

# Elections, Protest, and Alternation of Power\*

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## Abstract

Despite many examples to the contrary, most models of elections assume that rules determining the winner will be followed. We present a model where elections are solely a public signal of the incumbent popularity, and citizens can protest against leaders that do not step down from power. Compliance with electoral rules is possible when citizens are well-informed enough about the preferences of others to coordinate on either massive protests or supporting the incumbent for close election results. Such coordination is possible when the election result is highly informative. Less informative elections can also induce the incumbent to step down independent of the electoral rules if she performs poorly, but unlike the case of rule-based alternation this often requires citizens to actually take to the streets. Thus the information generated by elections is required for peaceful and rule-based alternation of power. An extension provides an explanation of why reports of electoral fraud are often central to post-election protests, and election monitoring may be required for electoral rules to be enforceable.

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# 1 Introduction

Why do incumbent politicians ever cede office voluntarily? After all, most models of elections begin with the assumption that politicians are office seeking, so why do we then assume that these same actors will simply give up power when an electoral result indicates they should do so? Of course, we know incumbents often do give up power after elections, as George H.W. Bush (United States, 1992), Nicholas Sarkozy (France, 2012), Eduard Shevardnadze (Georgia, 2003) and Slobodan Milosevic (Serbia, 2006) can readily attest.

However, the first two cases differ from the second two on several crucial and inter-related dimensions. First, Bush and Sarkozy stepped down following clear electoral defeat, and there was no serious doubt that they would accept this result; in contrast Milosevic and Shevardnadze initially claimed victory. Second, Milosevic and Shevardnadze did not step down until citizens took to the streets to protest, while the turnover in the first two cases was immediate and entirely peaceful.<sup>1</sup> Third, these protests were in part spurred by accusations of massive electoral fraud, while the U.S. and French elections were widely considered to be clean. Highlighting the importance of these distinctions, when Shevardnadze's successor Mikheil Saakashvili immediately conceded defeat for his party following Georgia's (relatively clean) 2012 parliamentary election, it was hailed as the first democratic transfer of power in the Caucasus.

Given these differences, it is not surprising that the political science and game theoretic literatures have approached these kinds of turnover in separate research agendas. The study of democratic political competition is one of the most prominent and successful applications

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<sup>1</sup>The protests that ousted Milosevic and Shevardnadze were relatively peaceful compared to other electoral revolutions such as Kyrgyzstan in 2005 and even more extreme examples of post-election violence such as the civil war in the Ivory Coast in 2010-2011. While acknowledging these differences, we use "peaceful" and "without protest" interchangeably, as these are the outcomes our model is more suited to differentiate between.

of game theory to political science, but has largely ignored fundamental questions like why only some elections present a real chance of the incumbent party losing, why losing parties comply with electoral rules, and when citizens (or elites) are able to oust leaders who do not follow the rules. More recent work on “authoritarian elections” and post-election protest does address such questions, but generally treats these elections as a separate animal from the more familiar democratic cases. This bifurcation is problematic in light of the large number of contemporary regimes that hold elections which do not fit neatly into the democratic or autocratic bin.

In this paper, we present a single model that encompasses both “democratic” (e.g., Bush, Sarkozy) and “semi-democratic” (e.g., Milosevic, Shevardnadze) incumbent turnover, where all three distinctions highlighted above emerge in equilibrium. Perhaps surprisingly, we are able to recover all of these properties with an extremely minimal treatment of the role of elections. Specifically, we assume the election is nothing but a public signal of the incumbent popularity, an idea which has recently been used in many recent and contemporaneous game theoretic models of non-democratic elections (Londregan and Vindigni, 2006; Cox, 2009; Egorov and Sonin, 2011; Gehlbach and Keefer, 2011; Little, 2012, 2011*b*; Rozenas, 2012; Svobik and Chernykh, 2012). The incumbent can step down immediately following the revelation of the election result, and if she does not step down citizens can take to the streets to protest. Following the protest, the incumbent has a second opportunity to step down.

Our argument hinges on when citizens are well informed enough about the beliefs of others to coordinate on either supporting or ousting the incumbent; i.e., when there are multiple equilibria in the stage of the game following the incumbent decision to cling to power. When the election generates a high level of information about the incumbent popularity, citizens are better able to coordinate on either supporting or protesting against the incumbent for close election results. So, it is possible that a codified legal threshold – say, 50% of the vote – acts as a focal point determining whether citizens support the incumbent should she claim

victory (when the result is above the legal threshold) or coordinate on massive protest when she earns less than 50% of the vote but attempts to stay in power. We say that equilibria of this form exhibit “rule-based” or “democratic” turnover.<sup>2</sup>

When citizens can not coordinate on multiple strategies that make it possible to keep or oust the incumbent – which tends to happen when the election result is not too informative – the incumbent may still step down following the protest or even to pre-empt protest if the election result signals that she is sufficiently weak. However, we argue that turnover in this manner should not be called democratic – even if the incumbent steps down immediately – as the critical election result that determines whether this occurs is a function of the exogenous parameters of the model (e.g., the cost of protest, the prior belief about the incumbent popularity). Thus the election result that determines who wins will not necessarily and generically won’t correspond to a codified electoral rule. This contrasts with the case of democratic turnover, where an equilibrium in which the citizens choose a protest strategy high enough to induce the incumbent to step down if and only if the codified rule is reached can be robust to changes in exogenous parameters like the cost of protest. As a result, we term turnover in this manner “semi-democratic.”

The second important contrast between these cases lies in the amount of protest on the equilibrium path. In the case with a high level of uncertainty and semi-democratic turnover, the incumbent will often “wait things out” and see how big the protest is before stepping down, consistent with the motivating examples of Shevardnadze and Milosevic. In the equilibrium with rule-based alternation of power, protest tends to be minimal on the equilibrium path, as citizens either coordinate on a low level of protest or credibly threaten to protest enough that the incumbent steps down before this happens. That is, peaceful alternation of power is possible when it is common knowledge that not complying with

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<sup>2</sup>Of course by nearly any definition compliance with electoral rules is a necessary but not sufficient condition for elections to be deemed democratic, but since this is the aspect of elections we focus on we use the phrase “democratic” turnover to describe this phenomenon.

electoral rules would trigger a massive protest, and this common knowledge is facilitated by the public information generated by elections.

Finally, we present an extension to the model which includes electoral fraud, a key component of many post-election protests. To do so, we incorporate uncertainty about how much of the election result was driven by electoral fraud, but give citizens a public signal of the level of fraud, which could correspond to media coverage or reports from international monitoring groups. Consistent with observed behavior, signals indicating high levels of fraud make citizens more apt to take to the streets. Further, if the uncertainty induced by fraud renders the electoral signal less informative, then monitoring may be required to facilitate compliance with electoral rules.

In sum, our central results are that (1) democratic and semi-democratic alternation of power can emerge from the same model, (2) the amount of information conveyed by the election result is a key parameter determining when democratic alternation is possible, (3) democratic alternation tends to be peaceful, while semi-democratic alternation often requires citizens to actually take to the streets, and (4) reports of rampant electoral fraud will lead to higher levels of protest and more accurate documentation of fraud – potentially by external actors – can help facilitate rule-based alternation of power.

The rest of the paper is organized as follows. Section 2 places our argument in the context of existing work on rule of law, coordination under uncertainty, and models of elections as public signals. Section 3 presents the main model and core results on when rule-based alternation of power is possible. Section 4 examines when protest occurs on the equilibrium path, and section 5 contains the extension that incorporates electoral fraud. Section 6 concludes. Proofs and other technical derivations can be found in the appendix

## 2 Past Work

The possibility or realization of leaders relinquishing power following an election plays a central role in prominent theoretical and operational definitions of democracy (Przeworski, 1991; Przeworski et al., 2000; Cheibub, Gandhi and Vreeland, 2010). However, most formal theories of democratization do not emphasize alternation of power, but decisions such as expanding the franchise (Acemoglu and Robinson, 2000; Bueno de Mesquita and Smith, 2009) or the decision to hold elections (Cox, 2009; Little, 2011*b*).

Two prominent explanations for compliance with electoral rules are that parties are willing to concede defeat when they will have a chance to compete for power again in the future (Przeworski, 1991, 2005) or if citizens can coordinate on mass protest against those who don't follow the rules (Fearon, 2011). We build on the latter by showing how uncertainty can undermine citizens ability to coordinate on punishing those who don't step down, highlighting the informational nature of elections (and the threat posed by fraud) as a central feature making them self-enforcing. In doing so, we show how elections may lead to leaders stepping down *not* as the rules dictate, building a tighter connection between “self-enforcing” democratic elections and less-than-democratic elections.

Our model brings together three lines of argument that have seen little overlap. First, we draw on the idea that laws – whether related to elections or not – can be interpreted as an equilibrium selection device in a coordination game (Calvert, 1995; Weingast, 1997; Mailath, Morris and Postlewaite, 2001; Hardin, 2003; Myerson, 2008, 2013; Dragu and Polborn, 2013). Put succinctly, “self-enforcing rules for a political system can be constructed arbitrarily from the equilibrium selection problem” in a simple, complete information coordination game (Myerson, 2013). In our context, even an “election result” that spat out a random number could be obeyed under the self-enforcing belief that others will support whichever candidate is associated with the observed random number.

While the simplicity of this argument is appealing, the second major idea we utilize – from a literature generally falling under the umbrella term “global games” – is that that without the assumption of complete information coordination games can have a unique equilibrium (two seminal references are Carlsson and van Damme 1993; Morris and Shin 2003; for related applications to political science see Dewan and Myatt 2008; Bueno De Mesquita 2010; Shadmehr and Bernhardt 2011; Little 2012; Hollyer, Rosendorff and Vreeland 2013; Tyson and Smith 2013).<sup>3</sup> If uncertainty undermines citizen’s ability to coordinate on supporting on different leaders, an election result that is unrelated to any fundamental political parameter such as how many citizens dislike the incumbent would be ignored when citizens decide which party to support.

The third major literature we draw on argues that elections – even in less-than-democratic settings – *do* generate public information (Magaloni, 2006; Londregan and Vindigni, 2006; Cox, 2009; Blaydes, 2011; Egorov and Sonin, 2011; Little, 2011*b*, 2012; Gehlbach and Simpser, 2013), including information about the incumbent strength or popularity.<sup>4</sup> Closest to our model, Egorov and Sonin (2011) and Little (2012) begin with the same premise, where an election result is a public signal that affects a group of citizens’ decision to coordinate on anti-regime action.<sup>5</sup> However, both of these models do not consider the case where the election result is informative enough that the model exhibits multiple equilibria, which is

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<sup>3</sup>There is no commonly accepted technical definition for what constitutes a global game. To point to a common reference, the payoff structure of our model meets the listed assumptions in section 2.2.1 of Morris and Shin (2003), but given the prior and election result citizens do not have a Laplacian belief about the incumbent popularity before receiving their private signal. In fact, this public information is precisely the reason why we do not always obtain uniqueness unlike the main result in that section.

<sup>4</sup>This literature has primarily focused on aspects of elections we treat as exogenous, such as why elections are held (Magaloni, 2006; Cox, 2009; Egorov and Sonin, 2011; Little, 2011*b*, 2012), the degree to which elections are manipulated (Gandhi and Przeworski, 2009; Simpser, 2011; Little, 2011*a*; Gehlbach and Simpser, 2013), or the degree of international or domestic monitoring of elections (Hyde, 2012; Little, 2011*a*). Several recent models examine post-election protest in this framework (Kuhn, 2012; Svobik and Chernykh, 2012; Rozenas, 2012), but treat the opposition and/or citizenry as a unitary actor, abstracting away from the coordination problem central to our argument.

<sup>5</sup>For a related argument about transparency of economic data see (Hollyer, Rosendorff and Vreeland, 2013)

precisely what we argue is required for electoral rules to be enforceable.<sup>6</sup> Further, these models do not treat the incumbent decision to step down as a strategic choice, so there is minimal overlap with our main conclusions.

In a sense, our approach turns the initial impetus of the global games literature on its head. Rather than treating multiplicity of equilibria as a “straitjacket” (Morris and Shin, 2003, pg. 58) to be escaped by introducing incomplete information, we seek to identify how public information generated by elections can bring back the multiplicity of equilibria required to make rule of law – in particular, electoral rules – enforceable. This highlights how the existence of multiple equilibria is necessary for “self-enforcing democracy”, which is implicit but not directly acknowledged in the seminal work on this topic (Przeworski, 1991, 2005; Fearon, 2011). In doing so, we also demonstrate why the informativeness of elections – which has been more prominently linked to those outside democratic regimes – is particularly important for this phenomenon.

The model also contributes to a debate over whether poor electoral showings can cause the fall of authoritarian regimes and democratization more generally (e.g., Lindberg, 2006; Bunce and Wolchik, 2011) or simply correlated with weak regimes that would fall in the absence of elections (Brownlee, 2007). While some of the association between the election result and protest size (and hence the likelihood of the incumbent stepping down) predicted by the model comes from both the result and protest size being affected by the underlying anti-regime sentiment, the election result does have two effects that can be comfortably deemed causal. First, the coordination dynamics of the model imply that moderate citizens will be more apt to protest when they think others will join. Since a low election result indicates more citizens will be protesting, moderates with less extreme anti-regime beliefs will be willing to take to the streets as well when the incumbent does poorly. Second, by

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<sup>6</sup>Others have argued that public information plays an important role in coordination games outside of the global games literature (e.g., Kuran, 1989; Lohmann, 1994; Chwe, 1998).



generating common knowledge about whether other citizens dislike the regime, citizens are better able to coordinate on ousting the incumbent than they would be in the absence of the electoral signal.

We draw most of our substantive examples from the “colored revolutions” in post-communist countries over the last decade, which have been a source of much scholarship on the relationship between elections, electoral fraud, and protest (Bunce and Wolchik, 2011). In particular, Tucker (2007) illustrates how electoral fraud can be a potent focal point for solving collective action problems among citizens living under abusive regimes, thus helping to justify our focus in this model on *post-election* protests.<sup>7</sup> Moreover, in comparative assessments of the factors that lead to successful colored revolutions, multiple authors highlight the importance of election results as either galvanizing or deflating an opposition movement (McFaul, 2005; Bunce and Wolchik, 2011). Even more recently, the reaction of the Russian opposition to Putin’s unexpectedly strong election results in 2012 (as opposed to weaker results in the 2011 parliamentary elections from the ruling United Russia party) has been noted by academic bloggers as having a potential deflationary effect on the nascent Russian opposition movement.<sup>8</sup>

### 3 The Model

The actors in the model are an incumbent denoted  $I$ , and  $N$  citizens, indexed by  $j$ .<sup>9</sup> We refer to the incumbent with the pronoun “she” and citizens with “he.”

Practically speaking, even large post-election protests include only a fraction of the total population, so the citizens modeled here are better conceptualized as those that could plau-

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<sup>7</sup>Meirowitz and Tucker (2013) add a dynamic element to this approach, but they model the protester as a single agent and not as a collective of agents as we do here.

<sup>8</sup><http://themonkeycage.org/blog/2012/03/05/russia-2012-presidential-election-post-election-report/>

<sup>9</sup>We could also think of this group of citizens as a collection of elites who have the power to oust the regime or bring other citizens (followers) out into the street. For the sake of clarity, however, we refer to these people simply as citizens.

sibly protest.<sup>10</sup> Still, the size of protests are generally large enough that citizens are unlikely to instrumentally affect the outcome of the protest (i.e., “pivotality” type concerns), so to abstract away from this consideration we analyze the model for a finite but arbitrarily large number of citizens, i.e., as  $N \rightarrow \infty$ .

Whether citizens want to protest or not depends in part on how much they like or dislike the current government, which is modeled by giving each citizen a *regime sentiment*  $\theta_j$ . We write individual regime sentiments as  $\theta_j = \omega + \nu_j$ , where  $\omega$  is the average assessment of the leader – analogous to an approval rating in a public opinion poll – and  $\nu_j$  an idiosyncratic component capturing whether or not that particular individual likes the leader more or less than average. Citizens with negative or anti-regime sentiment ( $\theta_j < 0$ ) will generally want to protest to force the leader to step down if necessary, while those with positive or pro-regime sentiment ( $\theta_j > 0$ ) will not want to protest against the regime. As citizens only observe their own regime sentiment, we will often refer to  $\theta_j$  as citizen  $j$ ’s *private signal*.

To capture the notion that citizens are uncertain about how much others like to dislike the regime in a tractable fashion, we assume that the citizens share a common prior belief over the average incumbent popularity that is normally distributed with mean  $\mu_0$  and precision  $\tau_0$  (i.e., variance  $1/\tau_0$ ), and that the idiosyncratic terms ( $\nu_j$ ’s) are normally distributed with mean 0 and precision  $\tau_\theta$ . The prior on the incumbent popularity could reflect information from previous elections as well as other information about the regime’s performance from economic data or media reports.<sup>11</sup>

By standard rules of Bayesian updating, citizens that personally like the regime (high  $\theta_j$ ) will tend to think that others like the regime as well while those observing low private signals will tend to think the regime is unpopular. Still, since citizens do not know exactly how

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<sup>10</sup>Consider for example the recent Russian post-election protests of 2011-12. Even if we accept the high end of estimates for the number of Russians who participated in these protests at between 250,000-300,000 people, that is still less than 1% of the population of a country with over 140 million people.

<sup>11</sup>See Hollyer, Rosendorff and Vreeland (2013) for a related model of transparency

much their belief deviates from the average, they will always have some uncertainty about the degree to which others dislike the regime, and, of more direct relevance, are willing to take to the streets to protest.

The election result ( $e$ ) is simply a noisy public signal of the average popularity, given by  $e = \omega + \nu_e$ .<sup>12</sup> To keep the focus on when citizens protest and when the incumbent steps down, we do not provide a micro-foundation for why the election result is informative, which can result from combination of citizens voting sincerely or voting strategically to signal their discontent and help spur protest.<sup>13</sup>

The random component ( $\nu_e$ ) accounts for any factors that might affect the election result independent of incumbent popularity, e.g., uncertainty over how closely those turning out to vote resemble the population at large (or the potential protesters). In an extension we consider the possibility that the election result is also affected by electoral fraud – which is a key source of uncertainty about how the election result reflects reality and a driving force behind post-election protest – but to keep the interpretation of our main model as simple and general as possible we do not yet specify exactly what causes the election to be noisy. As with the other random components of the model we assume  $\nu_e$  is normally distributed, with mean 0 and precision  $\tau_e$ . The jointly normal information structure has convenient properties derived below.<sup>14</sup>

Just as citizens learn about whether other citizens like the regime by their private signal,

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<sup>12</sup>An unsatisfying aspect of this specification is that the election result can be any real number, as opposed to representing something more concrete like the incumbent vote share on  $[0, 1]$ . However, all of the analysis here would apply (with clunkier notation) should the citizens instead observe a vote share  $s(e)$  where  $s : \mathbb{R} \rightarrow [0, 1]$  is a strictly increasing function.

<sup>13</sup>For more detailed discussion of this point and related arguments see (Lohmann, 1993; Meirowitz and Shotts, 2009; Little, 2011*b*; Fearon, 2011; Egorov and Sonin, 2011).

<sup>14</sup>The assumption that the idiosyncratic component to the private signals ( $\nu_j$ 's) is independent across individuals is potentially problematic. We might expect that these signals would be more strongly correlated among citizens that are “close” to each other, either geographically (especially when there is a strong regional component to politics, such as in Ukraine) or through social ties (e.g., some candidates may be more popular among adherents of particular religions); see Dahleh et al. (2012) for a global games model incorporating such social networks of information exchange. Still, we do not see how a more complicated information structure would change our main conclusions, and hence proceed with the more tractable formulation.

they also learn about other citizens' preferences by the election result. In fact, this is the only direct role played by the election in the model. However, as we will demonstrate, generating public information about the preferences others – and the precision of this information – can have important effects on qualitative nature of the model's equilibrium.

After observing the election result, the incumbent has her first opportunity to step down (alternatively, “yield”), in which case the game ends. The incumbent makes this decision regardless of whether or not she “wins” the election. In fact, a major feature of the model is that we do not assume any notion of winning into the payoffs or any other aspect of the model; when compliance with an electoral rule emerges, it will be a result of equilibrium behavior. If the incumbent does not step down, the game proceeds and citizens decide whether or not to protest. Let  $a_j$  denote the decision to protest or not for citizen  $j$ , where  $a_j = 1$  means protesting and  $a_j = 0$  not protesting. Denote the proportion of protesting citizens with  $\rho \equiv \sum_{j=1}^N a_j/N$ . After observing the protest level, the incumbent has a second opportunity to step down. If the incumbent steps down at the second opportunity, we say the protest succeeds.

The incumbent payoff is:

$$u^I(\rho) = \begin{cases} y^I & \text{if stepping down before protest} \\ y^I - \gamma\rho & \text{if stepping down after protest} \\ 1 - \rho & \text{otherwise} \end{cases}$$

That is, the incumbent payoff is a function of whether she yields and the size of the protest. She gets a partial payoff of  $y^I$  from stepping down and a payoff normalized to 1 when not stepping down. If protest occurs, the incumbent pays a cost that is linearly increasing in the size of protest:  $-\gamma\rho$  if she steps down after protest and  $-\rho$  if she does not step down at all.

We assume that  $0 < y^I < 1$  and  $0 < \gamma < y^I$ . This implies that the incumbent always prefers to stand firm if the amount of protest is small ( $1 - \rho > y^I > y^I - \gamma\rho$  for small  $\rho$ ) and prefers to step down if the (anticipated) level of protest is sufficiently high. When the  $\gamma$  parameter is low, the incumbent can greatly lessen the impact of the protests by stepping down – say, she will not be prosecuted – while if  $\gamma$  is higher the incumbent still harmed by the level of protest when stepping down, though still less so than when not yielding at all. For a fixed level of protest, the incumbent always gets a higher payoff from stepping down right away rather than after the protest. However, as demonstrated below, she will be uncertain about the size of protest and hence will sometimes “wait things out” and see how many citizens take to the streets before stepping down.

The citizens’ utility function if the incumbent does not step down immediately after the election are similar to those in Egorov and Sonin (2011) and are a special case of the utility function in Little (2012).<sup>15</sup> These payoffs depend on their regime sentiment ( $\theta_j$ ), protest decision ( $a_j$ ), and whether or not the protest succeeds. In particular, the citizen payoffs are summarized in the following table:

	Protest Fails	Protest Succeeds
Stay Home ( $a_j = 0$ )	0	$-b_2\theta_j$
Protest ( $a_j = 1$ )	$-b_1\theta_j - c$	$-b_3\theta_j - c$

The payoff to staying home when the incumbent does not step down is normalized to 0. If a citizen stays home and the incumbent yields he gets a payoff of  $-b_2\theta_j$  for some  $b_2 \geq 0$ . This reflects the fact that citizens who like the regime ( $\theta_j > 0$ ) will not want to see the incumbent lose power, while those who dislike the regime ( $\theta_j < 0$ ) want to see the incumbent step down even if they do not themselves protest. Citizens that protest pay a fixed cost  $-c < 0$ .<sup>16</sup>

<sup>15</sup>All of the results would require only minor technical caveats when using the more general payoffs. We use this less general formulation for simplicity, and to make comparative statics on the cost of protest more clear.

<sup>16</sup>If the cost of protest is decreasing in the number of participants, this will only reinforce the coordination dynamics analyzed here.

Protesting against a regime that does not ultimately step down gives a payoff of  $-b_1\theta_j - c$  for some  $b_1 > 0$ , where the first term reflects the “expressive” value of protest. That is, citizens with more intense anti-regime beliefs like protesting more (compared to staying home) even if the protest does not succeed. Finally, participating in a successful protest gives payoff  $-b_3\theta_j - c$ , where the  $b_3 > 0$  term again reflects the fact that citizens that dislike the regime more get a higher benefit from joining a successful protest.<sup>17</sup> If the incumbent steps down right away the citizens get the same payoff as they would if the incumbent steps down while they stay home ( $-b_2\theta_j$ ), though this does not affect equilibrium behavior.

It is natural to assume that  $b_3 > b_2$  – i.e., the benefit to participating (compared to staying home) for a citizen that dislikes the regime (negative  $\theta$ ) is increasing in the magnitude of his distaste. Similarly, a natural assumption is that  $b_3 > b_1$ , which implies that value of protesting for citizens that dislike the regime is larger when the incumbent eventually steps down. Further, we assume that  $b_3 > b_1 + b_2$ , which implies that the relative value of protesting (compared to staying home) is increasing more in the citizen’s anti-regime sentiment when the protest indeed succeeds. This is one way to capture the coordination dynamics we wish to model: as more citizens take to the streets, the incentives to join the protest increase for those that dislike the regime.

We include the arguably nonstandard expressive payoff (i.e., the  $-b_1\theta_j$  term) for a combination of substantive and technical reasons. First, considerable amounts of psychological research indicate that people do derive intrinsic benefits from participating in political activity for causes they believe in (e.g., Schuessler, 2000). Second, while some protesters in democratic countries may believe their actions may oust the standing government, many others have no illusion that their actions will have such dramatic consequences. Given our

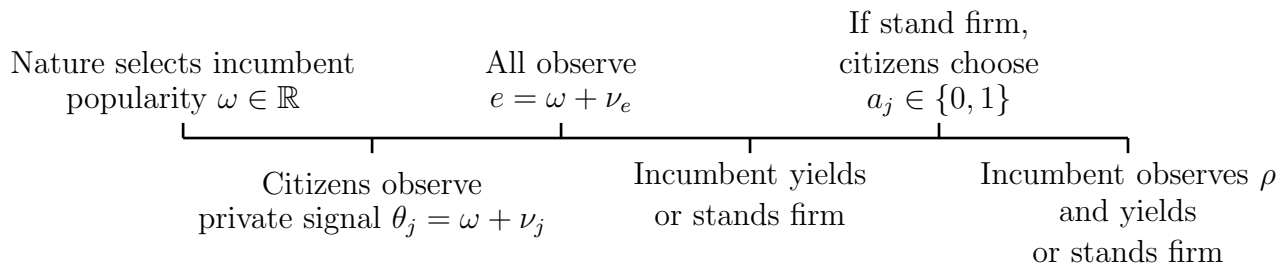
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<sup>17</sup>Some important aspects of protest that we abstract from include the role of opposition elites’ strategies (Bunce and Wolchik, 2011), how events other than elections can signal incumbent popularity (Tucker, 2007) or the possibility of learning across successive rounds of protest in a single country (Meirowitz and Tucker, 2013) or cross nationally (Bunce and Wolchik, 2011; Ash, 1999).

goal of creating a unified model of post-election protest in democratic and less than democratic contests, we would like citizen’s protesting without an expectation that it will induce the incumbent to step down to arise from the model as well. Our formulation does not imply that the citizens are insensitive to the material costs and benefits of protest – as we would expect, if the cost gets sufficiently high the proportion protesting always approaches zero – just that there is an expressive component to their decision as well. On the technical end, including this term (as well as the fact that  $b_3 > b_2$ ) implies a useful property of “two-sided limit dominance”: i.e., citizens who sufficiently dislike the regime always protest and those who sufficiently like the regime never protest.<sup>18</sup>

Formally, the citizen strategy is a mapping from the election result and his regime sentiment to the decision to protest or not. As is standard, we assume at the outset that for a fixed election result, citizens use a symmetric strategy of the form “protest if and only if  $\theta_j < \hat{\theta}(e)$ ”; i.e., if and only if their personal distaste for the regime is sufficiently strong. The incumbent strategy is the set of election results for which she steps down at the first opportunity and protest sizes for which she steps down at her second opportunity.<sup>19</sup>

To recap, the sequence of moves is:



<sup>18</sup>A more common way to attain this property is for citizens receiving extreme signals to be certain that the protest will “succeed” by reaching some critical mass even if none of the actors join. However, in the context of protesting after an election, we find it more intuitively plausible that citizens with extreme anti-regime beliefs protest because they derive an intrinsic benefit from participation that is not outweighed by its cost.

<sup>19</sup>The incumbent could condition this decision on the election result as well, but she has full information about her payoffs at this point since  $\rho$  is known and hence the election result cannot affect her behavior except in the probability 0 event that she is indifferent between stepping down and not.

We solve for Perfect Bayesian Equilibria with some additional restrictions elaborated below. Relying on sequential rationality, we first determine whether the incumbent would step down after observing the protest size, then solve for the protest size for each election result, and finally determine when the incumbent steps down before the protest can occur.<sup>20</sup>

### Stepping Down After Protest

If the incumbent does not step down immediately and observes protest level  $\rho$ , she knows with certainty that she will get payoff  $y^I - \gamma\rho$  if stepping down at the second opportunity and  $1 - \rho$  for standing firm. As a result, she steps down if and only if:

$$y^I - \gamma\rho \geq 1 - \rho \implies \rho > \frac{1 - y^I}{1 - \gamma} \equiv \rho^*$$

That is, the incumbent steps down if and only if the protest level is above a critical threshold  $\rho^*$ . As we assumed that  $\gamma < y^I < 1$ , this critical threshold is strictly less than 0 and strictly greater than 1. So, there are always some protest levels for which she steps down and some for which she does not. The incumbent is more apt to step down ( $\rho^*$  is lower) when she gets a higher payoff from stepping down (high  $y^I$ ) and when stepping down effectively mitigates the harm of being the target of protest (low  $\gamma$ ).

### The Protest Stage

When deciding whether to take to the streets, each citizen is uncertain about the actions of others, and hence whether the the protest will be large enough to succeed. If a citizen assigns probability  $q(\cdot)$  to the event that the protest is large enough to induce the incumbent

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<sup>20</sup>As the incumbent has no private information, citizens do not make inferences about her popularity from her decision.



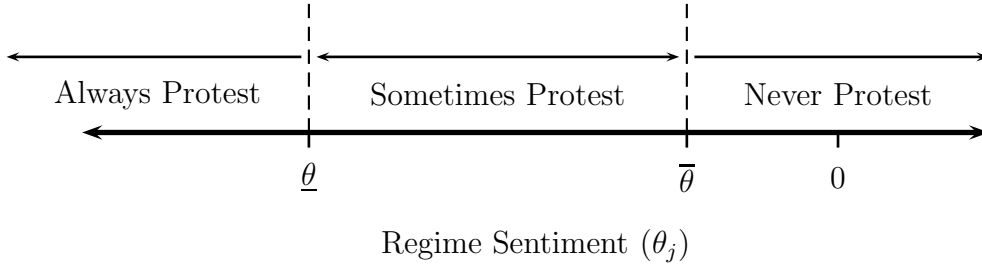
to step down, then he will protest if and only if:

$$q(\cdot)(-b_3\theta_j - c) + (1 - q(\cdot))(-b_1\theta_j - c) \geq q(\cdot)(-b_2\theta_j)$$

$$\theta_j \leq -\frac{c}{q(\cdot)(b_3 - b_2) + (1 - q(\cdot))b_1} \quad (1)$$

Figure 1 illustrates what we can say about the citizen  $j$ 's protest behavior regardless of what other citizens do (i.e., by eliminating dominated strategies). Since  $b_3 - b_2 > b_1$  and  $q(\cdot)$  is a probability and hence between 0 and 1, the RHS of equation 1 is negative, increasing in  $q(\cdot)$ , and bounded by  $[-\frac{c}{b_1}, -\frac{c}{b_3 - b_2}]$ . Let  $\underline{\theta} \equiv -\frac{c}{b_1}$  and  $\bar{\theta} \equiv -\frac{c}{b_3 - b_2}$ . So, citizens that like the regime ( $\theta_j \geq 0$ ) or have only lukewarm anti-regime sentiment ( $\bar{\theta} \leq \theta_j \leq 0$ ) will never protest and those who sufficiently dislike the regime ( $\theta_j < \underline{\theta}$ ) always protest.

Figure 1: Citizens with dominant strategies



For a citizens with a regime sentiment between  $\underline{\theta}$  and  $\bar{\theta}$ , the optimal protest decision depends on his belief about the likelihood that the protest succeeds. We refer to such citizens as *moderates*. In particular, the equilibrium condition is that when all other citizens use cutoff rule  $\hat{\theta}(e)$ , a citizen observing a private signal equal to the cutoff rule – or, the *marginal citizen* – assigns a probability of success that makes him indifferent between protesting and

not. Rearranging equation 1, write this as:

$$q(\theta_j = \hat{\theta}(e); \hat{\theta}(e)) \geq \frac{c + b_1 \hat{\theta}(e)}{\hat{\theta}(e)(b_1 + b_2 - b_3)} \quad (2)$$

The derivation of the probability the marginal citizen assigns to the protest succeeding (i.e., the LHS of equation 2) follows a standard calculation, which can be found in the appendix. In words, a fixed cutoff rule for election result  $e$  gives a critical regime popularity  $\omega'(e)$  such that the protest succeeds if and only if  $\omega < \omega'(e)$  as  $N \rightarrow \infty$ .<sup>21</sup> By the jointly normal structure of the prior and signals, citizens belief about  $\omega$  given  $e$  and their private signal is normal is well. So the probability of success  $q(\cdot)$  is then given by the probability that  $\omega < \omega'(e)$  for the citizen observing exactly  $\theta_j = \hat{\theta}(e)$ , which gives the following equilibrium condition:

$$\underbrace{\Phi \left( \frac{\hat{\theta}(e) - \frac{\tau_0 \mu_0 + \tau_e e + (\tau_0 + \tau_e + \tau_\theta) \tau_\theta^{1/2} \Phi^{-1}(\rho^*)}{\tau_0 + \tau_e}}{(\tau_0 + \tau_e + \tau_\theta)^{1/2} (\tau_0 + \tau_e)^{-1}} \right)}_{\text{Probability Protest Succeeds}} = \underbrace{\frac{c + b_1 \hat{\theta}(e)}{\hat{\theta}(e)(b_1 + b_2 - b_3)}}_{\text{Indifference Probability}} \quad (3)$$

where  $\Phi(\cdot)$  is the cumulative density function (CDF) of a standard normal random. As described above, the LHS of this equation is the probability that the marginal citizen attaches to the protest succeeding. This is continuous and increasing in the proposed equilibrium threshold  $\hat{\theta}(e)$ , as a higher threshold means citizens are more apt to protest. In particular, the LHS as a function of  $\hat{\theta}(e)$  is a normal CDF, so as  $\hat{\theta}(e) \rightarrow -\infty$  the probability of success for the marginal officers approaches 0 and as  $\hat{\theta}(e) \rightarrow \infty$  it approaches 1.

The RHS of equation 3 represents the probability of the protest succeeding at which a

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<sup>21</sup>The  $N \rightarrow \infty$  assumption greatly simplifies this step. For a fixed  $\omega$ , the probability that a citizen protests is  $\pi = Pr(\omega + \nu_j < \hat{\theta}(e))$ , and the number of protesters is a binomial random variable with  $N$  trials at probability  $\pi$  (which, from the perspective of the actors in the model is itself a random variable). However, for large  $N$  the proportion of protesters converges exactly to  $\pi$ , rendering this part of the calculation unnecessary. For small  $N$ , slightly more moderate citizens will protest as they would like to increase the chance of protest succeeding.

citizen with regime sentiment  $\hat{\theta}(e)$  is indifferent between protesting and not.<sup>22</sup> This function is also increasing, as moderates that dislike the regime less (higher  $\theta_j$ ) require a higher chance of success to join the protest. To intersect with the RHS this function must be on  $(0, 1)$ , and a consequence of the analysis above is that this function is strictly increasing from 0 to 1 on the interval  $[\underline{\theta}, \bar{\theta}]$ , and is not between 0 and 1 otherwise.

So, equation 3 can only be met for  $\hat{\theta}(e) \in [\underline{\theta}, \bar{\theta}]$  and must be met for at least one  $\hat{\theta}(e)$  in this interval. Such an intersection is an equilibrium strategy because the marginal citizen (i.e, a citizen observing  $\theta_j = \hat{\theta}(e)$ ) is indifferent between protesting and not. Citizens that dislike the regime more (lower  $\theta_j$ ) will get a higher relative payoff from protest and believe that more citizens are going to protest, making protest optimal. Citizens observing a higher private signal will get a lower relative payoff from protest and believe that the incumbent is less likely to step down, and hence stay home.

There is sometimes a unique threshold meeting the equilibrium condition and sometimes more than one. As is typical in related models, there can be multiple equilibria when the citizens have a large amount of public information about the incumbent popularity, which will be true when the election result is very informative. A broad intuition for why this is true is that when the election result is very informative, citizens have a better idea of what others think about the regime, making it easier for the moderates to coordinate on different protest levels. When citizens are in a highly uncertain environment, such coordination is more difficult, resulting in a single “intermediate” propensity to protest.

More precisely, consider the case when the election is completely uninformative and the prior on the regime’s popularity is very imprecise (i.e.,  $\tau_e \rightarrow 0$  and  $\tau_0 \rightarrow 0$ ). When this is true, citizens’ only knowledge about the regime’s popularity comes from their private signal. So, the marginal citizen will always think that he received a typical signal, and hence the

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<sup>22</sup>This interpretation is only valid on  $[\underline{\theta}, \bar{\theta}]$  where this expression is between 0 and 1, but this is the only range where an equilibrium is possible.

probability he assigns to the protest succeeding is fixed at some probability,  $\pi \in (0, 1)$ . As a result, there will be a unique equilibrium where moderates who are willing to protest if the probability of success is greater than  $\pi$  will do so and other moderates stay home.

Now, consider the other extreme where the election result is perfectly informative, meaning all citizens share the same belief about the regime's popularity and hence the distribution how much the other citizens like or dislike the regime. Therefore it is common knowledge how many citizens have a dominant strategy to protest or and how many have a dominant strategy to stay home. If the number of citizens with a dominant strategy to protest is enough to force the incumbent to step down (i.e. greater than  $\rho^*$ ) – which will happen when the election result is low – then the equilibrium is still unique as all of the moderates know the protest will succeed and hence join. If it is common knowledge that enough citizens have a dominant strategy to stay home that the protest will fail, then there is a unique equilibrium where all of the moderates stay home. However, when not enough citizens have a dominant strategy to guarantee the outcome – which will happen when the election result is intermediate – then there is an equilibrium where the moderates all join the protest and it succeeds as well as an equilibrium where the moderates all stay home and the protest fails.

Our first technical result generalizes these observations for the case where the election is informative but noisy. When the election result is not very informative, there is a unique equilibrium in the protest stage. When the election result *is* a very precise indicator of the regime's popularity and is not extreme enough to guarantee that the protest succeeds or fails, there are multiple equilibria; some where the moderates generally protest under the expectation that other moderates will join and some where the moderates generally stay home. That is:

**Lemma 1.** *i. There exists a  $\tau^* > 0$  such that there is a unique equilibrium in the protest stage for all election results if  $\tau_0 + \tau_e < \tau^*$  (i.e., there is little public information),*  
*ii. there is always a unique equilibrium for sufficiently low and high election results, and*

iii. there exists a  $\tau_e^* \geq 0$  such that if  $\tau_e \geq \tau_e^*$  then there exists an interval  $(\underline{e}, \bar{e})$  such that there are multiple equilibria for  $\underline{e} < e < \bar{e}$ .

**Proof** See the appendix for a more complete characterization of the citizen's equilibrium strategies and a proof.

The aim of the next two sections is to show why this result is important for understanding rule-based alternation of power. Before doing so, we present a useful result about the behavior in the protest stage:

**Proposition 1.** *When there is a unique equilibrium in the protest stage, the average size of protest and probability of the protest succeeding are:*

*i) decreasing in the cost of protesting against the incumbent ( $c$ ), and*

*ii) decreasing in the election result ( $e$ )*

*In the case where there are multiple equilibria for some election results:*

*iii) both of the above comparative statics also hold within the equilibrium with the highest and lowest level of protest.*

**Proof** See the appendix.

Increasing the cost of protest ( $c$ ) has two effects. First, from the perspective of an individual citizen, increasing the cost terms for a fixed expected level of protest makes participation less appealing. Second, the fact that other citizens experience the same higher cost means that they too are less likely to protest. Since the number of others taking to the streets will go down, the citizen in question becomes even more reluctant to join the protest.

The election result similarly has two effects. From the perspective of the incumbent (and analyst), lower election results indicate that she is less popular, and hence even keeping the citizen strategy fixed the expected protest level increases. This association is in line with a

central argument of Brownlee (2007), who claims poor electoral showings simply reflect the incumbent weakness, but do not have a causal effect on the downfall of regimes.

However, contrary to Brownlee’s argument, the election result also has a causal effect on protests levels by changing the citizens’ equilibrium strategy. Since citizens know the election result is correlated with the incumbent strength, upon observing a low result they expect more moderates will protest, making participation more attractive. That is, a lower-than expected result generates common knowledge among the citizens that the incumbent may be vulnerable, which will make moderate citizens more apt to join the protest. Note that both of these effects are entirely informational: the election result doesn’t directly affect payoffs but provides information about what others are apt to do, potentially having a large effect on the moderates’ behavior.

This result does not necessarily translate to the multiple equilibrium case, because the citizens could arbitrarily switch between the high and low protest equilibria for election results with both high and low protest strategies. However, we can rule out this behavior by restricting attention to equilibria where the citizens propensity to protest is monotone – i.e., either always increasing or always decreasing – in the election result. Lemma 1 and proposition 1 state that there is always a unique equilibrium for some election results, and that the protest threshold must be decreasing in these ranges, so in any monotone equilibrium citizens’ willingness to protest must be decreasing in the election result.

Since we are concerned with when the citizens can force the incumbent to step down, we also restrict attention to equilibria where citizens always play the highest or lowest protest equilibrium (see the appendix for a discussion of why this is innocuous on technical grounds as well). That is, we consider equilibria of the form “highest protest threshold for  $e < e^*$  and lowest protest threshold for  $e > e^*$ ” for some  $e^*$  in the range of election results with multiple equilibria.

## Stepping Down Before Protests

We now turn to the incumbent the incumbent decision to step down before the protest. By stepping down immediately, she is guaranteed a payoff of  $y^I$ . If she does not step down, she will observe the protest size (which she is uncertain about at the time of her pre-protest decision) and then make her second decision to step down or not as specified above. Formally, the expected payoff for standing firm and then making the optimal decision at the second chance to step down derived above is:

$$u_{SF}^I(e) = \underbrace{Pr(\rho < \rho^* | e)(1 - \mathbb{E}[\rho | \rho < \rho^*, e])}_{\text{Payoff When Standing Firm}} + \underbrace{Pr(\rho > \rho^* | e)(y^I - \gamma \mathbb{E}[\rho | \rho > \rho^*, e])}_{\text{Payoff When Stepping Down}}$$

If the incumbent is nearly certain that protest will be minimal, then she will not step down as her expected payoff for standing firm before the protest decision is near 1 versus a payoff of  $y^I < 1$  for stepping down. If the incumbent expects a very high protest level, she is better off stepping down right away as this guarantees a payoff of  $y^I$ , which is always better than what she will get if stepping down after protest ( $y^I - \gamma\rho$ ) and is better than the payoff for standing firm as well ( $1 - \rho$ ) if the protest is large.

For intermediate results, the incumbent faces a tradeoff. She can guarantee a moderate payoff by stepping down right away, but forgoes the opportunity to stay in power if the protests are in fact small. Since the expected level of protest is always decreasing in the election result, we can characterize the initial stepping down decision as follows:

**Proposition 2.** *i. The payoff for standing firm prior to protest when choosing the optimal second yielding decision is strictly increasing in the election result, and approaches  $y^I - \gamma$  as  $e \rightarrow -\infty$  and 1 as  $e \rightarrow \infty$ , and*

*ii. the incumbent steps down if and only if the election result is sufficiently low, and this event occurs with positive probability.*

**Proof** See the appendix.

This result holds for both the unique equilibrium case and the multiple equilibrium case (with the monotonicity restriction). Before contrasting these cases, we highlight some common properties to both.

First, whenever the incumbent steps down after protest she regrets not stepping down right away as this would guarantee a higher payoff. However, this does not imply the decision to stand firm was incorrect, as the protests may have been smaller, making attempting to stay in power worth the gamble.

Further, there are levels of protest where the incumbent does not step down at either chance, but given the realized level of protest would have preferred to step down immediately. In particular, if  $y^I - \gamma\rho < 1 - \rho$  but  $1 - \rho < y^I$ , which occurs if  $\rho \in \left(1 - y^I, \frac{1 - y^I}{1 - \gamma}\right)$  then the incumbent prefers standing firm to the payoff to stepping down after protest, but not to the payoff from stepping down immediately. Again, this does not imply that the incumbent made a poor decision to stand firm before the protest was observed, only that her gamble did not pay off. In a sense, leaders can become “locked in” to clinging to power if some of the costs to facing protests are sunk.

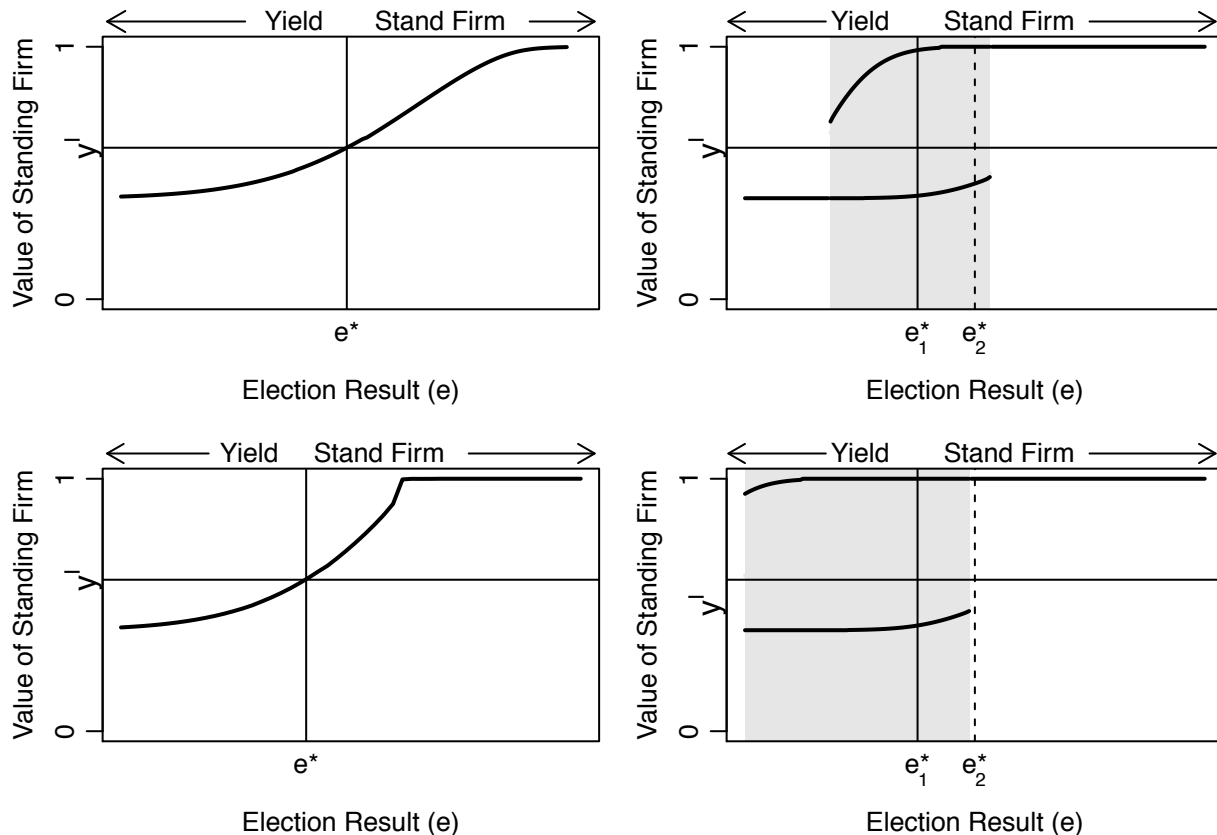
However, there are important distinctions between these cases illustrated by figure 2. In all panels, the curve traces the expected payoff for standing firm (and making the optimal second yielding decision) as a function of the election result.<sup>23</sup> The incumbent steps down immediately after the election if and only if the curve is below the value of stepping down  $y^I$ , represented by the horizontal line. The left panels show a case where the election result is noisy enough that there is always unique equilibrium, and the right panels show a similar case where the election is more informative and hence there are multiple equilibria for intermediate election results. For both the left and right panels, there is a higher cost of conflict in the bottom panel than the top panel.

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<sup>23</sup>The  $x$  axis ranges between the 5th and 95th percentile election results.



Figure 2: Illustration of decision to step down in unique equilibrium case (left panels) and multiple equilibrium case (right panels). The bottom panels have a higher cost of protest than the top panels.



Starting with the unique equilibrium case, for both costs of conflict there is a unique  $e^*$  such that the incumbent steps down before protest if and only if  $e < e^*$ . However, this critical election threshold is lower when the cost of conflict is higher (bottom panel), indicating the incumbent stays in office while getting a lower vote share. That is, it may be the case that the incumbent would step down if and only if she gets less than 50% of the vote when the cost of protest is low, but would cling to power even when getting 40% of the vote if the cost of protests increases.

This illustrates one of the central ideas of the paper: we would not consider the incumbent

stepping down in the unique equilibrium case democratic as the election result that determines who whether she does so varies based on the exogenous parameters. In general, the critical election result *won't* correspond to a formal rule. The critical threshold may happen to fall at a 50% vote share in a given election, but even if so changes in any of the model's parameters – e.g., the cost of protest, or the prior incumbent popularity, which would surely be different in subsequent elections – will shift this critical threshold away from the legally codified rule. So, some “losers” of elections will remain in office while some “winners” end up stepping down. We call this pattern “semi-democratic” alternation of power.

For reasons elaborated in the following section, leaders that eventually step down despite claiming victory in the election (as in the Colored Revolutions) generally do not do so right away. Cases where the incumbent does not step down from power despite losing according to rules are less common, likely because in such cases the regime is able to commit enough fraud to officially win the election. One example which generally does fit this pattern is the 1991 Algerian parliamentary elections. The Islamist *Islamic Salvation Front* won more than twice as many votes as any other party in the first round of these elections. By any democratic standards, the party would have taken control of the government following the second round of the election.<sup>24</sup> However, the military stepped in to prevent a change of government and cancelled the second round. While the importance of the military as an actor in this case is not formally included in our model, military leaders do take cues in whether to support or abandon regimes based on popular protest (Tyson and Smith, 2013), and could learn from the election result as well. In other words, if the Islamists had even more popular support and won the election result more convincingly, then the military may

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<sup>24</sup>We use the term “majority” loosely here as this was not a presidential election. However, if we think of the indirect translation of “majority” support in a parliamentary election as being able to win enough seats to form a government, then the forces supporting the Algerian military clearly were not going to win enough votes in this election to hold power in the parliament following the election. Thus the model highlights that winning the election by official rules was not the critical result needed for a transfer of power, though this may have happened if the Islamic forces has won even more convincingly.

have allowed their victory to stand.

Now consider the case where the election is more informative, and hence there are multiple equilibria for some election results (right panels). In both panels, the shaded area indicates the election results for which there is a high protest equilibrium (which gives a lower payoff to standing firm) and a low protest equilibrium (higher payoff to standing firm). Where the payoff for standing firm is above the payoff from the high protest equilibrium but below the value of standing firm in the low protest equilibrium, whether the incumbent steps down depends on which strategy the citizens select. So, there is a range of electoral results where the citizens can either coordinate on an equilibrium that ousts the incumbent or one where the incumbent claims victory. As a result, if a legally codified “democratic” rule specifying the incumbent wins if and only if their vote total lies in this range (e.g.,  $e < .50$ ), the rule can be enforced in equilibrium.

The top right panel illustrates two election results where the citizens can either induce the incumbent to step down or not, labelled  $e_1^*$  and  $e_2^*$ . For the higher cost of protest, (bottom right panel)  $e_1^*$  is still enforceable, as the incumbent would step down under the high protest equilibrium but stands firm if the citizens play the low protest equilibrium for this result. So, if this corresponds to a codified rule, it is not only enforceable for the parameters in the top right panel, but for a range of scenarios. The result is equilibrium behavior that *mimics rule-based behavior* without turning elections into a contract in our model, and therefore a single model that explains alternations of power in both “non-democratic” and “democratic”

However, with the higher cost of protest the election result  $e_2^*$  only has an equilibrium with a low level of protest, so it is not enforceable with the higher cost of protest. So, if  $e_2^*$  corresponds to the electoral law, then in the stylized example in Figure 2, the increase in the cost of protest going from the top to bottom panel would render this rule no longer enforceable. This allows the model to capture the fact that if exogenous conditions shift too much – e.g, the cost of protest increases too dramatically, something we might associate

with a country becoming less democratic – then the apparently “rule-based” democratic behavior in equilibrium will eventually collapse.

While we have argued repeatedly (and with precedent) that our model must have multiple equilibria to endogenize compliance with electoral rules, an obvious drawback is that other equilibria could be played as well. However, compliance with a majoritarian electoral rule and not some other electoral threshold is particularly appealing for various reasons. One argument is that following an electoral rule, particularly one at a natural threshold like a majority vote share is a natural focal point in sense coined by Schelling (1960, ch. 3). Citizens could threaten to protest against leaders who don’t achieve the electoral threshold plus five percent, or on any publicly observed “sunspot”, but these simply seem less natural than coordinating against law breakers. Further, given the fact that protest is costly and citizens want to protest against unpopular rules but not popular ones, such a rule may be optimal by putting popular leaders in office peacefully.<sup>25</sup>

In sum, electoral rules may be unenforceable in both the unique and multiple equilibria cases, but it is only possible for rules to be enforceable in a stable sense if there are multiple equilibria. So, alternation in power resembling a democracy and semi-democratic turnover in office can arise from the same model, and the key theoretical variable dividing these cases is how informative the election is.

## 4 When is Protest on the Equilibrium Path

The ability to enforce electoral rules has an additional important implication tied to the informativeness of elections. Combining propositions 1 and 2 tells us that protest only occurs on the equilibrium path when the expected level is not too high: otherwise the incumbent would step down immediately. That is, the actual amount of protest is highest

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<sup>25</sup>A full technical analysis of the optimal equilibria quickly becomes complex. Further issues like making fraud endogenous could complicate this argument, but we leave a thorough analysis of these questions to future work.

when the expected level of protest that the incumbent would face when clinging to power is intermediate: when the expected level of protest is low this incumbent will stay in power and face little protest, and when it is too high she steps down and there is *no* protest in equilibrium.

When the election is noisy and there is a unique equilibrium in the protest stage, there are always intermediate election results where the incumbent knows she might face a large protest but stays in power anyway on the chance that the protest will turn out to be small. This is less likely to occur in the multiple equilibrium case for two reasons. First, when the election result is more informative, the incumbent is better informed about her popularity and what the resulting level of protest will be, and can do a better job of knowing when to step down. Second, in the multiple equilibrium case the expected level of protest will tend to be very low for election results where the citizens select the low protest equilibrium and very high for election results with the high protest equilibrium. This can be seen in the right panels of figure 2: the incumbent payoff for standing firm after the election result is generally very close to  $y^I - \gamma$  (meaning nearly all protest) for low election results and near 1 (meaning almost no citizens protest) for high election results.

As a result, the intermediate expected levels of protest that lead to the highest levels of protest in equilibrium will not happen in the multiple equilibrium case. That is, there is little gain to waiting and seeing the actual protest size, which will mean the incumbent will step down immediately after “losing” the election based on a codified rule, and faces minimal protest (and hence almost never steps down) upon winning.

In the extreme case where the election fully reveals the incumbent popularity, the only citizens that protest are those with a dominant strategy to do so:

**Proposition 3.** *As  $\tau_e \rightarrow \infty$ , citizens with  $\theta_j \in (\underline{\theta}, \bar{\theta})$  (i.e., moderates) never protest on the equilibrium path.*

This highlights the second main insight of the model: in cases with democratic turnover

there will be little protest on the equilibrium path, while semi-democratic turnover often requires citizens to actually take to the streets. Further, in the democratic case, those taking to the streets do so not because they actually expect to oust the incumbent, but because they simply dislike the regime enough that the expressive component to protest outweighs its cost. This intuitively corresponds to much protest in democratic countries: all but the most extreme do not believe their actions will lead to the president or prime minister stepping down, but they are still willing to bear the cost of protest for expressive (or other) reasons.

## 5 Fraud and Monitoring

Until this point we have not explicitly considered the fact that election results can be contaminated by fraud and other manipulation. Indeed, in many of our motivating examples, attempts to resolve uncertainty about how much fraud was committed played a key role in the post-election protests. Further, we have some idea about how citizens come to hold ideas about how much fraud was committed: in addition to their personal experiences and the experiences of those in their social network, citizens learn from visible public reports of fraud from domestic sources such as news media, information (such as videos) posted to the internet, or reports of parallel vote tabulations. They also learn (directly or indirectly) from reports issued by international observers. We generically refer to these public signals as *monitoring reports*.

Fraud can undermine democratic turnover for two reasons. An obvious effect is that if the election result with no fraud reflects sincere voting of the citizens and the citizens use a majoritarian rule as a focal point determining whether the incumbent wins as above, then the incumbent can “win” elections even if they do not truly have majority support.<sup>26</sup> However, we focus more here on a less obvious effect: when fraud induces uncertainty about

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<sup>26</sup>Properly analyzing this effect would require a model of voting behavior, so we leave this to future work.

what other citizens think about the regime, it can be harder to enforce *any* electoral rule.<sup>27</sup>

To take account of these considerations, let the election result be  $e = \omega + x$ , where  $x$  represents amount of electoral fraud. Assume  $x = \bar{x} + \nu_e$ , where  $\bar{x}$  is the (common knowledge) average level of fraud and  $\nu_e$  is a random variable reflecting whether there is more or less fraud than expected. That is, the noise of the electoral signal is now uncertainty about how much of the result was driven by fraud.<sup>28</sup> While fraud is at least in part a strategic decision made by the incumbent, we treat it as exogenous here to keep the extension tractable and as multiple other recent game theoretic papers analyze fraud as a strategic choice in a similar framework (e.g., Rozenas, 2012; Little, 2012, 2011*a*; Simpson, 2011; Egorov and Sonin, 2011)

This change alone would only be a relabeling of what the noise in the election represents, as citizens would simply subtract the average level of fraud from the election result when forming an unbiased signal of  $\omega$ . However, we also assume that citizens observe a monitoring report  $m$ , which is a noisy signal of the level of fraud. The monitoring report is normally distributed with mean  $x$  and precision  $\tau_m$ . That is,  $m = \bar{x} + \nu_e + \nu_m$  where  $\nu_m$  is independent of the other noise terms and normally distributed with mean 0 and precision  $\tau_m$ . So, when forming a belief about the popularity of the incumbent, the citizens account for the monitoring report in addition to the election result and their private signal.

As in the main model, the joint normality assumptions lead to a convenient characterization of the citizens' *interim* beliefs about the incumbent popularity, which summarize the public information citizens have about the incumbent popularity before observing their private signal.<sup>29</sup> In the baseline model this is normal with a mean that is a weighted average of the prior mean ( $\mu_0$ ) and election result ( $e$ ), and a precision that is the sum of the precisions

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<sup>27</sup>Fearon (2011) makes an analogous argument about how fraud can undermine accountability in elections and how monitoring may alleviate this problem.

<sup>28</sup>All the results here would hold as long as some of the uncertainty built into the election result comes from uncertainty about fraud.

<sup>29</sup>The sequence of moves presented above places the private signals before election result, but since no actor makes a decision before both are revealed this order does not affect the equilibrium behavior.

of the prior and election result. With the monitoring report, this belief is again a weighted average, with the election result now corrected for the expected level of fraud (i.e.,  $e - \bar{x}$  is an unbiased estimate of  $\omega$ ) an additional correction term that reflects whether the monitoring report indicates more or less fraud than expected ( $m - \bar{x}$ ). In addition, the information from the monitoring report increases the precision of the belief about the incumbent popularity, which is now  $\tau_0 + \tau_e + \tau_m$ . Thus the analysis of the model is essentially the same with additional terms added to the beliefs. The first implications of adding the monitoring report are:

**Proposition 4.** *If there is a unique equilibrium in the protest stage or within the highest and lowest protest equilibrium when there are multiple equilibria, the level of protest is increasing in the monitoring report (i.e., as the report claims more fraud), and the incumbent steps down before protest if and only if the monitoring report claims sufficiently high levels of fraud.*

Just as stronger election results for the incumbent lead to less protest because they indicate that she is popular, reports of more fraud lead to more protest because they indicate the incumbent is, for a fixed election result, less popular.<sup>30</sup> This prediction is consistent with recent empirical work (Hyde and Marinov, 2012; Rozenas, 2012), who find that public reports of fraud are associated with more post-election protest.<sup>31</sup>

Further, a similar result about the relationship between the precision of the election result and the possibility of multiple equilibria holds for the precision of the monitoring report:

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<sup>30</sup>That is, this is not driven by citizens being angry at cheating incumbents (as in Tucker (2007)).

<sup>31</sup>This extension is also consistent with a more nuanced result in Rozenas (2012), which is that the effect of public reports of fraud on protest is larger when the margin of victory is small. In our model, the expected level of protest is an backwards (i.e., decreasing) s-shaped curve in the posterior belief about the incumbent popularity, which is a function of both the election result and monitoring report. When the election result is very high, and expected protest low, the posterior belief on the flat part of the s-curve and increasing beliefs about fraud has little effect on protest. However, for closer elections the posterior belief may be closer to the steep part of the s-curve, meaning changes in monitoring reports can have a large impact on the resulting amount of protest.



**Proposition 5.** *If the monitoring report is sufficiently informative ( $\tau_m$  is sufficiently high), then there exist multiple equilibria for some  $e$  and  $m$ .*

The intuition for this result is similar to that of proposition 1: one role that the monitoring report plays is to make the election result more informative as there is less uncertainty about the level of fraud. Thus institutions that detect electoral fraud can help consolidate democracy for a potentially unexpected reason: increasing the amount of public information and facilitating the coordination dynamics required to make electoral rules enforceable.<sup>32</sup>

## 6 Conclusion

The spread of elections globally is one of the most interesting developments in comparative politics over the past half-century. Nearly every country on earth now holds elections, and in the vast majority of cases there is at least the appearance of some competition; that is, most of these are not Soviet style 99% turnout and 99% incumbent vote share elections. And yet, as a discipline we tend to still want to dichotomize the electoral experience into “free” elections and (insert adjective here)<sup>33</sup> elections, with the former being imbued with an almost mythical contractual power that guarantees results will be respected and the latter treated as a separate animal if not dismissed as meaningless. Somewhat surprisingly, as a field we lack a general theory of elections. This paper is a step in that direction.

More specifically, the model presented above provides a general framework to capture how elections across regime types can facilitate alternation in power. In particular, it shows that a minimalist assumption about the role of elections – that they are a public signal of the incumbent popularity – can lead to equilibria that are qualitatively consistent with transfer

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<sup>32</sup>Svolik and Chernykh (2012) and Hyde and Marinov (2012) make a related argument that information generated by third parties like international monitors can help peaceful compliance with election results by alleviating information asymmetries between elites. Our results show that information can also facilitate peaceful transitions of power by alleviating information problems among the citizenry.

<sup>33</sup>E.g., not free and fair, quasi-democratic, semi-democratic, competitive authoritarian, unequal, etc.

of power due to (the threat of) post-election protest as well as peaceful and democratic transfer of power. This highlights that electoral rules can be complied with precisely when they are informative, and that it is in this democratic case that alternation of power tends to be peaceful.

More broadly, the results speak to the value of modeling elections primarily as information generating institutions. Here we have shown that this allows us to capture a phenomenon that at first may seem unrelated to information generation: peaceful and rule-based alternation of power. The universality of this approach may build a bridge between the study of democratic and less-than-democratic elections.

## Appendix: Derivation of Equilibrium Condition and Proofs

### Conditional distributions of $\omega$

Here we derive the most complex posterior belief about  $\omega$ : the conditional belief given  $e$ ,  $m$ , and  $\theta_j$ . The other posteriors follow from a similar calculation. By the assumptions in the main text, the joint distribution of  $\omega$ ,  $e$ ,  $m$ , and  $\theta_j$  is a multivariate normal with mean vector  $(\mu_0, \mu_0 + \bar{x}, \bar{x}, \mu_0)$  and covariance matrix:

$$\Sigma = \begin{matrix} & \begin{matrix} \omega & e & m & \theta_j \end{matrix} \\ \begin{matrix} \omega \\ e \\ m \\ \theta_j \end{matrix} & \begin{pmatrix} \tau_0^{-1} & \tau_0^{-1} & 0 & \tau_0^{-1} \\ \tau_0^{-1} & \tau_0^{-1} + \tau_e^{-1} & \tau_e^{-1} & \tau_0^{-1} \\ 0 & \tau_e^{-1} & \tau_e^{-1} + \tau_m^{-1} & 0 \\ \tau_0^{-1} & \tau_0^{-1} & 0 & \tau_0^{-1} + \tau_\theta^2 \end{pmatrix} \end{matrix} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

where  $\Sigma_{11} = \tau_0^{-1}$  (which uniquely determines the remainder of the partition). As a consequence, the desired posterior is normal (see Greene 2009, p 1014, or LaGatta 2013 for the general case). The conditional mean is

$$\begin{aligned} \bar{\mu}(e, m, \theta_j) &= \mu_0 + \Sigma_{12}\Sigma_{22}^{-1}(e - (\mu_0 - \bar{x}), m - \bar{x}, \theta_j - \mu_0) \\ &= \frac{\tau_0\mu_0 + (\tau_e + \tau_m)(e - \bar{x}) + \tau_\theta\theta_j - \tau_m(m - \bar{x})}{\tau_0 + \tau_e + \tau_m + \tau_\theta} \end{aligned}$$

and the conditional precision is given using the Schur complement:

$$(\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})^{-1/2} = \tau_0 + \tau_e + \tau_m + \tau_\theta.$$

## Derivation of the Equilibrium Condition

For a fixed  $\omega$  and cutoff rule  $\hat{\theta}(e)$ , the probability that a citizen protests is:

$$Pr(\omega + \nu_j < \hat{\theta}(e)) = \Phi\left(\tau_\theta^{1/2}(\hat{\theta}(e) - \omega)\right) \quad (4)$$

Using the Weak Law of Large Numbers, as  $N \rightarrow \infty$ , the actual protest level given  $\omega$  and  $\hat{\theta}(e)$  converges in probability to the expression on the right side of (4). Hence the protest size is continuous and increasing in  $\omega$ , approaches 0 as  $\omega \rightarrow -\infty$  and approaches 1 as  $\omega \rightarrow \infty$ . So for each  $e$  there will be a critical  $\omega'(e)$  such that the protest induces the incumbent to step down if and only if  $\omega < \omega'(e)$ , given by:

$$\begin{aligned} \Phi\left(\tau_\theta^{1/2}(\hat{\theta}(e) - \omega'(e))\right) &= \rho^* \\ \omega'(e) &= \hat{\theta}(e) - \tau_\theta^{-1/2}\Phi^{-1}(\rho^*) \end{aligned}$$

where  $\rho^*$  is derived in the main text. By an analogous calculation as above, the posterior of a citizen observing  $\theta_j = \hat{\theta}(e)$  and  $e$  has a posterior belief about the regime sentiment that is normally distributed with mean:

$$\bar{\mu}(e, \hat{\theta}(e)) = \frac{\tau_0\mu_0 + \tau_e e + \tau_\theta \hat{\theta}(e)}{\tau_0 + \tau_e + \tau_\theta}$$

and precision  $\tau_0 + \tau_e + \tau_\theta$ . So the probability this citizen assigns to the protest succeeding is:

$$\begin{aligned} Pr(\omega < \omega'(e)) &= \Phi((\tau_0 + \tau_e + \tau_\theta)^{1/2}(\omega'(e) - \bar{\mu}(e, \hat{\theta}(e)))) \quad (5) \\ &= \Phi\left((\tau_0 + \tau_e + \tau_\theta)^{1/2}\left(\hat{\theta}(e) - \tau_\theta^{-1/2}\Phi^{-1}(\rho^*) - \frac{\tau_0\mu_0 + \tau_e e + \tau_\theta \hat{\theta}(e)}{\tau_0 + \tau_e + \tau_\theta}\right)\right) \\ &= \Phi\left(\frac{\hat{\theta}(e) - \mu_{RHS}}{\sigma_{RHS}}\right), \end{aligned}$$

where

$$\begin{aligned}\mu_{RHS}(e) &= \frac{\tau_0\mu_0 + \tau_e e + (\tau_0 + \tau_e + \tau_\theta)\tau_\theta^{1/2}\Phi^{-1}(\rho^*)}{\tau_0 + \tau_e} \\ \sigma_{RHS} &= (\tau_0 + \tau_e + \tau_\theta)^{1/2}(\tau_0 + \tau_e)^{-1}.\end{aligned}$$

Consequently, the equilibrium condition is that  $\hat{\theta}(e)$  satisfies the following equation:

$$\Phi\left(\frac{\hat{\theta}(e) - \mu_{RHS}(e)}{\sigma_{RHS}}\right) = \frac{c + b_1\hat{\theta}(e)}{-\hat{\theta}(e)(b_3 - b_1 - b_2)} \quad (6)$$

### Proof of Lemma 1

First, note that  $\tau_{RHS} = \sigma_{RHS}^{-2}$  is strictly increasing in  $\tau_0 + \tau_e$  and approaches 0 as  $\tau_0 + \tau_e \rightarrow 0$ . Second, by standard properties of normal a normal CDF, the LHS of the equilibrium condition is equal to 1/2 and attains it's maximum slope of  $\sqrt{\tau_{RHS}/2\pi}$  at  $\hat{\theta}(e) = \mu_{RHS}$ . Third,  $\mu_{RHS}$  is an increasing affine function of  $e$ .

Let  $\underline{d} > 0$  be minimum slope of the RHS of the equilibrium condition on  $[\underline{\theta}, \bar{\theta}]$ . Let  $\theta_5$  be the value of  $\hat{\theta}(e)$  such that the RHS is equal to 1/2, and let and  $d_5$  be the slope of the RHS at  $\theta_5$ .

The above arguments imply that there exists a  $\tau^*$  such that if  $\tau_0 + \tau_e < \tau^*$  then  $\sqrt{\tau_{RHS}/2\pi} < d_0$ , which guarantees a unique solution to equation 3 and hence unique equilibrium.

If  $\tau_e$  is high enough that  $\sqrt{\tau_{RHS}/2\pi} > d_0$  – which may hold if  $\tau_e = 0$ , but will always require  $\tau_e > 0$  if  $\tau_0$  is sufficiently small – then when  $e$  is such that  $\mu_{RHS} = \theta_5$  there will be an intersection at  $\hat{\theta}(e) = \theta_5$  where the LHS is increasing faster than the RHS. Further, there will be an open interval  $(\underline{\mu}, \bar{\mu})$  and a corresponding  $(\underline{e}, \bar{e})$  such that there is a  $\hat{\theta}(e)$  “near”  $\mu_{RHS}$  where the equilibrium condition is met and the LHS is increasing faster than the RHS. Since the LHS is above the RHS at  $\underline{\theta}$  and vice versa at  $\bar{\theta}$ , this implies there must be two

additional intersections, and hence multiple equilibria.

### Proof of Proposition 1

First we need to determine how the equilibrium threshold changes as a function of  $c$  and  $e$ . Implicitly differentiating with respect to  $c$  gives:

$$\frac{\partial \hat{\theta}(e)}{\partial c} = \frac{1/(\hat{\theta}(e)(b_3 - b_1 - b_2))}{\sigma_{RHS}^{-2} \phi\left(\frac{\hat{\theta}(e) - \mu_{RHS}(e)}{\sigma_{RHS}}\right) + \left(\frac{b_1(-\hat{\theta}(e)(b_3 - b_1 - b_2)) - (c + b_1 \hat{\theta}(e))(- (b_3 - b_1 - b_2))}{(-\hat{\theta}(e)(b_3 - b_1 - b_2))^2}\right)}$$

The numerator is positive. The denominator is the derivative of the LHS of the equilibrium condition minus the RHS. Since the LHS is above the RHS at  $\underline{\theta}$  and below the RHS at  $\bar{\theta}$  and both sides are continuous, if the intersection is unique, the RHS must be increasing faster than the LHS at  $\hat{\theta}(e)$ , and hence this expression is negative. Similarly, the RHS is also increasing faster than the LHS at the lowest and highest intersection, and hence the expression is negative for these equilibria as well. So if there is a unique equilibrium, the threshold is decreasing (and hence fewer citizens protest) as  $c$  increases, and if there are multiple equilibria this comparative static holds in the equilibrium with the highest and lowest protest level. The protest threshold is decreasing in  $e$  in these two cases by a similar argument.

For a fixed  $\omega$  and  $e$ , the protest size as  $N \rightarrow \infty$  is:

$$\rho(\omega, e) = \Phi(\tau_\theta^{1/2}(\omega - \hat{\theta}(e)))$$

So, conditional on  $e$ ,  $\omega$  is normally distributed with mean  $\bar{\mu}(e)$  and precision  $\tau_0 + \tau_e$ , and hence  $\rho$  is a normal random variable with CDF:

$$Pr(\rho < x|e) = \Phi(\tau_\rho(\bar{\mu}(e) - x)) \tag{7}$$

where  $\tau_\rho = \frac{\tau_\theta(\tau_0 + \tau_e)}{\tau_\theta + \tau_0 + \tau_e}$ .<sup>34</sup> Differentiating with respect  $c$  gives that this expression has the same sign as  $\frac{\partial \hat{\theta}(e)}{\partial c}$ , which as shown above is negative in the unique equilibrium case and the equilibria with the most and least protest in the multiple equilibrium case. So, for any  $c_1 < c_2$ , the distribution of  $\rho$  under  $c_1$  first order stochastically dominates the distribution of  $\rho$  under  $c_2$ . Evaluating at  $x = \rho^*$  gives the likelihood of protest succeeding is decreasing in  $c$ , the first order stochastic dominance property implies the expected level of protest is decreasing in the cost of protest as well. An analogous argument applies when differentiating with respect to  $e$ .

### Discussion of Monotonicity and Lowest/Highest Equilibrium Restriction

As stated in the main text, since the protest threshold is decreasing for some election results, in any monotone equilibrium the threshold must be decreasing in the election result everywhere. This also implies that in every equilibria of the form “highest protest equilibrium if  $e < e^*$  and lowest protest equilibrium is  $e > e^*$ ” the citizen protest threshold is monotone in the election result.

In the case where there are at most 3 equilibria, all monotone equilibria are essentially of this form. This follows from the fact that when there are 3 equilibria, the “middle” equilibrium must be one where the LHS of the equilibrium condition is increasing faster than the RHS, so the equilibrium threshold is *increasing* in the election result in this equilibrium. Hence this middle equilibrium can not be played for more than a single election result in a monotone equilibrium, and hence can not be played with strictly positive probability.

When there are 5 or more equilibria it would be possible to have a monotone equilibrium where an protest threshold other than the highest or lowest possible is played, but substantial numerical analysis indicates there are never more than 3 equilibrium thresholds.

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<sup>34</sup>This follows from a property that is more commonly stated about random variables that are normally distributed with mean  $\mu_0$  and variance  $\sigma_A^2$ , where  $\mu_0$  is also random and normally distributed with mean  $m$  and variance  $\sigma_B^2$ . The unconditional distribution (i.e., not knowing  $\mu_0$ ) of such a random variable has mean  $m$  and variance  $\sigma_A^2 + \sigma_B^2$ .

## Proof of proposition 2

The payoff to standing firm at the first opportunity as a function of the realization of  $\rho$  is:

$$u_{SF}^I(\rho, e) = \begin{cases} 1 - \rho & \rho < \rho^* \\ y^I - \gamma\rho & \rho > \rho^* \end{cases}$$

Which is strictly decreasing in  $\rho$ . Since for any  $e_2 > e_1$ , the distribution of  $\rho$  under  $e_1$  first order stochastic dominates the distribution under  $e_2$ , the expected payoff for standing firm is strictly increasing in  $e$ . The limiting payoffs follow from the fact that  $Pr(\rho < \rho^*|e)$  approaches 1 and  $\mathbb{E}[\rho|\rho < \rho^*|e]$  approaches 0 as  $e \rightarrow \infty$  and  $Pr(\rho > \rho^*|e)$  approaches 1 and  $\mathbb{E}[\rho|\rho > \rho^*, e]$  approaches 1 as  $e \rightarrow -\infty$ . Part ii follows immediately from part i.

## Proof of proposition 3

Consider the equilibrium condition as  $\tau_e \rightarrow \infty$ . First,  $\sigma_{RHS}$  approaches 0, indicating that the LHS of the equilibrium condition approaches a step function equal to 0 for  $\hat{\theta}(e) < \mu_{RHS}(e)$  and 1 for  $\hat{\theta}(e) > \mu_{RHS}(e)$ . Write  $\mu_{RHS}(e)$  as:

$$\frac{\tau_e e + \tau_0 \mu_0}{\tau_0 + \tau_e} = \frac{\tau_0 + \tau_e + \tau_\theta}{\tau_0 + \tau_e} (\tau_\theta)^{-1/2} \Phi^{-1}(\rho^*)$$

And hence:

$$\lim_{\tau_e \rightarrow \infty} \mu_{RHS}(e) = e + \tau_\theta^{-1/2} \Phi^{-1}(\rho^*)$$

Which is linearly increasing in  $e$ . This implies that as  $\tau_e \rightarrow \infty$ , the equilibrium threshold is unique at  $\hat{\theta}(e) = \bar{\theta}$  for  $e < \underline{\theta} - \tau_\theta^{-1/2} \Phi^{-1}(\rho^*)$  and unique at  $\hat{\theta}(e) = \underline{\theta}$  for  $e > \bar{\theta} - \tau_\theta^{-1/2} \Phi^{-1}(\rho^*)$ . For  $e$  in between these bounds, there are intersections at both  $\hat{\theta}(e) = \underline{\theta}$  and  $\hat{\theta}(e) = \bar{\theta}$ .



If the election is fully informative, than the marginal citizen has no additional private information about  $\omega$  and hence the LHS of the equilibrium condition is the probability that the protest will induce the incumbent to step down. So if the citizens select  $\hat{\theta}(e) = \underline{\theta}$  then no citizens without a dominant strategy protest and the incumbent stays in power with probability 1, and if  $\hat{\theta}(e) = \bar{\theta}$  all citizens without a dominant strategy to protest would do so and the incumbent steps down prior to protest, hence the moderates do not protest on the equilibrium path.

### Proof of propositions 4-5

With the addition of the monitoring report the interim mean belief for citizens before observing their private signal is:

$$\bar{\mu}(e, m) = \frac{\tau_0 \mu_0 + (\tau_e + \tau_m)(e - x) + \tau_m(m - x)}{\tau_0 + \tau_e + \tau_m}$$

Both are linearly increasing in  $e$  and the belief with the monitoring report is linearly decreasing in  $m$ .

**Lemma 2.** *i. We can write the equilibrium threshold rule as the mean interim popularity:  $\hat{\theta}(\bar{\mu}(e))$  or  $\hat{\theta}(\bar{\mu}(e, m))$ , and*

*ii. In both the baseline model and extension, all of the results referring to the election result can be rewritten with respect to the interim mean popularity*

**Proof** The citizen posterior belief about the incumbent popularity with the monitoring report and their private signal has mean:

$$\bar{\bar{\mu}}(e, m, \theta_j) \equiv \frac{(\tau_0 + \tau_e + \tau_m)\bar{\mu}(e, m) + \tau_\theta \theta_j}{\tau_0 + \tau_e + \tau_m + \tau_\theta} = \frac{\tau_0 \mu_0 + (\tau_e + \tau_m)(e - x) - \tau_m(m - x) + \tau_\theta \theta_j}{\tau_0 + \tau_e + \tau_m + \tau_\theta}$$

and precision  $\tau_0 + \tau_e + \tau_m + \tau_\theta$ . Replacing  $\bar{\mu}(e, \theta_j)$  with  $\bar{\bar{\mu}}(e, m, \theta_j)$  and  $\tau_0 + \tau_e + \tau_\theta$  with

$\tau_0 + \tau_e + \tau_m + \tau_\theta$  in equation 5 gives an analogous equilibrium condition for  $\hat{\theta}(e, m)$ , which is decreasing in  $m$  in the unique equilibrium case (and the high and low protest equilibria with multiple equilibria) by an analogous argument. For part ii, it is clear from definition that  $\bar{\mu}(e)$  is an invertible function of the election result  $e$ . So, as long as there is a unique equilibrium for all  $e$  there will be a unique equilibrium for all  $\bar{\mu}(e)$  and a one-to-one correspondence between these descriptions of the equilibrium threshold.<sup>35</sup> So all comparative statics with respect to  $e$  can be equivalently derived with respect to  $\bar{\mu}(e)$  in the baseline model and  $\bar{\mu}(e, m)$  with the monitoring report. ■

The proposition follows from the fact lemma 2 and the fact that  $\bar{\mu}(e, m)$  is decreasing in  $m$ .

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<sup>35</sup>The uniqueness caveat is only necessary because when there are multiple equilibria the mapping from the election result to equilibrium thresholds is not a function.

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