Chemistry 236 -- Quiz 2

Lab Day ____

September 29, 2010 — Least Squares and Probability Distributions

Pledge and signature:

Note: If you want your paper returned folded (*i.e.*, score concealed), please print your name on the back.

1. (7) Consider the probability distribution, $P(x) = c\,(1 - x)$, defined over the range $0 \le x \le 1$. For this distribution, calculate: (a) the normalization constant, (b) the mean, (c) the variance, and (d) the standard deviation.

   (a) 2 (b) 1/3 (c) 1/18 (d) $1/18^{1/2}$

2. (6) Suppose you generate 10^5 random numbers using this distribution.

   (a) How many would you expect to fall within the x range 0.40–0.50? And what is the standard deviation of this value?

   11000 $(11000)^{1/2}$ (Poisson)

   (b) If you now generate 10^5 *averages* of 8 such random numbers, what is the expected mean and standard deviation for these averages?

   1/3 $\sigma/8^{1/2} = 1/12$

3. (5) Briefly sketch on the provided grid the *shapes* of the following probability distributions (*i.e.*, don't worry about the y-axis scale). Label your curves clearly.

   (a) $P(x)$ from Prob. 1 above; straight line, from (x,y) = (0,2) to (1,0)

   (b) what you expect if you average 20 random deviates from this same $P(x)$;
   narrow normal distribution centered at x = 1/3

   (c) what you observed in KG4 when you histogrammed averages of 2 uniform random deviates.

   histogram approximates a triangular distribution, peak at x = 1/2

4. (8) **Least Squares and KaleidaGraph.**

   (a) We often fit data to polynomials in x or in $(x - x_0)$ (x_0 a constant) to achieve a smooth representation of data. Suppose you have fitted data, unweighted, to such a function. When do these results tell you that dropping a term will yield a smaller s_y^2, hence a statistically better fit?

   If the absolute value of any parameter is smaller than its statistical error, setting it = 0 will reduce s_y^2.

   (b) Suppose you fit thermistor calibration data for the region 19-34° to a quadratic polynomial. How can you define this fit so as to easily obtain the calibration correction at 29° *and* its uncertainty?

   Fit to $a + bz + cz^2$, w/ $z = (x - 29)$. Then when x = 29, the correction = a and its error s_a.

   (c) Write **exactly** what you should enter in the Define Fit box to fit your sums of 12 uniform random deviates to a Gaussian function. [gf(x) NOT adequate here!] What values should you obtain (approximately) for the parameters that govern the *location* and *width* of the distribution?

   a*exp(-(x-b)^2/c^2); a=1000; b=6; c=1. [or a*exp(-(x-b)^2/2/d^2)] The location parameter (b) should be about 6, and the width — d = 1 or c = sqrt(2).