-transfer proposal results in a positive probability that both legislators vote for the

[14] See Bagnoli and Bergstrom (2005) for a list of distribution functions that satisfy the monotone hazard rate property and references to some of the seminal papers that assume it.

proposal, which is "wasteful" for the chair because she needs only one other vote to pass her proposal. Under increasing hazard rate property, the waste is sufficiently high that it is not optimal for the chair to give transfer to both legislators.

The next two lemmas establish some properties of no-transfer proposals and one-transfer proposals, which are useful in equilibrium characterization.

Lemma 4. *Suppose $z = (y; x)$ is elicited in an equilibrium in $\Gamma^{\{1,2\}}$ with $x_1 = x_2 = 0$. Then (i) $y = e(\hat{y}_1)$. (ii) $u_1(z, \theta_1) = u_1(s, \theta_1)$ for any θ_1. (iii) If $e(\hat{y}_1) = e(\hat{y}_2)$, then $u_2(z, \theta_2) = u_2(s, \theta_2)$ for any θ_2. (iv) If $e(\hat{y}_1) < e(\hat{y}_2)$, then $u_2(z, \theta_2) < u_2(s, \theta_2)$ for any θ_2 and legislator 1 is pivotal.*

Lemma 4 says that a no-transfer proposal z must make the ideologically closer legislator just willing to accept. Therefore, if $e(\hat{y}_1) = e(\hat{y}_2)$, then legislator 2 is indifferent between z and s and both legislators accept z, but if $e(\hat{y}_1) < e(\hat{y}_2)$, then legislator 2 rejects z and legislator 1 is pivotal. Henceforth, we denote the optimal no-transfer proposal $(e(\hat{y}_1); c, 0, 0)$ by z^{NT}.

Lemma 5. *Suppose $z = (y; x)$ is elicited in an equilibrium in $\Gamma^{\{1,2\}}$ and $x_i > 0$, $x_j = 0$. Then any type of legislator j strictly prefers the status quo s to z, but some types of legislator i strictly prefers z to s. Hence legislator j rejects z and legislator i is pivotal.*

Lemma 5 says that the legislator who is excluded in a one-transfer proposal rejects it, making the legislator who is included pivotal.

If F_1 and F_2 satisfy the increasing hazard rate property, then by Lemma 3, any proposal elicited in a monotone equilibrium is either a no-transfer proposal or a one-transfer proposal. This greatly simplifies the problem of characterizing elicited proposals in a monotone equilibrium. Specifically, recall that $t_i(z)$ is the highest θ_i willing to accept z if some $\theta_i \in \Theta_i$ prefers z to s and $t_i(z) = \underline{\theta}_i$ otherwise. Suppose the chair's posterior is $G = (G_1, G_2)$. Let $\beta(z) = 1 - [1 - G_1(t_1(z))][1 - G_2(t_2(z))]$ and

$$z(G) \in \arg\max_{z \in Y \times X} u_0(z)\beta(z) + u_0(s)(1 - \beta(z)).$$

That is, $z(G)$ is an optimal proposal for the chair under belief G. Let $U_0(G)$ be the associated value function, i.e., $U_0(G)$ is the highest expected payoff for the chair under belief G.

Denote by $z^{-i}(G_{-i})$ a proposal that gives the chair the highest expected payoff among all the proposals that exclude legislator i, under belief G_{-i}, and let $U_0^{-i}(G_{-i})$ denote the associated value function. Note that $z^{-i}(G_{-i})$ does not depend on G_i because for any proposal that excludes i, either every θ_i accepts it or no θ_i accepts it. Fix a monotone equilibrium (γ, μ, π). Let $H(m) = (H_1(m_1), H_2(m_2))$ be the chair's posterior when receiving m. By Lemma 3, for any m sent in (γ, μ, π), $z(H(m))$ is not a two-transfer proposal and therefore $U_0(H(m)) = \max_{i=1,2} U_0^{-i}(H_{-i}(m_{-i}))$. Note that $U_0^{-i}(H_{-i}(m_{-i})) \geq u_0(z^{NT})$ for $i = 1, 2$. Hence, if $U_0^{-i}(H_{-i}(m_{-i})) > U_0^{-j}(H_{-j}(m_{-j}))$, then it is optimal for the chair to exclude i and include j. If $U_0^{-i}(H_{-i}(m_{-i})) = u_0(z^{NT})$ for $i = 1, 2$, then proposing z^{NT} is optimal when receiving m.

Since a babbling equilibrium is a monotone equilibrium, all the results established for monotone equilibrium apply. Specifically, suppose F_1 and F_2 satisfy the increasing hazard rate property. If $U_0^{-i}(F_{-i}) > U_0^{-j}(F_{-j}) \geq u_0(z^{NT})$, then the proposal elicited in a babbling equilibrium includes j and excludes i; if $U_0^{-1}(F_{-1}) = U_0^{-2}(F_{-2}) > u_0(z^{NT})$, then the proposal elicited in a babbling equilibrium is a one-transfer proposal that includes either 1 or 2; if $U_0^{-1}(F_{-1}) = U_0^{-2}(F_{-2}) = u_0(z^{NT})$, then there exists a babbling equilibrium in which the no-transfer proposal z^{NT} is elicited.

5.2 Informative equilibria

In this section, we characterize equilibria in $\Gamma^{\{1,2\}}$ in which some information is transmitted. Throughout this section, we assume that F_1 and F_2 satisfy the monotone hazard rate property. Fix a monotone equilibrium (γ, μ, π) and consider the proposals $\pi(m')$ and $\pi(m'')$ where $m'_i = m''_i$. Suppose both $\pi(m')$ and $\pi(m'')$ exclude legislator j. As shown in the previous section, this implies that $z^{-j}(H_{-j}(m'_i))$ is an optimal proposal when the chair receives m' and $z^{-j}(H_{-j}(m''_i))$ is an optimal proposal when she receives m''. If $z^{-j}(H_{-j}(m'_i))$ is unique, then, since $m'_i = m''_i$ and both $\pi(m')$ and $\pi(m'')$ exclude j, we must have $\pi(m') = \pi(m'') = z^{-j}(H_{-j}(m'_i))$. If $z^{-j}(H_{-j}(m'_i))$ is not unique, however, then conceivably $\pi(m') \neq \pi(m'')$, but this requires that the chair chooses different proposals – none of which include legislator j – for different messages sent by legislator j, although she has the same belief about legislator i.

Call a monotone equilibrium (γ, μ, π) a **simple monotone equilibrium** (SME) if the following condition is satisfied: for any m' and m'' such that $m'_i = m''_i$, if both $\pi(m')$ and $\pi(m'')$ exclude legislator j, then $\pi(m') = \pi(m'')$. We find this to be a reasonable refinement because when the chair optimally excludes legislator j, her proposal depends only on her belief about legislator i's type, which has nothing to do with what legislator j says. This refinement is also automatically satisfied if $z^{-j}(H_{-j}(m'_i))$ is unique. (Uniqueness of $z^{-j}(H_{-j}(m'_i))$ holds under some familiar functional forms: Lemma 7 in Supplementary Appendix implies that if $H_i(m_i')$ is a uniform distribution and $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, then $z^{-j}(H_{-j}(m_i'))$ is unique.)

Say that μ_i is a *size-one message rule* if $\mu_i(\theta_i) = \mu_i(\theta_i')$ for all $\theta_i, \theta_i' \in \Theta_i$. Say that μ_i is a *size-two message rule* if there exists a set $A_i \subset \Theta_i$ with $P_i(\theta_i \in A_i) \in (0,1)$ such that (i) $\mu_i(\theta_i) = \mu_i(\theta_i')$ if either $\theta_i, \theta_i' \in A_i$ or $\theta_i, \theta_i' \in \Theta_i \backslash A_i$ and (ii) $\mu_i(\theta_i) \neq \mu_i(\theta_i')$ if $\theta_i \in A_i$ and $\theta_i' \in \Theta_i \backslash A_i$.

Recall that $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2)$ is the outcome for type profile (θ_1, θ_2) under (μ, γ, π). Fix an equilibrium (μ, γ, π). Say that μ_i is *equivalent to* μ_i' if for almost all $(\theta_1, \theta_1) \in \Theta_1 \times \Theta_2$, we have $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = \phi^{\mu',\gamma,\pi}(\theta_1, \theta_2)$ where $\mu_j' = \mu_j$. The message rule μ_i is equivalent to μ_i' in the sense that the joint distributions on type profiles and outcomes are the same under μ_i and μ_i', holding the other strategies in (μ, γ, π) fixed.

Say that legislator i is *uninformative* in equilibrium (μ, γ, π) if there exists a size-one message rule μ_i^I such that μ_i is equivalent to μ_i^I and legislator i is *informative* in equilibrium (μ, γ, π) otherwise. Say that (μ, γ, π) is an *informative equilibrium* if at least one legislator is informative in (μ, γ, π).

For any $z \in Y \times X$, let $I_i(z) = 1$ if z includes legislator i and $I_i(z) = 0$ if z excludes legislator i. Let $q_1(m_1) = \int_{\Theta_2} I_1(\pi(m_1, \mu_2(\theta_2)))\, dF_2$ be the probability that legislator 1 is included when sending m_1 in (μ, γ, π). Similarly, let $q_2(m_2) = \int_{\Theta_1} I_2(\pi(\mu_1(\theta_1), m_2))\, dF_1$.

Proposition 3. *Fix a simple monotone equilibrium (μ, γ, π). If legislator i is informative in this equilibrium, then there exist m_i', $\hat{m}_i \in M_i$ such that $q_i(m_i') > 0$, $q_i(\hat{m}_i) = 0$ and μ_i is equivalent to a size-two message rule μ_i^{II} with the property that there exists $\theta_i^* \in (\underline{\theta}_i, \bar{\theta}_i)$ such that $\mu_i^{II}(\theta_i) = m_i'$ for $\theta_i < \theta_i^*$ and $\mu_i^{II}(\theta_i) = \hat{m}_i$ for $\theta_i > \theta_i^*$.*

Proposition 3 says that in any SME, legislator i can convey only a limited amount of information in that even when informative, his message rule is equivalent a size-two message rule. To give a sketch of the proof, we first show that in (μ, γ, π), there exists at most one m_i sent by a positive measure of θ_i such that $q_i(m_i) > 0$; when such a message exists, the types who send this message forms an interval at the lower end of Θ_i. We also show that there exists at most one message m_i sent by a single type such that $q_i(m_i) > 0$. So at most two m_i's have the property that $q_i(m_i) > 0$ and one of them is sent by only a single type. Consider the following two possibilities: (a) Suppose there exists no m_i sent with positive probability such that $q_i(m_i) > 0$. Then with probability 1, $q_i(\mu_i(\theta_i)) = 0$. Since the proposal and the resulting outcome does not depend on m_i if legislator i is excluded in a SME, it follows that μ_i is equivalent to a size-one message rule such that every θ_i sends the same messsage that results in probability 1 that legislator i is excluded. (b) Suppose there exists m_i' sent with positive probability that such that $q_i\left(m_i'\right) > 0$. If legislator i is informative, then there exists $\hat{m}_i$ sent by some type such that $q_i\left(\hat{m}_i\right) = 0$. Since in (μ, γ, π), the types who send m_i' form an interval at the lower end of Θ_i, there exists a threshold θ_i^* such that any type below θ_i^* send m_i' and almost every type above θ_i^* sends a message that results in probability 1 that i is excluded. Hence μ_i is equivalent to μ_i^{II} such that $\mu_i^{II}\left(\theta_i\right) = m_i'$ for $\theta_i < \theta_i^*$ and $\mu_i^{II}\left(\theta_i\right) = \hat{m}_i$ for $\theta_i > \theta_i^*$.

Proposition 4. *Fix a simple monotone equilibrium (μ, γ, π). (i) At most one legislator is informative in (μ, γ, π). (ii) If $e\left(\hat{y}_1\right) < e\left(\hat{y}_2\right)$, then legislator 2 is uninformative in (μ, γ, π).*

To gain some intuition for Proposition 4, imagine that both legislators are informative in (μ, γ, π). Then, by Proposition 3, both legislators are included with positive probability. By Lemma 5, a legislator's payoff is weakly higher than his status quo payoff when included, but strictly lower than his status quo payoff when the other legislator is included. So, independent of his type, each legislator has an incentive to send the message that generates the highest probability of inclusion. But as shown in Proposition 3, if a legislator is informative, then with positive probability, he sends a message that results in zero probability of inclusion, a contradiction. As to why legislator 2 is uninformative when $e\left(\hat{y}_1\right) < e\left(\hat{y}_2\right)$, note that in this case, under the no-transfer proposal z^{NT}, legislator 2's payoff is *strictly* lower than his status quo payoff. Therefore, between m_2' and $\hat{m}_2$ as described in Proposition 3, every type of legislator 2 strictly prefers to send m_2' (with $q_2\left(m_2'\right) > 0$) than $\hat{m}_2$ (with $q_2\left(\hat{m}_2\right) = 0$), again a contradiction.

What are the proposals elicited in an informative equilibrium? Consider an SME (μ, γ, π) in which legislator i is informative. For simplicity, assume $\mu_j\left(\theta_j\right) = m_j^*$ and $\mu_i\left(\theta_i\right) = m_i'$ for $\theta_i < \theta_i^*$ and $\mu_i\left(\theta_i\right) = \hat{m}_i$ for $\theta_i > \theta_i^*$ where $q_i\left(m_i'\right) > 0$ and $q_i\left(\hat{m}_i\right) = 0$. Since $q_i\left(\hat{m}_i\right) = 0$, $\pi(\hat{m}_i, m_j^*)$ excludes legislator i. Suppose $\pi(\hat{m}_i, m_j^*)$ includes legislator j. Then, by Lemma 5, legislator j is pivotal with respect to $\pi(\hat{m}_i, m_j^*)$ and accepts it with positive probability. This implies that by sending $\hat{m}_i$, type θ_i's payoff is strictly lower than $u_i\left(s, \theta_i\right)$. Since $q_i\left(m_i'\right) > 0$, $\pi(m_i', m_j^*)$ includes legislator i. By Lemma 5, type θ_i's payoff by sending m_i' is weakly higher than $u_i\left(s, \theta_i\right)$. Therefore any type $\theta_i > \theta_i^*$ has an incentive to deviate and send m_i', a contradiction. It follows that $\pi(\hat{m}_i, m_j^*)$ excludes j as well as i and $\pi(\hat{m}_i, m_j^*) = z^{NT}$. As to $\pi(m_i', m_j^*)$, it must have $y < e\left(\hat{y}_1\right), x_i > 0$ and $x_j = 0$. To summarize:

Proposition 5. *Fix an informative simple monotone equilibrium* (μ, γ, π) *in which legislator* i *follows a size-two message rule and legislator* j *follows a size-one message rule. Then there exists* $\theta_i^* \in (\underline{\theta}_i, \bar{\theta}_i)$ *such that for any* $\theta_j \in \Theta_j$*, if* $\theta_i > \theta_i^*$*, then* $\pi(\mu_i(\theta_i), \mu_j(\theta_j)) = z^{NT}$*; and if* $\theta_i < \theta_i^*$*, then* $\pi(\mu_i(\theta_i), \mu_j(\theta_j)) = (y; x)$ *with* $y < e(\hat{y}_1)$*,* $x_i > 0$*, and* $x_j = 0$*.*

Similar to the one-sender case, we can interpret the message sent by types below θ_i^* as the "compromise" message, and the message sent by types above θ_i^* as the "fight" message. The chair responds to the "compromise" message with a proposal that gives legislator i some private benefit and moves the policy towards her own ideal and responds to the "fight" message with a proposal that involves minimum policy change and gives no private benefit to either legislator.

Our analysis has focused on monotone equilibria. Similar to $\Gamma^{\{1\}}$, non-monotone equilibria may exist in which legislator j babbles, types $\theta_i < \theta_i^*$ of legislator i elicit $(y; x)$ with $y < e(\hat{y}_1)$, $x_i > 0$, and $x_j = 0$, and types $\theta_i > \theta_i^*$ of legislator i either elicit z^{NT} and accept it, or elicit $(y; x)$ and reject it. Note that similar to $\Gamma^{\{1\}}$, these non-monotone equilibria are not robust to "trembles" by either legislator at the voting stage, i.e., if either legislator might not carry out a planned rejection, then legislator i's best message rule is to safely elicit z^{NT} when $\theta_i > \theta_i^*$.

To illustrate what an informative equilibrium looks like, we provide the following example.

Example 1. *Suppose* $\tilde{y} = 0$*,* $\hat{y}_0 = -1$*,* $\hat{y}_1 = -0.2$*,* $\hat{y}_2 = 0.5$*,* $c = 1$*. Assume that player* i*'s utility function is* $x_i - \theta_i (y - \hat{y}_i)^2$*,* $\theta_0 = 1$*, and* θ_1, θ_2 *are both uniformly distributed on* $[\frac{1}{4}, 4]$*.*

Suppose $\mu_1(\theta_1) = m_1'$ if $\theta_1 \in [\frac{1}{4}, 1]$ and $\mu_1(\theta_1) = \hat{m}_1$ if $\theta_1 \in (1, 4]$;[15] $\mu_2(\theta_2) = m_2^*$ for all θ_2. Given the message rules, when the chair receives m_1', she infers that $\theta_1 \in [\frac{1}{4}, 1]$. Calculation shows that $\pi(m_1', m_2^*) = (-0.6; 0.88, 0.12, 0)$, a proposal that gives legislator 1 positive transfer and moves the policy towards the chair's ideal.[16] When the chair receives $\hat{m}_1$, she infers that $\theta_1 \in (1, 4]$. Calculation shows that it is optimal to propose $z^{NT} = (-0.4; 1, 0, 0)$. Intuitively, it is too costly for the chair to move the policy closer to her ideal because legislator 1 is too intensely ideological and legislator 2 holds an ideological position that is too far away.

[15] Here we let $\theta_1^* = 1$, but there are many other equilibria given by different thresholds.

[16] In this example, the proposal that the chair makes in response to (m_1', m_2^*) is accepted with probability 1 by legislator 1. This is a feature of the example and does not hold in general. For example, suppose the distribution of θ_1 is a truncated exponential distribution on $[\frac{1}{4}, 4]$ with the parameter $\lambda = 4$, i.e., $F_1(\theta_1) = (e^{-1} - e^{-4x}) / (e^{-1} - e^{-16})$. Keep all the other parametric assumptions unchanged and assume $\mu_1(\theta_1) = m_1'$ if $\theta_1 \in [\frac{1}{4}, 2]$ and $\mu_1(\theta_1) = \hat{m}_1$ if $\theta_1 \in (2, 4]$ and $\mu_2(\theta_2) = m_2^*$ for all θ_2. Then $\pi(m_1', m_2^*) = (-0.585; 0.883, 0.117, 0)$ and it is rejected by all types of legislator 2 and accepted by legislator 1 if and only if $\theta_1 \leq 1.076$. Hence it is rejected with strictly positive probability.

Existence of informative equilibria: We provide sufficient conditions for the existence of SME in which legislator 1 is informative. The conditions are similar for SME in which legislator 2 is informative, with the additional requirement that $e(\hat{y}_1) = e(\hat{y}_2)$.

The existence conditions for informative equilibria in $\Gamma^{\{1,2\}}$ are analogous to those for size-two equilibria in the one-sender game, but with the additional necessary condition that it is optimal for the chair to exclude legislator 2. This is guaranteed if $U_0^{-1}(F_{-1}) = u_0\left(z^{NT}\right)$. To see this, recall that $U_0^{-1}(F_{-1})$ is the highest payoff the chair gets by excluding legislator 1. If $U_0^{-1}(F_{-1}) = u_0\left(z^{NT}\right)$, then no proposal that includes legislator 2 gives the chair a higher payoff than z^{NT} and therefore it is optimal for the chair to exclude 2. Recall that $z^1(\theta_1)$ is the chair's optimal proposal when facing only legislator 1 with known θ_1. We have the following result (the proof is omitted since it is similar to the proof of Proposition 2):

Proposition 6. *Suppose F_1 and F_2 satisfy the property of increasing hazard rate. A monotone simple equilibrium in which legislator 1 is informative exists if (i) $z^1\left(\bar{\theta}_1\right) = (e(\hat{y}_1); c, 0)$, (ii) $z^1(\underline{\theta}_1) = (y; x)$ where $y < e(\hat{y}_1)$ and $x_1 > 0$, and (iii) $U_0^{-1}(F_{-1}) = u_0\left(z^{NT}\right)$.*

5.3 Comparative statics

Two comparisons seem especially interesting. The first is the comparison between informative and uninformative equilibria in $\Gamma^{\{1,2\}}$. The second is the comparison of equilibria in $\Gamma^{\{1,2\}}$ and those in $\Gamma^{\{1\}}$, which allows us to answer: is the chair always better off bargaining with more legislators? Surprisingly, we show below that although the chair needs only one legislator's support to pass a proposal, she may be worse off when facing two legislators than just one.

Comparing informative and uninformative equilibria: Let E^u be an uninformative equilibrium and E^I be an SME in which legislator i is informative in $\Gamma^{\{1,2\}}$. Since the chair benefits from information transmission, she is better off in E^u than in E^I. The welfare comparison for the informative legislator is similar to that in the one-sender case (page 13); in particular, the informative legislator benefits from information transmission as well. The uniformative legislator j, however, may be made worse off when legislator i is informative. To illustrate, suppose the proposal elicited in E^u is z^{NT}. Since in an informative SME the elicited proposals are z^{NT} and $(y; x)$ with $y < e(\hat{y}_1)$ and $x_j = 0$, and legislator j prefers $e(\hat{y}_1)$ to any $y < e(\hat{y}_1)$, he is better off in E^u. Intuitively, in E^I, when legislator i signals willingness to compromise, the chair moves the policy towards her ideal and gives legislator i some private benefit in exchange for his support. Since legislator j is excluded, he is made worse off.

Does it benefit the chair to face more legislators? Under complete information, the chair is clearly better off bargaining with two legislators than only one because she gains flexibility as to who to make a deal with, as shown at the end of section 3. Under asymmetric information, however, the answer is less clear. As illustrated in the following example, having two legislators may result in less information transmitted in equilibrium and this hurts the chair.

Example 2. *Suppose $c = 1, \tilde{y} = 0$, $u_0(z) = x_0 - \theta_0 (y - \hat{y}_0)^2$ where $\theta_0 = 1$, $\hat{y}_0 = -1$ and $u_1(z, \theta_1)$ $x_1 - \theta_1 (y - \hat{y}_1)^2$ where $\hat{y}_1 = -0.2$ and θ_1 is uniformly distributed on $[\frac{1}{4}, 4]$.*

Size-two equilibria exist in $\Gamma^{\{1\}}$, in which the chair faces only legislator 1. For instance, analogous to Example 1, a size-two equilibrium exists in which $\mu_1(\theta_1) = m_1'$ if $\theta_1 \in [\frac{1}{4}, 1]$ and $\mu_1(\theta_1) = \hat{m}_1$ if $\theta_1 \in (1, 4]$. The chair's payoff in this equilibrium is 0.656. Now consider $\Gamma^{\{1,2\}}$ in which the chair faces legislator 2 as well as legislator 1.[17] Suppose $u_2(z, \theta_2) = x_2 - \theta_2 (y - \hat{y}_2)^2$ where $\hat{y}_2 = -0.201$ and θ_2 is uniformly distributed on $[5, 10]$. Since $e(\hat{y}_2) < e(\hat{y}_1)$, by Proposition 4, legislator 1 is not informative in any SME in $\Gamma^{\{1,2\}}$. Calculation shows that $z^2(\underline{\theta}_2) = (y; x)$ where $y = e(\hat{y}_2)$ and $x_2 = 0$. So condition (ii) in Proposition 6 (adapted to legislator 2) fails, and legislator 2 is not informative in any SME either. In an uninformative equilibrium in $\Gamma^{\{1,2\}}$, the proposal $z^{NT} = (-0.402; 1, 0, 0)$ is elicited and the chair's payoff is 0.642 , lower than 0.656 , the payoff in the size-two equilibrium that we identified in $\Gamma^{\{1\}}$.

In the preceding example, the chair is made worse off when we add legislator 2 whose position is closer to the chair's (making it impossible for 1 to be informative) but who is intensely ideological (making it impossible for himself to be informative). What happens if we add a legislator whose position is further away from the chair's? Can it still result in information loss? The next example shows that the answer is yes. Suppose $u_2(z, \theta_2) = x_2 - \theta_2 (y - \hat{y}_2)^2$ where $\hat{y}_2 = -0.1$ and θ_2 is uniformly distributed on $[\frac{1}{4}, \frac{4}{5}]$. Since $e(\hat{y}_1) < e(\hat{y}_2)$, by Proposition 4, legislator 2 is uninformative in any SME in $\Gamma^{\{1,2\}}$. Calculation shows that conditional on excluding 1, the chair's optimal proposal includes 2. In particular, $z^{-1}(F_{-1}) = (y; x)$ where $y = -0.6$ and $x_2 = 0.192$. So condition (iii) in Proposition 6 fails and it is not possible for legislator 1 to be informative in any SME in $\Gamma^{\{1,2\}}$ either.[18] The chair's proposal includes 2 in

[17] Although earlier we assumed that $\hat{y}_1 \leq \hat{y}_2$ for expositional convenience, in this example, in order to discuss all possibilities, we allow $\hat{y}_1 > \hat{y}_2$.

[18] To see this, note that if there exists an SME in which legislator 1 is informative, then the chair responds to his "fight" message by including legislator 2 making legislator 1 strictly worse off than the status quo and giving him an incentive to deviate.

any uninformative equilibrium in $\Gamma^{\{1,2\}}$, resulting in a payoff of 0.648, still lower than 0.656. So, the chair is again worse off when she faces two legislators than one.

To summarize, under asymmetric information, the chair may be better off bargaining with only one legislator when the information loss resulting from having two legislators is sufficiently high. This contrasts with Krishna and Morgan (2001b), in which the decision maker is never worse off when facing two experts than one. In their model, for any equilibrium in the one-expert case, there exists an equilibrium when another expert is added which gives the decision maker a payoff at least as high as his original equilibrium payoff.

5.4 Benefits of bundled bargaining

Since the legislators bargain over both an ideological dimension as well as a distributive dimension, a natural question is whether the proposer is better off bundling the two dimensions or negotiating them separately. In the model considered so far, they are bundled because the chair makes a proposal on both dimensions and the two dimensions are accepted or rejected together by the legislators. (In the following discussion, we call this the "bundled bargaining" game.) Alternatively, we can consider a game in which the chair, after receiving the messages sent by the legislators, makes a proposal on only the ideological dimension and another on only the distributive dimension. The legislators vote on each proposal separately and majority rule determines whether a proposal passes or the status quo (on that dimension) prevails. In this "separate bargaining" game, it is possible that a proposal on one dimension passes while the proposal on the other dimension fails to pass.

The chair is better off in the bundled bargaining game. To see why, note that in the separate bargaining game, the legislators' private information is irrelevant since it is about how they trade off one dimension for the other, not about their preferences on either dimension. The resulting unique equilibrium outcome is z^{NT}. In the bundled bargaining game, z^{NT} is still feasible and will pass with probability 1 if proposed, and this immediately implies that the chair cannot be worse off. In fact, for the chair, there are two advantages from bundling: (1) Useful information may be revealed in equilibrium, as seen in Proposition 5. (2) Given the information she has, the chair can use private benefit as an instrument to make better proposals that exploit the difference in how the players trade off the two dimensions. Because of these two advantages, if the chair could choose between bundled bargaining and separate bargaining, the chair would choose bundled bargaining. As to the other legislators, they get their status quo payoffs in the separate bargaining game, but in the bundled bargaining game, the informative legislator is better off than the status quo whereas the other, uninformative legislator, is worse off than the status quo. This result is reminiscent of the finding in Jackson and Moselle (2002), who also show that legislators may prefer to make proposals for the two dimensions together despite separable preferences, but their model has no asymmetric information or communication.

6 Concluding remarks

In this paper, we develop a new model of legislative bargaining that incorporates private information about preferences and allows speech making before a bill is proposed. Although the model is simple, our analysis generates interesting predictions about what speeches can be credible even without commitment and how they influence proposals and legislative outcomes.

We believe that both private information and communication are essential elements of the legislative decision making process. Our paper has taken a first step in understanding their roles in the workings of a legislature. There are many more issues to explore and many ways to generalize and extend our model and what follows is a brief discussion of some of them.

Our motivation for incorporating private information into legislative bargaining is that individual legislators know their preferences better than others. Another possible source of private information is that some legislators may have better information (perhaps acquired through specialized committee work or from staff advisors) regarding the consequences of certain policies, which is relevant for all legislators. Although the role of this kind of "common value" private information in debates and legislative decision making has been studied in the literature (e.g. Austen-Smith (1990)), it is only in the context of one-dimensional spatial policy making. It would be interesting to explore it further when there is tradeoff between ideology and distribution of private benefits.

In our model the chair does not have private information about her preference, consistent with the observation that bill proposers are typically established members with known positions. But sometimes legislators can be uncertain about what exactly the legislative leaders' goals are, in particular, how much compromise the leaders are willing to make to accommodate their demands in exchange for their votes. In this case, apart from speeches, the proposal that the chair puts on the table may also reveal some of his private information. This kind of signaling effect becomes especially relevant when the legislators have interdependent preferences or when the proposal is not an ultimatum but can be modified if agreement fails.

We have focused on a specific extensive form in which the legislators send messages simultaneously. It would be interesting to explore whether and how some of our results change if the legislators send messages sequentially. In that case, the design of the optimal order of speeches (from the perspective of the proposer as well as the legislature) itself is an interesting question. Another design question with respect to communication protocol is whether the messages should be public or private. Although this distinction does not matter for the model we analyzed because we assume simultaneous speeches and one round of bargaining, it would matter if either there were multiple rounds of bargaining or the preferences were interdependent.

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Appendix A

Proof of Lemma 1. (i) Since type θ_1 weakly prefers z' to z, we have $x_1' + \theta_1 v(y', \hat{y}_1) \geq x_1 + \theta_1 v(y, \hat{y}_1)$, which implies that $x_1' - x_1 \geq \theta_1 (v(y, \hat{y}_1) - v(y', \hat{y}_1))$.

Suppose $v(y, \hat{y}_1) - v(y', \hat{y}_1) \leq 0$. Since $x_1' - x_1 > 0$ and $\theta_1' > 0$, it follows that $x_1' - x_1 > 0 \geq \theta_1' (v(y, \hat{y}_1) - v(y', \hat{y}_1))$, i.e., $x_1' + \theta_1' v(y', \hat{y}_1) > x_1 + \theta_1' v(y, \hat{y}_1)$.

Suppose $v(y, \hat{y}_1) - v(y', \hat{y}_1) > 0$. Then $\theta_1 (v(y, \hat{y}_1) - v(y', \hat{y}_1)) > \theta_1' (v(y, \hat{y}_1) - v(y', \hat{y}_1))$ for $\theta_1 > \theta_1'$ and hence $x_1' - x_1 > \theta_1' (v(y, \hat{y}_1) - v(y', \hat{y}_1))$, i.e., $x_1' + \theta_1' v(y', \hat{y}_1) > x_1 + \theta_1' v(y, \hat{y}_1)$.

(ii) Since type θ_1 weakly prefers z'' to z, we have $x_1'' + \theta_1 v(y'', \hat{y}_1) \geq x_1 + \theta_1 v(y, \hat{y}_1)$, which implies that $\theta_1 (v(y'', \hat{y}_1) - v(y, \hat{y}_1)) \geq x_1 - x_1''$. Since $x_1 - x_1'' > 0$, we have $v(y'', \hat{y}_1) - v(y, \hat{y}_1) > 0$. So, for $\theta_1'' > \theta_1$, we have $\theta_1'' (v(y'', \hat{y}_1) - v(y, \hat{y}_1)) > \theta_1 (v(y'', \hat{y}_1) - v(y, \hat{y}_1)) \geq x_1 - x_1''$, i.e., $x_1'' + \theta_1'' v(y'', \hat{y}_1) > x_1 + \theta_1'' v(y, \hat{y}_1)$. ■

Proof of Lemma 2. Let (μ, γ, π) be an equilibrium in Γ_1 in which z' and z'' are elicited where $x_1' > 0$ and $x_1'' > 0$. Let $\alpha(z) = \{\theta_1 : \pi(\mu_1(\theta_1)) = z \text{ and } \gamma_1(z, \theta_1) = 1\}$. Since any proposal elicited in an equilibrium is accepted by some type who elicits it (page 2), $\alpha(z')$ and $\alpha(z'')$ are nonempty. Let $\theta_1' = \sup \alpha(z')$ and $\theta_1'' = \sup \alpha(z'')$.

Let $o(\theta_1) = \pi(\mu_1(\theta_1))$ if $\gamma_1(\mu_1(\theta_1), \theta_1) = 1$ and $o(\theta_1) = s$ otherwise. Also, let $u_1^e(\theta_1) = u_1(o(\theta_1), \theta_1)$. That is, $o(\theta_1)$ is type θ_1's outcome and $u_1^e(\theta_1)$ is type θ_1's payoff in (μ, γ, π).

Claim 1. $u_1^e(\theta_1') = u_1(z', \theta_1') = u_1(s, \theta_1')$ *and* $u_1^e(\theta_1'') = u_1(z'', \theta_1'') = u_1(s, \theta_1'')$.

Proof. We show that $u_1^e(\theta_1') = u_1(z', \theta_1') = u_1(s, \theta_1')$. A similar argument shows $u_1^e(\theta_1'') = u_1(z'', \theta_1'') = u_1(s, \theta_1'')$.

To show that $u_1^e(\theta_1') = u_1(z', \theta_1')$, first note that $u_1^e(\theta_1') \geq u_1(z', \theta_1')$ since type θ_1' can elicit z' and accept it. Suppose $u_1^e(\theta_1') > u_1(z', \theta_1')$. Note that for any $\varepsilon > 0$, there exists type

$\theta_1 \in \alpha(z')$ such that $\theta_1' - \theta_1 < \varepsilon$. Since $u_1(o(\theta_1'), \theta_1) - u_1(z', \theta_1)$ is continuous in θ_1, there must exist type $\theta_1 \in \alpha(z')$ suffciently close to θ_1' such that $u_1(o(\theta_1'), \theta_1) > u_1(z', \theta_1)$. Since for any $\theta_1 \in \alpha(z')$, $u_1^e(\theta_1) = u_1(z', \theta_1)$, this is a contradiction. Hence $u_1^e(\theta_1') = u_1(z', \theta_1')$.

To show that $u_1^e(\theta_1') = u_1(s, \theta_1')$, first note that for any θ_1, $u_1^e(\theta_1) \geq u_1(s, \theta_1)$ since type θ_1 can reject the proposal it elicits, resulting in s. Suppose $u_1^e(\theta_1') > u_1(s, \theta_1')$. Since $u_1^e(\theta_1') = u_1(z', \theta_1')$, we have $u_1(z', \theta_1') > u_1(s, \theta_1')$. Since $x_1' > 0$, there exists proposal $\tilde{z}$ where $\tilde{y} = y'$ and $0 < \tilde{x}_1 < x_1'$ such that $u_1(\tilde{z}, \theta_1') > u_1(s, \theta_1')$. Lemma 1 implies that for any $\theta_1 \in \alpha(z')$, $u_1(\tilde{z}, \theta_1) > u_1(s, \theta_1)$ and therefore $\gamma_1(\tilde{z}, \theta_1) = 1$ for any $\theta_1 \in \alpha(z')$. It follows that for any message sent by $\theta_1 \in \alpha(z')$, the chair is strictly better off by proposing $\tilde{z}$ than by proposing z', a contradiction. Hence, $u_1^e(\theta_1') = u_1(s, \theta_1')$. ∎

Suppose $z' \neq z''$. Consider the following two possibilities. (a) Suppose $x_1' = x_1''$ and $y' \neq y''$. Without loss of generality, suppose $y' < y''$. Since both z' and z'' are elicited in equilibrium, we have $y' < y'' \leq e(\hat{y}_1) \leq \hat{y}_1$. Since $x_1' = x_1''$ and $u_1(z, \theta_1)$ is increasing in y for $y < \hat{y}_1$, we have $u_1(z', \theta_1) < u_1(z'', \theta_1)$ for all $\theta_1 \in \Theta_1$, contradicting that z'' is elicited in equilibrium. (b) Suppose $x_1' \neq x_1''$ and without loss of generality, assume $x_1' > x_1'' > 0$. Note that for any $\theta_1 \in \alpha(z')$, $u_1^e(\theta_1) = u_1(z', \theta_1) \geq u_1(z'', \theta_1)$ and for any $\theta_1 \in \alpha(z'')$, $u_1^e(\theta_1) = u_1(z'', \theta_1) \geq u_1(z', \theta_1)$. It follows from Lemma 1 that any $\theta_1 \in \alpha(z')$ is lower than any $\theta_1 \in \alpha(z'')$. Hence $\theta_1' < \theta_1''$. Since $u_1(z'', \theta_1'') = u_1(s, \theta_1'')$, by Lemma 1, $u_1(z'', \theta_1') > u_1(s, \theta_1')$, contradicting $u_1^e(\theta_1') = u_1(s, \theta_1')$. Hence $z' = z''$. ∎

Proof of Proposition 2. Define $\tau(z, \theta_1', \theta_1'') = \max\{\theta_1 \in [\theta_1', \theta_1''] : u_1(z, \theta_1) \geq u_1(s, \theta_1)\}$ if $u_1(z, \theta_1') \geq u_1(s, \theta_1')$ and $\tau(z, \theta_1', \theta_1'') = \theta_1'$ if $u_1(z, \theta_1') < u_1(s, \theta_1')$. Let $k(\theta_1', \theta_1'')$ be the set of proposals that are optimal for the chair if she knows that $\theta_1 \in [\theta_1', \theta_1'']$, i.e.,

$$k\left(\theta_1', \theta_1''\right) = \arg\max_z u_0(z)\left(F_1\left(\tau\left(z, \theta_1', \theta_1''\right)\right) - F_1\left(\theta_1'\right)\right) + u_0(s)\left[F_1\left(\theta_1''\right) - F_1\left(\tau\left(z, \theta_1', \theta_1''\right)\right)\right].$$

Let $k(\theta_1, \theta_1) = z^1(\theta_1)$. We first establish the following claim.

Claim 2. *Let $l_1', l_1'', h_1', h_1'' \in \Theta_1$ be such that $l_1' < l_1''$, $h_1' < h_1''$, $l_1' \leq h_1'$ and $l_1'' \leq h_1''$. (i) If $\check{z} \in k(l_1', l_1'')$, then $\check{z} \in k(h_1', h_1'')$. (ii) If $k(h_1', h_1'') \neq \{\check{z}\}$, then $k(l_1', l_1'') \neq \{\check{z}\}$.*

Proof. For any $z \in Y \times X$, let $p(z) = (F_1(\tau(z, h_1', h_1'')) - F_1(h_1')) / (F_1(h_1'') - F_1(h_1'))$ and let $q(z) = (F_1(\tau(z, l_1', l_1'')) - F_1(l_1')) / (F_1(l_1'') - F_1(l_1'))$. Note that $q(\check{z}) = p(\check{z}) = 1$. We first show that $q(z) \geq p(z)$ for any z. If $\tau(z, l_1', l_1'') = l_1''$, then $q(z) = 1 \geq p(z)$. If $\tau(z, h_1', h_1'') = h_1'$

then $p(z) = 0 \leq q(z)$. If $\tau(z, l_1', l_1'') < l_1''$ and $\tau(z, h_1', h_1'') > h_1'$, then by Lemma 1, we have $\tau(z, l_1', l_1'') = \tau(z, h_1', h_1'')$ and again $q(z) \geq p(z)$.

Part (i): Since $\check{z} \in k(l_1', l_1'')$, we have $u_0(\check{z}) \geq u_0(z) q(z) + u_0(s)(1 - q(z))$ for any $z \in Y \times X$. Since $q(z) \geq p(z)$, for any z such that $u_0(z) \geq u_0(s)$, we have $u_0(\check{z}) \geq u_0(z) p(z) + u_0(s)(1 - p(z))$, which implies that $\check{z} \in k(h_1', h_1'')$.

Part (ii): If $k(h_1', h_1'') \neq \{\check{z}\}$, then there exists $z \neq \check{z}$ such that $u_0(z) p(z) + u_0(s)(1 - p(z)) \geq u_0(\check{z})$. Since $q(z) \geq p(z)$, we have $u_0(z) q(z) + u_0(s)(1 - q(z)) \geq u_0(\check{z})$ and therefore $k(l_1', l_1'') \neq \{\check{z}\}$. ■

Let $t_1' = \sup\{\theta_1 \in [\underline{\theta}_1, \bar{\theta}_1]$ such that $k(\underline{\theta}_1, \theta_1) \neq \{\check{z}\}\}$ and $t_1'' = \inf\{\theta_1 \in [\underline{\theta}_1, \bar{\theta}_1]$ such that $\check{z} \in k(\theta_1, \bar{\theta}_1)\}$. Under conditions (i) and (ii) in Proposition 2, t_1' and t_1'' are well defined. We next show that $t_1' \leq t_1''$. Suppose to the contrary $t_1' > t_1''$. Then there exists $\theta_1 \in (t_1'', t_1')$ such that $k(\underline{\theta}_1, \theta_1) = \{\check{z}\}$ and $\check{z} \notin k(\theta_1, \bar{\theta}_1)$, contradicting Claim 2. Hence $t_1' \leq t_1''$. Fix $\tilde{\theta}_1 \in [t_1', t_1'']$ and let $\mu_1(\theta_1) = m_1^1$ if $\theta_1 \leq \tilde{\theta}_1$ and $\mu_1(\theta_1) = m_1^2$ if $\theta_1 > \tilde{\theta}_1$, $\pi(m_1^1) \in k(\underline{\theta}_1, \tilde{\theta}_1)$ such that $\pi(m_1^1) \neq \check{z}$, $\pi(m_1^2) = \check{z}$, $\pi(m_1) \in \{\pi(m_1^1), \pi(m_1^2))\}$ for any other $m_1 \in M_1$. Also, let γ_1 satisfy (E1). Since this is an equilibrium profile, a size-two equilibrium exists. ■

Proof of Lemma 3. Fix a monotone equilibrium (μ, γ, π) and a message profile m sent in this equilibrium. Let G denote the posterior of the chair when receiving m and g denote the associated density. We show below that $\pi(m_1, m_2) = (y^*; x^*)$ is not a two-transfer proposal.

Case (i): Suppose $\mu_i^{-1}(m_i)$ is a singleton for at least one legislator i. Without loss of generality, suppose $\mu_1^{-1}(m_1)$ is a singleton, and let $\theta_1 = \mu_1^{-1}(m_1)$. Suppose to the contrary that $x_1^* > 0$ and $x_2^* > 0$. If $u_1(\pi(m_1, m_2), \theta_1) \geq u_1(s, \theta_1)$, then type θ_1 accepts $\pi(m_1, m_2)$, and the chair is strictly better off by proposing $z' = (y^*; x_1^*, 0)$, a contradiction. If $u_1(\pi(m_1, m_2), \theta_1) < u_1(s, \theta_1)$, then type θ_1 rejects $\pi(m_1, m_2)$, and the chair is strictly better off by proposing $z' = (y'; x')$ where $y' = y^*$, $x_1' = 0$ and $x_2' = x_2^*$, a contradiction.

Case (ii): Suppose $\mu_i^{-1}(m_i)$ is a proper interval for $i = 1, 2$. Recall that for any proposal z, $t_1(z)$ denotes the highest type who is willing to accept z. Likewise, $t_2(z)$ is defined analogously. Then the probability that z passes is $\beta(z) = 1 - (1 - G_1(t_1(z)))(1 - G_2(t_2(z)))$. For any $d > 0$, consider the following problem

$$z^d \in \arg\max_{z \in Y \times X} (c - d + \theta_0 v(y, \hat{y}_0))\beta(z) + \theta_0 v(\tilde{y}, \hat{y}_0)(1 - \beta(z))$$

subject to $x_1 + x_2 = d$. That is, $z^d = (y^d; x^d)$ is an optimal proposal for the chair when receiving m, subject to $x_1 + x_2 = d$. To show that $\pi(m)$ is not a two-transfer proposal, it is sufficient to show that for any $d > 0$, either $x_1^d = 0$ or $x_2^d = 0$.

Fix $d > 0$ and suppose to the contrary that $x_1^d > 0$ and $x_2^d > 0$. Let $v_i^d = v(\tilde{y}, \hat{y}_i) - v(y^d, \hat{y}_i)$. Since $x_i^d > 0$, it must be the case that $v_i^d > 0$. Also $G_i\left(t_i(z^d)\right) \in (0,1)$. This is because, if $G_i\left(t_i(z^d)\right) = 0$, then $x_i^d = 0$; and if $G_i\left(t_i(z^d)\right) = 1$, then $x_j^d = 0$. In the rest of the proof, abusing notation, we let $G_i = G_i\left(t_i(z^d)\right)$ and $g_i = g_i\left(t_i(z^d)\right)$.

Substituting x_2 with $(d - x_1)$ in the maximization problem, first order necessary conditions for an interior maximum requires:

$$\Big(\theta_0 v(y^d, \hat{y}_0) - \theta_0 v(\tilde{y}, \hat{y}_0)\Big) \frac{\partial \beta(z^d)}{\partial x_1} = 0. \tag{2}$$

Since $d > 0$, we have $v(y^d, \hat{y}_0) - v(\tilde{y}, \hat{y}_0) > 0$, and therefore $\partial\beta(z^d)/\partial x_1 = 0$. Since $\partial\beta(z^d)/\partial x_1 = g_1(1 - G_2)/v_1^d - g_2(1 - G_1)/v_2^d$ and $G_i \in (0,1)$, we can rearrange (2) to obtain $v_d^1 = g_1(1 - G_2)v_d^2/(g_2(1-G_1))$. The second order necessary condition for an interior maximum requires that $\partial^2\beta(z^d)/\partial x_1^2 \leq 0$. Substituting for v_1^d, direct calculation shows that the second order condition requires that,

$$(\frac{g_2}{g_1})^2 \left(g_1'\,(1 - G_1) + (g_1)^2\right) + \left(g_2'\,(1 - G_2) + (g_2)^2\right) \leq 0. \tag{3}$$

By Corollary 5 in Bagnoli and Bergstrom (2005), a truncation of a distribution preserves the property of increasing hazard rate. Since F_i satisfies increasing hazard rate property, it follows that G_i satisfy the increasing hazard rate property, which implies that $g_i'(t_i(z^d))(1 - G_i(t_i(z^d)) + g_i(t_i(z^d))^2 > 0$ for $i = 1, 2$. But this violates equation (3), a contradiction. Hence, for any $d > 0$, either $x_1^d = 0$ or $x_2^d = 0$. ■

Proof of Lemma 4. (i) Since $x_1 = x_2 = 0$ and $e\left(\hat{y}_1\right) \leq \hat{y}_1 \leq \hat{y}_2$, it follows that if $y < e\left(\hat{y}_1\right)$, neither legislator accepts z, and if $y \geq e\left(\hat{y}_1\right)$, at least legislator 1 accepts z. Since $\hat{y}_0 < e\left(\hat{y}_1\right) \leq \tilde{y}$, $v\left(y, \hat{y}_0\right)$ reaches its maximum at $y = \hat{y}_0$ and $v_{11} < 0$, it is optimal for the chair to propose $y = e\left(\hat{y}_1\right)$. (ii) Since $x_1 = 0$ and $y = e\left(\hat{y}_1\right)$, we have $u_1\left(z, \theta_1\right) = u_1\left(s, \theta_1\right)$ for any θ_1. (iii) Similarly, if $e\left(\hat{y}_1\right) = e\left(\hat{y}_2\right)$, we have $u_2\left(z, \theta_2\right) = u_2\left(s, \theta_2\right)$ for any θ_2. (iv) If $e\left(\hat{y}_1\right) < e\left(\hat{y}_2\right) \leq \hat{y}_2$, however, we have $u_2\left(z, \theta_2\right) < u_2\left(s, \theta_2\right)$ for any θ_2 and hence legislator 1 is pivotal. ■

Proof of Lemma 5. Suppose to the contrary that there exists type θ_j' who prefers $z = (y; x)$ to s, i.e., $u_j(z, \theta_j') \geq u_j(s, \theta_j')$. Since $x_j = 0$, this implies that $v\left(y, \hat{y}_j\right) \geq v\left(\tilde{y}, \hat{y}_j\right)$ and therefore $u_j(z, \theta_j) \geq u_j(s, \theta_j)$ for any $\theta_j \in \Theta_j$. Consider $z' = (y'; x')$ with $y' = y$, $x_i' = x_j' = 0$. We have $u_j(z', \theta_j) \geq u_j(s, \theta_j)$ for any $\theta_j \in \Theta_j$ and therefore every type of legislator j accepts z. Since

$x_i' < x_i$, we have $u_0(z') > u_0(z)$, which implies that z is not a best response for the chair, a contradiction. So every type of legislator j rejects z and legislator i is pivotal. ■

Proof of Proposition 3. We first state and prove the following lemma.

Lemma 6. *Fix a simple monotone equilibrium (μ, γ, π). Let $\theta_i' < \theta_i''$, $m_i' = \mu_i(\theta_i')$, and $m_i'' = \mu_i(\theta_i'')$. Suppose $q_i(m_i'') > 0$. (i) If $\mu_i^{-1}(m_i'')$ is not a singleton, then $m_i' = m_i''$. (ii) If $\mu_i^{-1}(m_i'')$ is a singleton, then $\mu_i^{-1}(m_i')$ is not a singleton.*

Proof. We prove the lemma for $i = 1$ and discuss how to modify the proof for $i = 2$ at the end.

Part (i): Suppose to the contrary that $m_1' \neq m_1''$. We first prove the following claim: for any m_2 sent by some $\theta_2 \in \Theta_2$, if $\pi(m_1'', m_2)$ includes 1 then $\pi(m_1', m_2)$ also includes 1. This claim implies that $q_1(m_1') \geq q_1(m_1'')$. We then use this inequality to establish a contradiction.

Proof of the claim: Suppose $\pi(m_1'', m_2) = (y''; x'')$ includes 1. By Lemma 3, $\pi(m_1'', m_2)$ excludes 2. Since $\mu_1^{-1}(m_1'')$ is not a singleton, it must be a proper interval. By Lemma 5, legislator 1 is pivotal with respect to $\pi(m_1'', m_2)$. Since any proposal elicited in equilibrium is accepted with positive probability, $\pi(m_1'', m_2)$ is accepted by a positive measure of $\theta_1 \in \mu_1^{-1}(m_1'')$, i.e., $P_1\left(\theta_1 \in \mu_1^{-1}(m_1'') | u_1(\pi(m_1'', m_2), \theta_1) \geq u_1(s, \theta_1)\right) > 0$. By Lemma 1, for any θ_1 and $\tilde{\theta}_1$ such that $\tilde{\theta}_1 < \theta_1$, if $u_1(\pi(m_1'', m_2), \theta_1) \geq u_1(s, \theta_1)$, then $u_1(\pi(m_1'', m_2), \tilde{\theta}_1) > u_1(s, \tilde{\theta}_1)$. Hence $P_1\left(\theta_1 \in \mu_1^{-1}(m_1'') | u_1(\pi(m_1'', m_2), \theta_1) > u_1(s, \theta_1)\right) > 0$.

Given any $\varepsilon > 0$, let $z_\varepsilon = (y''; x_0'' + \varepsilon, x_1'' - \varepsilon, 0)$. Since $x_1'' > 0$, and u_1 is continuous in x_1, it follows that $P_1\left(\theta_1 \in \mu_1^{-1}(m_1'') | u_1(z_\varepsilon, \theta_1) > u_1(s, \theta_1)\right) > 0$ for ε sufficiently small. Hence $u_1(z_\varepsilon, \theta_1) > u_1(s, \theta_1)$ for $\theta_1 = \inf\{\mu_1^{-1}(m_1'')\}$.

Since $\theta_1' < \theta_1''$ and μ_1 is part of a monotone equilibrium, $m_1' \neq m_1''$ implies that $\sup\{\mu_1^{-1}(m_1')\} \leq \inf\{\mu_1^{-1}(m_1'')\}$. By Lemma 1, $u_1(z_\varepsilon, \theta_1) > u_1(s, \theta_1)$ for any $\theta_1 \in \mu_1^{-1}(m_1')$. Since $u_0(z_\varepsilon) > u_0(\pi(m_1'', m_2))$ and z_ε is accepted by all $\theta_1 \in \mu_1^{-1}(m_1')$, we have $U_0^{-2}(H_1(m_1')) > U_0^{-2}(H_1(m_1''))$.

Since $\pi(m_1'', m_2)$ includes 1, we have $U_0^{-2}(H_1(m_1'')) \geq U_0^{-1}(H_2(m_2))$. Hence $U_0^{-2}(H_1(m_1')) > U_0^{-2}(H_1(m_1'')) \geq U_0^{-1}(H_2(m_2))$ and therefore $\pi(m_1', m_2)$ includes 1 as well. This completes the proof of the claim.

The claim implies that $q_1(m_1') \geq q_1(m_1'')$, and $\Theta_2 = \Theta_2^a \cup \Theta_2^b \cup \Theta_2^c \cup \Theta_2^d$ where $\Theta_2^a = \{\theta_2 \in \Theta_2 :$ both $\pi(m_1', \mu_2(\theta_2))$ and $\pi(m_1'', \mu_2(\theta_2))$ exclude $1\}$; $\Theta_2^b = \{\theta_2 \in \Theta_2 : \pi(m_1', \mu_2(\theta_2))$ includes 1 and $\pi(m_1'', \mu_2(\theta_2))$ includes 2 $\}$, $\Theta_2^c = \{\theta_2 \in \Theta_2 : \pi(m_1', \mu_2(\theta_2))$ includes 1 and $\pi(m_1'', \mu_2(\theta_2))$ excludes both 1 and $2\}$, $\Theta_2^d = \{\theta_2 \in \Theta_2 :$ both $\pi(m_1', \mu_2(\theta_2))$ and $\pi(m_1'', \mu_2(\theta_2))$ include $1\}$. By the claim above, $\Theta_2^d = \{\theta_2 \in \Theta_2 : \pi(m_1'', \mu_2(\theta_2))$ include $1\}$, and $P_2(\Theta_2^d) = q_1(m_1'') > 0$.

Let $\theta_1^* = \sup \mu_1^{-1}(m_1')$ and $\theta_1^{**} = \sup \mu_1^{-1}(m_1'')$. In what follows we show that either θ_1^* or θ_1^{**} has a profitable deviation. Note that $\theta_1^{**} > \theta_1^*$ since $m_1' \neq m_1''$ and $\mu_1^{-1}(m_1'')$ is not a singleton. Further note that if $\pi(m_1', \mu_2(\theta_2))$ includes legislator 1, then type θ_1^* weakly prefers s to $\pi(m_1', \mu_2(\theta_2))$. To see this, let $\pi(m_1', \mu_2(\theta_2)) = z' = (y'; x')$ and suppose to the contrary that $u_1(z', \theta_1^*) > u_1(s, \theta_1^*)$. Since $x_1' > 0$, there exists $\varepsilon > 0$ and $\tilde{z} = (y'; x_0 + \varepsilon, x_1 - \varepsilon, 0)$ such that $u_1(\tilde{z}, \theta_1^*) > u_1(s, \theta_1^*)$. Since $\theta_1 \leq \theta_1^*$ for any $\theta_1 \in \mu_1^{-1}(m_1')$, by Lemma 1, $\tilde{z}$ is accepted with probability 1 in response to $(m_1', \mu_2(\theta_2))$, contradicting the optimality of $\pi(m_1', \mu_2(\theta_2))$. Hence type θ_1^* weakly prefers s to $\pi(m_1', \mu_2(\theta_2))$. Since $\theta_1^{**} > \theta_1^*$, type θ_1^{**} strictly prefers s to $\pi(m_1', \mu_2(\theta_2))$ by Lemma 1. Similarly, if $\pi(m_1'', \mu_2(\theta_2))$ includes legislator 1, then there exists a type $\theta_1 > \theta_1^*$ who is indifferent between $\pi(m_1'', \mu_2(\theta_2))$ and s. Therefore θ_1^* strictly prefers $\pi(m_1'', \mu_2(\theta_2))$ to s by Lemma 1.

Fix the strategies of the chair and legislator 2, and consider legislator 1's payoff from sending m_1' and m_1'' followed by his optimal acceptance rule. In an SME, $\pi(m_1', \mu_2(\theta_2)) = \pi(m_1'', \mu_2(\theta_2))$ for any $\theta_2 \in \Theta_2^a$. So conditional on $\theta_2 \in \Theta_2^a$, legislator 1 is indifferent between sending m_1' and m_1'' regardless of his type. By Lemma 5, legislator 1 is strictly better off when the proposal includes him than when the proposal includes the other legislator. So conditional on $\theta_2 \in \Theta_2^b$, legislator 1 is strictly better off sending m_1' than sending m_1'' regardless of his type. By the discussion in the previous paragraph, conditional on $\theta_2 \in \Theta_2^c$, both θ_1^* and θ_1^{**} get their status quo payoffs by sending m_1' (followed by their optimal acceptance rule). By Lemma 4, sending m_1'' yields legislator 1 his status quo payoff conditional on $\theta_2 \in \Theta_2^c$. So both θ_1^* and θ_1^{**} are indifferent between sending m_1' and m_1'' if $\theta_2 \in \Theta_2^c$. Lastly, conditional on $\theta_2 \in \Theta_2^d$, type θ_1^* is strictly better off sending m_1'', while type θ_1^{**} gets his status quo payoff when sending either m_1' (followed by optimally rejecting $\pi(m_1', \mu_2(\theta_2))$) or m_1''.

If $P_2\left(\Theta_2^b\right) > 0$, then type θ_1^{**} receives a strictly higher payoff by deviating and sending m_1', a contradiction. If $P_2\left(\Theta_2^b\right) = 0$, then, since $P_2(\Theta_2^d) > 0$, type θ_1^* receives a strictly higher payoff by deviating and sending m_1'', a contradiction. Hence $m_1'' = m_1'$.

If $i = 2$ and $e(\hat{y}_1) = e(\hat{y}_2)$, then the proof is identical. If $e(\hat{y}_1) < e(\hat{y}_2)$, then the proof is identical except when $P_1\left(\Theta_1^b\right) = 0$. ($\Theta_1^b$ and all other sets are defined analogously.) This is because conditional on $\theta_1 \in \Theta_1^c$, any type of legislator 2 strictly prefers sending m_2' to sending m_2''. In this case, if $P_1(\Theta_1^c) = P_1(\Theta_1^b) = 0$, then, since $P_1(\Theta_1^d) > 0$, type θ_2^* receives a strictly higher payoff by sending m_2'', a contradiction. If $P_1(\Theta_1^b) = 0$ and $P_1(\Theta_1^c) > 0$, then type θ_2^{**} has an incentive to deviate and send m_2', a contradiction.

Part (ii): Proof is similar that of Part (i). Suppose to the contrary that $\mu_i^{-1}(m_i')$ is a singleton. As in Part (i), suppose $\pi(m_1'', m_2)$ includes 1. Then, since $\mu_i^{-1}(m_1'')$ is a singleton, $\pi(m_1'', m_2)$ is accepted by θ_1'', i.e. $u_1(\pi(m_1'', m_2), \theta_1'' \geq u_1(s, \theta_1'')$. Since $\theta_1' < \theta_1''$, by Lemma 1, $u_1(\pi(m_1'', m_2), \theta_1') > u_1(s, \theta_1')$. Given any $\varepsilon > 0$, let $z_\varepsilon = (y''; x_0'' + \varepsilon, x_1'' - \varepsilon, 0)$. Since $x_1'' > 0$, and u_1 is continuous in x_1, it follows that $u_1(z_\varepsilon, \theta_1') > u_1(s, \theta_1')$. The rest of the proof is the same as that of Part (i). ■

We next show that if $P_i(q_i(\mu_i(\theta_i)) = 0) = 0$ or $P_i(q_i(\mu_i(\theta_i)) > 0) = 0$, then legislator i is uninformative in (μ, γ, π). Suppose $P_i(q_i(\mu_i(\theta_i)) = 0) = 0$. Then, $P_i(q_i(\mu_i(\theta_i)) > 0) = 1$, and by Lemma 6, there exists m_i' such that $q_i(m_i') > 0$ and $P_i(\mu_i(\theta_i) = m_i') = 1$. Hence μ_i is equivalent to the size-one message rule $\mu_i^I(\theta_i) = m_i'$ for all θ_i. Next, suppose $P_i(q_i(\mu_i(\theta_i)) > 0) = 0$. Then $P_i(q_i(\mu_i(\theta_i)) = 0) = 1$. Consider $\hat{\theta}_i$ such that $q(\mu_i(\hat{\theta}_i)) = 0$ and a size-one message rule $\mu_i^I(\theta_i)$ such that $\mu_i^I(\theta_i) = \mu_i(\hat{\theta}_i)$ for all $\theta_i \in \Theta_i$. To see that μ_i is equivalent to μ_i^I, consider any θ_i such that $q_i(\mu_i(\theta_i)) = 0$. Note that in an SME, for any m_j, $\pi(\mu_i(\hat{\theta}_i), m_j) = \pi(\mu_i(\theta_i), m_j)$ if both $\pi(\mu_i(\hat{\theta}_i), m_j)$ and $\pi(\mu_i(\theta_i), m_j)$ exclude legislator i. Since $q_i(\mu_i(\hat{\theta}_i)) = q_i(\mu_i(\theta_i)) = 0$, it follows that $\pi(\mu_i(\hat{\theta}_i), \mu_j(\theta_j)) = \pi(\mu_i(\theta_i), \mu_j(\theta_j))$ for almost all θ_j. Since $P_i(q_i(\mu_i(\theta_i)) = 0) = 1$, it follows that μ_i is equivalent to μ_i^I.

Hence, if legislator i is informative, then $P_i(q_i(\mu_i(\theta_i)) = 0) > 0$ and $P_i(q_i(\mu_i(\theta_i)) > 0) > 0$. Again, by Lemma 6, there exists m_i' and type $\theta_i^* \in (\underline{\theta}_i, \bar{\theta}_i)$ such that $q_i(m_i') > 0$ and $\mu_i(\theta_i) = m_i'$ for all $\theta_i < \theta_i^*$ and $q_i(\mu_i(\theta_i)) = 0$ for almost all $\theta_i \geq \theta_i^*$. Pick any $\hat{\theta}_i$ such that $q_i(\mu_i(\hat{\theta}_i)) = 0$, and let $\hat{m}_i = \mu_i(\hat{\theta}_i)$. Then μ_i is equivalent to μ_i^{II} such that $\mu_i^{II}(\theta_i) = m_i'$ for $\theta_i < \theta_i^*$ and $\mu_i^{II}(\theta_i) = \hat{m}_i$ for $\theta_i > \theta_i^*$. ■

Proof of Proposition 4. Proposition 3 implies that if legislator i is informative, then there exist $m_i' \in M_i$ such that $q_i(m_i') > 0$, $P_i(\theta_i | \mu_i(\theta_i) = m_i') \in (0,1)$ and $P_i(\theta_i | \mu_i(\theta_i) = m_i') + P_i(\theta_i | q_i(\mu_i(\theta_i)) = 0) = 1$. Let $\Theta_i^* = \{\theta_i \in \Theta_i : q_i(\mu_i(\theta_i)) = 0\}$.

To prove part (i), suppose to the contrary that both legislators 1 and 2 are informative in (μ, γ, π), and consider the following two cases.

(a) Suppose $\pi(m_1', m_2')$ excludes 1. Consider any $\tilde{m}_1 \in M_1$ such that $q_1(\tilde{m}_1) = 0$. Since $P_2(\theta_2 \in \Theta_2 | \mu_2(\theta_2) = m_2') > 0$, $\pi(\tilde{m}_1, m_2')$ excludes 1. Thus, in an SME, $\pi(m_1', m_2') = \pi(\tilde{m}_1, m_2')$. Note that this is true for any $\tilde{m}_1$ with $q_1(\tilde{m}_1) = 0$. Since $q_2(m_2') > 0$ and $P_1(\theta_1 | \mu_1(\theta_1) = m_1') + P_1(\theta_1 | q_1(\mu_1(\theta_1)) = 0) = 1$, we have $q_2(m_2') = 1$.

Since $P_1(\theta_1 | \mu_1(\theta_1) = m_1') > 0$, we have $\pi(m_1', \mu_2(\theta_2))$ excludes 2 for all $\theta_2 \in \Theta_2^*$, and therefore $\pi(m_1', \mu_2(\theta_2))$ is the same for all $\theta_2 \in \Theta_2^*$. Since $\pi(m_1', m_2')$ excludes 1 and $q_1(m_1') > 0$, we have $P_2(\theta_2 | \theta_2 \in \Theta_2^*$ and $\pi(m_1', \mu_2(\theta_2))$ includes $1) > 0$. Hence $\pi(m_1', \mu_2(\theta_2))$ includes 1 for any $\theta_2 \in \Theta_2^*$. Consider the payoff of type $\theta_2 \in \Theta_2^*$ by sending m_2' and $\mu_2(\theta_2)$. Lemma 4 and Lemma 5 imply that if legislator 2 is included, his payoff is weakly higher than $u_2(s, \theta_2)$ and if he is excluded, his payoff is weakly lower than $u_2(s, \theta_2)$. Since $q_2(m_2') = 1$, by sending m_2', type θ_2's expected payoff is weakly higher than $u_2(s, \theta_2)$. Since $\pi(m_1', \mu_2(\theta_2))$ includes 1 and $P_1(\theta_1 | \mu_1(\theta_1) = m_1') > 0$, by Lemma 5, the expected payoff type θ_2 by sending $\mu_2(\theta_2)$ is strictly lower than $u_2(s, \theta_2)$, and hence he has an incentive to deviate and send m_2', a contradiction.

(b) Suppose $\pi(m_1', m_2')$ includes 1. Then the same arguments as in case (a) show that $q_1(m_1') = 1$ and any $\theta_1 \in \Theta_1^*$ is strictly better off by sending m_1' than $\mu_1(\theta_1)$, a contradiction.

To prove part (ii), suppose to the contrary that legislator 2 is informative. Consider the payoff of type $\theta_2 \in \Theta_2^*$ by sending m_2' and $\mu_2(\theta_2)$. Pick any m_1. If both $\pi(m_1, m_2')$ and $\pi(m_1, \mu_2(\theta_2))$ exclude 2, then $\pi(m_1, m_2') = \pi(m_1, \mu_2(\theta_2))$ and type θ_2 gets the same payoff by sending m_2' and $\mu_2(\theta_2)$. If $\pi(m_1, m_2')$ includes 2 and $\pi(m_1, \mu_2(\theta_2))$ excludes 2, then by Lemma 5, legislator 2 is pivotal with respect to $\pi(m_1, m_2')$ and gets a payoff weakly higher than $u_2(s, \theta_2)$. Since $e(\hat{y}_1) < e(\hat{y}_2)$, by Lemmas 4 and 5, $u_2(\pi(m_1, \mu_2(\theta_2)) < u_2(s, \theta_2)$. Since $q_2(m_2') > 0$ and $q_2(\mu_2(\theta_2)) = 0$, we have $q_2(m_2') = P_1(\theta_1 | \pi(\mu_1(\theta_1), m_2')$ includes 2, $\pi(\mu_1(\theta_1), \mu_2(\theta_2))$ excludes $2) > 0$. Hence any type $\theta_2 \in \Theta_2^*$ is strictly better off by sending m_2' than by sending $\mu_2(\theta_2)$, a contradiction.

■

Supplementary Appendix

Lemma 7. *Suppose $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, $c > 0$ and θ_1 is uniformly distributed on $[\underline{t}_1, \bar{t}_1] \subseteq \Theta_1$, where $\bar{t}_1 > \underline{t}_1$. Let G_1 be the cumulative distribution function of θ_1 and let $W(\theta_1) = V(\theta_1) G_1(\theta_1) + u_0(s)(1 - G_1(\theta_1))$.*

(i) If $\hat{y}_1 \leq \tilde{y}$, then $\bar{t}_1 = \arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$.

(ii) If $\hat{y}_1 > \tilde{y}$, then the solution to $\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$ is generically unique.

Proof. Without loss of generality, let $\tilde{y} = 0$. Recall that $z^1(\theta_1) = \left(y^1(\theta_1); x^1(\theta_1)\right)$ denotes the chair's optimal proposal under complete information when she faces legislator 1 only. When $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, it is straightforward to show that $y^1(\theta_1) = \min\{(\theta_0 \hat{y}_0 + \theta_1 \hat{y}_1)/(\theta_0 + \theta_1), e(\hat{y}_1)\}$ and $x_1^1(\theta_1) = \max\{\theta_1 \left(v(\tilde{y}, \hat{y}_1) - v(y^1(\theta_1), \hat{y}_1)\right), 0\}$. Hence, if $(\theta_0 \hat{y}_0 + \theta_1 \hat{y}_1)/(\theta_0 +$

$\theta_1) > e(\hat{y}_1)$, then $V(\theta_1) = c - \theta_0(e(\hat{y}_1) - \hat{y}_0)^2$, and $V'(\theta_1) = 0$; and if $(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) \leq e(\hat{y}_1)$, then

$$V(\theta_1) = c - \theta_1(-\frac{\theta_0\hat{y}_0 + \theta_1\hat{y}_1}{\theta_0 + \theta_1})(2\hat{y}_1 - \frac{\theta_0\hat{y}_0 + \theta_1\hat{y}_1}{\theta_0 + \theta_1}) - \theta_0(\frac{\theta_0\hat{y}_0 + \theta_1\hat{y}_1}{\theta_0 + \theta_1} - \hat{y}_0)^2.$$

In this case $V'(\theta_1) < 0$. This follows because

$$V'(\theta_1) = -\left(v(\tilde{y}, \hat{y}_1) - v(y^1(\theta_1), \hat{y}_1)\right) < -\left(v(\tilde{y}, \hat{y}_1) - v(e(\hat{y}_1), \hat{y}_1)\right) = 0.$$

Here the first equality is by the envelope theorem, and the inequality is true because $v(y^1(\theta_1), \hat{y}_1) < v(e(\hat{y}_1), \hat{y}_1)$, and the second equality follows from the definition of $e(\hat{y}_1)$.

Note that

$$W'(\theta_1) = \frac{V'(\theta_1)\,\theta_1 - V'(\theta_1)\,\underline{t}_1 + V(\theta_1) - u_0(s)}{\bar{t}_1 - \underline{t}_1}.$$

Part (i): Suppose $\hat{y}_1 \leq \tilde{y}$. It suffices to show that $W'(\theta_1) > 0$ for all $\theta_1 \in [\underline{t}_1, \bar{t}_1]$.

Since $V'(\theta_1) \leq 0$, to show that $W'(\theta_1) > 0$, we only need to show that $V'(\theta_1)\,\theta_1 + V(\theta_1) - u_0(s) > 0$. If $(\theta_0\hat{y}_0+\theta_1\hat{y}_1)/(\theta_0+\theta_1) > e(\hat{y}_1)$, then $V'(\theta_1)\,\theta_1+V(\theta_1)-u_0(s) > 0$ since $V'(\theta_1) = 0$ and $V(\theta_1) - u_0(s) > 0$. Hence $\bar{t}_1 = \arg\max W(\theta_1)$. If $(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) \leq e(\hat{y}_1)$, then $y^1(\theta_1) = (\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) \leq e(\hat{y}_1)$ and

$$V'(\theta_1)\,\theta_1 + V(\theta_1) - u_0(s) = c + \theta_0 y^1(\theta_1)^2 + 2\theta_1 y^1(\theta_1)\hat{y}_1.$$

Since $c > 0, \theta_0 > 0$, $\theta_1 > 0$, and $y^1(\theta_1) \leq e(\hat{y}_1) \leq \hat{y}_1 \leq \tilde{y} = 0$ it follows that $V'(\theta_1)\,\theta_1 + V(\theta_1) - u_0(s) > 0$ and therefore $W'(\theta_1) > 0$.

Part (ii): Suppose $\hat{y}_1 > \tilde{y}$. Since $c > 0$, if $V'(\theta_1) = 0$, then $W'(\theta_1) > 0$, and therefore if $\theta_1 \neq \bar{t}_1$, then $\theta_1 \neq \arg\max W(\theta_1)$. We next show that for θ_1 such that $V'(\theta_1) < 0$, the second derivative of $W(\theta_1)$ crosses 0 only once, from below, which implies that there is at most one interior maximum. It is straightforward to verify that

$$\begin{aligned} W''(\theta_1) &= \frac{V''(\theta_1)\,(\theta_1 - \underline{t}_1) + 2V'(\theta_1)}{\bar{t}_1 - \underline{t}_1} \\ &= \frac{2\left(\theta_1\hat{y}_1^2\left(3\theta_0\theta_1 + 3\theta_0^2 + \theta_1^2\right) + C\right)}{(\theta_0 + \theta_1)^3\,(\bar{t}_1 - \underline{t}_1)} \end{aligned}$$

where C does not depend on θ_1. Hence, if $W''(\theta_1) > 0$, then $W''(\theta_1') > 0$ for any $\theta_1' > \theta_1$, i.e., $W''(\theta_1)$ crosses 0 at most once and from below. Consider the following two possibilities.

(a) Suppose $W''(\theta_1) > 0$ for all $\theta_1 \in [\underline{t}_1, \bar{t}_1]$ such that $V'(\theta_1) < 0$. Then $W(\theta_1)$ does not have an interior maximum, and therefore $\bar{t}_1 = \arg\max W(\theta_1)$.

(b) Suppose $W''(\theta_1) = 0$ for some $\theta_1 \in [\underline{t}_1, \bar{t}_1]$ such that $V'(\theta_1) < 0$. Then there is at most one interior maximum of $W(\theta_1)$ at $\tilde{\theta}_1$ where $W'(\tilde{\theta}_1) = 0$ and $W''(\tilde{\theta}_1) < 0$. Unless $W(\tilde{\theta}_1) = W(\bar{t}_1)$, which happens only non-generically, $W(\theta_1)$ has a unique maximum either at $\tilde{\theta}_1$ or at $\bar{t}_1$. ■