Fragmented Trade and Manufacturing Services

- Examples for a Non-convex General Equilibrium

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Abstract

This paper applies the Jones - Kierzkowski model to the contract manufacturing service industry. Stylized facts of that industry imply a theory of non-convex general equilibrium. The cost structure combines a constant marginal cost and a positive fixed cost; Marshalian free entry - free exit prevails. This implies a distinct market structure (which is nether perfect nor monopolistic competition, nor the usual Bertrand oligopoly), and a *generalized* equilibrium concept, based on the 'full employment' and 'competitive profit' conditions.

In a class of examples where the technology is Ricardian for fabrication and Leontief for assembly, with fixed costs for 'service links', it is proved that there always exists Pareto optimal allocations, supported by a concept of *generalized* equilibrium (but – as shown by Koopmans – not by the Walras equilibrium, where the firms with increasing returns operate as price takers). Implications on specialization and cross country income distribution are noted.

Key words:

Contract manufacturing
Division of labor
Fixed cost
Fragmentation
General equilibrium
International trade
Nonconvexity

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1. Motivation

The recent rise of the contract manufacturing industry shows that managing supply chains is not cost free. Otherwise, manufacturers would never pay for such service. Clearly, the fragmentation studies of Jones and Kierzkowski ("JK") forthcoming, has captured the essence of international division of labor, in the era of globalization. To explore its full implications, this study casts their pioneering studies as a non-convex general equilibrium, based upon assumptions distilled from empirical observation. There may be other and better interpretations of both the observed reality and what JK really meant than what appear in this study. But for analysis, we study models, one at a time.

In the study of increasing returns, real life relevance is a proper concern. It is now well known that there may never be one theory satisfactory for all applications (see for example, Quinzii, 1992). Attention must be focused on the more relevant applications.

The next section distills observations on contract manufacturers into stylized facts, before interpreting JK and proposing a new equilibrium concept which confirm 'market supports efficiency'. It has novel implications on *what* one may observe in resource allocation and income distribution, and *how* we may apply JK to real life issues. Some digression is made about the foundations of micro-economic analysis of the firm to shed light on the equilibrium concept we introduced.

In a class of simple cases, we shall first prove the existence of Pareto optimal allocations, then establish their distinct properties. We show that even with scale economy, the suggested new equilibrium concept implies that decentralized decisions support optimality (though the Walras equilibrium does not). Novel properties of such solutions are then noted. This study also clarifies some classical issues of international economics and comments on possible generalizations.

2. Facts: Old and New

The advantage of mass production has been recognized since the Industrial Revolution. Adam Smith (1776), a contemporary observer, noted that (a) efficient pin-making took 18 operations, *and* (b) the division of labor is limited by the extent of the market.[1] For economists, how to choose the *degree* of division of labor almost became a lost art, once Ricardo made the division of labor an either-or choice of 'to trade or not to trade'. Yet Smith's concern about scale economy remains clear. If it is desirable and costless, one should always observe the fullest division of labor. But if the division of labor is desirable but costly (say, in managing the supply chains), a larger market is then needed to justify this management. Hence this is a situation with increasing returns.

The above discussion is even more relevant today than in the days of Smith. Traditional trade theory adapts well to such traded inputs like crude oil and pig iron – bulky goods with standard specifications, transacted at arms' length. Today, goods like clothing, automobiles, and electronic products not only have complex composition but also experience frequent style change. To manage the large number of parts and components of diverse origin becomes an independent activity: they must meet specifications, be ready on time, in large volumes, and at a competitive cost. Such management tasks have become the basis of a separate industry: the contract manufacturing service.

We find the information for the sub industry, contract electronic manufacturing, particularly valuable.[2] Documented observations about the industry may be distilled into stylized facts:

(a) (Cost structure) Firms enjoy internal increasing returns to scale (such as those due to fixed costs).[3]

(b) (Industry structure) A few large dominant firms operate with awareness of each other.[4]

(c) (Market position) Profit margin is thin due to the presence of many potential entrants.[5]

(d) (Mode of operation) Firms manufacture for contracts won by price quotations.[6]

Point (a) is consistent with the JK studies, where fabrication is under constant returns but service links incur a fixed cost. With average cost falling steadily at all outputs, this is not a form of the contestable market of Baumol, et al. (1988). Point (b) implies that this industry is not monopolistically competitive, with no firm having appreciable market share. For the same reason, it does not fit the mold of Novshek and Sonnenschein (1987). Point (c) signifies free entry: a flat demand curve for output faces each firm. Since no individual firm can raise the market price by reducing output, this market is not under Cournot competition. Point (d) shows the firms in the industry are not profit maximizing price-takers in the Walrasian sense. Nor do they operate in the fashion described by Allen and Hellweg (1986).

Thus, firms in the JK model form a distinct industry structure of their own. The *modus operandi* of this industry suggests a novel type of market adjustment as well as a novel form of equilibrium. These are critically important to our task of casting the JK model as a non-convex general equilibrium, and deducing distinct implications for today's world.

To close the model, all one needs to add is:

(e) Relative prices are flexible, and

(f) All markets are cleared.[7]

3. Market Adjustment and Equilibrium

Based on these stylized facts, one can describe the workings of the market as follows:

(a) The market provides all firms both (i) information on input prices, and (ii) contracts for output quantities.

(b) The firms submit to the market both (i) quotations on (unit) output prices, and (ii) plans for procurement of input quantities.

(c) Only those firms who submitted the lowest price quotations for output may be awarded contracts (some of these firms may not; the aggregate requirements of inputs by those firms won contracts must equal the aggregate supplies of inputs from the households).

(d) The market is in equilibrium if all input markets are cleared and outputs are paid to the owners of the inputs.

Firms are grouped by types. Each type always has potential entrants competing for any positive profit. Thus zero profit is always observed.

The information flow is shown in Figure 1.

[INSERT FIGURE 1 HERE]

The above discussion suggest that a generalized Walrasian equilibrium to encompass a sector with internal increasing returns where firms produce for contract.

Let w be a vector of input prices, q_k and c_k be the output and the average cost for firm k, then,

$$c_k = c_k(w, q_k), \tag{3.1}$$

Let p_k and p be the price quotation of firm k and the equilibrium price. Then at the equilibrium, this increasing returns sector satisfies two triplets of complementary slack conditions:

I. The Bertrand condition (winning price quotations):

$$p_k - p \geq 0, \tag{3.2}$$

(Equilibrium price is the lowest price quotation)

$$q_k \geq 0, \quad \text{(Output is non-negative)} \tag{3.3}$$

$$q_k (p_k - p) = 0. \tag{3.4}$$

(At the equilibrium, only firms quoting the lowest price produce)

II. The Marshall condition (free entry):

$$p - c_k(w, q_k) \leq 0 \quad \text{(No firm makes positive profit} \tag{3.5}$$

$$q_k \geq 0, \quad \text{(Output is non-negative)} \tag{3.6}$$

$$q_k [p - c_k(w, q_k)] = 0. \tag{3.7}$$

(Any output is produced at zero profit)

Together with the Walrasian conditions for firms under non-increasing returns and the households, we have a Bertrand-Marshall-Walras Equilibrium.

The discussion of equilibrium concepts is important in international trade, which is a structured general equilibrium. At the center of trade theory, the characterization of the patterns of specialization and price-income configuration have always been viewed as attributes of a model in equilibrium.

In the following sections, it will be seen that this empirically-based definition of equilibrium (a) is logically self-consistent, (b) exists for a class of examples rising out of the JK theory, and (c) resolves the issue of whether the market supports Pareto efficiency, a conclusion which cannot be reached in the pure Walrasian model of price takers under internal increasing returns (Koopmans, 1957, p. 50).

4. A Graphic Treatment: Efficiency and Supportability

In Figure 2, we consider an example taken from Jones and Kierzkowski, ibid., where there is one input (labor) and one output, with two production processes exhibiting respectively constant and increasing returns. Considering the fixed supply of labor, the social planner will choose Q, the point for the maximum attainable output.

In the upper portion of Figure 2, the aggregate production set (with the envelope of the constant returns and increasing returns processes as its boundary), is depicted with hatched borders. It is non-convex and unbounded. Yet the non-convex 'attainable aggregate production set' (defined in Debreu, 1959, p.76, and shown here as a shaded set) is bounded. It is also supported by price lines like OQ and SQ. The Marshallian condition of 'no entry, no exit' would exclude SQ and isolate OQ. The existence of a price support for the social planner raises the question: can the optimal resource allocations be supported under decentralized decisions in a market economy. The worldwide abandonment of the failed centrally planned economy makes this question relevant.

In the 'classical' sense as in Koopmans, ibid, the answer must be no. There, we come up against,

The Koopmans Dilemma

With an increasing returns technology, an optimal allocation, Q, may fail to be decentralized to the decisions of one price-taking consumer, and one price-taking producer.

In Figure 2, the price-taking producer will reject Q as it is seen to be inferior to points like R, even though the latter is unfeasible due to the full employment constraint. The superiority of R over Q is due to the fact that large volume lowers the average cost by spreading the fixed costs.

Note here that the marginal cost - pricing approach is not very valuable. In the upper panel, it focuses on the line through QR. Thus, it only '*supports*' Q, without '*singling out'* Q, the 'corner maximum'. In the lower panel, marginal cost is irrelevant. In contrast, under the Marshallian industry equilibrium of no entry and no exit, it is 'average cost-pricing' that supports first-best optimal allocation! In the JK model, there is the simple formula:

(4.1) average cost/the marginal cost = the total cost/(the total cost - the fixed cost).

[INSERT FIGURE 2 HERE]

The Koopmans dilemma can be seen even more clearly in the lower half of Figure 2. A price-taking firm enjoying internal increasing returns would not only favor point q over θ, which is fine, but also regard q as inferior to a point like r (because a larger volume spreads the fixed overhead more). But in view of the full employment constraint, r is certainly 'a point too far'. Thus, under the unconstrained price-taking behavior of the firms, the Walrasian assumption cannot rule out the unfeasible. Thus one desires another equilibrium concept that can do the job.

The Bertrand-Marshall-Walras equilibrium fits reality. Contract manufacturing firms submit price bids for contracts with a *given quantity*. They optimize by selecting the method to produce that output at minimum cost. In Figure 2, they reject point θ (under the constant returns process) in favor of point q (under the increasing returns process), for the quantity contracted for.

Under this novel equilibrium concept, decentralization to the consumer-producer pair poses no problem. The Bertrand firm quotes the price only for itself, not a magnitude for the entire market. Equilibrium quantity is decided with the aid of the Marshallian condition. It allows the entry of efficient firms (who suffer no loss) at the equilibrium price. Unlike what Koopmans encountered, point Q (or q) is now rejected by noone.

Actually, the concept of the Bertrand-Marshall-Walras equilibrium in Figure 2, corresponds to the twin pillars of the trade literature: 'the full employment condition' and 'the competitive profit condition' in the terminology made popular by (Caves and Jones, 1973):

(i) The upper diagram reflects the Walras half (the full employment condition). Through the aggregate production function (shown in heavy line), the market-clearance level for the labor input is translated into the market clearance level for output.

(ii) The lower diagram reflects the Bertrand-Marshall half (the competitive profit condition). Through the cost function (also shown in heavy line), the zero profit price-wage configuration (for the active firms) becomes associated with their aggregate output.

The key fact here is, firms having no direct role in selecting its output. This is the crucial distinction between the Bertrand-Marshall-Walras firm and the Walrasian firm. This also shows how the Koopmans Dilemma is resolved.

For the classic convex economy, the three principal theorems of general equilibrium are: (a) the existence of a market equilibrium; (b) the First Fundamental Theorem of Welfare Economics: the market equilibrium is Pareto efficient; and (c) the Second Fundamental Theorem of Welfare Economics: Pareto efficiency can be supported by a market equilibrium. These serve as a benchmark for the non-convex general equilibrium of the JK model.

Results have been obtained for a class of examples for the JK model. (i) In Section 6 below, it will be established that Pareto efficient allocations exist. (ii) The resolution of the Koopmans Dilemma mirrors the Second Fundamental Theorem of Welfare Economics. It will be seen in Section 7, that Pareto efficient allocations are supportable with a Bertrand-Marshall-Walras equilibrium.

Parallel to the First Fundamental Theorem of Welfare Economics, one may ask whether every Bertrand-Marshall-Walras equilibrium Pareto is efficient. That is an open question today. Instead of the conditions of marginal equalities, what can safeguard efficiency is free entry. The key seems to be how to define 'no entry'. Most would agree that at an equilibrium, one should rule out the entry of a single firm to displace, say, two firms operating under an optimal size. But must it rule out the correlated simultaneous entry of a set of firms which displaces all firms engaging collectively in some non-optimal allocation?

5. A Digression on the Theory of the Firm

How should economists decide the manner in which firms operate? As building blocks of any general equilibrium model, firms must be cast as decision makers with their choice variables. To make the model tractable, theorists take shortcuts that suit their purpose. Walrasian firms pay no attention to their small but non-zero ability to influence market prices. Firms in the monopolistic competition model ignore altogether the fact that their rivals are not part of an abstract market force, but are individual decision makers just like themselves. Cournot oligopolists set only outputs but not prices.

Here supply chains are cast as Bertrand oligopolists, choosing their own type of chain and setting price quotations for contracts, but not deciding their own outputs. Although their scale economy is due to their fixed costs, they cannot select their output to spread those costs. Whether their price quotations win contracts is decided not by themselves but the market force, under free entry. The fact that supply chains lack monopoly power is due to a relatively blurred line between the supply chains and the suppliers of parts and components.[8] In principle, the latter can hire away the managerial labor force of an existing chain and take over its business.

In some sense, our use of facts from the contract manufacturing industry to study the inter-firm relations underlying the JK model is to follow the steps by Milgrom, North, and Weingast (1990), and Greif (1993) in deducing inter-personal economic relations from the record of medieval merchants. Available data does not lend the material to be studied with formal statistical methods.[9] Nonetheless, it is true in micro-economic no less than in other sciences, a careful scrutiny of the facts provides grist for the theorists' mill.[10]

6. A Class of Examples - The Existence of Efficient Allocations

It is time to concentrate now on the analytic content of this study. Since non-convex general equilibrium is not a familiar problem for most economists, the class of examples here is chosen for its technical transparency, using the elementary framework of linear programming. These examples constitute only a single special case of the JK model. It is believed however, that once the common principles are known, extensions into far richer contexts should be an easier task.

For simplicity, we adopt the following,

Assumptions:

1. There are m countries (or 'regions'[11]) each with a fixed supply of homogeneous labor

$$L_i, i = 1,...,m.$$

2. The unit productivity of labor in country i for the n kinds of produced inputs (simply, inputs, from now on, if there is no likely confusion) form the m - by - n matrix of constant, *strictly positive* entries,

$$A = [a_{ij}],$$

so that the *m* - by - *n* matrix of input *j* produced by country *i* is:

$$V = [v_{ij}] = [L_{ij} a_{ij}],$$

L_{ij} being the country i labor assigned to produce input j.

3. There is a finite class S of $K = 2^m - 1$ non-empty subsets of countries, (sk), (also called here as supply chains), such that each k corresponds to a 'type' of firms which can produce and use the inputs within that subset k of countries.

4. A supply chain k is 'active', only if a m - vector of required labor $L_k = (L_{ki})$ is assigned to manage the chain, where:

if $s(k)$ is a 'singleton' (consists of one country), then L_k is a null vector, (6.1)

and

if $s(k)$ includes two or more countries, then, (6.2)

$$L_{ki} > 0 \; for\, i \in s(k) \; and \; L_{ki} = 0 \; for\, i \notin s(k).$$

Remark.

A one-country supply chain k, $s(k) = \{i\}$, $i = 1, \ldots, m$ needs no managerial labor. A multi-country supply chain only needs labor for management in those country i within the subset $s(k)$ of countries. Let w_i be the wage in country i, the fixed cost for the service link in the JK model is regarded as managerial labor cost:

$$\sum_{i \in s(k)} L_{ki} w_i .$$

5. There is one *universal final* good q.

6. Only an active type-k firm can produce the universal final good.

7. The production function of any active type-k firm for the universal final good is the Leontief production function:

$$q_k = Min_j \{v_{kj} : 1 \leq j \leq n\}, \tag{6.3}$$

where

q_k is the output of the universal final good produced (that is, assembled) by firm k, and

v_{kj} is the total input *j* acquired from countries *i* in *s(k)*.

The formulation above is a variation of the simple Ricardian framework, with the following exceptions:

(a) Assumptions 4 and 6 on the fixed costs for managing the supply chain, which follow closely the JK model,

(b) Assumption 5, the postulation of a universal final product, under which Pareto optimum is equivalent to the maximization of the final output, is a special case of:

Assumption 5' All final goods are perfect substitutes for each other, and

(c) Assumption 7, the perfect complementary between the intermediate products. These simplify the calculation of examples and approximates the reality that the substitutability among various components and subcomponents is rather limited.

Lemma 6.1

Assumptions 2 and 7 mean every country can produce some output even in autarky.

To show how the above concepts and notations operate, we shall first apply the model to the problem facing a firm, and then address a question we alluded to earlier: what did Adam Smith mean when he treated division of labor not as an either-or choice, but as something which could vary by degree.

Price Quotations of a Firm

To show the relationship between Figure 2 and this class of examples, and to illustrate the workings of various parts of our model, we shall make some routine calculations regarding how firms decide their price quotations.

Let L_{ij}^{k} be the labor from country $i \in s(k)$ used to produce input *j* by firm *k*, then

$$v_{kj} = \sum_{i \in s(k)} L_{ij}^{k} a_{ij},$$

and under efficient production, the Leontief technology implies:

$$q_k = \sum_{i \in s(k)} L_{ij}^k a_{ij},$$

for all $j = 1, \ldots, n$,

and the total direct production cost is:

$$\sum_{j=1^n} [\sum_{i \in s(k)} (L_{ij}^k w_i)] = \sum_{i \in s(k)} (\sum_{j=1^n} L_{ij}^k) w_i,$$

so that the total cost for production and management is:

$$C_k = \sum_{i \in s(k)} [L_{ki} + \sum_{i \in s(k)} (\sum_{j=1^n} L_{ij}^k)] w_i.$$

Given q_k and w, cost minimization for a firm of type k has then the problem of finding:

$$Min \sum_{i \in s(k)} [L_{ki} + \sum_{i \in s(k)} (\sum_{j=1^n} L_{ij}^k)] w_i = \sum_{i \in s(k)} L_{ki} w_i + Min \sum_{i \in s(k)} (\sum_{j=1^n} L_{ij}^k) w_i$$

subject to:

$$\sum_{i \in s(k)} L_{ij}^k a_{ij} = q_k \qquad j = 1, \ldots, n.$$

This should yield a solution $\{(L_{ij}^{k}*): i \in s(k), j = 1, \ldots n\}$ and a payoff of $C_k*(w, q_k)$, say. This is an affine function, taking a form of $L'Q$ in Figure 2.

Given the vector of wages $w = (w_1, \ldots, w_m)$ and a contract for output q_i, a firm of type k has formally the problem of:

$$Max[p_k q_k - C_k^*(w, q_k)].$$

Now operating under the free entry condition, firms must avoid to be undercut by potential entrants within the same type. To win contract, any price quotation must satisfy the condition:

$$p_k - C_k^*(w, q_k) / q_k = 0. \qquad \text{(Firms at most breakeven)}$$

Next, one can shift from the operation of a firm to the configuration of a market. To understand how Adam Smith interpreted as the degrees of division of labor, consider,

Example 1 (A problem inspired by Adam Smith).

Product *Q* is assembled out of one unit each of both a component *x* and another component, made of one unit each of two *sub*components, *y* and *z*. These can be supplied from *regions* A and *B* within a country, or in another country, *C*. The configuration of unit labor productivity for fabrication is displayed in Table 1.

[PLACE TABLE 1 HERE]

For *unrestricted* efficient resource allocation, the i - i assignment criterion of Jones (1961) shows that some final goods will be assembled with *x* produced by *A*, *y* by *C* and *z* by *B*. But this pattern may be blocked by restrictions that either (a) the division of labor is only possible inside the national boundary, or (b) each *component* must be produced entirely within a single *region*. While in his efforts against the Corn Law, Ricardo focused on the restrictions due to Mercantilist laws, the insight of Smith goes also over to the case of heavy managerial costs of running supply chains, in the form of a fixed charge, as in the JK model. Certainly, it is costly to ascertain the comparative advantage of supplying a particular item over diverse locations, or down to the minute item. Yet, oftentimes, the cost of collecting information and confirming its continued validity does not rise appreciably with the scale of operations, at least over the relevant range. Thus, more efficient production assignment may prevail only for large markets.

Now not all types of supply chains would (or could, conceivably, for that matter) be simultaneously active. Let I_k be the active indicator for type *k*, so that:

$$I_k = 0 \;\; if\ type\ k\ is\, inactive, and\ \ I_k = 1, \;\; if\ type\ k\ is\, inactive, \qquad (6.4)$$

There is then a finite class $\prod$ *of* $2^K - 1$ distinct patterns of activity indices, each pattern π signifies the distinct subset of active supply chains k under it.

Next, one defines a pattern as *feasible* only if (i) each country belongs to some active supply chain and (ii) after the labor requirement of managing all the active supply chains, there is still some positive amount of labor input left in each country for production:

$$L_i^{\pi} = L_i - \sum_{k\in\pi} (I_k L_{ki}).0, \quad \forall i. \tag{6.5}$$

The class of feasible patterns is defined as $\prod^{o}$, which depends on vector, $L = \{L_i: \forall i\}$.

Lemma 6.2

For any strictly positive vector L, $\prod^{o}$ is not empty.

Proof.

There exists the pattern of universal autarky, $\pi^{o} = \{k = \{i\}: \forall i\}$,

with

$$\sum_{k\in\pi^0} (I_k L_{ki}) = 0, \; \pi^0 \in \prod^0 (L).$$

Next, consider

The output-maximization problem of the social planner:

Find:

$$q_{\max} = Max_{\pi\in\prod^0} q_{\max}^{\pi} \tag{6.6}$$

the maximum maximorum over a finite class of maximization problems, where, for each pattern π, there is the maximization problem:

$$q_{\max}^{\pi} = Max \sum_{k\in\pi} Min_j \{v_{kj} : 1 \le j \le n\}, \tag{6.7}$$

with

$$v_{kj} \ge 0, \forall k \in \pi, j = 1,\dots, n, \tag{6.8}$$

being the input *j* produced for *k*;

for each *j*, each $k \in \pi$,

$$v_{kj} = \sum_{i \in s(k)} L_{ij}^{k} a_{ij};\qquad (6.9)$$

$$L_{ij}^{k} \geq 0, \forall k \in \pi, j = 1,\ldots,n, i = 1,\ldots,m,\qquad (6.10)$$

being the country *i* labor producing input *j* for chain *k*;

also,

for each $i = 1, \ldots, m,$

$$\sum_{j=1^n} \sum_{\{k \in \pi:\ i \in s(k)\}} L_{ij}^{k} = L_i^{\pi}. \quad \text{(The full employment condition).} \qquad (6.11)$$

For each $\pi \in \prod^{o}$, a collection with finite many members, (6.7 - 11) defines a well posed linear programming problem, [and so does (6.6)], one has now proved,

Proposition 6.1 (Existence)

There exists a well defined social optimum for the JK model.

Proof. By the fact some output is surely producible (Lemma 6.2) and by construction, there is always a best solution among a finite set of linear programing problems.

Corollary 6.1

The Proposition remains valid should Assumption 7 be replaced with the more general Assumption 7'.

$$q_k = f(v_{k1},\ldots,v_{kn})\qquad (6.12)$$

where *f* is a strictly increasing, strictly quasi-concave, first order homogeneous function with all inputs being essential.

Sketch of a proof.

Use generalized linear programing rather than linear programing (Dantzig, 1963).

Remark.

Components usually can be redesigned to vary the input requirement to some degree.

To substantiate Corollary 6.1 and gain further insight into the model, it is convenient to consider the following particular example.

Example 2

Three countries *A*, *B*, and *C* produce two type of parts *x* and *y* which are assembled into a universal final product *q*. A unit of *q* requires both *x* and *y*, in fixed number of units, but the costliness of parts vary by design in a 'putty-clay' kind of process, so when parts are measured by their 'input content', the input requirement is indicated by:

$$q = x^{1/2} y^{1/2}.$$

The productivity per worker (again measured in 'input content') in the three countries are 1 unit of *x*, and 1/2 unit of *y* for *A* and *C*, but 1/2 unit of *x* and 1 unit of *y* for *B*. The cost of managing a supply chain is zero if it covers one country only, one unit of labor in each country for a two-country chain, and two units each for a three-country (global) chain.

The social planner's problem is well defined: to find $q(L_A, L_B, L_C)$, the maximum output, given the labor supply, choosing first what subset of all the $7 = 2^3 - 1$ supply chains to be kept active, and then the corresponding labor force allocation.

In the special case where the fixed labor supplies of countries A, B, and C are 4, 8 and 4, one can show that the optimal allocation of the social planner is to select the pattern $\pi^* = \{\{A, B\}, \{B, C\}\}$, say, with countries *A* and *C* allocating 3 units of labor to produce *x*, and country *B* allocating 6 units of labor to produce *y*. The total output is 6.

In comparison, no pattern containing the supply chain $\{A, C\}$ need be considered, since such a chain only diverts manpower from production for management and makes no economic sense.

The pattern π° of universal autarky has a total output of $16/3 < 6$. The global supply chain $\{A, B, C\}$ yields a total output of $2(6)^{1/2} < 6$. A pattern with a supply chain including A and B, but no supply chain including C, also its mirror image, (as well as their equivalents) are not optimal, since the output is: $(21)^{1/2} + 4/3 < 6$.

This example demonstrates two facts:

First, there are socially optimal allocations where the labor of a country works for two distinct supply chains.

Second, with substitutability between parts, the wage rates among countries are uniquely determined. In this case, workers in all countries receive a wage rate of 3/8 = 6/16 per unit of labor.

7. Some More Examples - Characterization of the Optimal Allocations[12]

Having shown that for the JK model, there always exist socially optimal allocations, the next example demonstrates the novel aspects of that optimal set.

Example 3

There are four countries in the world, each with 18 units of labor $(L_i = 18, \forall i)$. To assemble a unit of the universal final good, q, one requires one unit of each of the three components, v_1, v_2, and v_3. The values of labor productivity in fabrication are as follows.

[PLACE TABLE 2 HERE]

For any supply chain k over two or more countries, then two units of labor must be employed in each country for management $(L_{ki} = 2, \forall i \in k$ if k is not a singleton).

For illustration, under the pattern of universal autarky , $\pi = (\{1\}, \{2\}, \{3\}, \{4\})$, it can be easily shown that labor allocations and outputs should be as follows:

[PLACE TABLE 3 HERE]

In contrast, if there are two supply chains, under the alternative pattern of $\pi = (\{1, 2\}, \{3, 4\})$, labor allocations and outputs then should be:

[PLACE TABLE 4 HERE]

But globally efficient patterns come in two alternative forms, as shown below: either with the pattern of $\{\{1, 2, 3\}, \{4\}\}$, or with the pattern of $\{\{1\}, \{2, 3, 4\}\}$, where these two are mirror images of each other, but the pattern is quite asymmetric in how countries are grouped together. The full implications of such patterns will be explored in ensuing discussions

.[PLACE TABLE 5 HERE]

[PLACE TABLE 6 HERE]

Basically, the input - output relation is a function from a 16 dimensional space for labor allocation to the one dimension of total output. Therefore, one has arrived at

Proposition 7.1. (Characterization)

(i) The socially optimal allocation may neither be unique, nor form a convex set;[13]

(ii) In either globally optimum allocation, countries apparently symmetric to each other (namely, 1 and 4) play asymmetric roles: one in a supply chain; the other under autarky.

Proof. By Construction.

There are *four* countries and only *three* perfectly complementary intermediary goods. The non-matching of these numbers and the zero substitutability among the intermediate goods may seem to be the sources of the rather odd results.above. But the following comparative static analysis suggests otherwise. What causes the 'anomaly' is the costliness of the service link.

A Comparative Static Analysis

Suppose in the above example, assume the fixed cost for managing an *international* supply chain is not 2 units but any $z > 0$ units of labor. Clearly, if z is large, there would be no international supply chain, and if z is zero, one all-encompassing chain will emerge as in Table 7. But if z is somewhere in between, say, 2 (or any value above 4/3) then one returns to the situation of Tables 5 and 6 above: some international supply chain can form, but not the all-encompassing chain, and one gets the result in Proposition 7.1. Among countries who are symmetric *ex ante*, who will be included in and who will be excluded from the international chain becomes arbitrary. That may imply a non-convex set of multiple solutions for the social planner.

[PLACE TABLE 7 HERE]

Corollary 7.1.

For $z < 4/3$, there will be one solution for the social planner's problem: an all-encompassing supply chain;

for $2 \geq z > 4/3$, there are two solutions for the social planner's problem, each has one active three country supply chain;

for $z = 4/3$, all the above three are isolated solutions: for the social planner, with the formation of the all-encompassing supply chain a matter of indifference for the social planner.

8. Decentralized Implementation of the Optimal Allocation

Proposition 6.1 has already established that there is at least one optimal solution for the social planner. The question is, can this allocation be decentralized in implementation? One may gain some intuition from Figure 2. Essentially, we look for an average cost - pricing equilibrium. Here we can offer:

Proposition 8.1

The solution of the social planner in Proposition 6.1 can be supported with a Bertrand-Marshall-Walras equilibrium.

Proof.

The Walras part is easily seen as satisfied by,

(a) the full employment of labor in each country and

(b) all the produced universal final goods will be used to reward workers for consumption.

The proof will be complete if, for the Bertrand-Marshall part, one can find wage rates for all countries such that:

The maximum aggregate output is produced by those active firms,

(a) at the minimum cost for that output and

(b) with zero profit.

This is done by construction.

Note, the solution of Proposition 6.1 under an optimal pattern π^* for the supply chains is a solution of linear programing. This formulation implies:

a) There will be at most n active firms (or supply chains),

b) each of this is associated with output levels, also the input demands of the n country-specific types of labor for both management and production.

Each active chain corresponds to one equality form of the 'competitive profit' condition on the *n* country-specific wage levels. The latter take strictly positive values, since the presence of the autarky solution for each country sets a positive minimum for that country specific wage.

Corollary 8.1

The equilibrium wage configuration is often non-unique.

To illustrate that the two allocations in Tables 5 and 6 can be supported by market (as claimed in Proposition 8.1), the interested reader may set the equilibrium price of the final good to 1, and try the vector of wages $w = (5/27,\ 19/54,\ 19/54,\ 5/27)$, where the contracts are 10/3 for the autarkic country, and 16 for the three-country supply chain, and the average cost functions are $c(w;\ 10/3) = (27/5)w_i,\ i = 4$ or 1, for the former, and $c(w;\ 16) = (9/8)\ (w_1 + w_2 + w_3)$, *or* $c(w;\ 16) = (9/8)\ (w_2 + w_3 + w_4)$.

9. Recapitulation of the JK Model

In casting the JK model as a non-convex general equilibrium, we treat trade as happening between 'middle products' in the form of parts following the tradition of Sanyal and Jones (1982) while all assembly may be perceived as 'happening in household production' on a do-it-yourself ('DIY') basis. Resource allocation takes the form of worldwide specialization in the production of parts and the terms of trade are manifested in the worldwide distribution of wages (in terms of the universal final product). Figure 3 below supplies the visual representation.

[INSERT FIGURE 3 HERE]

We are now ready to highlight the strong and novel implications of the JK model on international distribution of wage income.

10. A Value Function for 'Supply Chains' and its Implications

The JK model as interpreted here has distinct and novel implications on the topical issue of world income distribution: with labor forces possessing apparently similar skills, economies may differ widely in levels of wage income.

To begin with, we introduce the concept of a value function *V(k)* over a supply chain, *k*, as the maximum output that can be assembled within those countries covered by chain *k*. Supposing that chain is active, then the competitive profit condition provides an equation relating this to the value of wage rates covered by that supply chain.

Using data for Example 3, the results for all the 15 types of supply chains follow:

$$V\{1\} = V\{2\} = V\{3\} = V\{4\} = 10/3, w_1 \geq 5/27, w_2 \geq 5/27, w_3 \geq 5/27, w_4 \geq 5/27;$$
$$V\{1,2\} = V\{3,4\} = 8, w_1 + w_2 = w_3 + w_4 \geq 4/9;$$
$$V\{1,3\} = V\{2,4\} = 171/23, w_1 + w_3 = w_2 + w_4 \geq 19/46;$$
$$V\{2,3\} = 80/11, w_2 + w_3 \geq 40/99;$$
$$V\{1,4\} = 32/5, w_1 + w_4 \geq 16/45;$$
$$V\{1,2,3\} = V\{2,3,4\} = 16; w_1 + w_2 + w_3 = w_2 + w_3 + w_4 \geq 8/9;$$
$$V\{1,2,4\} = V\{1,3,4\} = 272/23, w_1 + w_2 + w_4 = w_1 + w_3 + w_4 \geq 136/207.$$
$$V\{1,2,3,4\} = 96/5, w_1 + w_2 + w_3 + w_4 \geq 16/15.$$

The analysis below resembles the study of the core in non-cooperative game theory.

As we have seen, the worldwide supply chain *V{1, 2, 3, 4}* is not active for the Pareto efficient resource allocations. The two optimal patterns are patterns are:

$$\pi' = (\{1,2,3\}, \{4\}), \tag{10.1}$$

and

$$\pi'' = (\{1\}, \{2,3,4\}). \tag{10.2}$$

implying either

$$w_1 + w_2 + w_3 = 8/9, w_4 = 5/27,$$

or,

$$w_1 = 5/27, w_2 + w_3 + w_4 = 8/9.$$

Now, any firm in a market equilibrium (rather than under a social planner) must avoid being undercut by any potential entrant.

Thus, we have

Lemma 10.1

For Example 3, equilibrium wage income distribution must have the pattern:

$$w_1 = w_4 = 5/27, \tag{10.3}$$

$$w_2 + w_3 = 19/27. \tag{10.4}$$

Proof.

If in π' (π'''), $w_1 > 5/27$ $(w_4 > 5/27)$, then $w_2 + w_3 < 19/27$, opening the door to an entrant of type π'' (π') who can pay a higher combination of wages than the supply chain in question and still undercut the latter in unit price.

Furthermore, similar consideration implies

Lemma 10.2

The values of w_2 and w_3 must be within the following bounds:

$$178/621 \le w_2, w_3 \le 259/621. \tag{10.5}$$

Proof.

The possible entry of supply chains of types {1, 2, 4}, {1, 3, 4} would assure the lower limit for these two wages. The higher limit comes following (10.4).

We may summarize the above information into:

Proposition 10.1

The fixed cost of implementing international division of labor can introduce an element of inequality into the distribution of wage income, such as between countries 1 and 4 on one side and countries 2 and 3 on the other, whether or not these countries are served by the same supply chain in the equilibrium.

Remarks.

As Example 2 has shown, the indeterminacy of w_2, w_3 is the result of lacking substitutability between parts under the Leontief technology. In contrast, the sharply defined values for w_1, w_4 is rather unexpected, as a result of the free entry assumption.

Why is there a gap between w_1 and w_4 on one side, and w_2 and w_3 on another? Study of Table 2 is suggestive. Although the three parts are entirely symmetric to each other in their demand side, on the supply side, there exists asymmetry. In the optimal matching by the i - i assignment, there are two countries suitable to produce part 1, namely, countries 1 and 4, but one country each (2 and 3) to produce parts 2 and 3, and all countries have the same size of labor force. Thus, the possession of skills in relatively greater demand is the ultimate reason why wages are higher in some locations than others.

11. Concluding Comments

Three points may be made, on the observed facts, the future theoretic development and the applications.

First, the 'service links' in the JK model of fragmentation is identified with the free-standing firms operating in the manufacturing service industry. In the theoretic model we presented, under the Marshallian assumption of no entry, no exit, these firms managing the supply chains neither earn positive profit nor wield monopoly power on their own. In real life, those providing manufacturing services mostly operate out of headquarters in high income (or industrialized) economies and may also undertake some fabrication on the side, or vice versa. Hence the reported high net income of some of these firms and the assumption of a Marshallian industry equilibrium at zero profit are not necessarily mutually contradictory.

Second, due to the highly complementary nature of parts and components, labor inputs across countries often are not readily substitutable to each other, so that income divergence is expected. Costly work in carrying out division of labor will only accentuate the situation, and our exercise seems to confirm it.

To concentrate on characterizing the implications of the JK model in a general equilibrium context, we have attempted to avoid technical complexities in our examples. It seems that a simple reinterpretation should allow the input endowment of each country to be a vector and not a number, entering a linear homogeneous production function for each part. But then through the Ricardo-Viner form, studied in Jones (1971), the extension to decreasing returns to scale is obvious, and as Jones and Scheinkman (1977) taught us, an even fuller generalization is in sight. The general approach used here remains valid if the internal scale economy is such that the total cost function does not take the affine form with an additively separable term for fixed cost.[14]

Our draconian assumption of a single universal consumption good has served this purpose well. For generalization, the natural next step is to allow the presence of different final goods, producible from various parts under constant returns, but consumed by all households sharing the same homothetic utility index, as in a Heckscher-Ohlin-Vanek model. Again the problem of the social planner can be solved as an exercise of mathematical programing.

Many comparative static studies on the implications of outsourcing can be done at this stage.

Still another round of generalization is to allow households of each country to have their own kind of homothetic utility index. A tentative line of inquiry is as follows. Each normalized price vector in the price simplex generates both a vector of final goods as a supply response, and a corresponding input price vector, then a vector of country-specific incomes, and finally a vector of final goods as a demand response. The agreement between these two quantity vectors in their relative proportions forms a fixed point in some suitably defined mapping, along the work of McKenzie (1954). Further afield lies the task of exploring the limit of supporting Pareto efficient allocations with average-cost pricing.

Last but not least, this theory of trade was developed not merely for its generality and elegance, but for its relevance in policy issues. The JK study has opened the door to analyzing important questions of our day, related to globalization. Like the reduction of artificial impediments to trade when tariff barriers are removed at the formation of customs unions, reduced cost for the service links opens up new supply chains and shifts the pattern of trade within existing ones, with inherent changes to absolute and relative income levels. To clinch such issues, a general equilibrium foundation helps so that over fewer variables and parameters, researchers need to assume their values as staying equal. Although we have not considered applications here, the present study may point to directions where more may be done.

Appendix Notes on the derivation of examples in Section 7

1. For Table 3.

Let $s(k) = \{i\}$, $i = 1, 2, 3$, or 4, solve:

$$
\begin{aligned}
&\quad Maximize\, q_k \\
&s.t. \\
&\forall\, j, \\
&\quad q_k \le v_{ij} \\
&\quad v_{ij} \le a_{ij} L_{ij} \\
&\quad \sum_j L_{ij} \le L_i \\
&\quad L_{ij} \ge 0.
\end{aligned}
$$

Replacing ≤ with = in the first three in equations,

then, one gets may be called, the 'Rule of the Harmonic Mean',

$$L_{ij} = q_i / a_{ij} = L_i[(1/a_{ij})/\sum_{j'}(1/a_{ij'})$$

and

$$q_k = L_i / \sum_{j'}(1/a_{ij'})$$

Thus, from Table 2, for country *1 (i = 1)*,

$$L_{11} : L_{12} : L_{13} = 1/a_{11} : 1/a_{12} : 1/a_{13} = 1:2:12/5.$$

With $L_1 = 18$, optimal labor allocated to the easiest, the medium and the hardest activities are respectively,

$$L_{11} = 18(5/27) = 10/3, L_{12} = 18(10/27) = 20/3, L_{13} = 18(12/27)) = 8,$$

and the output is 10/3.

2. For Table 4.

Let *s(k) = {1, 2}*, solve:

$$
\begin{aligned}
&\quad Maximize\, q_k \\
&s.t. \\
&\forall j, \\
&\quad q_k \le v_{ij} + v_{2j} \\
&\forall i, \\
&\quad q_k \le v_{ij} \\
&\quad v_{ij} \le a_{ij} L_{ij} \\
&\quad \sum L_{ij} \le L_i - L_{ki} \\
&\quad L_{iy} \ge 0.
\end{aligned}
$$

By the 'Criterion of Maximum Continuous Product' in Jones (1961), optimal allocation requires the three i - i assignments: $\{L_{11} > 0, L_{23} > 0\}$, $\{ L_{12} > 0\}$, and $\{L_{22} > 0\}$.

(The first satisfies the Jones criterion, as $a_{11} a_{23} > a_{13} a_{21}$)

One can then compute the allocation of the non-managerial labor to intermediate inputs, to keep the total parts balanced, also the aggregate final output. For example, in country *1*, the labor allocation is on the two *produced* parts, where it enjoys the most and the next largest comparative advantage, but none to part 3, where it has the least comparative advantage. The labor allocation for country *2* can be deduced likewise.

Similar computation applies for $s(k) = \{3, 4\}$.

3. For Table 5 (as well as its mirror image, Table 6), the optimal pattern includes two chains, each consists both a single i - i assignment, a 3-by-3 case for $k = \{1, 2, 3\}$, where each country only does what it does best, and a singleton for country 4, which operates under autarky.

4. Table 7 corresponds to a pattern which contains four supply chains, in two pairs symmetric to each other, each containing a 3-by-3 assignment and a 1-by-1 assignment. In particular, one pair is $(\{L_{11} > 0, L_{22} > 0, L_{33} > 0\}, \{L_{12} > 0\})$ and the other is $(L_{22} > 0, L_{33} > 0, L_{41} > 0\}, \{L_{43} > 0\})$, both satisfy the Criterion of Maximum Continuous Product. $(a_{11}a_{22}a_{33} = a_{22}a_{33}a_{41})$ is the largest continuous product in the array A. The labor allocation is such to keep the parts balanced.

How does the costliness of managing the supply chain affect the Pareto optimal allocation? Consider any international chain requires a managerial staff of z units of labor per economy, instead of 2 units. for the patterns in Table 5 and Table 6, the total output is then:

$$10/3 + 16(18 - z)/16 = 10/3 + (18 - z),$$

and for Table 7, the total output is:

$$(108/5)(18 - z)/18 = (6/5)(18 - z).$$

These two values become equal when:

$$z = 4/3.$$

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Notes

[1] This the title of Chapter 3 of his Book I, which explains how had water carriage reduced the cost of the 'service link' (in JK terms), thus enlarging the market and facilitating the division of labor. This resembles how has the Internet ushered in the outsourcing of backroom service jobs of corporate America to South Asia, today.

[2] This is an industry of some importance. PRC today has become the world's second largest exporter of electronic products, and the largest exporter of electronics from PRC is Hon Hai, a contract electronics manufacturer of Taiwan (Clough, 2003).

[3] An official of Hon Hai, a leading firm said, 'Our advantage is large economies of scale…' (*Reuters*, Taipei, August 31, 2004).

[4] In 2000, the two largest firms had a combined market share of 25%, and no other's exceeded 10%. (*PC Fab*, Sept. 2001, 24:9). Recently, Flextronics decided to manufacture its own parts, moving closer to the successful strategy of the Taiwanese manufacturers. (*Electronics News*, September 20, 2004).

[5] About the thinness of profit, the new general manager of Flextronics said, 'This will not be a 2% company anymore.'.(*Electronics News*, ibid). Regarding entry, Hon Hai used to be a supplier of plastic switches for TVs (*Reuters*, ibid). Presently, among the 13 on the 'all competitors' list for Flextronics there is SYNNEX, which is controlled by MiTAC, International, a Taiwanese computer maker. (Hoover's Online, September 29, 2004). The crossover from a producer of parts and components to a 'contract electronics manufacturing' seems to be an open door.

[6] Contract electronics manufacturing refers to companies 'that manufacture electronic products for computer and electronic companies on a contract basis (Hoover's Online, ibid).

[7] A macroeconomist may interpret that the labor market stays at the natural rate of unemployment.

[8] In our model below, we assume for simplicity, all parts are fabricated by the supply chain. In real life, many parts are outsourced. During the late 1990s, computers are sold on the build - to - order (BTO) basis, and the parts suppliers are required to store the parts or components in warehouses near the market (Kuo and Wang, 2001). This may lead some of them to cross over as supply chain managers.

[9] As noted in footnote 4, there are only 13 firms listed as all competitors for the leading enterprise.

[10] In biology, the theory of the double helix arose only after facts appeared on the X-ray plates.

[11] Which may be sub-national, like Hong Kong or supra-national, like Belgium-Luxembourg.

[12] For detaied computation, see Appendix.

[13] Thus, the existence of equilibrium cannot be established as the fixed point of some convex-valued mapping.

[14] Thus, the analysis is extended beyond the framework pioneered by Oi (1971).

Table 1 A 3-by-3 problem inspired by Adam Smith				
Labor productivity		Component	Component	
		x	y	z
Country	A	1	1/2	5/12
	B	5/12	1/2	1
Country	C	5/12	1	1/2

Table 2 The basic 4-by-3 problem			
Labor productivity a_{ij}	j = 1	j = 2	j = 3
i = 1	1	1/2	5/12
i = 2	5/12	1/2	1
i = 3	5/12	1	1/2
i = 4	1	5/12	1/2

Table 3 Labor allocations and outputs - pattern of autarky: $\pi = (\{1\}, \{2\}, \{3\}, \{4\})$

	L_{ki}	L_{i1}	L_{i2}	L_{i3}	q_k
i = 1	0	10/3	20/3	8	10/3
i = 2	0	8	20/3	10/3	10/3
i = 3	0	8	10/3	20/3	10/3
i = 4	0	10/3	8	20/3	10/3
Total output					40/3

Table 4 Labor allocations and outputs - a two chians pattern: $\pi = (\{1, 2\}, \{3, 4\})$

	L_{ki}	L_{i1}	L_{i2}	L_{i3}	q_k
i = 1	2	8	8	0	8
i = 2	2	0	8	8	
i = 3	2	0	8	8	8
i = 4	2	8	0	8	
Total output					16

Table 5 Labor allocations and outputs - optimal pattern 1: π = ({1, 2, 3}, {4})					
	L_{ki}	L_{i1}	L_{i2}	L_{i3}	q_k
i = 1	2	16	0	0	16
i = 2	2	0	16	0	
i = 3	2	0	0	16	
i = 4	0	10/3	8	20/3	10/3
Total output					58/3

Table 6 Labor allocations and outputs - optimal pattern 2: π = ({1}, {2, 3,4})					
	L_{ki}	L_{i1}	L_{i2}	L_{i3}	q_k
i = 1	0	10/3	20/3	8	10/3
i = 2	2	0	16	0	16
i = 3	2	0	0	16	
i = 4	2	16	0	0	
Total output					58/3

Table 7 Labor allocations and outputs – the pattern of a single chain: π = ({1, 2, 3, 4})					
	L_{ki}	L_{i1}	L_{i2}	L_{i3}	q_k
i = 1	0	54/5	36/5	0	108/5
i = 2	0	0	18	0	
i = 3	0	0	0	18	
i = 4	0	54/5	0	36/5	
Total output					108/5

Figure 1 Market with Contract Manufacturers

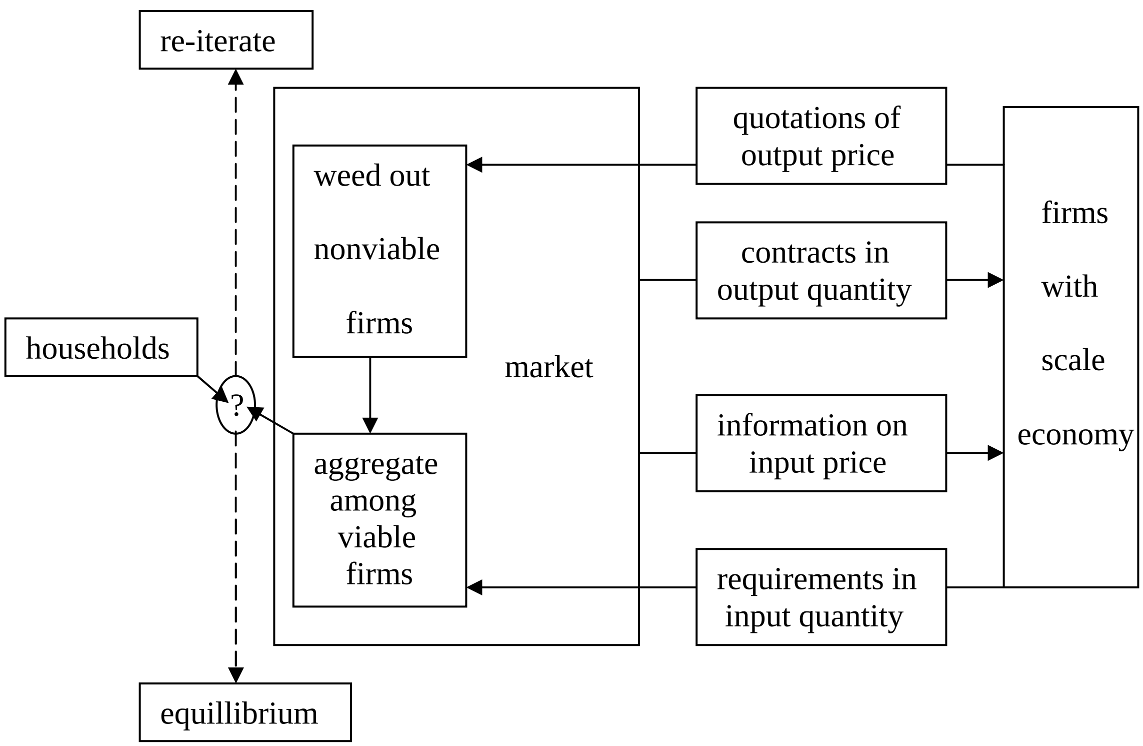
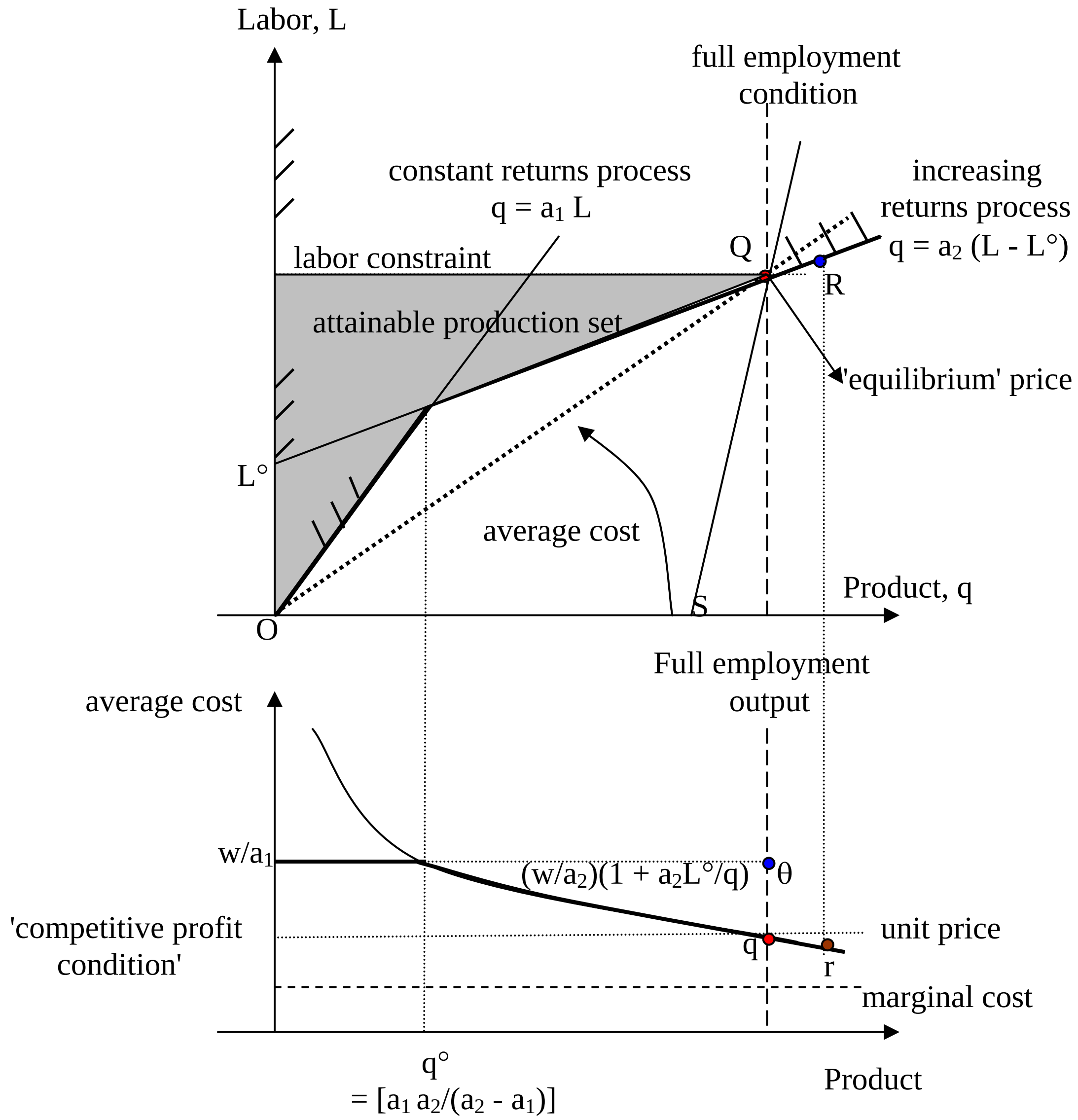


Figure 2 Total and Average Cost



labor productivity

$a_2 > a_1$

wage: w

Figure 3 Produc tion and Trade