Section 2.3.2 Proca Mass Term

Exercise: Short pulses of microwave radiation are emitted from the Crab nebula, a distance of 2 kpc (kiloparsecs), or about \( L = 6 \times 10^{19} \) m, from the earth. When they reach the earth the frequency components of the pulses are dispersed so that higher frequencies around \( \nu = 115 \) MHz arrive first, after a travel time \( T \), while lower frequencies around \( \nu = 110 \) MHz appear about \( \Delta T = 1.5 \) s later. Although the observed dispersion is believed to be due to the intervening interstellar plasma, it nevertheless places an upper bound on the photon mass, if any. The classical description of the dispersive spreading of electromagnetic pulses is discussed in detail in Chapter 5, but for now we can take the quantum mechanical point of view and argue that the observed dispersion reflects group velocity dispersion since the photons travel as particles at their group velocity.

(a) Show that in terms of these observations the Proca constant is bounded by

\[
\mu_\gamma \leq 2\pi \sqrt{\frac{\nu^3}{Lc}} \frac{dT}{d\nu}
\]

(b) Use the data from the Crab nebula to show that the photon mass is bounded by \( m_\gamma \leq 6 \times 10^{-12} \) eV.