Along the New Keynesian Phillips Curve with Nominal and Real Rigidities

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Abstract

The new Keynesian Phillips curve (NKPC) has become central to monetary theory and policy. A seemingly benign NKPC prediction is that trend shocks dominate price level fluctuations at all forecast horizons. Since the NKPC cycle of the U.S. GDP deflator peaks at each of the last seven NBER dated recessions, support for the NKPC is limited. We develop monetary business cycle models that contain different combinations of nominal (sticky price) and real (labor market search) rigidities to understand this puzzle. Simulations indicate a model that combines labor market search and flexible prices is better able to match actual price level movements, than do sticky price models. This represents a challenge to claims sticky prices are a key part of the monetary transmission mechanism.

Key Words : New Keynesian Phillips Curve; Sticky Prices; Labor Market Search; Common Cycle; Common Trend.

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1. Introduction

Of all the comebacks of the 1990s, it seems a revival in Phillips curve research was the least anticipated. Unlike earlier Phillips curve research that focused on aggregate demand shocks, recent work aims to identify inflationary expectations. The way inflationary expectations are formed matters for business cycle theory and monetary policy. For example, a Phillips curve dependent more on forward- than backward-looking expectations allows policymakers to disinflate with few costs. This favorable trade-off appears at odds with empirical evidence and the views of policymakers.

Yun (1996) constructs a rational expectations-monetary business cycle model consistent with a revivalist Phillips curve. He assumes monopolistically competitive firms maximize their expected discounted profit stream subject to a sticky price constraint that reflects a nominal rigidity. The solution to the firms' problem can be cast as the new Keynesian Phillips curve (NKPC) in which price expectations are forward-looking and real marginal cost is the fundamental.

The forward-looking NKPC implies a present-value (PV) relation for the price level. The NKPC-PV relation predicts that trend shocks dominate price level movements. For example, if consumption has economically important cyclical fluctuations, the permanent income hypothesis is rejected. Likewise, the NKPC null is that only trend shocks matter for price level fluctuations.

This paper uses the NKPC-PV relation to test the economic importance of the nominal rigidity of sticky prices for theoretical NKPCs generated by dynamic stochastic general equilibrium (DSGE) monetary models. The link between the empirical and theoretical NKPCs is a common trend-common cycle decomposition of the price level predicted by the NKPC-PV relation, that is based on Beveridge and Nelson (1981), Stock and Watson (1988), and Vahid and Engle (1993).

Our Beveridge, Nelson, Stock, Watson-Vahid and Engle (BNSW-VE) decomposition relies on the NKPC-PV relation and employs *nominal* marginal cost as the $I(1)$ fundamental.

The NKPC common trend-common cycle decomposition provides us with three "moments". The moments are (i) the fraction of price constrained firms, (ii) the NKPC common trend-common cycle decomposition, and (iii) the associated forecast error variance decomposition (FEVD). We use these moments to test the implications of the NKPC-PV restrictions for DSGE models.

Sample NKPC moments are based on U.S. GDP deflator and nominal unit labor cost data that runs from $1960Q1$ to $2001Q4$. Our estimate of NKPC moment (i) has about half of final goods firms being price constrained, which is similar to Sbordone (2002), but smaller than those Gali and Gertler (1999) report. NKPC sample moment (ii) is an economically important cycle because it peaks at each of the last seven NBER dated recessions. NKPC sample moment (iii) reveals trend shocks explain 60 (85) percent or more of price level variation only at forecast horizons of two (four) years or more. Thus, NKPC sample moments $(ii)-(iii)$ reject the NKPC-PV predictions.

We solve and simulate a version of the Yun (1996) DSGE model to understand the sources and causes of NKPC sample moments $(i)-(iii)$.[1] Given Yun's results, it is not a surprise the theoretical NKPC of his Calvo (1983) staggered prices-DSGE model is dominated by trend shocks, which places it at odds with NKPC sample moments $(i)-(iii)$.

Chari, Kehoe, and McGratten (2000) and Ellison and Scott (2000) also cast doubt on sticky prices alone causing actual real and nominal fluctuations.[2] Ball and Romer (1993), Jeanne (1998),

[1] Greenwood and Huffman (1986), Chéron and Langot (1999), Cooley and Quadrini (1999), Walsh (2002), and Trigari (2003b) represent a tradition in which monetary DSGE models generate the unconditional Phillips curve observation that real economic activity and inflation are positively correlated. Our study is conditional on the NKPC.

[2] Other monetary models fit different aspects of U.S. price dynamics. Ireland (1999) and Ruge-Murcia (2003) esti-

Gali and Gertler (1999), Dotsey and King (2001), Walsh (2002), Trigari (2003b), and Ireland (2003) argue real rigidities solve this problem. For example, Ireland shows that a persistent, exogenous real demand shock is needed for his sticky price model to match U.S. business cycle fluctuations.

Solow (1976) points out that traditional Phillips curves invoke a real rigidity, labor market imperfections, rather than sticky prices, to identify unemployment with the state of aggregate demand. These arguments motivate us to combine the Yun-sticky price model with Mortensen and Pissarides (1994) labor market search in the way Andolfatto (1996), Merz (1995), and den Haan, Ramey, and Watson (2000) add it to real business cycle (RBC) models. Since Walsh (2002) and Trigari (2003b) obtain economically interesting results with similar models, it is a surprise the labor market search-sticky price model behaves about as well as the Yun-sticky price model.

We study a labor market search-flexible price model in response to the poor results with sticky prices. Data from this model produces synthetic NKPC moments $(i) - (iii)$ that match their sample counterparts. This suggests economically important price level fluctuations reside with the real rigidity of labor market imperfections. These imperfections are a potential monetary transmission mechanism, which creates a role for monetary policy.

The next section presents the NKPC-PV relation, its BNSW-VE decomposition, and reports empirical results. Section 3 reviews the Yun-sticky price model. Model calibration, labor market search, and Monte Carlo results are discussed in section 4. Section 5 concludes.

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mate game-theoretic monetary policy models that capture long-run inflation. Nason and Cogley (1994) show a flexible price-DSGE monetary model replicates short-run price dynamics under a long-run monetary neutrality identification.

2. The New Keynesian Phillips Curve

NKPC estimates are controversially. Sbordone (2002), Gali and Gertler (1999), and Rabanal and Rubio-Ramírez (2001) report empirical success with the NKPC. Fuhrer and Moore (1995), Fuhrer (1997), Roberts (1995, 1997, and 2001), and Rudd and Whelan (2001), among others, test backward-looking Phillips curves against the forward-looking NKPC and reject it. We come at this debate differently because we identify the trend and cyclical components of the NKPC.

2.1 A NKPC Specification

Roberts (1995) shows that several sticky-price models yield the NKPC. Typical is the Calvo (1983) staggered price setting mechanism. Sticky prices arise because only a fraction, $1 - \mu$, of monopolistically competitive final goods firms are able to set and commit to a new price, $P_{C,t}$, between dates $t-1$ and t. Aggregate price, P_t, dynamics are restricted by

$$(1) \qquad P_t = [(1-\mu)P_{C,t}^{1-\xi} + \mu\left(\frac{m^*}{\gamma^*}P_{t-1}\right)^{1-\xi}]^{1/(1-\xi)}, \quad 1 < \xi,$$

where ξ, m^*, and γ^* are the demand elasticity, steady state money growth, and non-stochastic growth rate of labor augmenting technology change, respectively. Assume the aggregator of final demand (in physical units) firms face is $Y_{D,t} = \left[\int_0^1 y_{D,j,t}^{(\xi-1)/\xi} dj\right]^{\xi/(\xi-1)}$, where $y_{D,j,t}$ represents the demand firm j faces. This implies the demand schedule of the jth firm is

$$(2) \qquad y_{D,j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\xi} Y_{D,t}$$

where firm j charges $P_{j,t}$ for its output and all firms take $Y_{D,t}$ and P_t as given. Subsequent to cost minimization, profit maximization leads to the forward-looking optimal commitment price

$$(3)\quad P_{C,t} \;=\; \left(\frac{\xi}{\xi\,-\,1}\right)\left[\frac{\mathbf{E}_t\left\{\sum_{i=0}^{\infty}\left(\beta\mu\left[\frac{m^*}{\gamma^*}\right]^{-\xi}\right)^{i}\Gamma_{t+i}\phi_{t+i}Y_{D,t+i}P_{t+i}^{\xi}\right\}}{\mathbf{E}_t\left\{\sum_{i=0}^{\infty}\left(\beta\mu\left[\frac{m^*}{\gamma^*}\right]^{1-\xi}\right)^{i}\Gamma_{t+i}Y_{D,t+i}P_{t+i}^{\xi\,-\,1}\right\}}\right],\quad 0\;<\;\beta\;<\;1,$$

where $\mathbf{E}_t\{\cdot\}$, $\beta^i\Gamma_{t+i}$, and ϕ_t represent the mathematical expectations operator conditional on date t information, the date $t+i$ (stochastic) discount rate all firms face, and real marginal cost.

Sticky price dynamics force monopolistically competitive firms to be forward-looking when price setting. This gives the NKPC its forward-looking character, which is developed by linearizing the price aggregator (1) and the optimal price rule (3), subsequent to detrending, to construct

$$(4)\quad ln[P_t] \;=\; \mu ln[P_{t-1}] \;+\; (1-\mu)\left(1\,-\,\frac{\mu}{\mathcal{B}}\right)\sum_{j=0}^{\infty}\left(\frac{\mu}{\mathcal{B}}\right)^{j}\mathbf{E}_t ln[\Phi_{t+j}],\quad \mathcal{B}\;\equiv\;\frac{m^*}{\beta\gamma^*},$$

where constants are ignored. The equilibrium law of motion (4) shows the price level is driven by the trend of the "annuity value" of the expected future path of *nominal* marginal cost.

The price dynamics of (4) shows that the price level and nominal marginal cost share a common trend or cointegrating relation, $ln[\phi_t] = \varpi_{CT}(ln[P_t]\;\; ln[\Phi_t])'$, where $\varpi_{CT} = [1\;\, -1]$, if nominal marginal cost is $I(1)$. Subtract $ln[\Phi_t]$ from both sides of equation (4), apply the usual PV algebra, and multiply the result through by minus one to produce

$$ln[\phi_t] \;=\; \left(\frac{\mu}{1-\mu}\right)\Delta ln[P_t] \;-\; \sum_{j=1}^{\infty}\left(\frac{\mu}{\mathcal{B}}\right)^{j}\mathbf{E}_t\Delta ln[\Phi_{t+j}].$$

Real marginal cost is endogenous and forward-looking because it equals (a multiple of) inflation net of the expected PV of nominal marginal cost growth. Note also that real marginal cost is stationary and acts as the cointegrating relation or error correction mechanism in a vector error correction model (VECM) of inflation and nominal marginal cost growth. Given Engle and Issler (1995) refer to cointegrating relations as cycle generators, real marginal cost approximates the NKPC cycle.

Another NKPC prediction is that inflation and nominal marginal cost growth share a serially correlated common feature, in the sense of Engle and Kozicki (1993). The NKPC common feature exists if inflation and nominal marginal cost growth form a linear combination whose residual is unpredictable. The NKPC-PV relation (4) produces the linear combination

$$(5) \quad \Delta ln[P_t] - \mu_{\mathcal{B}} \Delta ln[\Phi_t] = \mu^{-1} \mathbf{E}_t \Delta ln[P_{t+1}] - \mathcal{E}_{\Phi,t}, \quad \mu_{\mathcal{B}} \equiv \frac{(1-\mu)(\mathcal{B}-\mu)}{\mu^2},$$

where $\mathcal{E}_{\Phi,t} = \mu_{\mathcal{B}} \{\mathbf{E}_{t-1} \Delta ln[\Phi_t] + (\mathbf{E}_t - \mathbf{E}_{t-1}) ln[\Phi_t] + \sum_{j=1}^{\infty} (\mu/\mathcal{B})^j (\mathbf{E}_t \Delta ln[\Phi_{t+j}] - (\mu/\mathcal{B}) (\mathbf{E}_t - \mathbf{E}_{t-1}) ln[\Phi_{t+j-1}])\}$. The regression (5) yields the NKPC common feature relation if $\mathcal{E}_{\Phi,t}$ annihilates serial correlation in expected inflation to generate (unpredictable) innovations.

2.2 NKPC Common Trend Prediction: Estimates and Tests

The proxy for nominal marginal cost is *nominal* unit labor costs, ULC_t, which is measured as the ratio of hourly compensation to output per hour.[3] The price level, P_t, is the GDP deflator. The sample period is $1960Q1$–$2001Q4$, $T = 168$, with lags available beginning with $1955Q1$.

We test for a common trend in P_t and ULC_t using Johansen (1988, 1991) likelihood ratio (LR) tests based on a third-order VECM, case 1* model of Osterwald-Lenum (1992).[4] The LR-max and LR-trace statistics are [8.23, 12.98] and [8.23, 21.20], respectively. The former test rejects a

[3]Sbordone (2002) and Gali and Gertler (1999) show marginal cost equals ULC using the labor demand elasticity of a Cobb-Douglas technology of a monopolistic competitive firm. The Federal Reserve Bank of St.Louis' FRED databank labels the index of hourly compensation (output per hour), non-farm business sector `compnfb` (`ophnfb`).

[4]A LR test for the lag length – beginning with 12 lags – of the VAR of the logs of the price level and ULC cannot reject a four lag model. The AIC gives the same result.

common trend in the price level and ULC and the latter does not based on MacKinnon, Haug, and Michelis (1999) five percent critical values of [9.17, 15.88] for the LR-max test and [9.17, 20.25] for the LR-trace test. The maximum likelihood estimate (MLE) of ϖ_{CT-MLE} is [1 -1.08 6.82].[5]

We report two additional cointegration tests because Johansen's are inconclusive and can be unreliable. The Engle and Granger (1987) cointegration test yields a $t-$ratio of -3.34, which rejects its null at the five percent level, according to MacKinnon (1991). Boswijk (1994) constructs a Wald test from a simultaneous equations VECM, based on a two-stage least squares (2SLS) estimator. The Wald statistic of 11.82 falls between the ten and five percent critical values Boswijk tabulates (his table B.3). These tests lend support to a model in which $ln[P_t]$ and $ln[ULC_t]$ cointegrate.

2.3 NKPC Common Feature Prediction: Estimates and Tests

Two tests for a serially correlated common feature employ the canonical correlations, λ, of inflation and ULC growth, conditional on the VECM(3) information set. Inflation and ULC growth share a serially correlated common feature if the smallest $\lambda = 0$. Vahid and Engle (1993) develop a χ^2-common feature test, but a $F-$test exists due to Rao (1973) that has superior small sample properties, according to Engle and Issler (1995). The λ^2s equal 0.0513 and 0.8243, with associated p-values of 0.19 (0.21) and 0.00 (0.00) for the χ^2 ($F-$)test.

Vahid and Engle show a 2SLS regression provides a common feature test and recovers $\varpi_{CC} = [1 \quad -\mu_{\mathcal{B}}]$.[6] The 2SLS estimate of $\mu_{\mathcal{B}}$ equals 0.8203, with a standard error of 0.0800, given

[5]The Johansen (1991) test of the theoretical ϖ_{CT} against ϖ_{CT-MLE} is not rejected at standard significance levels.

[6]Gali and Gertler (1999) use generalized method of moments to estimate the NKPC, which imputes inflationary expectations to the instrument vector. Sbordone (2002) minimizes the distance between price level dynamics restricted by a NKPC and the actual price level. This is akin to the instrumental variables estimator of West (1989).

the VECM(3) regressors are instruments. The LM test of instrument validity cannot reject the null, given a p-value of 0.16 for the statistic 9.32.

We have to calibrate $\mathcal{B}$ to calculate μ_{2SLS}, NKPC sample moment (i), from $\mu_{\mathcal{B},2SLS}$. We set $\beta = 1.03^{-0.25}$, $\gamma = 0.0047$, and $m^* = \exp\{0.0167\}$, where m^* and γ are taken from U.S. data; see section 4.1 for details. The calibration implies the NKPC sample moment (i), μ_{2SLS}, is 0.5292 with a standard error of 0.0081.[7] Thus, firms change prices twice a year, on average.

Another test compares a VECM(3) restricted by the common feature against an unrestricted VECM(3). This LR test has ten degrees of freedom and a p-value of 0.55. Along with tests of the squared canonical correlations and 2SLS instrument validity, the LR test provides evidence that favors the NKPC common feature of (5).

2.4 The BNSW-VE Decomposition of the NKPC

The BNSW-VE decomposition relies on levels data and ϖ_{CT} and ϖ_{CC}, according to Vahid and Engle (1993). Partition the columns of the inverse of the stack of these vectors into the matrix

$$[\pi_{\cdot,1} \;\; \pi_{\cdot,2}] \quad = \quad \begin{bmatrix} \varpi_{CT} \\ \varpi_{CC} \end{bmatrix}^{-1},$$

to recover the NKPC cycle from $\pi_{\cdot,2} \times \varpi_{CT}(ln[P_t] \;\; ln[ULC_t])'$. The trend follows.

Figure 1 contain the NKPC trend and cycle, or NKPC sample moment (ii). The top window of figure 1 shows that the NKPC trend supports traditional views of recent U.S. aggregate price history. The tight labor markets of the mid-1960s coincide with an increase in the NKPC trend.[8]

[7] Vahid and Engle describe a MLE that stacks the common feature regression on top of the ECM(3) of ULC growth. The MLE of $\mu_{\mathcal{B}}$ equals 0.8927, with a standard error of 0.1080. The gives $\mu = 0.5192$ and a standard error of 0.0153.

[8] The price level is less volatile than the NKPC trend because the covariance equals -0.13.

The trend falls with the recession that begins in late 1969. The 1970s sees a rising trend at the time of the first oil price shock. The contractionary monetary policy initiated late in 1979 pushes the NKPC trend below the price level from 1980 until the economic expansion of the mid-1990s. The NKPC trend dips below the price level just before the NBER peak dated 2001 $Q1$.

The NKPC cycle and NBER dated business cycle peaks (vertical dash lines) and troughs (vertical dot-dash lines) appear in the bottom window of figure 1. It shows NKPC cycle peaks at the last seven NBER dated recessions. Since the cycle is a negative (up to a scalar) of real unit labor cost, when it rises it signals recovery from recession. Thus, the NKPC cycle is economically important, consistent with prior views of the Phillips curve, but at odds with the NKPC-PV predictions.

The NKPC sample moment (iii) employs the BNSW-VE decomposition to gauge the contribution of the identified trend shock to movements in the GDP deflator.[9] The FEVDs with respect to the trend are 2.70, 8.80, 26.38, 60.12, 78.37, 86.55, 91.44, and 98.05 percent at 1, 2, 4, 8, 12, 16, 20, and 40 quarter forecast horizons, respectively.[10] Note that trend innovations are responsible for about a quarter of price level fluctuations at a one-year forecast horizon and 60 percent at the end of two years. It takes five years to reach 90 percent.[11]. This is evidence against the NKPC because cyclical shocks matter for the price level, at least, through a two-year forecast horizon.

[9] It is not possible to identify the trend (cyclical) shock as a supply (demand) shock.

[10] Engle and Issler (1995) and Issler and Vahid (2001) outline methods to calculate FEVDs, under the BNSW-VE decomposition. Trend innovation are a function of the common trend growth rate, lagged appropriately. Innovations to the cyclical component are the residuals of the cyclical component regressed on the information set of the VECM, lagged j times. Trend and cyclical innovations are orthogonalized by 'regressing' the latter on the former, which asserts trend innovations are prior to cyclical innovations; see footnote 11 and appendix C of Issler and Vahid for details.

[11] The trend shock takes longer to dominate ULC fluctuations, because its FEVDs are 0.28, 2.01, 10.79, 41.34, 65.00, 77.52, 85.41, and 96.65 percent. However, the NKPC places no restrictions on these FEVDs

To summarize, we study three NKPC moments: (i) the 2SLS estimate of the sticky price parameter, μ_{2SLS}, (ii) the NKPC common trend-common cycle decomposition, and (iii) its FEVD. Our evidence lends only weak support to the NKPC because cyclical shocks matter for price level moments. This raises the question of the role of nominal rigidities in DSGE monetary models. The next two sections study this question.

3. A Sticky Price DSGE Model

This section reviews the sticky-price DSGE model of Yun (1996). This model combines cash and credit goods, a cash-in-advance (CIA) constraint, and a Calvo-staggered price mechanism into a one-sector growth model.[12] Section 4 integrates a real rigidity into Yun's DSGE model with the labor market-search technology that Merz (1995), Andolfatto (1996), and den Haan, Ramey, and Watson (2000) use in a RBC setting. We also study a flexible price version of this DSGE model.

3.1 The Final Goods Sector

Monopolistically competitive final goods firms take addresses on the unit interval. Producing a differentiated good employs a constant returns to scale (CRS) technology, $F(k-\overline{K}, hZ) \equiv (k-\overline{K})^{\theta}(hZ)^{1-\theta}$, $\theta \in (0, 1)$, where k is capital, $\overline{K}$ is an exogenous minimum capital threshold (e.g., infra-structure) common to all final goods firms, and hZ is productivity augmented hours.[13]

Monopolistic competition in the final goods market forces the associated prices to depend

[12] A slew of sticky price specifications are used in monetary business cycle models. Examples are King and Wolman (1996), Nelson (1998), Ireland (2001), Kozicki and Tinsley (2001), Sbordone (2001), and Smets and Wouters (2002).

[13] Monopolistically competitive final goods firms must face period-by-period fixed costs. Below, we outline a labor market search structure that precludes fixed labor costs as in Yun (1996) because hours are not priced in a spot market.

on nominal marginal cost, Φ. The jth final good firm sets its price by minimizing its total cost, $\mathcal{TC}_j$ $= R_K k_j + W h_j$, subject to the CRS technology, where R_K is the nominal rental rate of capital. The first-order necessary conditions (FONCs) are $R_K = \Phi\theta y_j/k_j$ and $W = \Phi(1-\theta)y_j/h_j$. Place these optimality conditions into the cost function and exploit the CRS technology to show, $\mathcal{TC}_j =$ Φy_j - $R_K\overline{K}$. Hence, the net profit function of this firm is

$$\frac{D_j}{P} = \left(\frac{P_j}{P} - \phi\right)\left(\frac{P_j}{P}\right)^{-\xi} Y_D - \frac{R_K}{P}\overline{K}, \tag{6}$$

given demand schedule (2).

We study economies in which final goods prices are sticky and flexible. When final goods prices are flexible, real marginal cost is constant, $\phi = (\xi - 1)/\xi$, and prices are a constant markup over marginal costs. A final good firm whose behavior is restricted by the Calvo staggered price mechanism (1) solves the intertemporal profit maximization problem

$$\mathbf{E}_t\left\{\sum_{i=0}^{\infty}(\beta\mu)^i\Gamma_{t+i}\left[\left(\left[\frac{m^*}{\gamma^*}\right]^i \frac{P_{C,t}}{P_{t+i}} - \phi_{t+i}\right)\left(\left[\frac{m^*}{\gamma^*}\right]^i \frac{P_{C,t}}{P_{t+i}}\right)^{-\xi} Y_{D,t+i} - \frac{R_{K,t+i}}{P_{t+i}}\overline{K}_{t+i}\right]\right\}.$$

The FONC of $P_{C,t}$ leads to the forward-looking price-setting optimality condition (3).

Construction of aggregate dividend and production functions closes the final goods sector. Yun (1996) shows aggregate demand is connected to aggregate supply through the supply price aggregator, $P_{A,t}^{-\xi} \equiv \int_0^1 P_{j,t}^{-\xi}dj$.[14] The definition of aggregate output, $Y_{A,t} \equiv \int_0^1 y_{A,j,t}dj$, and the demand schedule (2) gives $Y_{D,t} = (P_t/P_{A,t})^{-\xi}\, Y_{A,t}$.[15] These facts lead to the aggregate real dividend function, $D_t/P_t = (P_t/P_{A,t})^{-\xi}\,[1-\theta\phi_t]Y_{A,t}$ - $(R_{K,t}/P_t)K_t$ - $(W_t/P_t)h_t$. Since technology

[14]The associated dynamics are $P_{A,t}^{-\xi} = (1-\mu)P_{C,t}^{-\xi} + \mu(m^*\exp\{-\gamma\}P_{A,t-1})^{-\xi}$.

[15]This eliminates $P_{C,t}$ from the state of the economy leaving only current and lagged aggregate prices.

is CRS, market clearing relative prices, $R_{K,t}/P_t$ and W_t/P_t, and the definitions of aggregate capital and hours result in $Y_{A,t} = (K_t - \overline{K}_t)^{\theta}\ (h_t Z_t)^{1-\theta}$, which is the aggregate production function.

3.2 The Household

Households decisions cover consumption, leisure, capital accumulation, and financial portfolios (to hold cash and government bonds). Felicity is summarized by

$$(7)\qquad u(c_{M,t},\ c_{L,t},\ \ell_t) \quad \equiv \quad \psi_1 ln[c_{M,t}] \ + \ (1-\psi_1) ln[c_{L,t}] \ + \ \psi_3 \frac{\ell_t^{1-\psi_2}}{1-\psi_2},$$

where $0 < \psi_1 < 1$, $\psi_2 \neq 1$, $0 \leq \psi_3$, $c_{M,t}$, $c_{L,t}$, and ℓ_t $(= 1 - h_t)$ are cash consumption, credit consumption, and leisure, respectively. The household faces the budget constraint

$$(8)\qquad D_t \ + \ R_{K,t}\, k_t \ + \ W_t\, h_t \ + \ (1 + R_{B,t})\, B_{G,t} \ + \ M_t \ - \ A_{t+1}$$

$$= \quad P_t[c_{M,t} \ + \ c_{L,t} \ + \ k_{t+1} \ - \ (1-\delta_K)\, k_t \ + \ T_t],$$

the CIA constraint

$$(9)\qquad M_t \quad \geq \quad P_t\, c_{M,t},$$

and the wealth constraint

$$(10)\qquad A_t \quad \geq \quad B_{G,t} \ + \ M_t \ - \ X_{t-1},$$

where $0 < \delta_K < 1$, and $D_t,\ B_{G,t},\ M_t,\ A_t,\ T_t$, and X_{t-1} denote dividends the household receives from final good firms, government bonds this household owns at the beginning of date t, cash the household carries over to date t from the end of date $t-1$, nominal wealth it takes from the end of date $t-1$ into the beginning of date t, a lump-sum tax levied on all households, and the total cash injection, respectively. The government pays $R_{B,t}$ on its one-period unit discount bond.

3.3 The Government

The government engages in monetary and fiscal operations. The latter activities involve expenditures, G_t, lump-sum tax collecting, T_t, and issuing one-period unit discount bonds, $B_{G,t+1}$. The monetary operation injects X_t units of cash into the household sector. Hence, the intertemporal budget constraint of the government is

$$(11) \qquad P_t T_t + (B_{G,t+1} - B_{G,t}) + (M_{t+1} - M_t) = P_t G_t + R_{B,t} B_{G,t} + X_t.$$

We let $T_t = G_t$ at each date t and assume the government spending-output ratio, $g_t = G_t / Y_{D,t}$, evolves exogenously. Government bonds are restricted to be in *zero* net supply, $B_{G,t+1} = 0$, along the equilibrium path. Cash injections obey $X_t = M_{t+1} - M_t$ and monetary base growth, m_t (= M_{t+1}/M_t), is assumed to be an exogenous stochastic process to avoid entangling the predictions of our DSGE models with an arbitrary monetary policy rule.

3.4 Household Optimality

The household maximizes its expected lifetime utility subject to $(8)-(10)$. Lifetime utility is the expectation of the infinite discounted sum of felicity,

$$\mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j u(c_{M,t+j},\ c_{L,t+j},\ \ell_{t+j}) \right\}, \quad \beta \in (0,\ 1).$$

This problem yields the consumption-based money demand function

$$\frac{M_t}{P_t} = C_t \left[\frac{\psi_1}{1 + (1 - \psi_1) R_{B,t}} \right],$$

where $C_t \equiv c_{M,t} + c_{L,t}$. Another implication is the household's stochastic discount factor (SDF)

(12) $$\frac{\Gamma_t}{P_t} \quad = \quad \beta \mathbf{E}_t \left\{ \frac{\psi_1}{P_{t+1}\, c_{M,t+1}} \right\}.$$

Firms and the government take the sequence of SDFs, $\{\Gamma_{t+j}\}_{j=0}^{\infty}$, as given, when they discount using (12). The SDF, the CIA constraint (9), and the FONC with respect to $c_{L,t}$ produces

(13) $$C_t \quad = \quad \frac{M_t}{P_t} \; + \; \frac{(1 \,-\, \psi_1)}{\Gamma_t\, P_t}.$$

which is the household "consumption function".

Optimal choice of employment hours by the typical household involves the usual trade-off between leisure and the rewards of labor market activity. The optimality condition of h_t is

(14) $$\frac{\psi_3}{(1 \,-\, h_t)^{\psi_2}} \quad = \quad \Gamma_t \frac{W_t}{P_t}.$$

The household supplies labor up to the point at which the dis-felicity of work equals the discounted real wage according to (14). This wage is determined in a perfectly competitive spot market.

The dynamic program the household solves produces two intertemporal optimality conditions. The Euler equation, $\Gamma_t / P_t \;=\; \beta \mathbf{E}_t \{ (\Gamma_{t+1} / P_{t+1})[1 \;+\; R_{B,t+1}] \}$, describes optimal intertemporal choice in the money market. It shows the interaction of the CIA constraint and next period's liquidity preference trade-off between consumption and the government's unit discount bond. The intertemporal trade-off between consumption and capital accumulation is given by

(15) $$\Gamma_t \quad = \quad \beta \mathbf{E}_t \left\{ \Gamma_{t+1} \left[\frac{R_{t+1}}{P_{t+1}} \;+\; (1 \,-\, \delta_K) \right] \right\},$$

which is determined by the FONC of K_{t+1} and the envelope condition for K_t. Euler equation (15) shows the household is willing to postpone a unit of date t consumption for the return additional capital is expected to yield during date $t+1$ for date $t+2$ consumption.

3.5 Aggregate Equilibrium and Optimality

Equilibrium requires the goods, capital, money, government bond, and labor markets to clear. Goods market equilibrium relies on the aggregate resource constraint

$$(16) \qquad \left(\frac{P_t}{P_{A,t}}\right)^{-\xi} Y_t \quad = \quad C_t \; + \; K_{t+1} \; + \; (1 - \delta_K)K_t \; + \; G_t.$$

The aggregate resource constraint (16) adds together the budget and wealth constraints, (8) and (10), of the household, the government's budget constraint (11), and the firm's dividend flow (6). Since the rental market for capital, the money market, the government bond market, and the labor market are perfectly competitive, agents treat the joint stochastic process that generates returns and the nominal wage, $\{R_{K,t+j},\ R_{B,t+j},\ W_{t+j}\}_{j=0}^{\infty}$, as given. The same holds for the exogenous shock process $\{Z_{t+j},\ \overline{K}_{t+j},\ G_{t+j},\ X_{t+j}\}_{j=0}^{\infty}$.

The Yun-sticky price model has optimality conditions for $C_t,\ h_t,\ P_t,\ \phi_t$, and K_{t+1}. The optimality condition for consumption is the aggregate resource constraint (16), given optimal choices elsewhere. Optimal labor market activity brings together the labor supply schedule embedded in (14) and a firm's FONC with respect to hours

$$(17) \qquad \frac{\psi_3}{(1 - h_t)^{\psi_2}} \quad = \quad \Gamma_t \left(\frac{P_t}{P_{A,t}}\right)^{-\xi} \phi_t(1 - \theta)(K_t - \overline{K}_t)^{\theta}\, h_t^{-\theta}\, Z_t^{1-\theta}.$$

Optimal price behavior requires the consumption function (13), the SDF (12), and $c_{M,t} = M_t/P_t$, which is the CIA constraint (9) in equilibrium, which define money market equilibrium. A flexible price regime equates aggregate consumption to real balances plus (the present-value of) the purchasing power of a dollar. The law of motion of the price level (1) and the optimal commitment price condition (3) restricts the optimal path of ϕ_t in a sticky-price economy. This forces money market adjustment onto C_t and Γ_t. Optimal capital accumulation arises from the Euler equation

$$(18) \qquad \Gamma_t \quad = \quad \beta \mathbf{E}_t \Big\{ \Gamma_{t+1} \Big[\theta \phi_{t+1} (K_{t+1} - \overline{K}_{t+1})^{\theta - 1} (h_{t+1} Z_{t+1})^{1-\theta} \;\; + \;\; (1 - \delta_K) \Big] \Big\},$$

which rests on the Euler equation (15) and the nominal rental rate of capital.

Any candidate equilibrium must satisfy the optimality conditions and the aggregate resource constraint. The sufficient conditions of any candidate equilibrium are the transversality conditions, where for capital $lim_{j\rightarrow\infty}\beta^j \mathbf{E}_t \left\{\Gamma_{t+j} K_{t+1+j}\right\} = 0$.

4. Comparing Sample and Theoretical NKPCs

This section reports on the calibration, solution strategies, and Monte Carlo experiments. Next, labor market search is placed in the sticky-price model to compare and contrast the implications for the NKPC of this real rigidity with the nominal rigidity of sticky-prices. We complete this study of the NKPC with a flexible price version of our monetary DSGE with labor market search.

4.1 The Calibration and Numerical Solution

We generate an approximate numerical solution of the Yun-sticky price model from the linearized stochastically detrended variants of its optimality conditions, laws of motion, and equilibrium conditions. Stochastic detrending is necessary because labor augmenting technology evolves as a random walk with drift, $Z_{t+1} = Z_t \exp\{\gamma + \varepsilon_{t+1}\}$, $0 < \gamma$, $\varepsilon_{t+1} \sim \mathbf{N}\left(0, \sigma_\varepsilon^2\right)$, and money growth is a AR(1), $m_{t+1} = m^{*(1-\rho_m)} m_t^{\rho_m} exp\left\{\eta_{m,t+1}\right\}$, $|\rho_m| < 1$, $\eta_{m,t+1} \sim \mathbf{N}\left(0, \sigma^2_{\eta,m}\right)$, where $ln[m_t] = ln[M_{t+1}/M_t]$ and $\mathbf{E}\{\varepsilon_{t+i}\eta_{m,t+j}\} = 0, \ \forall\, i, j$.[16] Real aggregates and prices are detrended

[16]We assume the transitory components of $\overline{K}_{t+1}$ and g_{t+1} are non-stochastic.

as $\widehat{U}_{Y,t} = U_{Y,t}/Z_t$ and $\widehat{U}_{P,t} = U_{P,t}Z_t/M_t$, respectively, where $U_{Y,t} = [Y_{D,t}\ Y_{A,t}\ C_t\ K_{t+1}\ G_t]$ and $U_{P,t} = [P_t\ P_{A,t}\ P_{C,t}]$. Detrending $\widehat{W}_t = W_t/M_t$, $\widehat{\Gamma}_t = \Gamma_t Z_t$, and $\widehat{R}_{K,t} = R_{K,t}/P_t$, follows.

The numerical solution begins by linearizing the detrended aggregate resource constraint (16), the hours schedule (17), the SDF (12), the consumption function (13), the law of motion of the price level that underlies the NKPC-PV relation (4), and the Euler equation of K_{t+1}, (18). The solution we conjecture is

$$(19) \qquad \mathcal{K}_{t+1} \quad = \quad \mu_{\mathcal{K}}\mathcal{K}_t \quad + \quad \mu_{\mathcal{E}}\mathcal{E}_t,$$

where $\mathcal{K}_{t+1} = [\widetilde{K}_{t+1}\ \aleph_{t+1}\ \Im_{t+1}]'$, $\widetilde{K}_{t+1} = ln[\widehat{K}_{t+1}/K^*]$, $\aleph_{t+1} = \mathbf{E}_t\widetilde{P}_{t+1}$, $\Im_{t+1} = \aleph_t$, and the exogenous state vector, $\mathcal{E}_t = [\varepsilon_t\ \ \eta_{m,t}]'$. We seek the unknown elements of the three-by-three matrix $\mu_{\mathcal{K}}$ and the three-by-two matrix $\mu_{\mathcal{E}}$ using methods Zadrozny (1998) and Sims (2000) develop to compute approximate numerical solutions.[17] Given solutions for $\mu_{\mathcal{K}}$ and $\mu_{\mathcal{E}}$, the control system is

$$(20) \qquad \mathcal{C}_t \quad = \quad \pi_{\mathcal{K}}\mathcal{K}_t \quad + \quad \pi_{\mathcal{E}}\mathcal{E}_t,$$

where $\mathcal{C}_t = [\widetilde{C}_t\ \widetilde{h}_t\ \ \widetilde{P}_t\ \widetilde{\phi}_t]'$, $\pi_{\mathcal{K}}$ is a four-by-three matrix, and $\pi_{\mathcal{E}}$ is a four-by-two matrix.

We employ sample data and choices made by other studies to calibrate model parameters. Preference parameters β and ψ_1 are 0.9950 and 0.8428, respectively. The latter implies an interest elasticity of money demand of one percent, given the federal funds rate sample mean. We take the other preferences parameters from Andolfatto (1996), $\psi_2 = 2.0$ and $\psi_3 = 2.08$. The technology parameter $\theta = 0.35$ and $\delta_K = 0.0195$. The steady state markup is 1.10, which yields $\xi = 11.0$. The sticky price parameter is calibrated to the NKPC sample moment (i), $\mu_{2SLS} = 0.5292$.

[17] Sticky price models have a singular leading coefficient matrix in its linearized stochastic difference equation. Sims (2000) describes a solution to this problem and provides software.

The calibration of the impulse structure relies on sample data from $1960Q1-2001Q4$. The deterministic growth rate $\gamma = 0.0047$ is the sample mean of measured total factor productivity and $\sigma_\varepsilon = 0.0117$ is the sample standard deviation of measured total factor productivity growth.

The parameters of the AR(1) process of money growth are estimated from sample data, which is the Federal Reserve Bank of St. Louis' monetary base series. The mean growth rate is 0.0166. OLS estimates of the AR(1) of money growth yield $\rho_m = 0.4456$ and $\sigma_{\eta,m} = 0.0068$.[18]

4.2 *Monte Carlo Design*

We generate 5000 replications of the monetary DSGE models. A replication is 168 observations of the price level and ULC.[19] Next, the MLE-cointegrating vector of the case 1^* VECM(3) is estimated. Conditional on this estimated cointegrating relation and three lags of artificial inflation and ULC growth, the 2SLS regression of inflation on a constant and ULC growth is calculated to produce the common feature vector and a synthetic estimate of μ_{2SLS}. Synthetic estimates of the ϖ_{CT} and ϖ_{CC} vectors are employed to construct the BNSW-VE decomposition and its FEVD. We report theoretical FEVDs in table 1. Figure 2 contains nonparametric densities of empirical distributions of μ_{2SLS} drawn from the DSGE models and the asymptotic 95 percent confidence interval of the sample μ_{2SLS}. Theoretical NKPC trends and cycles appear in figures 3, 4, and 5.

[18] As previously mentioned, the transitory components of the fixed capital component and government spending are assumed to be nonstochastic. We set g^* at its sample mean, 0.1878. Calibration of $\overline{K}^*$ is problematic. It cannot be constructed without observations on fixed capital. The closest notion is structures, but U.S. capital stock data reveals the ratio of structures to total capital is about 0.23 for the $1960-2000$ sample. We assume $\overline{K}^* = 0.025$. Experiments with values between 0.25 and 25 percent had little impact on the Monte Carlo experiments.

[19] We compute 372 artificial observations, but drop the first 204 to remove dependence on initial conditions.

4.3 Yun-Sticky Price Model Experiments

Simulations of the Yun (1996) model reveal it to be at odds with NKPC sample moments $(i)-(iii)$. The density of theoretical μ_{2SLS} estimates generated from this model (the dashed curve) and the asymptotic 95 percent confidence interval (vertical dotted lines) of the sample $\mu_{2SLS}(= 0.5293)$ appear in figure 2. Since the mean of theoretical estimates is 0.8264, the theoretical density is to the right of the asymptotic 95 percent confidence interval of the sample μ_{2SLS}. Although only about half of firms are truly price constrained, the Yun-sticky price model gives the appearance that aggregate price adjustment is more costly than supported by NKPC sample moment (i).

Figure 3 has the evidence the Yun (1996) sticky price DSGE model fails to replicate NKPC sample moment (ii). The top window of figure 3 shows that the theoretical NKPC trend falls on top of the sample GDP deflator. Thus, the theoretical NKPC cycle exhibits excess smoothness, which explains the theoretical one-standard deviation confidence bands of figure 3.

Table 1 contains the sample and theoretical FEVDs of the price level with respect to trend shocks and theoretical one-standard deviation confidence intervals of the latter FEVDs (in brackets). The FEVDs of the Yun-sticky-price model and its one-standard deviation confidence intervals are all greater than 97.8 percent, which indicate little uncertainty about these FEVDs. The price level FEVDs of this sticky price model matches the NKPC-PV prediction, but is far away from NKPC sample moment (iii) because actual U.S. price movements are not only driven by trend shocks.

4.4 Labor Market Search-Sticky Price Models Experiments

The failure of the Yun-sticky price model indicates the nominal rigidity of sticky prices *alone* cannot explain the NKPC sample moments $(i)-(iii)$. Gali and Gertler (1999), among others, suggest a real rigidity may resolve this problem. We add the real rigidity of Mortensen and Pissarides (1994) job-search that Andolfatto (1996), Merz (1995), and den Haan, Ramey, and Watson (2000) successfully place in RBC models. An appeal of labor market search is the restrictions it places on the Phillips curve, as discussed, for example, by Solow (1976).

Labor market search connects real and nominal activity together with the matching technology and contracting mechanism available to firms and households. Firms and households engage in job search because hours are bought and sold in the presence of labor market externalities related to the costs of posting vacancies and looking for work. A final good firm posts $v_{j,t}$ plant-job vacancies at a cost of υ per vacancy.[20] The not-employed devote S_t hours to job search effort, which imposes felicity and pecuniary costs on them.

We assume a final good firm operates multiple plants and identify an active plant with a job.[21] Firms with empty-plant jobs and the not-employed are brought together randomly, according to the den Haan, Ramey, and Watson(2000) CRS matching technology

$$\mathcal{M}(V_t,\,(1-N_t)S_t) \quad = \quad \frac{V_t[(1\,-\,N_t)S_t]}{(V_t^{\vartheta}\;+\;[(1\,-\,N_t)S_t]^{\vartheta})^{1/\vartheta}}, \quad 0\,<\,\vartheta, \tag{21}$$

where $V_t(\equiv \int_0^1 v_{j,t}dj)$ is total plant-job vacancies and N_t denotes aggregate employment (or the measure of active plant-jobs). However, the probability of a successful match is influenced indirectly by variation either in posted vacancies or in not-employed search effort. This reflects the labor market externalities associated with search; see the appendix for details.

[20] Total recruitment costs represent a drain on aggregate output. This forces us to assume that υ shares the technology trend, but has a non-stochastic transitory component.

[21] Andolfatto (1996) points out that a CRS production technology in the presence of job search equates a plant-job with an operating plant. Hence, the aggregate measure of plant-jobs and the measure of the employed are equivalent.

Job search alters aggregate household felicity (7). When a not-employed household gives up leisure to search a fraction S_t of its one unit of date t time endowment, this household suffers a felicity loss equal to $\psi_5\ (1-S_t)^{1-\psi_4}\ /(1-\psi_4)$, where $\psi_4\ \neq\ 1$ and $0\ \leq\ \psi_5$. Since complete income and wealth insurance creates an aggregate household that is a weighted average of employed and not-employed households, the leisure component of aggregate household felicity becomes $N_t\psi_3$ $(1-h_t)^{1-\psi_2}/(1-\psi_2)$ + $(1-N_t)\ \psi_5\ (1-S_t)^{1-\psi_4}(1\ -\ \psi_4)$; see the appendix for details.

The wealth constraint of the not-employed differs from that of the employed. The source is the transactions costs the not-employed face during job search. We assume these costs rise with search effort at rate $\varphi\,(>\ 0)$ and that the only resource available to pay these costs is the cash injection the not-employed receive from the government. If this cost is handled differently in the model, the money demand functions of the employed and not-employed do not aggregate because of their disparate labor market histories. The result is the aggregate wealth constraint becomes

$$(22)\qquad A_t \quad \geq \quad B_{G,t}\ +\ M_t\ -\ [1-\varphi(1-N_{t-1})S_{t-1}]X_{t-1}.$$

The final term reflects the cash injection net of transactions costs generated by the search effort of the not-employed. An interpretation is that the employed transfer cash to the not-employed to hold the latter harmless for their search costs. The appendix discusses these issues.

Firm and not-employed search frictions place demands on aggregate output. Given complete income and wealth insurance, these costs enter the aggregate resource constraint additively

$$(23)\qquad \left(\frac{P_t}{P_{A,t}}\right)^{-\xi} Y_t \quad = \quad C_t\ +\ K_{t+1}\ +\ (1\ -\ \delta_K)K_t\ +\ G_t\ +\ (1-N_t)\varphi\frac{X_t}{P_t}S_t\ +\ \upsilon_t V_t,$$

from the wealth constraint of the aggregate household and the aggregate dividend process.[22] The

[22] The appendix outlines the insurance schemes that give rise to the felicity function of the aggregate household and

last two terms on the right of the aggregate resource constraint (23) reflect the real resource loss that arises from job search by households and firms, respectively.

Job search precludes a spot market for labor. Rather than a Walrasian auctioneer, a final good firm and the aggregate household negotiate labor contracts over hours and the real wage to split match surplus during the life of the employment relationship. Match surplus is the sum of the capitalized value of an active plant-job and the net benefits the aggregate household receives from the ongoing job match. Merz (1995), Andolfatto (1996), and Cooley and Quadrini (1999) assume the aggregate household receives a fixed fraction, ζ, of the surplus during each date t. This equates the contribution of the aggregate household to the match to a fraction ζ of the capitalized value of an active plant-job.

The surplus splitting rule together with optimal firm and aggregate household behavior produces the (Nash) equilibrium real wage function

$$(24) \qquad \Gamma_t \frac{W_t}{P_t} h_t \;=\; \zeta \Gamma_t \left[\left(\frac{P_{A,t}}{P_t} \right)^{\xi} \left(1 - \left[\frac{\theta \phi_t}{1 - \overline{K}^*} \right] \right) \frac{Y_{A,t}}{N_t} + \frac{\upsilon_t V_t}{1 - N_t} \right] + (1 - \zeta) \mathcal{H}_{X,t},$$

where $\mathcal{H}_{X,t} = \psi_3 \, (1-\psi_2)^{-1} \, (1-h_t)^{1-\psi_2} - \psi_5 \, (1-\psi_4)^{-1} \, (1-S_t)^{1-\psi_4} - \varphi \Gamma_t X_t S_t / P_t$ and $\overline{K}^*$ is the steady state of $\overline{K}_t$.[23] Along the equilibrium path, discounted real labor income is a weighted average of that match's value-added to aggregate output and the alternative activities (e.g., non-employment

the rest of the economy-wide optimality and equilibrium conditions. Aggregation rests on the capital stocks, dividends received, cash held, and bonds owned by these households to be equal date-by-date. This assumes that employed and not-employed households hold equal endowments of capital and financial wealth at date zero. Further, we assume away any wealth disparities caused by ownership claims on final goods firms. If employed and not-employed households are initially given equal equity in final goods firms, the dividend flows will be equalized. Also, these results depend on the additive separability of felicity. Sims (1998) discusses related issues.

[23] The appendix constructs the equilibrium real wage process (24).

and search) available to the aggregate household. The marginal product of labor plus the foregone costs of fixed capital and firm job search represent the former. The latter is the net impact on felicity of an ongoing plant-job match and foregone transactions job-search costs. Note that the real wage is a function of real marginal cost, ϕ_t, in a sticky price regime. Unlike a spot market in which the real wage equals the intersection of labor supply and labor demand (productivity) schedules at each date t, labor market search creates persistence and volatility in the real wage that differs from labor productivity. This is the labor market search propagation mechanism.

Calibration of the labor market search models follows the process described in section 4.1. The not-employed preference parameters ψ_4 and ψ_5 equal two and 1.37, respectively. The exogenous fixed separation rate is set at 0.0848, which places δ_N within the range Merz (1995), den Haan, Ramey, and Watson (2000), and Andolfatto (1996) use. The calibration of ψ_4, ψ_5, and δ_N help guarantees aggregate hours and employment match their sample counterparts. Cooley and Quadrini (1999) fix $1/\vartheta$ at 0.6. We do the same.[24] The vacancy cost parameter $\upsilon = 0.1050$ is taken from Andolfatto.[25] We assume job-search transactions costs impose a 0.1 percent loss on velocity at the steady state (in terms of sample GDP and the monetary base). This yields $\varphi = 0.3060$.

We solve models with labor market search using methods described in section 4.1. The linearized aggregate household and firm job-search optimality conditions, the aggregate resource constraint (23), and the law of motion of aggregate employment, $N_{t+1} = (1 - \delta_N)N_t + \mathcal{M}_t$, add $\widetilde{N}_{t+1}$ to the state vector $\mathcal{K}_{t+1}$ and $\widetilde{S}_t$ to the control vector $\mathcal{C}_t$. The transversality condition for

[24]Since $(1 - \zeta) = 1/\vartheta$, the power the aggregate household exerts on contract negotiations equals the household's share of the match surplus. Thus, the equilibrium real wage is the same as the socially optimal wage; see Hosios (1990).

[25]The steady state is also constructed to make consistent with the den Haan, Ramey, and Watson calibration the probabilities that a vacant plant-job is filled and that someone not-employed finds work.

employment is $lim_{j\to\infty}\beta^{j}\mathbf{E}_t\left\{\Lambda_{t+j}N_{t+1+j}\right\}=0$, where Λ_t is the shadow price of a job match.

The theoretical density of μ_{2SLS} generated by the labor market search-sticky price model (dot-dash curve) appears in figure 2. This density is to the right of the NKPC sample moment (i) because only 58 of the 5000 estimates reside within the 95 percent asymptotic confidence interval of the sample estimate of μ_{2SLS}. An ensemble mean of 0.6607 also signals the labor market search-sticky price model cannot explain NKPC sample moment (i).

Figure 4 presents NKPC moment (ii), the common trend and common cycle, of the labor market search-sticky price model. The theoretical NKPC trend (the top window) closely follows the actual GDP deflator. This explains the smoothness of the one-standard deviation confidence bands of the theoretical NKPC cycle (the bottom window). Thus, the labor market search- and Yun-sticky price models have nearly identical price level fluctuations.

The FEVDs of the price level with respect to the trend shock and their one-standard deviation coverage intervals generated by the labor market search-sticky price model appear in the fourth column of table 1. Since the one-standard deviation coverage interval of the FEVD runs from 35 to 96.5 percent at the one-quarter forecast horizon, it suggests a short-run role for cyclical shocks. However, the theoretical one-quarter horizon FEVD of 69 percent and the one year-horizon of nearly 90 percent are closer to the NKPC prediction than to the relevant sample FEVDs. Thus, the labor market search-sticky price model finds it difficult to reproduce NKPC sample moment (iii).

In summary, NKPC sample moments $(i)-(iii)$ fail to be replicated by the Yun- and labor market search-sticky price models. The NKPC-PV restriction that only trend shocks matter for price level movements explains these rejections. This suggests the link between the price level and trend shocks needs to be broken for monetary DSGE models to fit NKPC sample moments $(i)-(iii)$.

4.5 Flexible Price-Labor Search Model Experiments

This section reports on a DSGE model that replaces the sticky price mechanism (1) with a flexible price regime. This eliminates the price expectation terms, $\mathbf{E}_t \widetilde{P}_{t+1}$ and $\mathbf{E}_{t-1} \widetilde{P}_t$, from the state vector, $\mathcal{K}_{t+1}$ in the solution of the labor market search-flexible price model. Although the NKPC-PV restrictions no longer govern the price level, it continues to exhibit "stickiness" because of the real rigidity of labor market search.

Figure 2 shows that the 95 percent asymptotic confidence interval of μ_{2SLS} falls within the density of theoretical μ_{2SLS} estimates produced by the labor market search-flexible price model (the solid line). More than 46 percent of these estimates are contained in the 95 percent asymptotic confidence interval, $[0.5133,\ 0.5450]$. The theoretical mean of μ_{2SLS} is 0.5300, compared to a sample mean of 0.5293. Thus, an econometrician who studies the labor market search-flexible price model would recover the NKPC sample moment (i). This is more evidence Solow (1976) is correct that labor market search helps to explains Phillips curve observations.

NKPC FEVDs of the labor market search-flexible price model appear in the last column of table 1. Theoretical FEVDs are larger than sample FEVDs at 1, 2, 4, and 8 quarter forecast horizons, but smaller beyond a two-year horizon. However, one-standard deviation confidence intervals of the theoretical FEVDs cover the sample FEVDs, except at one- and two-quarter horizons. Thus, the labor market search-flexible price model matches much of NKPC sample moment (iii) driven only by a random walk technology shock and a money growth process whose AR1 coefficient is 0.45.

The theoretical NKPC trend and cycle of the labor market search-flexible price model appear in figure 5. A feature that stands out is that the theoretical NKPC trend (the top window) and cycle (the bottom window) are more volatile than their sample counterparts. The relative volatility of the

NKPC cycle (trend) is 1.5098 (1.3622), which is a weakness of the labor market search-flexible price model, according to Walsh (2002) and Trigari (2003a). The labor market search-flexible price model is able to replicate the persistence of the sample NKPC cycle. The AR1 coefficients from the sample and the theoretical ensemble of the NKPC cycles are 0.9335 and 0.9476, respectively.

Figure 5 also shows that the differences between the empirical and theoretical NKPC trends are greatest around peaks and troughs. The one-standard deviation confidence bands cover the sample NKPC cycle, beginning with the mid-1970s. We conclude that the flexible price-labor search model has more success matching NKPC sample moment (ii), than do the sticky price models.

4.6 Price Level Fluctuations, Labor Market Search, and the NKPC

Labor market search has difficulties with several important business cycle facts. Cole and Rogerson (1999) point out labor market search models suffer from several weaknesses, among them incorrect predictions about job creation, job destruction, and unemployment flows.[26] Likewise, Walsh (2002) and Trigari (2003a) report that labor market search-monetary DSGE model produce too much nominal volatility, which we confirm with the labor market search-flexible price model. Nevertheless, the labor market search-flexible price model is better able to replicate the NKPC sample moments $(i) - (iii)$, than do the sticky price models we study.

Our results link the NKPC to previous Phillips curve research. For example, the labor market search-flexible price experiments explain the importance attached to sticky wage mechanisms, for example, by Jeanne (1998), Erceg, Henderson, and Levin (2000), and Rabanal and Rubio-Ramírez (2001). Since the labor market search-flexible price model relies on “stickiness” in the labor market,

[26]Trigari (2003b) argues that a combination of nominal rigidities and labor market search resolves these problems.

our results are also tied to King and Watson (1994, 1997). They show U.S. data supports a Phillips curve that allows for flexible prices, but in which labor market variables are sticky. This is consistent with our labor market search-flexible price model because the current price level is flexible with respect to date t shocks (price expectations are not an explicit element of equilibrium decision rules) and the equilibrium decision rule for N_{t+1} responds to shocks dated t, not $t+1$.

We show the impact of sticky and flexible prices on theoretical NKPCs using the equilibrium real wage equation (24). Divide its left-side by $\Gamma_t Y_{A,t}$ to obtain real unit labor cost, $W_t h_t / P_t Y_{A,t}$. Since this measure of real unit labor cost does not equal ϕ_t, a wedge is driven between theoretical marginal cost and unit labor costs. The mismeasurement of real marginal cost helps to create excess smoothness in the theoretical price level of the labor market search-sticky price model.

The equilibrium real wage equation (24) also impose restrictions on the NKCP trend and NKPC cycle of the labor market search-sticky price and -flexible price models. The labor market search models predict that real unit labor cost is driven by the wage contracting process because the right hand side of equation (24) equals $W_t h_t / P_t Y_{A,t}$, subsequent to dividing by $\Gamma_t Y_{A,t}$. The balanced growth conditions of the models we study force $W_t h_t / P_t Y_{A,t}$ to be a cointegrating relation. The theoretical version of the NKPC common feature relation (5) is

$$ln[P_t] - \mu_{\mathcal{B}} ln\left[\frac{W_t h_t}{Y_{A,t}}\right] = ln\zeta + (1-\mu_{\mathcal{B}})(ln[Z_t] - ln[M_t] + ln[\widehat{P}_t])$$

$$+ ln\Bigg[\left(\frac{P_{A,t}}{P_t}\right)^{\xi}\left(1 - \left[\frac{\theta\phi_t}{1-\overline{K}^*}\right]\right)\frac{1}{N_t} + \frac{\upsilon_t V_t}{(1-N_t)Y_{A,t}} + \frac{(1-\zeta)\mathcal{H}_{X,t}}{\zeta\Gamma_t Y_{A,t}}\Bigg],$$

which we derive from the equilibrium real wage generating equation (24).

Engle and Issler (1995) refer to common feature relations as trend generators because applying a common feature vector to levels data wipes out the serial correlation leaving only the trend.

Note that $ln[Z_t] - ln[M_t]$ is the difference of two trend processes. Thus, permanent movements in the level of technology and the money stock drive the theoretical NKPC trend.

Transitory movements in the theoretical NKPC cycle is "excessively" smooth under a sticky price regime because the stochastically detrended price level, $ln[\widehat{P}_t]$, is governed by restrictions embodied in the NKPC-PV relation (4). Within the linearized solutions of the sticky price models, this is reflected by the response of $\widetilde{P}_t$ to the price expectation variables $\mathbf{E}_{t-1}\widetilde{P}_t$ and $\mathbf{E}_{t-2}\widetilde{P}_{t-1}$, as, for example, in the control system (20) of Yun's model. Under the flexible price regime, $\widetilde{P}_t$ is not driven by price expectations, but instead by capital, employment, and shock dynamics restricted (in part) by the real wage process (24). Since labor market search and flexible prices do a better job of mimicking NKPC sample moments $(i) - (iii)$, it presents a challenge to models in which nominal price rigidities are the prime motivate of short-run monetary non-neutralities.

5. Conclusion

This paper develops a new Keynesian Phillips curve (NKPC) present-value relation, in which *nominal* unit labor cost is the fundamental, rather than real unit labor cost. An implication is that the NKPC predicts trend shocks dominate price level movements at all forecast horizons. We also show that the NKPC present-value relation has a common cycle-common trend decomposition that is based on Beveridge and Nelson (1981), Stock and Watson (1988), and Vahid and Engle (1993).

The last 40 years of U.S. GDP deflator and nominal unit labor costs data offers weak support of the NKPC. We estimate that about half of U.S. final goods firms are price constrained. These estimates are close to estimates reported elsewhere. The disparity between the data and the NKPC predictions arises in its common trend-common cycle decomposition and FEVDs. The NKPC cycle is economically important because it peaks during the last seven NBER dated recessions. The FEVDs show that trend shocks only begin to account for more than 60 percent of price level movements at forecast horizons of two years or more. Thus, the NKPC prediction that trend shocks dominate price level fluctuations at all forecast horizons is not supported by the data.

We also study the implication of the NKPC present-value prediction for several dynamic stochastic general equilibrium monetary models. Simulation experiments show that the Yun (1996) model with Calvo (1983) staggered price setting reproduce the NKPC predictions. Hence, a model with only the nominal rigidity of sticky prices is unable to match our estimated NKPC.

Earlier Phillips curve models invoke labor market imperfections to explain observed inflation dynamics. We pursue this idea by adding labor market search to the Yun-sticky price model. Monte Carlo experiments of the labor market search-sticky price model yield NKPC moments that are not qualitatively different from the model with only sticky prices. Unlike the labor market search-sticky price model, its flexible price cousin is better able to match U.S. price level movements.

Our results continue the challenge of Chari, Kehoe, and McGratten (2000), among others, to the new Keynesian notion that nominal rigidities generate short-run monetary non-neutralities. We broaden their research agenda by studying an alternative monetary transmission mechanism: the real rigidity of labor market search. We identify the real rigidity with labor market search because its externality suggests a role for monetary policy. However, our labor market search-flexible price model is only one specification within a wide class of real rigidity-DSGE monetary models. This points to the need to search for an economically meaningful monetary transmission mechanism within real rigidity-flexible price DSGE models that can be used for policy analysis. We leave this task for future research.

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Table 1. Forecast Error Variance Decomposition

One Standard Deviation Confidence Intervals
FEVDs of Price w/r/t Trend, Generated by DSGE Models

Horizon	Sample PGDP	Yun-Sticky Price Model	Search-Sticky Price Model	Search-Flexible Price Model
1	2.70	98.90	69.22	24.94
		[97.81 99.93]	[34.77 96.53]	[6.92 48.03]
2	8.80	99.33	80.87	37.90
		[98.67 99.96]	[61.21 98.72]	[13.09 69.48]
4	26.38	99.65	89.32	52.90
		[99.31 99.98]	[84.05 99.60]	[23.01 85.63]
8	60.13	99.85	94.32	67.48
		[99.70 99.99]	[94.80 99.88]	[38.70 94.08]
12	78.37	99.91	96.17	74.80
		[99.82 100.00]	[97.40 99.95]	[49.05 96.68]
16	86.55	99.94	97.15	79.18
		[99.88 100.00]	[98.48 99.97]	[56.35 97.77]
20	91.44	99.96	97.75	82.12
		[99.92 100.00]	[99.01 99.98]	[61.93 98.42]
40	98.05	99.99	98.97	89.21
		[99.97 100.00]	[99.76 99.99]	[76.92 99.51]

The values in brackets are the 16th and 84th percentiles of the FEVDs generated from 5000 replications of the DSGE models.

Figure 1: The U.S. Phillips Curve

The Phillips Curve Trend (solid line) and GDP Price Level (dotted line)

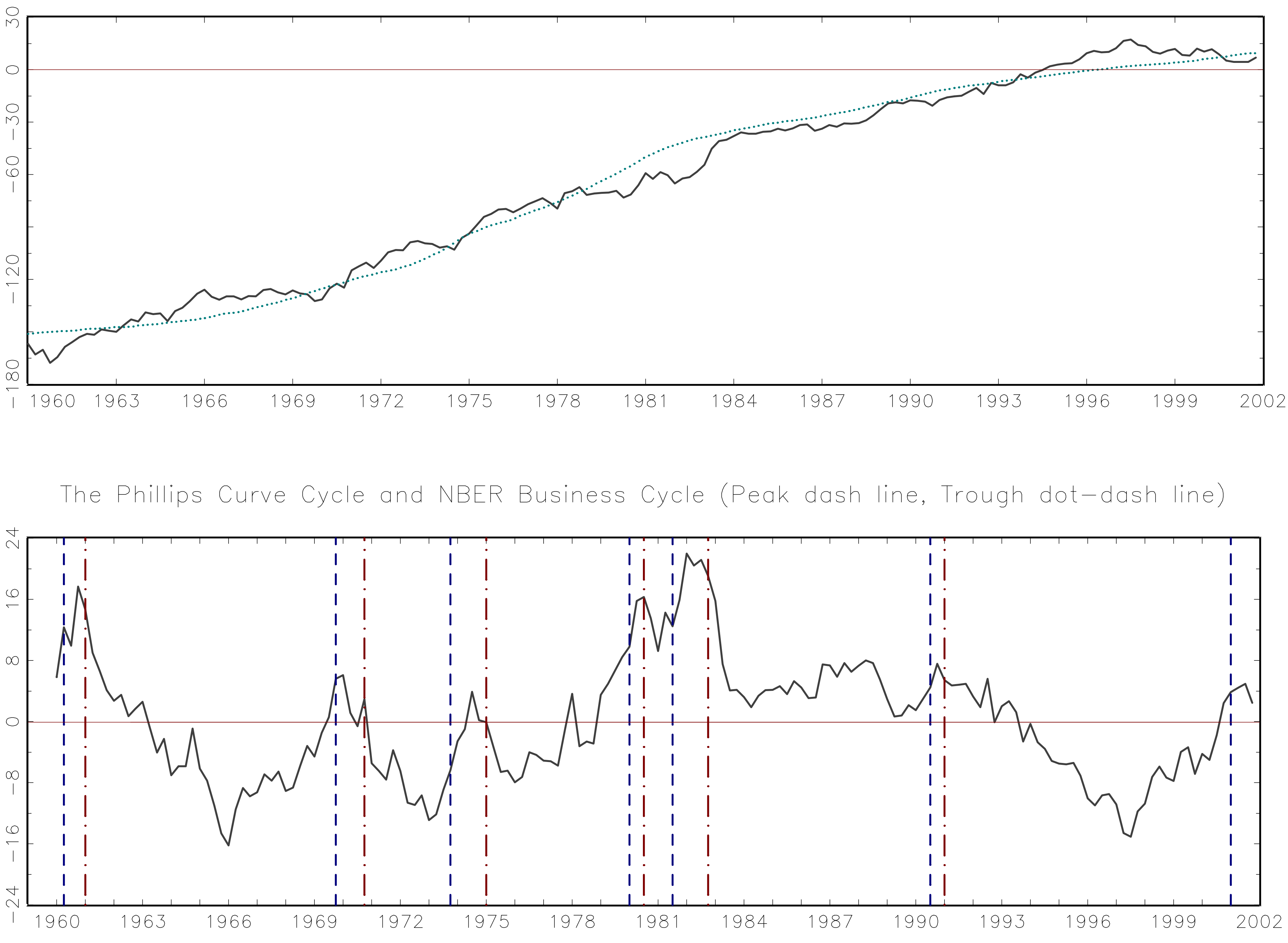


Figure 2: Theoretical Densities of μ

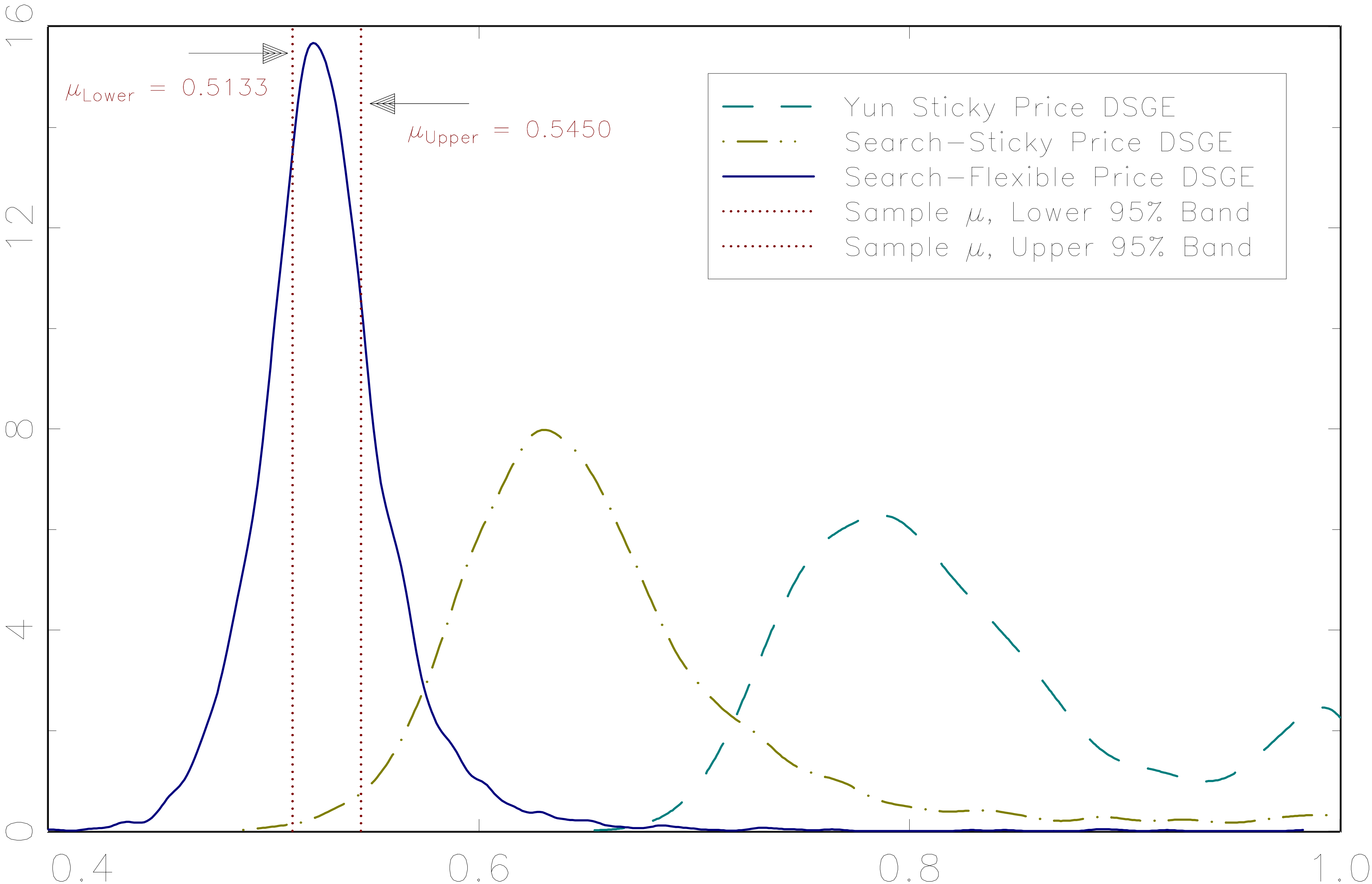


Figure 3: Yun–Sticky Price DSGE Model

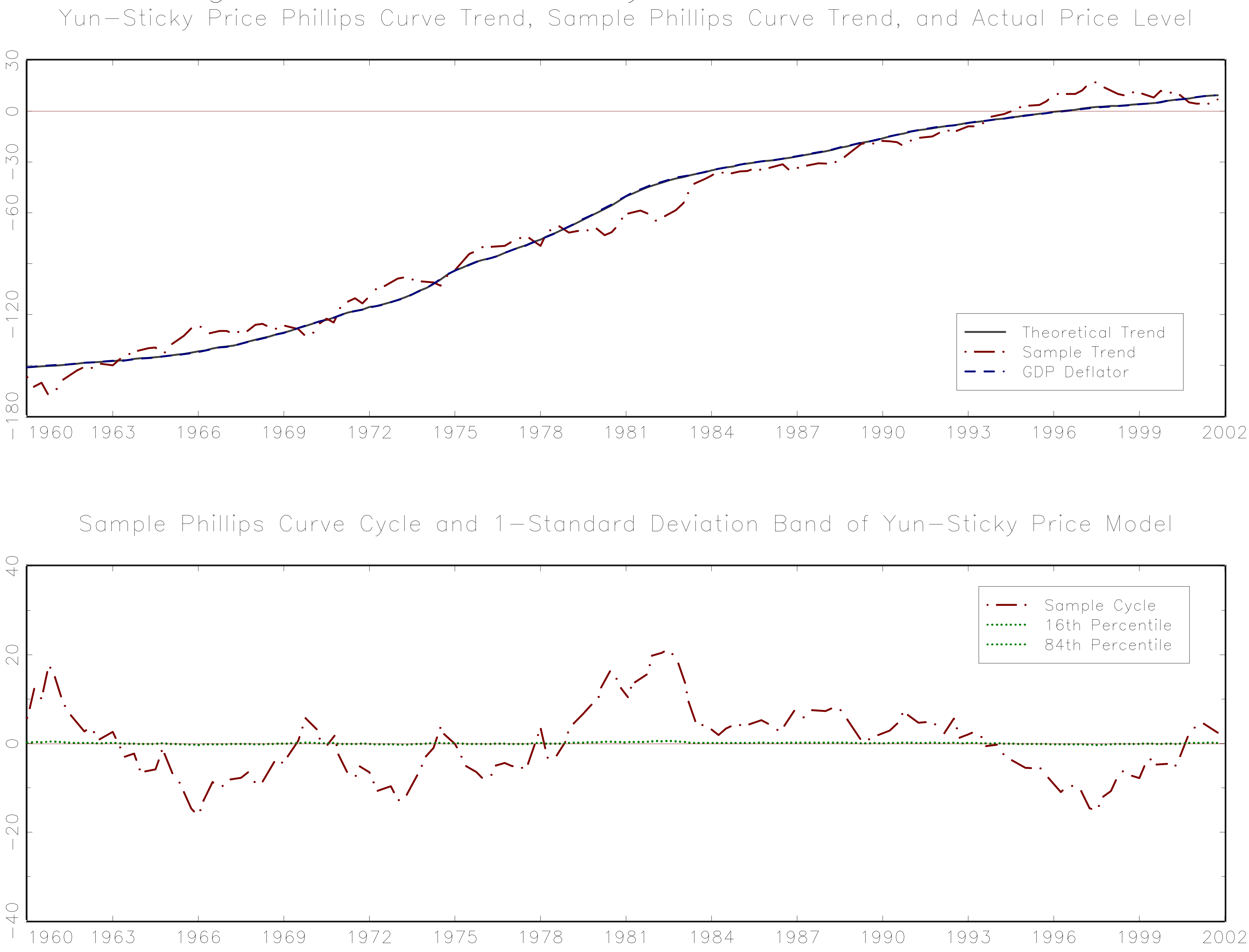
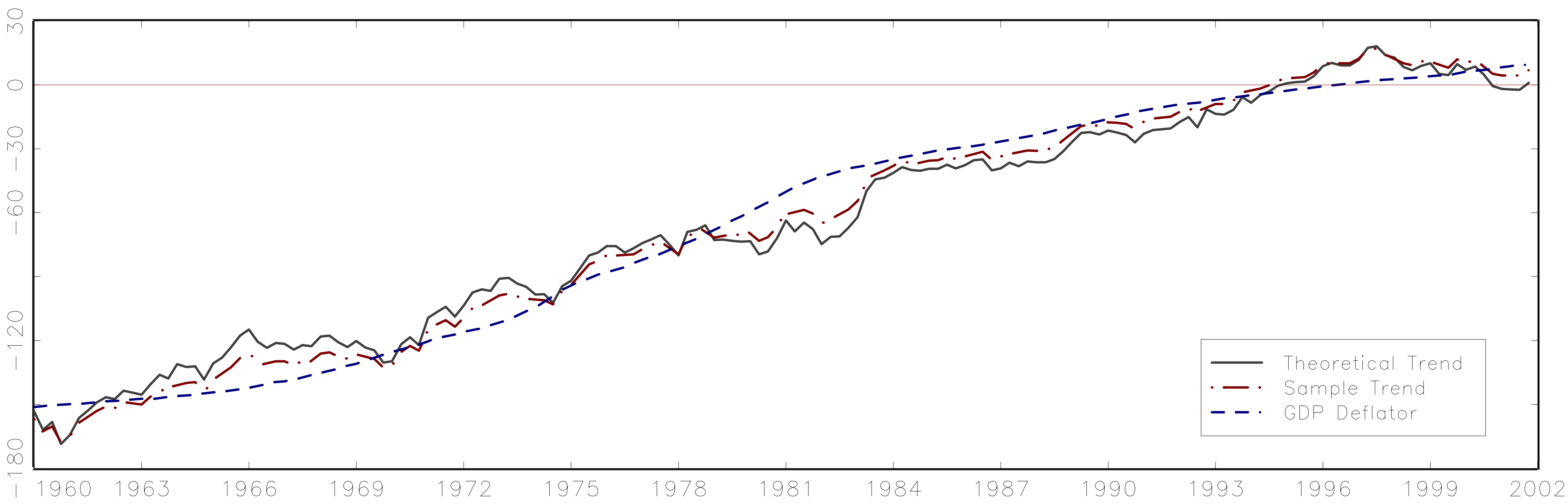


Figure 4: Search–Sticky Price DSGE Model

Figure 5: Search—Flexible Price DSGE Model

Search—Flexible Price Phillips Curve Trend, Sample Phillips Curve Trend, and Actual Price Level



Sample Phillips Curve Cycle and 1—Standard Deviation Band of Search—Flexible Price Model