

October 1, 2009 — Probability Distributions and Related Statistics

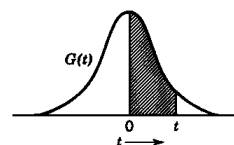
**Pledge and signature:****Note:** If you want your paper returned folded (i.e., score concealed), please print your name on the back.

1. (8) Consider the *uniform distribution* over the range  $-1 \leq x \leq 2$ . For this distribution, calculate:  
 (a) the normalization constant, (b) the mean, (c) the standard deviation, and (d)  $\langle x^3 \rangle$ .

2. (2) If you generate  $10^5$  random numbers using this distribution, how many would you expect to fall within the range 1.00–1.10? What is the standard deviation of this value?

3. (5) You now generate  $10^5$  *averages* of 16 such random numbers. (a) Calculate the expected mean and standard deviation for these averages; and (b) use the accompanying table to estimate how many of the  $10^5$  averages would fall between 0.9 and 1.2.

Table 4-4. Error Function  $\frac{1}{2} \operatorname{erf}(t)$  from 0 to  $t$  and Ordinate Values  
 $G(t) = (1/\sqrt{2\pi}) e^{-t^2/2}$



$t$	$\frac{1}{2} \operatorname{erf}(t)$ Area	$G(t)$ Ordinate	$t$	$\frac{1}{2} \operatorname{erf}(t)$ Area	$G(t)$ Ordinate
0	0	0.3989	2.0	0.4773	0.0540
0.1	0.0398	0.3970	2.1	0.4821	0.0440
0.2	0.0793	0.3910	2.2	0.4861	0.0355
0.3	0.1179	0.3814	2.3	0.4893	0.0283
0.4	0.1554	0.3683	2.4	0.4918	0.0224
0.5	0.1915	0.3521	2.5	0.4938	0.0175
0.6	0.2258	0.3332	2.6	0.4953	0.0136
0.7	0.2580	0.3123	2.7	0.4965	0.0104
0.8	0.2881	0.2897	2.8	0.4974	0.0079
0.9	0.3159	0.2661	2.9	0.4981	0.0060
1.0	0.3413	0.2420	3.0	0.4987	0.0044
1.1	0.3643	0.2179	3.1	0.4990	0.0033
1.2	0.3849	0.1942	3.2	0.4993	0.0024
1.3	0.4032	0.1714	3.3	0.4995	0.0017
1.4	0.4192	0.1497	3.4	0.4997	0.0012
1.5	0.4332	0.1295	3.5	0.4998	0.0009
1.6	0.4452	0.1109	3.6	0.4998	0.0006
1.7	0.4554	0.0941	3.8	0.4999	0.0003
1.8	0.4641	0.0790	4.0	0.5000	0.0001
1.9	0.4713	0.0656	4.4	0.5000	0.0000

4. (3) You have generated  $10^4$  sums of 12 random numbers (as in KG 4) and now want to fit the resulting histogrammed data to the normal distribution. Using **a** for the scaling parameter, **b** for  $\mu$ , and **c** for  $\sigma$ , write **exactly** what you must enter in the KaleidaGraph Define Fit box to obtain the desired estimates of the mean and standard deviation.
5. (3) In one of your KG4 exercises, you calculated and histogrammed  $10^4$  averages of **two** uniform random deviates. Sketch the approximate appearance of your histogram. (Include values along the  $x$  axis and approximate values along the  $y$ -axis.)
6. (6) Suppose  $t$ ,  $u$ , and  $v$  have % uncertainties of 2.0%, 4.0%, and 5.0%, respectively. If  $x = 5v^{-1/2}$ ,  $y = 7v/u$ , and  $z = 10t^2v/u^3$ , what are the % uncertainties in  $x$ ,  $y$ , and  $z$ ?