Physics-340a course overview:

low-energy nuclear structure and reactions

Volker Oberacker
Topics

- Nuclear chart, the frontiers of neutron-rich and superheavy nuclei, astrophysics connection (r- and rp- process)

- low-energy accelerators: radioactive ion beam (RIB) facilities

- The nucleus as a quantum many-body problem, static nuclear structure theory (HF), nuclei at the neutron dripline (HFB)

- Time-dependent mean field theory for nuclear reactions (TDHF) studies of deep-inelastic reactions and fusion of exotic neutron-rich nuclei
Topic 1

- Nuclear chart, the frontiers of neutron-rich and superheavy nuclei, astrophysics connection (r- and rp- process)
Nuclear chart, “magic” proton and neutron numbers, measured half-life ranges
Nuclear chart and the frontier of neutron-rich nuclei
Ref: Isotope Science Facility proposal, MSU (Nov. 2006)
Nuclear astrophysics connection

Rapid neutron capture process (= $r$-process) following supernova explosion goes through “terra incognita” of very neutron-rich nuclei
Second frontier: superheavy elements in heavy-ion fusion reactions
Ref: National Nuclear Data Center, Brookhaven
Topic 2

• low-energy accelerators: radioactive ion beam (RIB) facilities
Radioactive Ion Beam Facilities worldwide

Figure 4.6. Representative distribution of projected major facilities for RI beams. The location of a FRIB within the United States has not been determined and is therefore placed arbitrarily in the center of the country.
Facility for Rare Isotope Beams (FRIB)
under construction at Michigan State University
http://www.frib.msu.edu/
Topic 3

- The nucleus as a quantum many-body problem, static nuclear structure theory, nuclei in vicinity of neutron dripline
Nuclear many-particle Hamiltonian
(in coordinate representation)

\[
H = \sum_{i=1}^{A} \frac{-\hbar^2}{2m} \nabla_i^2 + \frac{1}{2} \sum_{i,j=1}^{Z} u_{ij}^{(2)\text{Coul}} + \frac{1}{2} \sum_{i,j=1}^{A} u_{ij}^{(2)\text{nucl}} + \frac{1}{6} \sum_{i,j,k=1}^{A} v_{ijk}^{(3)\text{nucl}}
\]

Formal structure

\[
H = T^{(1)} + V^{(2)} + V^{(3)}
\]

- kinetic energy of all nucleons (1-body operator)
- 2-body Coulomb interactions between protons
- 2-body and 3-body strong nuclear interactions

many-particle Schrödinger equation (in coordinate representation)

\[
(H - E_n) \Psi_n(x_1, x_2, \ldots, x_A) = 0 \quad \text{with} \quad x = (\vec{r}, s_z, t_z)
\]
Microscopic theories of nuclear structure

Nuclear Landscape

- Ab initio
- Configuration Interaction
- Density Functional Theory

known nuclei
stable nuclei
r-process
terra incognita

protons
neutrons
Mean-field concept

(Ref: Isotope Science Facility at Michigan State University, MSUCL-1345, p. 41, Nov. 2006)
Static Hartree-Fock: numerical implementation

Represent wave functions for all A nucleons on 3-D Cartesian lattice

Grid points:

\[ x_\alpha \quad (\alpha = 1, \ldots, N_x) \]
\[ y_\beta \quad (\beta = 1, \ldots, N_y) \]
\[ z_\gamma \quad (\gamma = 1, \ldots, N_z) \]

Wave function on the lattice becomes a complex-numbered array of dimension

\[ \psi(x, y, z; \sigma_z, t_z) \rightarrow \psi(x_\alpha, y_\beta, z_\gamma; \sigma_z, t_z) \]

\[ \psi_i(N_x, N_y, N_z, 2, 2) \]
$^{22}\text{Ne}$: note prolate quadrupole deformation
140 iterations, CPU time = 15 s
Nuclear structure theory

Main Goal
Input: nucleon-nucleon 2-body and 3-body interaction
Output: quantum many-body calculation of all observables, e.g.

1) Properties of nuclear ground state
   total binding energy energy
   n/p separation energies $\rightarrow$ dripline location, Q-values for nuclear reactions
   Proton and neutron density distribution
   equi-density contours $\rightarrow$ “nuclear shape” (spherical or deformed)
   moments of the density (root mean square radii)
   n/p single-particle energy levels
   pairing density $\rightarrow$ pairing gap, pairing energy

2) Excited states, in particular “collective” states
   surface vibrations, rotations, giant multipole resonances, pygmy resonance, ...
   quasiparticle excitations (pairing)
Nuclear collective excitations: surface vibrations

vibr. energy $\hbar \omega \approx 1.5 \text{MeV}$ $\rightarrow$ vibr. frequency $\omega \approx 2 \times 10^{21} \text{s}^{-1}$

quadrupole  octupole  hexadecapole
Nuclear collective excitations: rotations

rot. energy \( \hbar \omega \approx 100 \text{keV} \) \( \rightarrow \) rot. frequency \( \omega \approx 1.5 \times 10^{20} \text{s}^{-1} \)
Nuclear collective excitations: giant multipole resonances
proton / neutron density oscillations

Isovector giant dipole resonance

proton deficiency    proton surplus
Exotic collective modes (neutron-halo and neutron-skin nuclei) “pygmy” resonance and “scissors” mode
Cooper pair formation

Condensed matter physics: electron-phonon interaction

\[ +m_s \quad e^- \quad +k \]
\[ -m_s \quad e^- \quad -k \]

Nuclear physics: short-range residual interaction

\[ +m_j \]
\[ p/n \quad m_j \quad p/n \]
Stable isotope $^{120}\text{Sn}$ ($Z=50$, $N=70$)
VU-HFB code (2-D grid)

neutron mean field: $^{120}\text{Sn}$

Fermi energy = -7.99 MeV

proton mean field: $^{120}\text{Sn}$

Fermi energy = -8.16 MeV
Neutrons at Fermi energy are almost unbound. Neutron wave functions spread out much more than those of protons: “neutron skin” (“neutron halo” for light nuclei)
Strong continuum coupling near the neutron dripline

Figure 2. Schematic picture depicting correlations in nuclei near the drip-line, for which correlations involving the loosely bound and unbound continuum orbits play essential roles.
2n-dripline nucleus $^{122}$Zr ($Z=40$, $N=82$), HFB on 2-D grid

• Time-dependent mean field theory for nuclear reactions (TDHF),
  studies of deep-inelastic and fusion reactions of exotic
  neutron-rich nuclei
Heavy-ion reactions as function of impact parameter $b$

- **large** impact parameter $b$
  - Elastic collision, inelastic collision (Coulomb excitation)
  - Coulomb + nuclear inelastic excitation
  - Few-nucleon transfer

- **medium**
  - Multi-nucleon transfer

- **small**
  - Deep-inelastic collisions ($E^* \approx 30$ MeV), 2 fragments emerge
  - Fusion reactions, 1 fragment in exit channel
$^{48}\text{Ca} + ^{132}\text{Sn}, \ E_{\text{cm}} = 130 \text{ MeV}, \ b = 4.6 \text{ fm (deep-inelastic)}$

TDHF, SLy4 interaction, 3-D lattice (50*42*30 points)
$^{48}\text{Ca} + ^{132}\text{Sn}, \ E_{\text{cm}} = 130 \text{ MeV}, \ b = 4.45 \text{ fm (fusion)}$

TDHF, SLy4 interaction, 3-D lattice (50*40*30 points)
Heavy-ion fusion leading to superheavy element Z=112

spherical + spherical: “cold fusion” (E* small)

spherical + deformed: “hot fusion” (E* large)