As shown on our Website, many heavy nuclei decay by spontaneous $\alpha$-emission. Example: radium isotope $^{224}_{88}\text{Ra}$ [notation is $A=Z+N \frac{1}{2} X$] decays into radon:

$$^{224}_{88}\text{Ra} \rightarrow \text{Th}_{12}=3.6d \rightarrow ^{220}_{86}\text{Rn} + ^{4}_{2}\text{He}$$

"parent" nucleus \hspace{2cm} "daughter" nucleus \hspace{2cm} $\alpha$-particle

We may think of radium nucleus as radon (Rn) "core" surrounded by weakly bound $\alpha$-particle.

Potential ( Coulomb + Strong nuclear) seen by $\alpha$-particle:

We simplify this potential so we can get an analytical result:

$$V(x) \rightarrow \text{We get} V_c = \text{Coulomb barrier}$$

Typical values: $-V_0 = -50 \text{MeV}$

$$x_0 \approx R = 1.2 \text{ fm} \times A^{1/3} = 7.3 \text{ fm} \text{ (for Ra)}$$

$$V(x \geq x_0) = V_{\text{core}}(x) = \frac{2e^2 x_0 e^2}{x}$$

$$(2z = 2, z_0 = 86)$$
Determine WKB barrier transmission coefficient

\[ D = e^{-R} \]

with

\[ R = \frac{2}{\hbar} \int_{x_0}^{x_e} \sqrt{2m_\alpha \left( \frac{2e^2}{x} - E_x \right)} \, dx \]

One finds for the integral (Shankar p. 445, Mayer-Kuckuk p. 77)

\[ y = \frac{2}{\hbar c} \sqrt{\frac{2(m_\alpha e^2)}{E_x}} \, \tan^{-1} \left( \frac{e^2}{x_0} \right) \]

with

\[ y = \frac{x_0}{x_e} = \frac{\frac{2e^2}{x} \, \text{ratio of } \alpha\text{-particle energy to Coulomb barrier height.}}{\tan^{-1} \left( \frac{e^2}{x_0} \right)} \]

\[ D = e^{-R} \]

for \( \alpha \)-decay is called the "Gamow factor".

All quantities in \( y \) are experimentally accessible; in particular, we need \( E_x \) and \( V_0 \).

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Estimate of \( \alpha \)-decay rate \( R \)

\[ R \left[ \frac{1}{\text{sec}} \right] = P_\alpha \times R_b \times D \]

with

\[ D = e^{-R} \]

\( P_\alpha = \) probability that \( \alpha \)-particle is formed in nuclear surface region of heavy nucleus \( (P_\alpha \approx 2) \)

\( R_b = \) rate of "barrier assaults"/second.

\( \alpha \)-particle moves back and forth inside potential well \( (\text{distance } 2x_0) \)

\[ R_b = \frac{V_\alpha}{2x_0} \]

with

\[ T_\alpha = E_x + V_0 \approx E_x + 50\text{MeV} \]

\[ m_\alpha \frac{V_\alpha^2}{2} = E_x + 50\text{MeV} \Rightarrow \sqrt{\frac{V_\alpha}{2}} \]
radioactive decay law: $N(t) = N_0 e^{-Rt}$

mean lifetime: $\tau = \frac{1}{R}$ (activity decays to 1/e)

half-life: $T_{1/2} = \tau \cdot \ln 2$ (activity decays to 1/2)

For $\alpha$-decay: $T_{1/2}^\alpha = \frac{\ln 2}{\frac{1}{2} \cdot R_k \cdot D} = \frac{\ln 2}{\left(\frac{\alpha}{2\lambda_0}\right) \cdot e^{-\tau}}$

Examples (Raney - Kreckuk, p. 77):

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$E_\alpha$ (MeV)</th>
<th>$D = e^{-\tau}$</th>
<th>$T_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{144}_{60}$Nd (neodymium)</td>
<td>1.8</td>
<td>$2.2 \times 10^{-42}$</td>
<td>$2 \times 10^{15}$ y</td>
</tr>
<tr>
<td>$^{224}_{88}$Ra (radium)</td>
<td>5.7</td>
<td>$5.9 \times 10^{-26}$</td>
<td>3.6 d</td>
</tr>
<tr>
<td>$^{212}_{84}$Po (polonium)</td>
<td>8.8</td>
<td>$1.3 \times 10^{-13}$</td>
<td>0.3 $\mu$s</td>
</tr>
</tbody>
</table>

We see that with increasing $E_\alpha$, the half-life for $\alpha$-decay decreases by many orders of magnitude!