Separation energy of last electron for neutral atoms

Note: separation energy decreases after the completion of each major shell (noble gases)
One-proton separation energy for nuclei

Ref: J.H.E. Mattauch, W. Thiele, and A.H. Wapstra, Nucl. Phys. 67 (1965) 1
One-neutron separation energy for nuclei
Ref: J.H.E. Mattauch, W. Thiele, and A.H. Wapstra, Nucl. Phys. 67 (1965) 1
Energies of first $2^+$ states in even-even nuclei
Spherical shell model with strong spin-orbit force (1949)

Maria Goeppert Mayer, U.S.A.  J. Hans D. Jensen, Germany
Nobel prize 1963
Mean-field concept

(Ref: Isotope Science Facility at Michigan State University, MSUCL-1345, p. 41, Nov. 2006)
Spherical shell model with spin-orbit interaction: quantum numbers

\[ n = 1, 2, 3, \ldots \]
\[ \ell = 0, 1, 2, \ldots \]
\[ N = 2(n - 1) + \ell \] determines 3D harmonic oscillator energy
\[ j = |\ell - 1/2| \quad \text{and} \quad j = \ell + 1/2 \]

\[ E_{n\ell j} = -V_0 + \hbar \omega (N + 3/2) - \alpha \ell \quad \text{for} \quad j = \ell + 1/2 \]
\[ E_{n\ell j} = -V_0 + \hbar \omega (N + 3/2) + \alpha (\ell + 1) \quad \text{for} \quad j = |\ell - 1/2| \]
Spherical shell model with spin-orbit interaction:
  single-particle quantum states

\[
\begin{align*}
\ell & = 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\
\ell & = s \ p \ d \ f \ g \ h \ i
\end{align*}
\]

spectroscopic notation
notation for single particle energy states, e.g.
\[
1s_{1/2} \quad 1p_{3/2} \quad 2d_{5/2} \quad 1h_{11/2} \quad 1i_{13/2}
\]

\[
| n\ell m_j \rangle 
\]
  single particle state vectors

\[
m_j = -j, -j+1, \ldots, +j
\]

\[\Rightarrow M = 2j + 1\]
degeneracy of single-particle energy level

Spherical shell model (Mayer & Jensen, 1955): part 1
Spherical shell model (Mayer & Jensen, 1955): part 2
spherical nuclear shell model
square of radial wave functions for $^{208}\text{Pb}$

- 1s ($n=1, l=0$)
- 1d ($n=1, l=2$)
- 2s ($n=2, l=0$)
- 3p ($n=3, l=1$)
Spherical shell model for $^{208}$Pb: $1s_{1/2}$ orbital ($m_j = +1/2$)
Spherical shell model for $^{208}\text{Pb}$: $1p_{3/2}$ orbital ($m_j = +3/2$)
Spherical shell model for $^{208}\text{Pb}$: 2d$_{3/2}$ orbital ($m_j = +3/2$)
Spherical shell model for $^{208}\text{Pb}$: 2d$_{3/2}$ orbital ($m_j = +1/2$)
Spherical shell model for $^{208}\text{Pb}$: $3p_{3/2}$ orbital ($m_j = +1/2$)
Comparison of one-particle shell models
atomic “orbitals” (left) vs. nuclear “orbitals” (right)
Note that atomic orbitals are more diffuse!

size: $10^{-10}$ m

size: $10^{-15}$ m
Measurement of single-particle proton energy levels: $^{16}\text{O}(p,2p)^{15}\text{N}$

Ref: Textbook by Ring & Schuck
Measured single-particle proton energy levels in $^{16}$O

Ref: Textbook by Ring & Schuck

Figure 2.13. Experimental $^{16}$O (p, 2p) $^{15}$N cross section [MHT 58].
Measured single-particle neutron energy levels in $^{209}$Pb

Ref: Ellegaard et al., Nucl. Phys. A129 (1969) 113

Fig. 6. Proton spectrum from the $^{208}$Pb(d, p)$^{209}$Pb reaction. The intense peaks are labelled by their shell-model assignment. The peaks labelled A . . . L are the peaks for which angular distributions are measured. The arrow labelled $\frac{3}{2}^-$ indicates the position of the known $\frac{3}{2}^-$ level.