I. (35) **Kinetic Theory of Gases.**

A. Calculate the total translational energy (in J) for (1) 1.50 mol of O\(_2\) at 25°C and 1.00 atm, and (2) 3.00 mol of CO\(_2\) at 1000 K and 11.11 atm.

B. For Ar atoms, calculate the ratios: (1) \(\langle \varepsilon_{\text{tr}}(1000 \text{ K}) \rangle / \langle \varepsilon_{\text{tr}}(100 \text{ K}) \rangle\), and (2) \(\langle \nu(1000 \text{ K}) \rangle / \langle \nu(100 \text{ K}) \rangle\).

C. Calculate the ratio, \(\nu_{\text{rms}}(\text{O}_2 \text{ at } 1000^\circ\text{C}) / \nu_{\text{rms}}(\text{H}_2 \text{ at } 100^\circ\text{C})\). [Take the atomic masses to be 16 and 1.]

D. For each pair of integrals given below, pick the most specific from among the following answers:

1. \(X = Y\) 
2. \(X = -Y\) 
3. \(X \neq 0; Y = 0\) 
4. Both \(\neq 0\) but no simple relation

(a) \(X = \int_0^\infty x^3 e^{-ax^2} \, dx \); \(Y = \int_0^\infty x^3 e^{-ax^2} \, dx\)

(b) \(X = \int_0^\infty e^{-ax^2} \, dx \); \(Y = \int_0^\infty e^{-ax^2} \, dx\)

(c) \(X = \int_{-\infty}^\infty x^2 e^{-ax^2} \, dx \); \(Y = \int_{-\infty}^\infty x \, e^{-ax^2} \, dx\)

E. Give the functional form (normalization constant unimportant) for the kinetic energy distribution in three dimensions [we called this \(H(\varepsilon_{\text{tr}})\)]; and determine the most probable \(\varepsilon_{\text{tr}}\) in units \(kT\).
F. Consider the normalized three-dimensional velocity distribution, \( P(\upsilon_x, \upsilon_y, \upsilon_z) = g(\upsilon_x) g(\upsilon_y) g(\upsilon_z) \). Give an expression for the fraction of molecules having \( a < \upsilon_x < b \), \( c < \upsilon_y < d \), and \( q < \upsilon_z < r \). Use this to evaluate: (1) the fraction of molecules having \( \upsilon_x > \langle \upsilon_x \rangle \) (irrespective of their \( \upsilon_y \) and \( \upsilon_y \)); and (2) the fraction of molecules having both \( \upsilon_x > \langle \upsilon_x \rangle \) and \( \upsilon_z > \langle \upsilon_z \rangle \) (irrespective of their \( \upsilon_y \)).

G. Indicate whether the items in each of the following pairs are the same or different:

1. The fraction of \( ^{20}\text{Ne} \) atoms having \( \upsilon > \upsilon_{\text{rms}} \) at 1000 K vs. the fraction of \( ^{4}\text{He} \) atoms having \( \upsilon > \upsilon_{\text{rms}} \) at 300 K.

2. The fraction of \( ^{20}\text{Ne} \) atoms with \( \varepsilon > kT \) at 1000 K vs. the fraction of \( ^{4}\text{He} \) atoms with \( \varepsilon > kT \) at 300 K.

3. The most probable kinetic energy \( \varepsilon_{\text{tr}} \) in the kinetic energy distribution \( H(\varepsilon_{\text{tr}}) \) vs. the kinetic energy of a molecule having the most probable speed \( \upsilon_{\text{mp}} \) in the M-B distribution \( G(\upsilon) \), both at the same \( T \).

II. (10) Surface Tension – Capillary Effect. For the Hg-air interface on glass, the contact angle is \( \theta = 140^\circ \). A glass capillary of diameter 0.3 mm is inserted into a beaker of Hg. Sketch a qualitatively correct diagram of the resulting capillary effect, including a clear indication of the levels of Hg in the capillary and in the beaker, and a graphical definition of the contact angle \( \theta \).
III. (30) **Electrical Conductivity.** Consider an aqueous solution of calcium acetate at 25°C and 1 atm. The \( \lambda^\infty_m \) values (\( \Omega^{-1} \) cm\(^2\) mol\(^{-1}\)) for the cation and anion are 118.0 and 40.8, respectively.

A. For this solution, calculate (1) \( \Lambda^\infty_m \), (2) the transport number \( t^\infty_- \), and (3) the mobility \( u^\infty_+ \).

B. Consider a solution of concentration \( c = 0.00350 \) c\(^-\) contained in a conductivity cell having electrodes 7.00 cm apart, to which a potential of 2.50 V is applied. Calculate (1) the electric field strength, (2) the drift velocity \( \nu_+ \), and (3) the conductivity \( \kappa \). (Continue to use the infinite dilution assumption where necessary.)

C. Now consider the weak acid, acetic acid. The conductivity of a 0.001028-c\(^-\) aqueous solution at 25°C is \( 4.95 \times 10^{-5} \) \( \Omega^{-1} \) cm\(^{-1}\). \( \lambda^\infty_m \) for H\(_3\)O\(^+\) is 350.0 \( \Omega^{-1} \) cm\(^2\) mol\(^{-1}\). Use this information to estimate the fractional dissociation \( \alpha \) and the acid ionization constant \( K_c \). (Take \( \gamma_\pm = 1 \) here.)
IV. (35) **Derivations.**

A. (15) Derive the Langmuir isotherm for adsorption of a gas on a solid. Clearly define all symbols and state the assumptions behind the model.

B. (10) Stokes’ Law for the frictional force on a solid sphere of radius $r$ moving at speed $\upsilon$ through a Newtonian fluid of viscosity $\eta$ is $F_{\text{fr}} = 6\pi \eta r \upsilon$. At terminal speed in a gravitational field, this force is equal and opposite to the gravitational force. Use this relationship to solve for the terminal speed in terms of the quantities already defined, the gravitational acceleration $g$, and the densities of the sphere and fluid, $\rho$ and $\rho_{\text{fl}}$, respectively.

C. (10) As was noted in class, the same Stokes model can be used to treat the drift motion of ions in solution experiencing an electric field $E$. The force of the field has magnitude $|z| e E$. This force is equal and opposite to the frictional force of an ion of effective radius $r_B$ drifting with speed $\upsilon_B$. Use this information to derive a relation giving the radius $r_B$ in terms of the ion’s mobility $\mu_B$ and the solvent viscosity $\eta$. 
V. (20) **Thermal Conductivity.** Glass has a thermal conductivity of about 0.01 W cm$^{-1}$ K$^{-1}$, while the thermal conductivity of air is about $3 \times 10^{-4}$ W cm$^{-1}$ K$^{-1}$. Calculate the rate of heat transmission into a house (in W) for a 1-m$^2$ window having two 4-mm panes separated by 1.2 cm of air. The inside of the house is maintained at a temperature of 22.0˚C, while the outside temperature is 36.0˚C.

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\int_0^\infty x^{2n} e^{-ax^2} \, dx = \frac{(2n)!}{2^{2n+1}} \frac{\pi^{1/2}}{n! \, a^{n+1/2}}.
\]

\[
\int_0^\infty x^{2n+1} e^{-ax^2} \, dx = \frac{n!}{2 \, a^{n+1}}.
\]

\[
\int_0^\infty x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}}.
\]