1. \[ l = l_r \exp[\alpha(T - T_r)] \approx l_r (1 + \alpha \Delta T) \quad \text{(for small } \Delta T)\]

\[ V = l^3 = l_r^3 \exp[3\alpha(T - T_r)] \approx l_r^3 (1 + 3\alpha \Delta T). \]

\((1 + \alpha \Delta T)^3 = 1 + 3\alpha \Delta T + 3\alpha^2 \Delta T^2 + \alpha^3 \Delta T^3 \approx 1 + 3\alpha \Delta T.\]

2. Volume of the water = 50.000 mL @ 20.0°C = 49.91035 g \(\rightarrow\) 50.30165 mL @ 40.0°C.

(a) \(V_{\text{cap}} = 0.30165 \) mL @ 40.0°C neglecting bulb expansion.

(b) \(V_{\text{bulb}} = 50.00975 \) mL @ 40.0°C \(\rightarrow\) \(V_{\text{cap}} = 0.2919\) mL.

Answer (a) is higher by 0.00975 mL, or 3.34%; the capillary length for 0.00975 mL is 15.3 mm, which is easily measured. Hence this error is significant.

3. The relative change is \(V_{25}/V_{40} = \rho_{40}/\rho_{25} = 0.99510\). The error is thus -0.49%. Since the assumption that the whole capillary is at 25°C is an extreme one, this problem may not be significant in typical situations. Still, the maximum error would be a measurable 0.7 mm on the capillary.

4. Since \(T = t + 273.15\), \(dV/dT = dV/dt\). Thus \(\alpha = (a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4)/V(t)\). At 32.5°C \(V = 1.00516 \) g/mL and \(\alpha = 3.248 \times 10^{-4} \) K\(^{-1}\).

5. (a) 50.127 g H\(_2\)O

(b) \(V = 50.2171\) mL

(c) 37.788 g alcohol

(d) \(\rho_{\text{alc}} = 0.75249 \) g/mL.

Air buoyancy = 0.062 g

(a’) 50.065 g

(b’) 50.1550 mL

(c’) 37.726 g alc.

(d’) \(\rho_{\text{alc}} = 0.75219 \) g/mL (0.040% error)

The error vanishes if the density of the unknown is the same as that of water.

6. For an ideal gas, \(PV = nRT\), and \(\alpha = 1/T\).

7. (a) \(x = t - 50°C = T - 323.15 \) K, so \(d\rho/dT = d\rho/dx\), so \(\alpha = -(b + 2cx + 3dx^2).\)

(b) \(\rho(30°C) = 0.78062 \) g/cm\(^3\); \(\alpha(30°C) = 0.001106 \) K\(^{-1}\).

(c) No.

8. All but mass.