HOW TO SUBSIDIZE EDUCATION: AN ANALYSIS OF VOUCHER SYSTEMS

BENJAMIN EDEN*

THE UNIVERSITY OF IOWA AND
TECHNION-ISRAEL INSTITUTE OF TECHNOLOGY

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ABSTRACT

It is argued that simple vouchers are not sufficient for successful decentralization: To achieve the socially desired outcome, the government must pay schools for the educational outputs in addition to payments for the employment of students. Once this achievement based system (ABS) is in place, the optimal amount of integration will arise voluntarily, because in the ABS schools face the correct shadow wages for the employment of students.

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Mailing address: Benjamin Eden, Faculty of Industrial Engineering and Management, Technion, Haifa, 32000, Israel.

1. INTRODUCTION

Chubb and Moe open their book by the following statement: "For America's public schools, the last decade has been the worst of times and the best of times. Never before in recent history have the public schools been subjected to such savage criticism for failing to meet the nation's educational needs-yet never before have governments been so aggressively dedicated to studying the schools' problems and finding the resources for solving them.” Elsewhere, they argue: "Of all the reforms that attract attention, only choice can address the basic institutional problems plaguing America's schools". (Chubb and Moe, 1990a, pp.1 and 1990b, pp.7.) Manski (1992) states the need for economic analysis instead of rhetoric partly because "during the past thirty years, the basic intellectual argument for systematic choice has not notably advanced beyond the classical economic ideas sketched by Friedman (1955, 1962)."[1] Here I attempt to advance the understanding of vouchers systems by applying well known results about external effects.

The paper makes two points: The first is that if education of individuals generates externalities for society, a system of market provision of education can be induced to provide the socially optimal

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[1] For a recent statement see Milton and Rose Friedman (1981, pp.140-78). For some discussion see Cohen and Farrar (1977), Coons and Sugarman (1978) and Lieberman (1989). An early voucher proposal, by Tom Paine is discussed in West (1967).

outcome. To achieve the desired social outcome the government must pay a subsidy per unit of educational achievement for each student type: the subsidy equals the difference between the social and private return per unit of educational achievement for that type. The second point is that voucher payments that are not contingent on educational achievements will yield the same outcome in a market system for providing education as would be achieved without the vouchers. Schools will simply pass the vouchers along to students in the form of stipends (in cash or in kind).

These points are made in the context of a model with two student types: high- and low-ability. This permits making the distinction between two types of externalities. The external benefit that high ability types convey on low ability types can be internalized by differential tuition charges.[2] By contrast individual and schools do not have incentives to take account of social externalities such as the benefits to democracy of an educated citizenry.[3] This latter type of externality leads to the divergence between private and social incentives that the achievement-based subsidy is shown to remedy. The model assumes no income effects so that the voucher payment does not alter the demand for education.

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[2] This point was made by White and Rothschild (1992) Eden and Klinov (1992) and Eden (1992).

[3] Another source of external effect is that some parents may fail to represent the interest of their children. Externalities seem to be important in elementry and high-school, where socialization takes place. It is often argued that external effects in higher education are less important. See Arrow (1973), for example.

The environment considered in the paper abstracts away from several important issues. First, it assumes that the government has well-defined social weights on the welfare of each student type thus abstracting from the political difficulties involved in determining such weights. Second, it assumes that the government can costlessly measure both student type and student educational achievement. In practice, these may be extremely difficult for a government to assess. Third, it assumes a perfect capital market.

Government backed loans can mitigate capital market imperfections and most economists will favor this type of intervention. The question is whether government direct production mitigates the first two problems. Direct government production requires some implicit social welfare function that govern the allocation of resources. The achievement based subsidy scheme (ABS) requires that this welfare function will be made explicit. In principle an explicit function has an advantage because it allows for healthier public debate. (On the other hand, it is possible that making the objective function explicit is not politically feasible, precisely because it allows such a debate). The second problem is more serious.

Measurement in education is difficult and may interfere with the educational process itself (Madaus, 1988). To overcome this difficulty government direct production relies mainly on the measurement of inputs

rather than educational outputs. If we have some knowledge of the production function, we can use the inputs as proxies for the outputs.[4]

In an ABS both the government and the parents monitor schools performance. The need for government monitoring arises because parents want to receive some of the education budget as consumption, while society wants to spend the entire budget on educational inputs.[5][6] Since parents choice acts as a monitoring device, the ABS has a clear

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[4] We may know for example, that if we put together a qualified history teacher, 10 low ability (L) students and 30 high ability (H) students, for one hour in a class room that meet a certain criteria, we will get on average 2 units of history knowledge per H student and 1 unit per L student. If the ABS calls for 1 dollar per unit of history knowledge achieved by H and 2 dollars per unit achieved by L, we pay the school 60 dollars for this history class.

[5] An analogy with subsidizing clean air may be useful. Assume, for example, that a factory and a school get money from the government. The factory is run and owned by the workers and the school is run and owned by the parents. In the absence of government monitoring, there are two options for spending the money. For the workers these are: pollution controls and subsidized lunches. For the parents these are: a library and subsidized lunches. In the absence of monitoring, both groups may go for subsidized lunches even when society favors the alternative.

[6] Asymmetric information is another argument for joint monitoring. But this is a more general problem: Car manufacturers are better informed then consumers about the quality of their products. The government intervention in this case is to impose certain safety standards for cars and, using analogy, minimum requirements for schools. The ABS requires the measurements of achievements, which is more than just monitoring minimum requirements. This additional monitoring is required because society and parents want different things.

advantage over direct government production, even when only inputs are measured.[7]

If zero tuition is an independent objective, the government can pay schools for the employment of students who will otherwise have to pay tuition and tax schools for the employment of students who will otherwise receive stipends.[8] In this case, students do not have out of pocket expenses and universal school attendance can be achieved.

The second point of the paper is that in the presence of externalities, simple vouchers will not work because schools will simply

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[7] The ABS allows for another indirect way of measuring educational outputs. If lifetime earnings is a good proxy for the external effects of education, we can measure the average lifetime earnings of the graduate student and credit the various schools that contributed to it. For example, a student that earned 1.2 million dollars during his lifetime, will credit his elementary school (classes 1-6) by 0.6 points, his Junior-high-school by 0.3 points and his high school by 0.3 points. A student that earned 0.6 million, will credit his three schools by 3, 1.5 and 1.5 points. After points are divided, funds are distributed according to a formula which depends on the social objectives. The payoffs for the school outputs, will thus be paid about 60 years later. Since schools in the ABS operate like firms, they can sell stocks or borrow against these future receipts. If capital markets are imperfect, we may use starting salaries as proxies for lifetime income.

[8] This requires the classification of students to many types. Indeed if we insist on no out of pocket expenses we may have to auction individual students among all potential schools. More realistically, we should expect that students will have small out of pocket expenses.

pass the vouchers along to students in the form of stipends.[9] This enforcement problem was noted by Manski (1992) who argues that in the absence of monitoring "students and schools can subvert social objectives by using the subsidies to further their own private interests." To appreciate the enforcement problem, consider first the case in which there are no restrictions on the amount of stipends. If students want to spend only $y on education out of a voucher of $x, then in equilibrium schools will offer $y worth of education + a stipend of x - y dollars. Schools that offer $y worth of education will not be able to compete because students prefer the additional x - y dollars as cash. If monetary stipends are not allowed, schools will pay stipends in kind by providing, for example, cheap lunches and clothing.[10] In what

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[9] There are proponents of choice who think that the external effects are relatively unimportant. They base their argument for vouchers on incentives. Levin (1991) notes that this incentive argument can be found as early as 1776, in Adam Smith's *The Wealth of Nations*. Smith (1937, p.737) argued that if the government pays all the cost of education the teacher "... would soon learn to neglect his business". The incentive problems can be solved by government backed loans and do not require vouchers.

[10] Food stamps is another example of transfer payments with an added restriction on spending. We expect a market for food stamps to develop whenever the amount of food stamps is larger than what the receipients want to spend on food: The receipients will use this market to sell any amount of food stamps which is above what they really want to consume. There are however, two important differences between food stamps and vouchers. Food stamps represent a transfer from the entire population to a small group of poor people and therefore the receipients get a significant increase in their wealth. In the case of

follows, I refer to everything that society does not wish to subsidize as stipend in kind or consumption.[11]

Since some of the voucher money will be spent on consumption a voucher system is an expensive way to increase the level of education.

2. THE MODEL

There are only two types of students: low and high ability. There are L low ability students and H high ability students in the economy. The capital letters L and H will also be used as indexes. Let E_L (E_H) stands for the quantity of education received by low (high) ability students in the economy and let m denotes the amount of money spent on education.[12] The production possibility set for the economy is denoted

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vouchers, the transfer is from families with small number of children to families with large number of children and the wealth effect is much smaller. In addition the elasticity of food consumption with respect to wealth for poor people must be relatively large but the wealth elasticity of education may be small because the consumption element in education may be small. These differences suggest that in spite of the enforcement problem food stamps will increase the demand of poor people for food, but vouchers may fail to have a significant effect on the demand for education.

[11] For example, some parents may value the knowledge of the bible more than society does. In this case spending the additional x - y dollars on bible studies, is equivalent from the social point of view to spending it on consumption because it does not generate external effects.

[12] The model can be easily extended to the case in which there are many types of students, many educational outputs and many other inputs. In

by Y. Thus $y = (E_H , E_L , l , h , m) \in Y$, if it is possible to produce E_H units of type H education and E_L units of type L education with the inputs of l low ability students, h high ability students and m dollars.

Factors of production are perfectly mobile. Thus as in Friedman (1962), I limit the analysis to large urban areas. It is assumed that the level of education per student depends on his class but not on his school. Adding a "school effect" will not change the main results.

I now introduce rather standard assumptions regarding the production possibility set.[13] Suppose that y can be achieved by a school system that uses l low ability students, h high ability students and m dollars as inputs, and y' can be achieved by a school system that uses l' low ability students, h' high ability students and m' dollars as inputs. Then we assume that it is possible to build both kinds of school systems. Thus,

Additivity (A1): $y \in Y$ and $y' \in Y$ implies $y + y' \in Y$.

I also assume that it is possible to reduce all inputs by the same percentage without affecting the level of education per student. This

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such a general model, E_j is a vector that may include knowledge of math, the ability to think and to create, achievements in sports and social skills. The "other" inputs may include teachers and facilities.

[13] See, for example, Arrow and Hahn (1971. ch.3).

seems a reasonable approximation when the relevant scale of operation is large.[14] Thus,

Divisibility (A2): $y \in Y$ implies $\lambda y \in Y$ for all $0 \leq \lambda \leq 1$.

The planner's problem: There are infinitely many potential classes. The maximum amount of education that a type t student can get in class i with the inputs $x_i = (m_i , h_i , l_i)$ is denoted by $G_t(x_i)$. It is assumed that $G_t(0) = 0$, G_t is an increasing function of m and a decreasing function of l. It may be either increasing or decreasing in h.

Let $\alpha_t > 0$ denotes the weight assigned by a social planner to type t education and let $x = [x_1, x_2 , x_3 \ldots]$ denotes an allocation of inputs among all potential classes. The planner values the outputs associated with the vector of inputs x, by:

$$(1) \qquad F(x) = \Sigma_i \, \alpha_H h_i G_H(x_i) + \alpha_L l_i G_L(x_i) \ .$$

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[14] To illustrate, suppose that initially we have 31 classes with 30 students per class. We are now asked to cut all inputs to a third of their initial level. In this case, we will have 10 classes with 31 students per class. We do not expect that changing the number of students per class from 30 to 31, while holding the amount of money spent per student constant, will have much effect on the level of education per student. To see that large scale operation is required, consider the case in which in the context of the above example, initially we have 4 classes rather than 31. In this case, if we cut the level of all inputs to a third of their initial level, we will end up with a class of 40 students. This is more likely to alter the level of education achieved by each student.

The assumptions (A1) and (A2), imply that F( ) is concave.[15] Let, $X = (M, H, L)$ denote the total quantities of available inputs. The planner solves:

$$(2) \qquad \max F(x) \quad \text{s.t.} \quad \Sigma_i x_i \leq X , \quad x_i \geq 0.$$

Assuming the constraint qualifications[16], there exist shadow prices (lagrangian multipliers) λ_j such that the solution to (2) is also the solution to:

$$(3) \qquad \max F(x) - \lambda_1\Sigma_i l_i - \lambda_2\Sigma_i h_i - \lambda_3\Sigma_i m_i ; \quad \text{s.t. } x \geq 0.$$

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[15] To show this claim we need to show that:

$F(\delta x' + [1-\delta]x) \geq \delta F(x') + (1 - \delta)F(x)$; for $0 < \delta < 1$.

From the definition of $G_t( )$ as the maximum amount of type t education possible given the input vector, it follows that

$y = [\Sigma_i h_i G_H(x_i) , \Sigma_i l_i G_L(x_i), \Sigma_i x_i] \in Y$ and

$y' = [\Sigma_i h'_i G_H(x'_i) , \Sigma_i l'_i G_L(x'_i), \Sigma_i x'_i] \in Y$. It therefore follows from (A2) that $(1-\delta)y \in Y$ and $\delta y' \in Y$. From (A1) it follows that $(1-\delta)y + \delta y' \in Y$. Thus the firm can produce the output $\delta F(x') + (1 - \delta)F(x)$, with the vector of inputs $\delta x' + (1 - \delta)x$. This completes the proof.

[16] I.e., the inputs vector which is available for the economy, X, is strictly positive and it is possible to produce some education with $X_0 << X$.

We can therefore choose units in a way that will make $\lambda_3 = 1$. Under this choice, α_t can be interpreted as the price in terms of tax dollars that the planner is willing to pay for a unit of type t education.

Students objective function: Let $U_t(E , w)$ denote the level of utility of a type t student who gets E units of education and is paid a current wage of w dollars. The current wage can be positive (a stipend) or negative (tuition). I assume[17]:

$$U_t(E , w) = E + w \ . \tag{4}$$

In the presence of perfect capital markets, we may think of E as the contribution of education to human capital. Following the labor contracts literature I assume that an individual school must promise a type t student W_t utils to attract him or her, where W_t is the level of utility that can be achieved by a type t student elsewhere. Thus,

$$U_t(E_{ti} , w_{ti}) = G_t(x_i) + w_{ti} \ \geq W_t \ . \tag{5}$$

The achievement based system (ABS): I start with a system in which tuitions and stipends are allowed: $w_t \gtrless 0$. The government pays P_t dollars per unit of type t education and does nothing else. Each firm

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[17] Assuming $U = \beta E + w$, with $\beta > 0$, will not change the main results.

owns one class and chooses the current wages, w_{ti} , and the number of students from each type to maximize:

$$(6) \qquad \max P_H h_i G_H(x_i) + P_L l_i G_L(x_i) - w_{Hi} h_i - w_{Li} l_i - m_i$$
$$\text{s.t. (5) and } x_i \geq 0.$$

Since I allow negative current wages, at the optimum (5) must hold with strict equality. Substituting $w_{ti} = W_t - G_t(x_i)$ the problem (6) becomes choosing $x_i \geq 0$, to:

$$(7) \qquad \max (P_H + 1) h_i G_H(x_i) + (P_L + 1) l_i G_L(x_i) - W_H h_i - W_L l_i - m_i.$$

The problem (7) uses full prices. The full price of education is $P_t + 1$. This is intuitive: For a unit of output the school gets P_t dollars from the government and a dollar increase in tuition.

Using $x = (x_1, x_2, \ldots)$, $P = (P_L , P_H)$ and $W = (W_H , W_L)$, I define equilibrium as follows.

Equilibrium for an output price vector P is a non-negative vector $[x(P) ; W(P)]$ that satisfies:

(a) given $[P, W(P)]$ the vector $x_i(P)$ solves (7) for all i ;

(b) market clearing:

$\sum_i h_i(P) \leq H$ with equality if $W_H(P) > 0$;

$\sum_i l_i(P) \leq L$ with equality if $W_L(P) > 0$.

Note that in the case of unemployment, the full wage must be zero.

A solution to the planner's problem (3) can be implemented by an ABS if we can find P such that the resulting equilibrium allocation coincide with the solution to (7).

Proposition 1: (a) Any solution to the planner's problem (3) can be implemented by an ABS by setting: $P_t = \alpha_t - 1$; (b) The resulting equilibrium full wages are given by: $W_H = \lambda_2$ and $W_L = \lambda_1$.

Note that when $\alpha_t = 1$, $P_t = 0$. In this case the private and social value of education are the same and there is no need to subsidize education.

Proof: Substituting $P_t = \alpha_t - 1$, $W_H = \lambda_2$ and $W_L = \lambda_1$ in (7) leads to:

(8) $\max \alpha_H h_i G_H(x_i) + \alpha_L l_i G_L(x_i) - \lambda_2 h_i - \lambda_1 l_i - m_i$; s.t. $x_i \geq 0$.

Since (3) can be written as:

(9) $\Sigma_i \{\max \alpha_H h_i G_H(x_i) + \alpha_L l_i G_L(x_i) - \lambda_2 h_i - \lambda_1 l_i - m_i$; s.t. $x_i \geq 0\}$,

it follows that the solution to (3) must also be a solution to (8) for all i. To show that the market clearing conditions are satisfied note that the solution to (3) is also the solution to (2) and must therefore satisfy the constraints in (9). Furthermore, the lagrangian multipliers are strictly positive only when the constraints are binding. Thus for

the suggested output price vector there exists an equilibrium and the equilibrium allocation coincides with the solution to (9). □

I now assume that the government rather than the students receives (pays) the equilibrium levels of the current wages w_t so that students do not have out of pocket expenses. I assume also that in equilibrium firms make zero profits. Under these assumptions:

Proposition 2: Government spending is not affected by the introduction of the ABS.

Proof: Note that zero profits imply:

(10) $$P_H h_i G_H(x_i) + P_L l_i G_L(x_i) - w_{Hi} h_i - w_{Li} l_i = m_i$$

Summing the left hand side of (10) over all i gives the government expenditure for the proposed scheme. Summing the right hand side of (10) is the total amount of money spent by the firms on material teachers' salaries, etc. Since Proposition 1 implies that $\sum_i m_i = M$, this is equal to the total amount spent by our planner on education. □

If in equilibrium firms make positive profits the government can collect these profits by imposing lump sum taxes on schools.

An ABS is necessary to solve the externality problem: Suppose that the government does not pay for outputs and pays only for the employment of

students: V_t dollars per type t student. I do not impose any restriction on current wages and therefore the firm faces the total wage cost of W - V ($\gtrless 0$) per student and choose non negative x_i , to solve:

(11) $\max h_i G_H(x_i) + l_i G_L(x_i) - (W_L - V_L)l_i - (W_H - V_H)h_i - m_i$.

Equilibrium for a non-negative voucher vector V = (V_L , V_H) is a non-negative vector [$\hat{x}(V)$, $\hat{W}(V)$] that satisfies:

(a) given [V , $\hat{W}(V)$] the vector [$\hat{x}_i(V)$] solves (11) ;

(b) market clearing[18]:

$\Sigma_i \hat{h}_i(V) \leq H$ with equality if $\hat{W}_H(V) > 0$;

$\Sigma_i \hat{l}_i(V) \leq L$ with equality if $\hat{W}_L(V) > 0$.

We can now state the effect of introducing a voucher system to a free market economy in which there is full employment: $\Sigma_i h_i(0) = H$ and $\Sigma_i l_i(0) = L$. In this case, the vouchers will end up in the pocket of the students as an increase in current wages without having an effect on the amount of education produced. Formally,

Proposition 3: If [x(0), W(0)] is an equilibrium for a price vector P = 0, then $\hat{x}(V) = x(0)$ and $\hat{W}(V) = W(0) + V$ is an equilibrium for a voucher vector V.

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[18] Student unemployment with $W_t > 0$, will create a downward pressure on students' current wage until either $W_t = 0$ or there is full employment. I therefore require market clearing whenever $W_t > 0$.

Proof: Choose $\hat{W}(V) = W(0) + V$. Given this choice and P = 0, the problem (7) is the same as (11). Since $x_i(0)$ is a solution to (7) it therefore follows that $\hat{x}_i(V) = x_i(0)$ is a solution to (11). Furthermore, the market clearing conditions: $\Sigma_i\ h_i(0) = H$; $\Sigma_i\ l_i(0) = L$ imply the market clearing conditions: $\Sigma_i\ \hat{h}_i(V) = H$; $\Sigma_i\ \hat{l}_i(V) = L$. □

Thus differential vouchers lead to the same outputs as the free market.

3. CONCLUDING REMARKS

In the absence of external effects an achievement based subsidy scheme (ABS) is the same as a system that use differential vouchers (DVS): In both schemes schools are paid on the basis of the number and mix of students. In the presence of external effects, an ABS pays schools for educational outputs in addition to payments for the employment of students. At a minimum, an ABS will achieve the same educational outputs as a centrally planned system, without coercion. More realistically, an ABS will improve productivity relative to a government run operation because parents choice acts as an effective monitoring device: Schools that do not use their inputs efficiently will eventually go bankrupt.

An ABS can work even when educational outputs are measured indirectly by the quantities of inputs used. An ABS creates the

incentives to spend the entire budget on educational inputs, while in a DVS some of the budget may be spent on stipends in kind.

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