Chemistry 236 -- Quiz 4

Lab Day ____

March 25, 2014 — Least Squares, Probability, and KaleidaGraph

1. (8) Consider the probability distribution, $P(x) = c\,(1 - x)$, defined over the range $0 \le x \le 1$. For this distribution, calculate: **(a)** the normalization constant, **(b)** the mean, **(c)** the variance, and **(d)** the standard deviation. Also, **(e)** give an equation that could be solved to obtain the median.

(a) $c = 2$ (b) $\mu = 1/3$ (c) $\sigma^2 = 1/18$ (d) $\sigma = \sqrt{1/18}$

(e) [CP Eq. (22) → $x_{1/2} = 0.2929$]

2. (7) (a) If you generate 10^4 random numbers having this distribution, how many are expected to fall within the x range 0.0–0.10? What is the standard deviation of this value?

$P(0 < x < 1.1) = 0.19$ counts = 1900 SD = 43.6

(b) Suppose you now generate 10^4 random quantities, each an *average* of 12 random deviates having this distribution, and you then histogram these results into 10 bins of width 0.1 each. Describe the resulting histogram, and give its approximate mean and standard deviation.

nearly Gaussian, with mean $\approx 1/3$ and SD $\approx 1\sqrt{18\times12}$

3. (6) We often fit data to polynomials in $(x - x_0)$ (x_0 a constant) to smoothly represent data.

(a) Using the General routine of KaleidaGraph in trial-and-error fashion for such fitting, under what conditions can you drop a term from the fit to obtain a statistically better fit (i.e., smaller s_y^2) having one fewer adjustable parameters?

If any adjustable parameter has |value| < its standard error (SE, the error reported by KG), that parameter can be set = 0 to improve the fit.

(b) Suppose you fit thermistor calibration data for the region 19-34° C to a cubic polynomial. What is an easy way to get the statistical uncertainty in the calibration correction at 29°C ?

Take $x_0 = 29$. Then when $x = 29$, the function value is equal to the value of the constant term, and its error is the SE for that parameter.

(c) Write **exactly** what you must enter in the Define Fit box to carry out such a cubic fit, taking x_0 to be 25.

a + b*(x-25) + c*(x-25)^2 + d*(x-25)^3; a=1; b=1; c=1; d=1

4. (6) Suppose t, u, and v have % uncertainties of 2.0%, 4.0%, and 5.0%, respectively. If $x = 5u^{-1/2}$, $y = 7\,v/u$, and $z = 10\,t^3\,v^2/u$, what are the % uncertainties in x, y, and z?

x: 2.0% y: 6.4% z: 12.3%