Section 10.3.2 Nonlinear Thomson Scattering

Exercise: Consider a laser pulse traveling in the direction $\hat{n}$, linearly polarized in the direction $\hat{e}$, described by the vector potential $A_0(\xi) = \hat{e}A(\xi)$, where $\xi = ct - \hat{n} \cdot \mathbf{r}$, incident on a particle of mass $m$ and charge $q$ traveling in the same direction with the initial velocity $c\beta_0 \hat{n}$. When the pulse strikes the particle, the particle oscillates and recoils from the incident field, as described in Chapter 4, and radiates nonlinear Thomson scattering. When the electrons are initially moving at a relativistic velocity in the direction opposite the laser pulse ($\beta_0 \approx -1$), the backscattered radiation is Doppler shifted to much shorter wavelengths, sometimes in the x-ray and gamma-ray regions, and can be used for a variety of laboratory experiments. This is called a “laser synchrotron” light source.

(a) Show that the radiation scattered in the backward direction $\mathbf{N} = -\hat{n}$ has the angular spectral fluence

$$
\frac{d^2\mathcal{W}}{d\omega d\Omega} = \frac{\mu_0 q^2 \omega^2}{16\pi^2 c^2} \frac{1-\beta_0}{1+\beta_0} \left| \int_{-\infty}^{\infty} e^{i\omega(1-\beta_0) \xi} \frac{q^2 A^2(\xi)}{mc^2} d\xi \right|^2
$$

(b) Consider a particle that is initially at rest, in the limit $|qA/mc| \ll 1$. Show that the total energy scattered in the backward direction is

$$
\frac{d\mathcal{W}}{d\Omega} = \int_0^\infty \frac{d^2\mathcal{W}}{d\omega d\Omega} d\omega = \frac{\mu_0 q^4}{16\pi^2 m^2} \int_{-\infty}^{\infty} A'^2(\xi) d\xi
$$

where $A'(\xi) = dA/d\xi$. Compare this with the backscattered energy predicted using the linear Thomson differential scattering cross section.