MODE-LOCKED LASERS

Reading: O’Shea, Chapter 5 on modifying the output of a laser oscillator

Although we have discussed the phenomenon of gain and oscillation in a laser as if all the transitions occurred in just the same way, in a real laser, the process of laser activity is extremely complicated because of inhomogeneous broadening of the atomic or molecular transitions. In this notebook, we explore the consequences of that inhomogeneous broadening for the generation of ultrashort laser pulses.

1. Review of laser gain and oscillation

The gain constant which prevails inside any laser oscillator at a given frequency n is clamped when the laser is operating in the steady state, at the value determined by the gain threshold equation:

\[ g_{\text{threshold}} = a - \frac{1}{L} \ln \left( \frac{R_1 R_2}{1} \right) \]  

where L is the length of the resonator, the distributed loss of the medium is a, and the mirror reflectivities in the laser cavity are R₁, R₂.

The gain constant in a spatially distributed laser medium is given by

\[ g(n) \int \frac{(N_2 - N_1)^2 g(t)}{8 \rho n^2 t_{\text{spont}}} \]  

The new twist about which we have not spoken up to now is that, in the wave picture, the laser oscillates when the gain is sufficient to support standing waves in the cavity. This means that in addition to the center frequency of the laser transition, other transitions can lase as well provided that their wavelengths are such as to add up to standing waves. These modes can exist in either the transverse or longitudinal directions. In the case of longitudinal oscillations, standing waves can be formed for any two frequencies satisfying the equation:

\[ n_{q+1} - n_q = \frac{2 \pi}{n L} \]  

where n is, as usual, the index of refraction of the laser medium. If the lineshape function for a particular wavelength is sufficiently broad, and if the threshold is sufficiently low for the laser to oscillate, it is quite possible that many different longitudinal modes will be lasing in competition with each other. However, there is no guarantee that these modes will oscillate in phase, so the coherence of the oscillation may be reduced. Mode locking is a way of making use of these longitudinal modes so that one can keep the laser coherent.

2. Beats and Wave Packets

Before going on with our mode-locking discussion, let us consider an analogy. When you learned about waves in elementary physics, you learned that adding two waves with only slightly differing frequencies localized the total wave amplitude in space, a localization that was reflected in the phenomenon of audible beats.
Notice that the beats continue to extend throughout space for many wave periods, and that the amplitude of the superimposed waves is twice that of either separately. Now notice what happens when we add a third wave, extending the range of times over which the wave is plotted: the localization is somewhat stronger, and the total wave amplitude is larger, but one still has substantial amplitude in the "intervening" space.

If we now plot ten waves extending over the same time or space domain, we see significant differences. Now the range over which the amplitude is quite small has been extended substantially, while the ratio of the central amplitude spike to the small-amplitude region is considerably larger.
This trend continues as we add in more and more waves with smaller and smaller phase differences between them. If we were to add a continuous range of phases, corresponding to the quantum-mechanical uncertainty in the momentum (wavelength) of the wave packet, we could expect the central amplitude to grow, and the region over which the waveform would be repeated to recede ever further away.

3. Physics of ultrashort pulses
Suppose we have an inhomogeneously broadened laser medium, in which oscillation takes place at many different frequencies separated by a common frequency difference

\[ w_q - w_{q-1} \int Dw = 2p \left( n_{q+1} - n_q \right) = \frac{2p}{nL} \frac{\Delta n}{nL} \quad (4) \]

A schematic of the situation in such a laser is shown in the figure above. The total optical field resulting from a multimode oscillation would then be given by

\[ E(t) = \sum_m E_m e^{i \left[ \left( w_m + nDw \right) t + f_m \right]} \quad (5) \]

where the index \( m \) runs over all the possible modes which can lase because they have gain above threshold.

Now consider the optical field at a time \( T=2L/c \) later. This field is given by

\[ E(t+T) = \sum_m E_m e^{i \left[ \left( w_m + nDw \right) \left( t+2L/c \right) + f_m \right]} \quad (6) \]

If we could find a physical means to arrange things so that the phases \( f_m \) could all be set to some common value (such as zero), \( E(t+T) \) would equal \( E(t) \) for all later round-trip times \( T \), and the total output would be equal to (assuming an even number of modes)

\[ E(t) = \sum_m e^{i \left( w_m + nDw \right) t} \quad (7) \]

**In-class exercise:** Where have you seen this equation before? Can you factor it in such a way that you can carry out this sum?

**Homework:** Use Mathematica to calculate the output for a realistic laser cavity length and for some appropriate number of modes in the visible part of the spectrum.

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**Figure 6-10** (a) Inhomogeneously broadened Doppler gain curve of the 6328 Å Ne transition and position of allowed longitudinal-mode frequencies. (b) Intensity versus frequency profile of an oscillating He–Ne laser. Six modes have sufficient gain to oscillate (After Reference [8].)
4. Practical mode-locking techniques

The most common mode-locked lasers used for ultrashort pulse generation are based on either broad-band fluorescent dyes or broad-band solid-state media such as Ti-doped sapphire crystals or Nd-doped glass (not Nd-doped YAG crystals!). All of these media have very broad emission bands. This is not necessarily an advantage for the more conventional lasers, but it is necessary for mode-locked lasers. Why?

The most common physical device for mode-locking the laser is an acousto-optic modulator, which drives a standing acoustic wave through the crystal at rf frequencies (70-100 MHz). This creates a Bragg grating from which the optical waves scatter; only those which arrive at a zero-crossing of the driving frequency are transmitted undisturbed. This has the effect of setting the "clock" or the phase of the waves to the same value, just as we did in our earlier discussion.