

SEIGNIORAGE AND THE DESIRABILITY OF NATIONAL MONIES

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This paper studies the choice of currency and monetary policy in a sequential trading model. It is shown that a stable demand country with low inflation rate may lose when an unstable demand country adopts its currency. Efficiency requires national monies with differential and low inflation rates but this is not the Nash equilibrium outcome. We get "inflation bias" and "partial dollarization" under perfect commitment. On the positive side, the model may account for the following observations: (a) the US is cheap relative to the prediction of income-price regressions, (b) US liabilities are in dollar terms (c) foreigners pay a "liquidity premium" when holding US liabilities and (c) a common currency increases trade.

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## 1. INTRODUCTION

Should a country discourage currency substitution? Should it have its own national currency? These questions have occupied monetary economics and international finance for a long time. Fischer (1982) argued that countries choose to have national money to avoid paying inflation tax to a foreign government. He notes that the problem of choosing between national and foreign money is related to the choice between fixed and flexible exchange rates, the optimum currency area and the optimal inflation tax. Fischer also notes that transaction costs and the lack of the ability to commit to a monetary policy could make the use of foreign money optimal.

Here I reexamine these issues using the Prescott (1975) "hotels" model. Versions of the Prescott model have been studied by, among others, Bryant (1980), Dana (1998) and Deneckere and Peck (2005). Here I use a flexible price version of the Prescott model: The uncertain and sequential trade (UST) model in Eden (1990) and Lucas and Woodford (1993).

The discrete version of the UST model uses the survival probability function of demand (one minus the cumulative distribution) to define a sequence of Walrasian markets. The number of markets that open for trade is an increasing function of the realization of demand. Only one market opens if the lowest possible realization of demand occurs. Additional markets open if demand is higher. At each stage of the trading process sellers know that they can sell at the "current" market but are not sure whether additional markets will open or not. In equilibrium they are indifferent between selling at the "current" price

to betting on the event that additional markets will open and they will be able to sell at a higher price.

I use an open economy version of the monetary overlapping generations model in Eden (1994) and assume that the demand in the home country (the US) is stable relative to the demand in the rest of the world (Japan). I focus on an equilibrium in which US sellers accept dollars only and sell at a relatively cheap price while Japanese sellers accept both currencies.

The main results are as follows.

1. Demand Uncertainty is "bad" because it leads to price dispersion and less than full capacity utilization.
2. The US may suffer from trade with the unstable demand country.
3. The foreign country may benefit from full or partial dollarization because it increases the probability of buying at the cheaper price offered by US sellers.
4. As a result the US may collect some inflation tax from the foreign country.
5. We get "inflation bias" and "partial dollarization" as a Nash equilibrium outcome.
6. Efficiency requires national monies with differential and low inflation rates.

On the positive side we get:

7. The US is cheap relative to the prediction of income-price regressions.
8. US liabilities are in dollar terms and foreigners who hold them pay a "liquidity premium".
9. A common currency increases trade.

Additional predictions are:

10. The dollar rate of inflation is higher than the yen rate of inflation because it promises a higher chance of buying at the cheaper price offered by US sellers.
11. US sellers strictly prefer dollars because they do not suffer from taste shocks and are therefore more likely to use dollars for buying at the cheaper price.
12. The advantage of the dollar is an increasing function of the fraction of the dollar supply held by US agents. Partial dollarization in Japan reduces this fraction, reduces the advantage of the dollar and reduces the "liquidity premium" on the dollar.

#### THE MODEL

I consider a single good overlapping generations model. There is a single asset called money. There may be one or two currencies. There are two countries. The demand in the home country is stable and the demand in the foreign country is unstable. Otherwise the countries are symmetric.

I start with the case of autarky.

#### The home country under autarky:

A new generation is born each period. Individuals live for two-periods. They work in the first period of their life and consume in the second period. The representative agent is risk neutral and his utility

function is:  $-v(L) + c$ , where  $L$  is the amount of first period labor and  $c$  is the amount of second period consumption.

The cost function is quadratic:  $v(L) = (\frac{1}{2})L^2$ .

Buyer  $h$  (an old agent) starts period  $t$  with  $M_t^h$  dollars and gets in addition, a perfectly anticipated lump sum transfer of  $G_t$  dollars.

The average per-buyer post transfer amount of money is:

$M_t = G_t + (\frac{1}{N}) \sum_{h=1}^N M_t^h$  dollars. The deterministic rate of change in the money supply is:  $M_{t+1}/M_t = 1 + \mu$ .

The representative young agent born at time  $t$  takes the dollar prices of the consumption good ( $P_t, P_{t+1}$ ) and the dollar amount of the transfer payment ( $G_{t+1}$ ) as given and solves:

$$(1) \quad \max_L -v(L_t) + (P_t L_t + G_{t+1})/P_{t+1}$$

The first order condition for this problem is:

$$(2) \quad v'(L_t) = P_t/P_{t+1}$$

We may think of  $P_t/P_{t+1}$  as a real price or a real wage. The first order condition (2) says that the marginal cost must equal the real price.

Market clearing requires:

$$(3) \quad P_t L_t = M_t (1 + \mu)$$

I focus on an equilibrium in which inflation is constant and the level of the money supply is therefore proportional to the post transfer

money supply. I thus assume that there exists a normalized price  $p$  such that:

$$(4) \quad P_t = pM_t(1 + \mu)$$

Substituting (4) in the first order condition (2) leads to:

$$(5) \quad v'(L_t) = P_t/P_{t+1} = pM_t(1 + \mu) / pM_t(1 + \mu)^2 = 1/(1 + \mu).$$

Thus by varying  $\mu$  the monetary authorities can vary  $L$ .

With the risk of repetition I now set the problem in normalized magnitudes. This will become useful later when more complicated economies are considered.

In general, normalized magnitudes are nominal magnitudes divided by the post transfer money supply,  $M_t(1 + \mu)$ . A normalized dollar (ND) is  $M_t(1 + \mu)$  regular dollars. The price of consumption is  $P_t$  dollars per unit or  $p = P_t / M_t(1 + \mu)$  ND per unit. The purchasing power of a normalized dollar is:

$$(6) \quad Z = 1/p$$

A normalized dollar (ND) in the current period that is carried to the next period will become  $M_t(1 + \mu) / M_{t+1}(1 + \mu) = (1 + \mu)^{-1}$  next period's NDs. I use  $\omega = (1 + \mu)^{-1}$  to convert current NDs into next period's NDs. A worker (young agent) who sells a unit for  $p$  NDs will therefore have in the next period  $p\omega$  NDs. The nominal wage in terms of next period's NDs

is therefore  $p\omega$  and the real wage is  $p\omega Z$ . Using (6), we can write the real wage as:

$$(7) \quad w = \omega p Z = \omega.$$

In addition to the wage income the worker will get a transfer payment of  $\omega\mu$  in terms of next period's NDs. The worker's problem is therefore:

$$(8) \quad \max_L wL + (\omega\mu)Z - v(L)$$

The first order condition to (8) requires that the marginal labor cost is equal to the real wage:

$$(9) \quad v'(L) = \omega.$$

Market clearing requires:

$$(10) \quad pL = 1.$$

Note that (9) and (10) are (2) and (3) expressed in normalized magnitudes. The solution for (9) and (10) is:  $L = \omega$ ,  $p = 1/\omega$ .

The welfare of the representative agent in the steady state is:

$$(11) \quad W = (1/p) - v(L)$$

Table 1 computes the equilibrium solutions for different values of  $\omega$ .

Table 1: Autarky in the home country

$\omega = 1/(1 + \mu)$	$p = 1/\omega$	$L = \omega$	Welfare
1	1	1	0.5
0.90	1.111	0.90	0.495

Thus, as in other models inflation reduces output and welfare.

The foreign country under autarky:

The representative agent in the foreign country experiences taste shocks and discounts future consumption. His utility function is  $\theta\beta c - v(L)$  where  $\beta$  is a discount factor and  $\theta$  is a random variable that can take the realizations 1 with probability  $\pi$  and 0 otherwise. The realization of the taste shock is known only after production has been made and therefore output produced will be sold only when  $\theta = 1$ . It is assumed that when  $\theta = 0$ , the old generation transfers the money it holds to the young generation as a bequest but does not derive any utility from that bequest. This may be thought as accidental bequest. An alternative formulation may assume that agents derive utility from consumption (in both periods) and from bequest but the weight they assign to bequest is random. I do not think that the main results will change if this more general specification is employed.

Except for the taste shock and discounting the two countries are completely symmetric. Here I use the same symbols to denote the analogous variables in the foreign country. (Later I will use stars to

distinguish between foreign and domestic variables). The money supply grows at the rate of  $\mu$ . The price of a unit is  $p$  NDs. When  $\theta = 1$  a ND will buy  $1/p$  units of consumption. Otherwise, it will not be used to buy goods. We can therefore define the expected utility of a ND held by the buyer (before he knows the realization of the taste shock) by:

$$(12) \quad Z = \pi(1/p).$$

When  $\theta = 1$  the worker sells his output ( $L$ ) and gets  $(\omega p Z)L$  units of consumption. In addition he gets a transfer payment of  $\omega \mu$  (in terms of next period's normalized dollars) that will buy on average  $(\omega \mu)Z$  units. His consumption when  $\theta = 1$  is therefore:  $(\omega p Z)L + (\omega \mu)Z$  units. When  $\theta = 0$  the worker does not sell his output but receive a bequest of  $\omega$  in terms of next period's ND. In addition to the bequest he receives a transfer payment of  $\omega \mu$  ND and his expected consumption when  $\theta = 0$  is therefore:  $\omega(1 + \mu)Z = Z$ . The foreign worker maximization problem is therefore:

$$(13) \quad \max_L \pi \omega (pL + \mu) \beta Z + (1 - \pi) \beta Z - v(L).$$

Using (12) the expected real wage is:

$$(14) \quad w = \beta \pi \omega p Z = \beta \pi^2 \omega.$$

The first order condition for (13) requires that the marginal cost equals the expected real wage:

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$$(15) \quad v'(L) = L = \beta\pi^2\omega.$$

We require that if the market is open (that is if demand is strictly positive) then the market is cleared:

$$(16) \quad pL = 1.$$

Steady state welfare is measured by:

$$(17) \quad \text{Welfare} = \beta\pi(1/p) - (\frac{1}{2})L^2.$$

Table 2 calculates the equilibrium magnitudes for different values of  $\omega$ . As in all subsequent numerical examples it assumes  $\pi = 0.9$  and  $\beta = 1$ .

Table 2: Autarky in the foreign country ( $\pi = 0.9$ ;  $\beta = 1$ )

$\omega = 1/(1 + \mu)$	p	L	Welfare
$1/\pi$	$1/\pi$	$\pi$	0.405
1	1.235	0.81	0.401
0.90	1.372	0.729	0.390

Note that labor supply is lower in the foreign country. This is because the uncertainty about demand leads to less than full capacity utilization and to a lower expected real wage.

I now turn to discuss efficiency. To maximize steady-state welfare a planner will solve:

$$(18) \quad \max \pi\beta L - v(L)$$

The first order condition for this problem is:

$$(19) \quad v'(L) = \pi\beta.$$

Since in equilibrium  $v'(L) = \beta\pi^2\omega$  efficiency requires:

$$(20) \quad \beta\pi^2\omega = \pi\beta \text{ or } \omega = 1/\pi.$$

Thus, when  $\pi < 1$ , efficiency requires deflation. This result is similar to the well-known result by Friedman (1969) but here, as in other OG models, the optimal deflation rate does not depend on the discount factor. The argument for deflation is however, analogous to Friedman's argument. When there is zero inflation there is a difference between the social and the private value of a unit produced. The social value of a unit produced is  $\pi\beta$  because it will be consumed, by an old agent, with probability  $\pi$ . From the individual's point of view a unit produced yields utility only if he sells and only if he wants to consume. This joint event occurs with probability  $\pi^2$ . Therefore when inflation is zero a unit produced is worth to the individual only  $\beta\pi^2$  units of consumption. Deflation is required to correct for the difference between the social and the individual's point of view.

In what follows I assume  $\beta = 1$ .

### 3. A FULLY INTEGRATED WORLD ECONOMY

I now allow for trade between the two countries under the assumption of costless transportation and travel. We may think, for example, of the market for resorts. Buyers from all over the world may make reservations on the internet. Those who make early reservations may get relatively cheap vacations.

I start by assuming a single world currency: The dollar. At the beginning of each period the buyers in the home country get a transfer payment and as a result the world money supply grows at the rate of  $\mu$ . Foreigners do not get a transfer payment.

After receiving the transfer payment the representative buyer in the home country holds  $m$  normalized dollars and the representative buyer in the foreign country holds  $1 - m$  normalized dollar, where as before, a normalized dollar (ND) is the post transfer supply of dollars per buyer. To simplify, I consider here only a steady state in which  $m$  does not change over time.

Trade occurs sequentially. At the beginning of the period buyers who want to buy form a line. When  $\theta = 0$ , only buyers from the home country get in line. When  $\theta = 1$  buyers from both countries get in line. The place in the line is determined by a lottery that treats all buyers symmetrically.

Buyers arrive at the market place one by one according to their place in the line. They see all prices and choose to buy at the cheapest available price.

The amount of money that will arrive at the market place is  $m$  ND if only the home country buyers want to consume and  $1$  ND if all buyers

want to consume. We say that the first  $m$  NDs buy in the first market at the price of  $p_1$  ND per unit. If  $\theta = 1$  an additional amount of  $1 - m$  NDs will arrive, open the second market and buy at the price  $p_2$ .

When only one market opens the probability of buying at the first market price is unity. When two markets open the probability of buying at the first market price is  $m$  (= the fraction of dollars that will buy in the first market). The expected purchasing power of a normalized dollar if exactly  $s$  markets open ( $z_s$ ) is therefore:

$$(21) \quad z_1 = 1/p_1 \text{ and } z_2 = m/p_1 + (1-m)/p_2$$

The unconditional expected purchasing power of a normalized dollar is:

$$(22) \quad Z = (1 - \pi)z_1 + \pi z_2, \text{ for a home country buyer and}$$

$$Z^* = \pi z_2, \text{ for a foreign country buyer.}$$

Note that a buyer in the home country will buy regardless of the realization of  $\theta$  and therefore  $Z$  is a weighted average of  $z_1$  and  $z_2$ . A foreign buyer will buy only if  $\theta = 1$ . In this case two markets will open and therefore  $Z^*$  is a weighted average between zero and  $z_2$ .

Sellers (workers) take prices as given. They know that they can sell (in the first market) at the price  $p_1$  with probability  $1$  and (in the second market) at the price  $p_2$  with probability  $\pi$ . I use  $k_s$  to denote the supply of the home country seller to market  $s$ . The home country seller solves:

$$(23) \quad \max_{k_s} - v(k_1+k_2) \\ + (1 - \pi)\omega(p_1k_1 + \mu)Z + \pi\omega(p_1k_1 + p_2k_2 + \mu)Z.$$

The first term in (23) is the cost of producing  $k_1 + k_2$  units. The last two terms are the expected discounted consumption. When only one market opens the seller sells only  $k_1$  units and his revenues is  $p_1k_1$  ND. In addition he gets a transfer payment of  $\mu$  ND so his next period money balances are  $\omega(p_1k_1 + \mu)$  NDs. When both markets open the sellers revenues are:  $p_1k_1 + p_2k_2$  and his next period balances are:  $\omega(p_1k_1 + p_2k_2 + \mu)$ . To convert next period's balances to expected consumption we multiply by  $Z$ .

The representative young agent in the foreign country solves:

$$(24) \quad \max_{k_s^*} - v(k_1^* + k_2^*) \\ + (1 - \pi)\omega[p_1k_1^* + (1 - m)]Z^* + \pi\omega(p_1k_1^* + p_2k_2^*)Z^*$$

There are two differences between the home agent's problem (23) and the foreign agent's problem (24). The expected purchasing power function is different ( $Z^*$  instead of  $Z$ ) because the foreign agent may experience  $\theta = 0$ . And the foreign agent does not get a transfer payment from the government but may get a bequest.

It is convenient to use:  $L = k_1 + k_2$  for the supply of labor in the home country and  $L^* = k_1^* + k_2^*$  for the supply of labor in the foreign country. I focus on a steady state in which the post transfer balances held by the buyer in the home country do not change over time. This occurs when the home country seller supplies to the first market only ( $L = k_1$ ). In this case:

$$(25) \quad m = \omega(p_1L + \mu)$$

Introducing small transportation costs may be used to motivate this equilibrium choice. See Eden (2005).

I now turn to describe the first order condition for the problems (23) and (24). The expected real revenue per unit for a home country seller is  $\omega p_1 Z$  if the unit is supplied to the first market and  $\pi \omega p_2 Z$  if it is supplied to the second market. At the optimum the marginal cost ( $v'[L] = L$ ) must be equal to the expected real wage:

$$(26) \quad L = \omega p_1 Z = \pi \omega p_2 Z$$

Similarly for the foreign seller:

$$(27) \quad L^* = \omega p_1 Z^* = \pi \omega p_2 Z^*$$

In addition to the first order conditions (26) - (27), a steady state equilibrium requires the clearing of markets that open. Thus,

$$(28) \quad p_1(L + k_1^*) = m ; p_2(k_2^* = L^* - k_1^*) = 1 - m$$

Solving for a Steady state equilibrium:

The first order conditions (26)-(27) imply:

$$(29) \quad p_1 = \pi p_2.$$

We substitute (29) in (21) and (22) to get:

$$z_1 = 1/p_1 ; z_2 = m/p_1 + (1-m)\pi/p_1 \text{ and}$$

$$(30) \quad \begin{aligned} z(m) &= (1/p_1)[1 - \pi + \pi^2 + \pi(1 - \pi)m] \\ z^*(m) &= (1/p_1)[\pi^2 + \pi(1 - \pi)m] \end{aligned}$$

Substituting (30) in (26) and (27) yields:

$$(31) \quad L = \omega[1 - \pi + \pi^2 + \pi(1 - \pi)m] ; \quad L^* = \omega[\pi^2 + \pi(1 - \pi)m]$$

We now have six equations (25), (28), (29), (31) with six unknowns ( $L, L^*, k_1^*, p_1, p_2, m$ ).

Claim 1: There exists a unique steady state equilibrium for the single currency world.

The proof of this and all other claims is in the Appendix.

Table 3 illustrates the steady state solutions for two values of  $\mu$ . The last two columns are the steady state welfare in each country computed by:  $W = c - (1/2)L^2$ ,  $W^* = \pi(L + L^* - c_2) - (1/2)(L^*)^2$ , where

$$c = (1 - \pi)c_1 + \pi c_2, \quad c_1 = m/p_1 \text{ and } c_2 = m[(m/p_1 + (1-m)/p_2)].$$

Table 3: The fully integrated single currency world ( $\pi = 0.9$ )  $m = 0.501$ 

$\mu$	L	L*	W	W*
0	0.955	0.855	0.456	0.447
0.05	0.912	0.817	0.498	0.404

Comparing Tables 3 and 1 reveals that when  $\mu = 0$  both employment and welfare in the home country are higher under autarky. The reason is that buyers in the home country suffer from the price dispersion introduced by the foreigners and are sometimes forced to buy at the more expensive price. Another way of describing this result is by looking at the level of demand uncertainty. This went up from the point of view of the residents in the home country and went down from the point of view of the residents in the foreign country. Since demand uncertainty is "bad" in our model, trade improves welfare in the foreign country and reduces welfare in the home country. Table 3 also shows that imposing a moderate inflation tax works in the direction of compensating the home country.

#### Two currencies:

I now introduce an additional currency: the yen. It is assumed that US sellers accept dollars only. Japanese sellers are willing to accept both yens and dollars. This assumption will be justified in equilibrium. It will be shown that in equilibrium US sellers strictly prefer dollars to an equivalent amount of yens while Japanese sellers are indifferent between the two. To simplify, I assume that Japanese who

sell in the first market accept dollars only but they accept both currencies for goods sold in the second market.

As before, in the steady state US residents hold a fraction  $m$  of the dollar money supply and Japanese sellers hold a fraction  $1 - m$  of the dollar money supply. In addition Japanese sellers hold an amount of yens that is equivalent to a fraction  $\alpha$  of the dollar supply. Not surprisingly, an increase in  $\alpha$  leads Japanese sellers to substitute away from the dollar and this leads to an increase in  $m$ .

Since supplier to the first market accept dollars only, dollars promise a higher chance of buying in the first market. Therefore Japanese sellers will accept both currencies only if the rate of inflation of the yen is lower than the rate of inflation of the dollar. Since in the steady state  $m$  ND buy in the first market, the "liquidity advantage" of the dollar is higher the higher  $m$  and  $\alpha$  are. A higher  $\alpha$  requires therefore a higher "liquidity premium".

Taking the inflation of the dollar as given, the Japanese central bank determines  $\alpha$  by an appropriate choice of the yen inflation rate (a higher  $\alpha$  requires a lower yen inflation rate). It is convenient to treat  $\alpha$  as the policy choice variable and the yen inflation rate as an endogenous variable. An alternative that treats the yen inflation rate as the policy choice variable will make no difference for the analysis.

The pre-transfer supply of dollars at time  $t$  is  $M_t$  and the pre-transfer supply of yens is  $M_t^*$ . At the beginning of each period the home country buyer gets a lump sum transfer of  $\mu M_t$  dollars and the foreign country buyer gets a lump sum transfer of  $\mu^* M_t^*$  yens.

There is a foreign exchange market that opens before the realization of the taste shock  $\theta$ . The dollar price of yens (the exchange rate) is denoted by  $e_t$  and it satisfies:

$$(32) \quad e_t/e_{t-1} = (1 + \mu)/(1 + \mu^*).$$

For now I treat (32) as an assumption. Later I show that under (32) the foreign exchange market is cleared and no one trade in this market. The rate of change (32) implies:

$$(33) \quad e_t M_t^* (1 + \mu^*) / e_{t-1} M_t^* = 1 + \mu;$$

$$\alpha = e_t M_t^* (1 + \mu^*) / M_t (1 + \mu) = e_{t-1} M_t^* / M_t.$$

Thus the dollar value of the yens supply grows at the rate of  $\mu$  and is a constant fraction  $\alpha$  of the dollar supply.

It is assumed that dollar prices are proportional to the dollar supply and therefore grow at the rate of  $\mu$ . Thus,  $P_{st} = p_s M_t (1 + \mu)$  where  $P_{st}$  is the regular dollar price and  $p_s$  is the normalized dollar price. Since both currencies are accepted in the second market we require:

$$(34) \quad P_{2t}^* = P_{2t} / e_t = p_2 M_t (1 + \mu) / e_t,$$

where  $P_{2t}^*$  is the yen price of goods supplied to market 2. This leads to:

$$(35) \quad P_{2t}^* / P_{2t-1}^* = (1 + \mu) (e_{t-1} / e_t) = 1 + \mu^*$$

Thus yen prices grow at the rate of  $\mu^*$  and there is a normalized yen price  $p_2^*$  such that:

$$(36) \quad P_{2t}^* = p_2^* M_t^* (1 + \mu^*).$$

The expected purchasing power of a normalized dollar is given by (22). The expected purchasing power of a normalized yen is:

$$(37) \quad X^* = \pi(1/p_2^*).$$

To derive (37) note that a foreign buyer who wants to consume can use yens to buy goods in the second market only.

Since a foreign seller accepts both currencies in the second market we require:

$$(38) \quad \omega^* p_2^* X^* = \omega p_2 Z^*.$$

The left hand side of (38) is the expected consumption that the seller will get if he sells a unit in the second market for yens. To elaborate, note that the revenues from selling a unit for yens is  $p_2^*$  in terms of current normalized yens or  $\omega^* p_2^*$  in terms of next period's normalized yens, where  $\omega^* = 1/(1 + \mu^*)$ . To convert it to expected consumption we multiply by  $X^*$ . Similarly, the right hand side of (39) is the expected consumption that the seller will get if he sells a unit in the second market for dollars.

I now modify the foreign seller's problem (24) to allow for the yen transfer payment and to allow for the choice of the fraction of goods ( $0 \leq \lambda \leq 1$ ) sold for dollars. The foreign seller's problem is now:

$$(39) \quad \max_{k_1^*, 0 \leq \lambda \leq 1} -v(k_1^* + k_2^*) \\ + (1 - \pi)\{\omega[p_1 k_1^* + (1 - m)]Z^* + X^*\} \\ + \pi[\omega p_1 k_1^* Z^* + \lambda \omega p_2 k_2^* Z^* + (1 - \lambda)\omega^* p_2^* k_2^* X^* + \omega^* \mu^* X^*].$$

The first term in the maximization problem is the cost of production. We then have the utility (expected consumption) when only the first market opens and then the expected consumption when both markets open. Note that under (38) the value of the objective function does not depend on the choice of  $\lambda$  and therefore the seller is willing to accept both currencies for the goods supplied to the second market.

Steady state equilibrium requires (25), (38), the first order conditions (26) - (27) and the market clearing conditions

$$(40) \quad p_1(L + k_1^*) = m ; p_2(L^* - k_1^*) = 1 - m + \alpha,$$

where  $\alpha = e_t M_t^* (1 + \mu^*) / M_t (1 + \mu)$  is the supply of yens in terms of normalized dollars. I require in addition that  $0 \leq m, \alpha \leq 1$ .

#### Solving for a steady state equilibrium:

We use (29) and (30) to get:  $\omega p_2 Z^*(m) = \omega[\pi + (1 - \pi)m]$ .  
Substituting this and (37) in (38) leads to:

$$(41) \quad \omega^*(m) = \omega \left( 1 + \frac{(1-\pi)m}{\pi} \right)$$

Condition (41) delivers useful intuition. It implies that  $\omega(m)$  is an increasing function and  $\omega^* > \omega$ . As was said before, a lower rate of yen inflation is required to compensate yen holders for the inability of the yen to buy in the first market. An increase in  $m$  increases the advantage of the dollar because the probability that a dollar will buy in the first market when both markets open is  $m$ . Therefore when  $m$  goes up, a higher  $\omega^*$  is required to compete with the dollar.

We now show (the proof is in the Appendix) the following Proposition.

Proposition 1: There exists a unique steady state equilibrium for the two currencies world with the following properties:

- (a)  $L \geq L^*$  and  $\mu^* \leq \mu$  with the inequalities being strict when  $\pi < 1$ ;
- (b) An increase in  $\alpha$  leads to an increase in  $m$ , and an increase in labor supplies in both countries;
- (c) Japanese sellers are indifferent between accepting dollars to accepting the equivalent yen amount;
- (d) US sellers strictly prefer dollars to the equivalent yen amount;
- (e) When  $\mu = 0$  and  $\alpha = 1$  the steady state equilibrium allocation solves the following planner's problem:

$$(42) \quad \max c - v(L) \quad \text{s.t.} \quad c + c^* = L + L^* ; \quad \pi c^* - v(L^*) \geq x.$$

I now repeat the intuition. Foreign workers may not want to consume and therefore have less incentive to work. The yen inflation must be lower to compensate for the inability to buy in the first market. When  $\alpha$  increases foreign buyers substitute yens for dollars and  $1 - m$  goes down. As a result the dollar promises a higher chance of buying in the first market and a lower yen inflation is required to compete. The lower yen inflation leads to a higher expected real wage in Japan. The expected real wage in the US also went up as a result of the increase in  $m$  and the increase in the probability that US buyers will buy at the cheaper price. As a result labor supply in both countries go up. By construction the "liquidity premium" on the dollar is sufficient to make Japanese sellers accept both currencies. US sellers are willing to pay a higher "liquidity premium" because they buy in both states and the advantage of the dollar is larger in the low demand state (where the dollar guarantees buying in the first market).

Note that since the sellers are happy with their choice of currencies there is no need for a separate foreign exchange market.

The planner's problem (42) is that of maximizing the welfare in the home country subject to a worldwide resource constraint and a requirement that the level of welfare in the foreign country is given. Similar to (19), the first order conditions for this problem are:

$$(43) \quad v'(L) = 1 ; v'(L^*) = \pi.$$

Since  $\alpha = 1$  means autarky, (e) says that an efficient outcome can be obtained under autarky with an appropriate choice of monetary policies. The choice  $\mu = 0$  insures efficiency in the home country. It turns out

that to support  $\alpha = 1$  the foreign country must choose the efficient rate of inflation  $\omega^* = 1/\pi$  or  $\mu^* = \pi - 1$ .

Will we observe the efficient policy choices? To answer this question I consider a game between the two policy makers. The policy maker in the home country chooses  $\mu$  and the policy maker in the foreign country chooses  $\alpha$ . The payoff of each policy maker is the resulting welfare in his country. We ask whether the choice  $(\mu = 0, \alpha = 1)$  is a Nash equilibrium. For this purpose we must compute the optimal reaction functions. It turns out that this computation is rather difficult. I therefore turn to a numerical example that assumes  $\pi = 0.9$ .

Table 4 computes the equilibrium magnitudes for various  $\mu$  and  $\alpha$ . The first four rows assume  $\mu = 0$  and allow for four different values of  $\alpha$  ( $\alpha = 0, 0.1, 0.8, 1$ ). Note that  $\alpha > 0$  requires deflation of the yen ( $\mu^* < 0$ ). An increase in  $\alpha$  reduces welfare in the foreign country and increases welfare in the home country. This occurs because an increase in  $\alpha$  reduces the probability that Japanese buyers will buy at the cheaper price.

When  $\mu > 0$ , increasing  $\alpha$  (and holding  $\mu$  constant) has an ambiguous effect on welfare. It reduces the inflation tax paid by foreigners and the probability that a foreign buyer will buy at the cheaper price. The first inflation tax effect works to improve welfare in the foreign country and reduce welfare in the home country. The second term of trade effect works in the opposite direction. The inflation tax effect dominates when  $\mu$  is large. This can be seen in the last four rows of Table 4 when  $\mu = 0.1$ .

Increasing  $\mu$  (and holding  $\alpha$  constant) has also two effects on welfare. It increases the inflation tax collected from foreigners (when

$\alpha < 1$ ) and it creates a distortion in the labor supply choice. When  $\alpha$  is low (say  $\alpha = 0.1$ ) the inflation tax effect dominates and therefore an increase in  $\mu$  increases welfare in the home country and reduces welfare in the foreign country. When  $\alpha$  is large (close to unity), the distortion effect dominates and an increase in  $\mu$  reduces welfare in both countries.

Table 4: The fully integrated world economy with two currencies

( $\pi = 0.9$ )

$\mu$	$\alpha$	m	$\mu^*$	L	L*	W	W*
0	0	0.501		0.955	0.855	0.456	0.447
0	0.1	0.551	-0.06	0.960	0.860	0.460	0.443
0	0.8	0.900	-0.09	0.991	0.891	0.491	0.414
0	1	1	-0.1	1	0.9	0.5	0.405
0.05	0	0.526		0.912	0.817	0.498	0.404
0.05	0.1	0.574	-0.01	0.916	0.821	0.495	0.407
0.05	0.8	0.905	-0.05	0.944	0.849	0.495	0.407
0.05	1	1	-0.05	0.952	0.857	0.499	0.404
0.1	0	0.549		0.872	0.781	0.531	0.366
0.1	0.1	0.594	0.03	0.876	0.785	0.522	0.375
0.1	0.8	0.910	-0.00	0.902	0.811	0.497	0.400
0.1	1	1	-0.01	0.909	0.818	0.496	0.402

\* The first two columns are the choice of the two policy-makers:  $\mu$ ,  $\alpha$ . We then have the following endogenous variables: the fraction of the post transfer dollar supply held by the buyers in the home country (m), the equilibrium rate of change in the yen supply ( $\mu^*$ ), labor supply in the home country (L), labor supply in the foreign country L\* and welfare in the two countries (W, W\*).

Figure 1 uses Table 4 to construct optimal reaction functions. The function  $\mu(\alpha)$  is the home country's optimal choice of  $\mu$  for any given  $\alpha$ . This is a decreasing function. When  $\alpha = 1$ , it is optimal for the home country to set  $\mu = 0$ . When  $\alpha = 0$ , it is optimal for the home country to set large  $\mu$  ( $\mu > 0.2$ ). The function  $\alpha(\mu)$  is the foreign country's optimal choice of  $\alpha$  ( $0 \leq \alpha \leq 1$ ). When  $\mu = 0$ , the best policy of the foreign government is to choose  $\alpha = 0$ . When  $\mu = 0.1$ , the best policy of the foreign government is to choose  $\alpha = 1$ . A Nash equilibrium is obtained at  $(\bar{\mu} > 0, 0 < \bar{\alpha} < 1)$ .

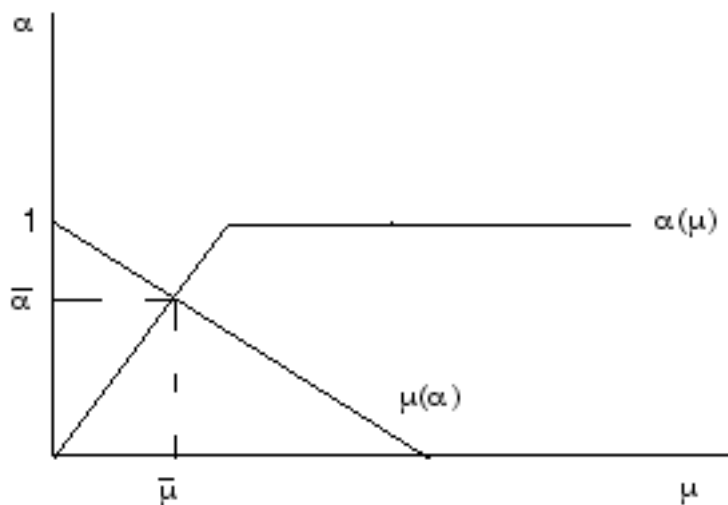


Figure 1: Equilibrium choice of  $\mu$  and  $\alpha$  in the fully integrated world economy

The foreign country trade-off is between the terms of trade (the probability of buying at the low price) and the inflation tax. When  $\mu = 0$  there is no inflation tax and therefore the foreign country focus on the terms of trade which are best when  $\alpha = 0$ . When  $\mu$  is positive a

higher  $\alpha$  means less inflation tax but also less favorable terms of trade. When  $\mu$  is sufficiently high (0.1 in our numerical example) the inflation tax dominates and the foreign country chooses  $\alpha = 1$ . The home country trade-off is between the inflation tax revenue received from foreigners and the labor supply distortion. When  $\alpha = 1$  it is not possible to impose inflation tax on foreigners and therefore the home country chooses to minimize distortion by setting  $\mu = 0$ . When  $\alpha$  is small the home country chooses some inflation tax revenue ( $\mu > 0$ ) at the price of creating a distortion in the labor market.

#### Net export in the steady state

Table 4 shows that when the dollar inflation is low and there is partial or full dollarization in Japan, the US suffers from trade. This is because of an adverse effect on the terms of trade: as a result of trade US buyers are sometimes forced to import at the high price.

Table 5 illustrates the adverse effect on the terms of trade by calculating measures of net exports for the home country. The real measure of net exports ( $x$ , measured in physical consumption units) varies with the states of nature while the nominal measure ( $p_1L - m$ ) does not. When  $\mu = 0$ , the nominal measure is always zero. But the real measure in the high demand ( $x_2$ ) is strictly positive and decreasing in  $\alpha$ . This occurs because in the high demand state, there is cross-hauling. The home country exports the good at the low price and pay the high price for some of its imports.

When  $\mu = 0.05$  the nominal measure of net export is negative and decreasing in absolute value with  $\alpha$ . This is the inflation tax imposed

on foreigners. But the real value of export in the high demand state is positive reflecting the terms of trade effect.

When  $\mu = 0.1$  the inflation tax effect dominates and all measures of net exports are negative. Note that net export are decreasing with  $\mu$  but are not monotonic in  $\alpha$ . Again this is because of the two effects of increasing  $\alpha$ : The inflation tax effect and the terms of trade effect.

Table 5 is consistent with the observation that the adoption of a common currency increases trade (Rose and Wincoop [2001]). To see this we may add the absolute value of exports from the home country in the two states:  $|x_1| + |x_2|$ . Holding  $\mu$  constant the Table reveals a negative correlation between  $\alpha$  and  $|x_1| + |x_2|$ .

Table 5: Net export for the home country ( $\pi = 0.9$ )

$\mu$	$\alpha$	$x_1 = L - c_1$	$x_2 = L - c_2$	Ex	$p_1L - m$
0	0	0	0.047	0.043	0
0	0.1	0	0.043	0.039	0
0	0.8	0	0.010	0.009	0
0	1	0	0	0	0
0.05	0	-0.043	0.002	-0.002	-0.024
0.05	0.1	-0.035	0.005	0.001	-0.021
0.05	0.8	-0.005	0.004	0.003	-0.005
0.05	1	0	0	0	0
0.1	0	-0.078	-0.035	-0.040	-0.045
0.1	0.1	-0.064	-0.026	-0.030	-0.041
0.1	0.8	-0.009	-0.001	-0.002	-0.009
0.1	1	0	0	0	0

\* The first two columns are the policy choices ( $\mu, \alpha$ ). We then have real net export in the low demand state ( $x_1 = L - c_1$ ) and real net export in the high demand state ( $x_2 = L - c_2$ ). The column that follows calculates the expected real net export:  $Ex = (1 - \pi)x_1 + \pi x_2$ . The last column is the normalized dollar measure of net export:  $p_1L - m$ .

To complete the picture, Table 6 calculates inflation tax revenues. The third column is the inflation tax collected by the foreign government from printing its own money. Then we have the inflation tax collected by the home government from its own residents and (in the fourth column) from foreign residents. An increase in  $\alpha$  reduces the inflation tax collected by the foreign government and reduces the

inflation tax paid by foreigners to the home country. An increase in  $\mu$  increases the inflation tax collected by the home country's government.

Table 6\*: Steady state inflation tax revenues

$\mu$	$\alpha$	$\omega^* \mu^* X^*$	$m\omega\mu Z$	$(1 - m)\omega\mu Z$
0	0	0	0	0
0	0.1	-0.086	0	0
0	0.8	-0.089	0	0
0	1	-0.09	0	0
0.05	0	-0.012	0.044	0.039
0.05	0.1	-0.018	0.044	0.032
0.05	0.8	-0.041	0.045	0.005
0.05	1	-0.045	0.045	0
0.1	0	0.050	0.083	0.068
0.1	0.1	0.040	0.082	0.056
0.1	0.8	-0.001	0.082	0.008
0.1	1	-0.007	0.083	0

\* The first two columns are the policy choice variables ( $\mu$ ,  $\alpha$ ). The third column is the inflation tax collected by the foreign government from printing its own money ( $\omega^* \mu^* X^*$ ). The fourth column is the inflation tax collected by the government in the home country from its own residents ( $m\omega\mu Z$ ) and the fourth column is the inflation tax collected by the government in the home country from foreign residents ( $(1-m)\omega\mu Z$ ).

Discussion:

We have considered the effect of partial and full dollarization from the point of view of the US and the foreign country. The welfare effect of dollarization is ambiguous. When inflation in the US is low a foreign country that adopts partial or full dollarization will "export" demand uncertainty to the US. In our model demand uncertainty is "bad" and therefore partial dollarization may increase welfare in the foreign country and reduce welfare in the US.

The model shed light on some related issues. It is shown that the optimal rate of inflation depends on the demand uncertainty parameter,  $\pi$ . From the private seller's point of view, labor pays if the current old generation experiences  $\theta = 1$  and he himself will experience  $\theta = 1$ . The probability that this joint event will happen is:  $\pi^2$ . But labor will have a social benefit if the current old generation experiences  $\theta = 1$  and regardless of the realization of  $\theta$  in the next period. Thus from the social point of view labor pays with probability  $\pi$ . An inflation rate of  $\pi - 1$  corrects for this discrepancy between the social and the private point of view and is therefore optimal.

It turns out that under portfolio autarky with  $\mu = 0$  and  $\alpha = 1$ , the allocation is Pareto efficient and each country's rate of inflation is optimal. However, when the US chooses the efficient zero inflation rate, it is optimal for the foreign country to "export" some of its demand uncertainty by choosing  $\alpha = 0$ . In a game in which countries want to maximize welfare in the steady state, we get a Nash equilibrium with  $\mu > 0$  and  $\alpha < 1$ . The Nash equilibrium steady state is not efficient.

The model has thus different implications from the literature that followed Friedman (1969) and Kydland and Prescott (1977). In Friedman (1969) the optimal rate of inflation depends only on the discount factor. Here it depends on the probability of "wanting to consume". Under perfect commitment the Nash equilibrium inflation rate is efficient in the Kydland and Prescott (1977) model. Here there is an "inflation bias" even under perfect commitment.

Assuming a broad definition of money, our model is consistent with the observation that in general the rate of return on US foreign assets has exceeded that on US foreign liabilities (Lane and Milesi-Ferretti [2005]) and that US assets are only partially linked to the dollar but US liabilities are almost entirely dollar-denominated (Tille [2003]).<sup>2</sup> In our model, US liabilities are in dollar terms and they earn a rate of return that is less than the rate of return on the foreign currency.

Foreigners are indifferent between holding dollars to holding yens but US sellers strictly prefer dollars. US sellers are willing to pay a larger premium for holding dollars because they consume also in the low demand state and in this state the "liquidity" advantage of the dollar is larger.

Our analysis explains deviations from PPP and is consistent with the observation that the US is cheap relative to the prediction of

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<sup>2</sup> The broad definition of money views any "over-priced" US government security as yielding inflation tax revenue. According to this view currency is just one example of an "over-priced" security. McGrattan and Prescott (2003) adopts this view arguing that short term US government bonds are over valued because they yield "liquidity".

income-price regressions. See Balassa (1964), Samuelson (1964) and Rogoff (1996).

Our model is also consistent with the observation that the adoption of a common currency increases trade (Rose and Wincoop [2001]). This correlation between the volume of trade and the stability of the exchange rate does not hold in all models. Recently, Bacchetta and van Wincoop (2000) used a cash-in-advance model to analyze the implications of a monetary union and demand uncertainty that arises as a result of money supply shocks. They find that exchange-rate stability is not necessarily associated with more trade. Devereux and Engel (2003) find that the implications of risk for foreign trade are highly sensitive to the choice of currency at which prices are set. In these models prices are rigid and firms satisfy demand. In the UST model used here prices can be changed during trade and sellers are not committed to satisfy demand (indeed, low price sellers maybe stocked out after the first stage of trade).

Our approach is related to the random matching models pioneered by Kiyotaki and Wright (1993). In both models uncertainty about trading opportunities plays a key role. In the random matching models agents are uncertain about whether they will meet someone that they can actually trade with. But whenever a meeting takes place it is bilateral. In the UST model sellers are also uncertain about the arrival of trading partners but whenever a meeting occurs there are a large number of agents on both sides of the market. As a result there is a difference between the assumed price determination mechanisms. In the random matching models prices are either fixed or are determined by bargaining

(as in Trejos and Wright [1995] and Shi [1995]). In the UST model prices clear markets that open.

At the end of their paper Kiyotaki and Wright (1993) consider an economy with two currencies: Red and Blue. The red one circulates with a higher probability and in equilibrium yields a lower rate of return. The high return asset is less acceptable or less liquid. Similarly, here the currency that promises the higher chance of buying at the low price yields a lower rate of return.

Matsuyama, Kiyotaki and Matsui (1993), Zhou (1997), Wright and Trejos (2001) and Liu and Shi (2005) use the random matching approach to study international currency. Wright and Trejos (2001) show that there can be three distinct type of equilibria, where in every case monies circulate locally, and either one, both, or neither circulate internationally. The assumed matching process plays a key role in determining the type of equilibria possible. For example, in the absence of inflation tax equilibrium with two national monies and no international money exists if the two countries are similar and the probability of meeting a foreigner is low. In our model the key difference between the two countries is in the probability of the taste shock. The example in Table 4 suggests that in the absence of inflation tax it is not possible to get equilibrium with national monies only (unless  $\pi = 1$  and the two countries are completely symmetric).

The difference in the taste shock probability limits the applicability of Gresham's law. In our model we get a steady state equilibrium with two monies that circulate internationally even when  $\mu \neq \mu^*$ . This is because dollars promise a higher chance of buying at the low price and as a result US sellers strictly prefer dollars to yens.

This is different from Karekan and Wallace (1981). In their model, there is no difference between the currencies. As a result there is a continuum of equilibria that differ in the nominal exchange rates. At any given equilibrium, the nominal exchange rate is constant over time and therefore the currency whose supply grows at a faster rate will represent an increasing fraction of the currency portfolio held by agents.

Is our analysis immune to the Lucas critique? As noted by Liu and Shi (2005) questions of optimal monetary policy in an international environment should not be analyzed in a model that specifies exogenously the currency that must be paid to each seller type. In our model we check that in equilibrium there is no incentive for agents to change their currency portfolio and in this sense our policy choice analysis is immune to the Lucas critique.

There is a lot more to do. One problem is that we have not investigate other steady state equilibria and out of steady state behavior. The second problem seems particularly relevant because policy makers are worried about the possibility that the rest of the world will stop accumulating US liabilities and that country like China may choose to use their accumulated US liabilities to buy goods.

## APPENDIX

Proof of Claim 1: We start by solving for the steady state level of  $m$ .

From (30) we get:  $p_1 L = m/\omega - \mu = m(1 + \mu) - \mu$ . Using (31) leads to:

$$(A1) \quad p_1 = [m(1 + \mu) - \mu] / \omega[1 - \pi + \pi^2 + \pi(1 - \pi)m].$$

Substituting  $p_1 L = m(1 + \mu) - \mu$  in the market clearing condition

$p_1(L + k_1^*) = m$  leads to:  $p_1 k_1^* = (1 - m)\mu$ . We now substitute this in the

market clearing condition  $p_1(L^* - k_1^*) = \pi(1 - m)$  to get:

$p_1 L^* = (\pi + \mu)(1 - m)$ . Using (31) leads to:

$$(A2) \quad p_1 = (\pi + \mu)(1 - m) / \omega[\pi^2 + \pi(1 - \pi)m].$$

Equating (A1) to (A2) leads to:

$$(A3) \quad \begin{aligned} & [\pi^2 + \pi(1 - \pi)m] / [1 - \pi + \pi^2 + \pi(1 - \pi)m] \\ & = (\pi + \mu)(1 - m) / (m + \mu m - \mu) \end{aligned}$$

Lemma: There exists a unique solution to (A3).

Proof: When  $L > 0$ , (25) implies  $m > \omega\mu$  and the right hand side of (A3) is positive. When  $m - \omega\mu$  is small the right hand side (RHS) is large and when  $m = 1$  the RHS is equal to zero. The left hand side (LHS) is  $\pi$  when  $m = 1$  and  $\pi^2/(1 - \pi + \pi^2)$  when  $m = 0$ . Furthermore, the RHS is decreasing in  $m$  and the LHS is increasing in  $m$ . Therefore there exists a unique solution  $\bar{m}$  in Figure A1.  $\square$

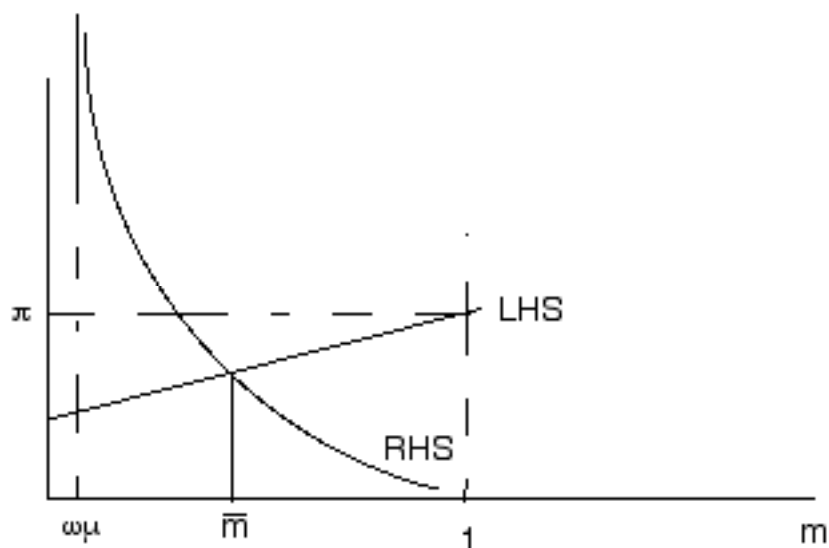


Figure A1

We now substitute the solution  $\bar{m}$  in (31) to solve for the steady state magnitudes  $L$  and  $L^*$ . We proceed by solving for  $p_1$  from (A2) and  $p_2 = \pi p_1$ . To solve for  $k_1^*$  we use the market clearing condition:

$$p_2(L^* - k_1^*) = 1 - m. \quad \square$$

Proof of Proposition 1: I start by solving for the steady state level of  $m$ . The only difference from the case  $\alpha = 0$  considered in Claim 1 is that now (40) replaces (28). Since we did not use (28) in the derivation of (31), this equation still hold. We did use (28) in the derivation of (A3). We now modify this equation.

As before we get:  $p_1 L = m(1 + \mu) - \mu$  from (25). Substituting this in the condition for clearing the first market,

$p_1(L + k_1^*) = m$ , leads to:  $p_1 k_1^* = (1 - m)\mu$ . We now substitute this in the second market clearing condition,  $p_1(L^* - k_1^*) = \pi(1 - m) + \pi\alpha$ , to get:

$p_1 L^* = (\pi + \mu)(1 - m) + \pi\alpha$ . Using this and (31) leads to:

$$(A2') \quad p_1 = [(\pi + \mu)(1 - m) + \pi\alpha] / \omega[\pi^2 + \pi(1 - \pi)m].$$

Equating (A1) to (A2') leads to:

$$(A3') \quad \begin{aligned} & [\pi^2 + \pi(1 - \pi)m] / [1 - \pi + \pi^2 + \pi(1 - \pi)m] \\ & = [(\pi + \mu)(1 - m) + \pi\alpha] / (m + \mu m - \mu) \end{aligned}$$

I now turn to show that there exists a unique solution to (A3'). The left hand side of (A3') is the same as the LHS of (A3) and therefore the LHS curve is unchanged. As before, when  $m - \omega\mu$  is small the right hand side (RHS) is large. But now when  $m = 1$  the RHS is equal to  $\pi\alpha$  rather than zero. Since  $0 \leq \alpha \leq 1$ ,  $\pi\alpha \leq \pi$  and there exists a unique solution,  $\bar{m}$  in Figure A2. We now use the solution  $\bar{m}$  to solve for the steady state magnitudes. We have thus shown existence and uniqueness.

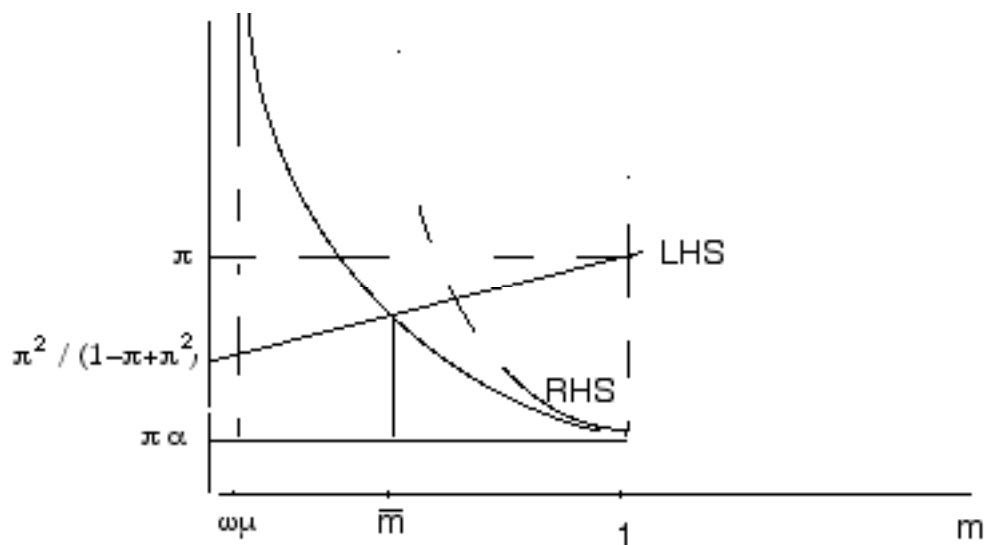


Figure A2

Part (a) follows directly from (31) and (41).

To show (b) note that an increase in  $\alpha$  increases the RHS of (A3') for all  $m$  and therefore shifts the RHS curve in Figure A2 to the right. This leads to an increase in the steady state level of  $m$ . Note also that (31) implies that  $L$  and  $L^*$  are monotonic in  $m$ . Therefore as  $\alpha$  grows and  $m$  grows and labor supplies in both countries grow.

To show (c) note that by construction the expected real wage for a Japanese seller who sell in the second market does not depend on the currency he accepts. We now show that this is also true for a Japanese seller who sells in the first market.

The equivalent yen amount is:  $p_1 M_t (1 + \mu) / e_t$ . The equivalent normalized yen amount is:

$$(A4) \quad p_1^* = p_1 M_t (1 + \mu) / e_t M_t^* (1 + \mu^*)$$

Substituting  $p_1 = \pi p_2$  in (A4) leads to:

$$(A5) \quad p_1^* = \pi p_2 M_t (1 + \mu) / e_t M_t^* (1 + \mu^*) = \pi p_2^*$$

The expected real wage for a Japanese seller who sells in the first market for dollars is:

$$(A6) \quad \omega p_1 (\pi z_2).$$

Substituting  $z_2 = m/p_1 + (1 - m)/p_2 = (1/p_1)[m + (1 - m)\pi]$  in (A6) leads to:

$$(A7) \quad \omega p_1(\pi z_2) = \omega\pi[m + (1 - m)\pi] = \omega\pi m + \omega\pi^2 - \omega\pi^2 m.$$

The expected real wage for a Japanese seller who sells in the first market for yens is:

$$(A8) \quad \omega^* p_1^* (\pi / p_2^*)$$

Substituting (A5) in (A8) leads to:

$$(A9) \quad \omega^* p_1^* (\pi / p_2^*) = \omega^* \pi^2$$

Substituting (42) in (A9) leads to:

$$(A10) \quad \omega^* \pi^2 = \omega\pi^2 \left(1 + \frac{(1 - \pi)m}{\pi}\right) = \omega\pi^2 + \omega\pi(1 - \pi)m = \omega\pi^2 + \omega\pi m - \omega\pi^2 m$$

Since (A10) is equal to (A7) the Japanese seller is indifferent between accepting dollars and the equivalent amount of yens in the first market.

I now turn to show (d) that the US seller strictly prefers dollars. A US seller that sells for dollars will have the expected real wage:

$$(A11) \quad \omega p_1 z = \omega[1 - \pi + \pi^2 + \pi(1 - \pi)m] = \omega - \omega\pi + \omega\pi^2 + \omega\pi m - \omega\pi^2 m$$

The expected real wage when selling in yens is:

$$(A12) \quad \omega^* p_1^* (1 / p_2^*)$$

Substituting (A5) in (A12) leads to:

$$(A13) \quad \omega^* p_1^* (1/p_2^*) = \omega^* \pi.$$

Substituting (37) in (A13) leads to:

$$(A14) \quad \omega^* \pi = \omega \pi \left( 1 + \frac{(1-\pi)m}{\pi} \right) = \omega \pi + \omega m - \omega \pi m$$

Subtracting (A14) from (A11) leads to:

$$(A15) \quad \omega(1-m)(1+\pi^2-2\pi)$$

When  $\pi < 0$ , this difference is decreasing in  $\pi$ . In the limit when  $\pi = 1$ , the difference (A15) is zero. It follows that for  $\pi < 1$ , the difference is strictly positive. We have thus shown that the US seller strictly prefers dollars to the equivalent yen amount. We have thus shown (c) and (d).

To show (e) note that  $m = 1$ ,  $\alpha = 1$  solve (A3'). Substituting  $m = 1$  in (31) leads to:  $L = 1$  and  $L^* = \pi$  that satisfy the first order condition (43).  $\square$

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