LIQUIDITY, EQUITY PREMIUM AND PARTICIPATION

Benjamin Eden*

Vanderbilt University

April 2008

I use price dispersion to model liquidity. Buyers may be rationed at the low price. An asset is more liquid if it is used relatively more in low price transactions and the probability that it will buy at the low price is relatively high. In equilibrium, government bonds are more liquid than stocks. Agents with a relatively stable demand are willing to pay a high "liquidity premium" for holding bonds and they specialize in bonds. The equity premium compensates agents with relatively unstable demand for the “illiquidity” of stocks and they hold both assets (stocks and bonds). The model offers a partial resolution of the equity premium puzzle and the participation puzzle.

JEL codes: E42, G12

Key Words: Liquidity, Sequential Trade, Equity premium puzzle, participation puzzle

---

* I would like to thank Maya Eden and Jacob Sagi for helpful comments and suggestions.

1. INTRODUCTION

The idea that liquidity is important for assets returns is not new. Recently, McGrattan and Prescott (2003) have argued that short term US government securities provide liquidity and are therefore overpriced. Amihud (2002) and Cochrane (2003) argued that some stocks are over-priced because they provide liquidity. For a survey of the recent literature see Amihud, Mendelson and Pederson (2005).

While many economists will agree that liquidity is important there is much less consensus on what exactly it means. The following are some dictionary definitions of liquidity: (a) The quality of being readily convertible into cash, (b) The ability or ease with which assets can be converted into cash, (c) The degree to which an asset or security can be sold or bought without affecting the asset's price, (d) Liquidity is characterized by a high level of trading activity.

The ease of converting an asset to cash may be measured by the average time it takes to do that. Houses for example are illiquid because they are typically on the market for a long time. But this measure is problematic: There are firms who will buy your house immediately at a price that is below the "market price".

Here I define liquidity in terms of a model with price dispersion. In the model there are two assets and four prices: Two prices in terms of asset 1 and two prices in terms of asset 2. A unit of asset i can thus buy relatively many units of the good at the low price or relatively few units of the good at the high price. Roughly speaking, an asset is liquid if it promises a high probability of making a buy at the low price. Although there is no asset market in the model, we may interpret the "ease" at which the asset is exchanged for low price goods as the "ease" of exchanging the asset for "cash" at the buying price. I elaborate on this interpretation later.

An important byproduct is a partial resolution of two puzzles: the equity premium puzzle and the participation puzzle. The equity premium puzzle originated with Mehra and Prescott (1985) who found a large difference between the average rate of return on equity and the average rate of return on Treasury bills. This paper led to a large literature surveyed by Kocherlakota (1996) who concluded that the equity premium is still a puzzle. Recently, Barro (2006) has argued for the importance of catastrophic events, previously suggested by Rietz (1988). Our explanation here borrows from Mankiw and Zeldes (1991), Attanasio, Bank and Tanner (2002) and Vissing-Jorgensen and Attanasio (2003) who observed that only a fraction of households actually hold stocks and the consumption of stockholders behave differently from the consumption of households who do not hold stocks. In particular, the standard deviation of the rate of consumption change is much higher (about 50% - 100%) for stockholders. They show that if we restrict the sample to stockholders (or households with the characteristics of stockholders) we do much better in terms of explaining the equity premium puzzle. The remaining question is why only a relatively small fraction of households hold stocks? This is the participation puzzle.

A related paper by Ait-Sahalia, Parker, and Yogo (2004) distinguishes between the consumption of basic goods and luxury goods. They find that the consumption of luxury goods is much more correlated with the stock market and argue that for the very rich, the equity premium is much less of a puzzle. But the equity premium is still a puzzle for the not so rich. They estimate a coefficient of risk aversion between 50 and 173 when using Personal Consumption Expenditures of non-durables and services. But when using their data on the consumption of luxury items the estimated coefficient is 7. These estimates leave the participation puzzle unanswered: Why only the "rich" hold stocks?

Here I address both puzzles. I use a flexible price version of Prescott (1975) "hotels" model: The Uncertain and Sequential Trade (UST) model in Eden (1990, 1994) and Lucas and Woodford (1993). I also use the ideas in Dana (1998) who considers a rigid price version of the model with heterogeneous agents.

Our approach borrows from the discussion of cashless economy in Woodford (2003). Woodford considers a technological advanced economy in which payments can be made in any asset. In Woodford's model money serves only as a unit of account and is priced correctly as an Arrow-Debreu security. Here we also allow payments in all assets but in our model, markets are incomplete.

Our approach is also related to the random matching models pioneered by Kiyotaki and Wright (1993). In both the random matching models and the UST model uncertainty about trading opportunities plays a key role. In the random matching models agents are uncertain about whether they will meet someone that they can actually trade with. But whenever a meeting takes place it is bilateral. In the UST model sellers are also uncertain about the arrival of trading partners but whenever a meeting occurs there is a large number of agents on both sides of the market. As a result there is a difference between the assumed price determination mechanisms. In the random matching models prices are either fixed or are determined by bargaining (as in Trejos and Wright [1995] and Shi [1995]). In the UST model prices clear markets that open.

2. OVERVIEW

I use an overlapping generations model. A new generation is born each period. Each individual lives for two periods, work in the first and if he wants to he consumes in the second. There are two types of agents in each generation: Agents who always want to consume (the stable demand type) and agents who want to consume only if they get a taste shock (the unstable demand type). Agents who do not want to consume leave their accumulated assets to their sons as accidental bequest.

I start with an exchange economy in which there are two types of government bonds (dollars and shekels) and then consider a production economy in which there are government bonds and stocks. Here I refer to the two assets as asset 1 and asset 2.

From the sellers' point of view purchasing power arrives sequentially. A minimum amount of asset i arrives with certainty and an additional amount of this asset arrives only if demand is high. Sellers expect to be able to sell the good for asset i at a low price if there is always a positive measure of active buyers who hold asset i. Sellers expect that they will be able to sell the good for asset i at a high price if in the high demand state there is a positive measure of active buyers who hold asset i and if demand is high.

It is useful to define four hypothetical markets: Two markets (indexed 1 and 3) for exchanging goods for asset 1 and two markets (indexed 2 and 4) for exchanging goods for asset 2. In market 1 (market 2) the good is exchanged for asset 1 (asset 2) at a low price. In market 3 (market 4) the good is exchanged for asset 1 (asset 2) at a high price. Market 1 (market 2) opens if there are always active buyers who hold asset 1 (asset 2). Market 3 (market 4) opens if in the state of high demand there are active buyers who hold asset 1 (asset 2) and demand is high. Thus the first asset 1 (asset 2) market opens if there are always active buyers who hold it and regardless of the realization of demand. The second asset 1 (asset 2) market opens if in the high demand state there are active buyers who hold it and demand is high. Sellers who supply to a given market can make a sale if the market opens.

It is assumed that individual agents cannot affect the prices in the four markets and cannot affect the probabilities that these markets will open. In equilibrium markets that open are cleared.

After the end of trade there are no buyers who wanted to exchange their asset for goods and could not do it. But there may be sellers who wanted but could not sell some of their supply. And there may be buyers who could not make a buy at the cheaper price.

An asset is liquid if sellers supply relatively more to its low price market and therefore the asset promises a high probability of making a buy at the low price.

Assuming that both assets are valued and excluding steady state equilibrium with knife-edge properties (to be described later) leads to a steady state equilibrium in which agents with stable demand choose to specialize in one asset and agents with unstable demand hold both assets. (We cannot have a steady state equilibrium in which the unstable demand type specializes in one asset and the stable demand type holds both assets. An equilibrium in which both types hold both assets has a knife-edge property).

In equilibrium the measured rate of return on the less liquid asset is higher than the measured rate of return on the more liquid asset. The equilibrium difference - the illiquidity premium – exactly compensates the unstable demand type and they hold both assets. The equilibrium illiquidity premium is not large enough to compensate the stable demand type and they specialize in the liquid asset.

3. AN EXCHANGE ECONOMY

I use an overlapping generations single good model. Two types of people are born each period. They live for two periods, work in the first and, if they want they consume in the second. A type h agent gets an endowment of λ^h units of the good. A type 1 that is born at time t will want to consume with probability 1 and his utility function is: $U^1(C_{t+1}) = C_{t+1}$, where C is his second period consumption. A type 2 wants to consume with probability π. His utility function is: $U^2(C_{t+1}) = \theta_{t+1} C_{t+1}$, where θ is a random variable that may take the realization $\theta = 1$ with probability π

and θ = 0 otherwise. Both types maximize expected utility. The number of agents from each type is normalized to 1 and a single agent represents each type.

There are two types of government bonds: dollars and shekels. A dollar promises $R = 1 + r$ dollars at the end of the period. A shekel promises $R^* = 1 + r^*$ shekels at the end of the period. Interest payments are financed by lump sum transfers at the end of the period: Seller h will receive g_s^{*h} shekels and g_s^h dollars if the current period state of demand is s, where $s = 1$ if in the current period $\theta = 0$ and $s = 2$ otherwise. In an international setting we may think of two governments: one that makes the dollar transfers and one that makes the shekel transfers. For our purpose a single government will do.

As in Abel (1985), it is assumed that if the type 2 old agents do not want to consume they leave their assets to type 2 young agents as accidental bequest. An alternative formulation may assume that agents derive utility from bequest as in Barro (1974), but the weight they assign to the utility of future generations is random. Unlike Diamond and Dybvig (1983) here it is important that the demand of type 2 agents is correlated with aggregate demand. An alternative formulation may assume a random tax instead of a taste shock and a household consisting to two types: Type 1 is responsible for basic consumption and always wants to buy. Type 2 is responsible for paying taxes and spends the money only if the government does not tax it (with probability π) on luxuries as in Ait-Sahalia, Parker, and Yogo (2004). Another possible scenario may assume that problems in the credit market (of the type we are currently experiencing) occasionally arise. When agents cannot get a loan they spend less on durables. I do not think that these more realistic alternative formulations will change the main results. I will stick with the simpler formulation.

The aggregate state of the economy is a description of the portfolios held by the old agents after the distribution of dividends and interest payments but before the beginning of trade in the goods market. The aggregate state is denoted by

y =(M^1, M^2, S^1, S^2), where M^h (S^h) is the amount of dollars (shekels) per type h old agent.

Trade occurs in a sequential manner. All agents who want to consume form an imaginary line. They then arrive at the market place one by one according to their place in the line. Upon arrival they see all prices, buy at the cheapest available offers and then disappear. Their place in line is determined by a lottery that treats all agents symmetrically. When θ = 0 only type 1 agents are in the line. When θ = 1 both types are in the line and in any segment of the line there is an equal number of agents from both types.

The young agents try to sell their endowments for one or both assets. From the young agents' point of view, demand arrives sequentially in batches. I distinguish between dollar demand and shekel demand. The minimum dollar demand is the amount held by type 1 old agents. Therefore from the sellers' point of view a first batch of M^1 dollars arrives with certainty. Additional dollar demand will arrive if demand is high and the second batch of buyers arrives: A second batch of M^2 dollars arrives if $\theta = 1$ with probability π. Similarly a first batch of S^1 shekels arrives with certainty and a second batch of S^2 shekels arrives if $\theta = 1$ with probability π.

The representative seller is a price-taker. He knows that if $M^1 > 0$, he can sell at the low price of $p_1(y)$ dollars to (buyers in) the first batch. He can sell at the higher price of $p_2(y)$ dollars to the second batch if it arrives and $M^2 > 0$. The seller can sell for $p_1^*(y)$ shekels to the first batch if $S^1 > 0$ and for $p_2^*(y)$ shekels to the second if it arrives and $S^2 > 0$. The seller chooses how much to sell to the first batch of buyers before he knows whether a second batch will arrive or not.

It may be helpful to think of sellers that put a price tag on each unit that they offer for sale. A price tag may specify the cost of the unit in terms of dollars or in terms of shekels (but not in term of both). Price tags may be different across units.

In the real world different markets use different currencies. In Israel you typically pay with shekels and in the US you typically pay with dollars. An American who exports to Israel may state his price in dollars or in shekels. In a recent paper Devereux and Engel (2003) distinguish between pricing in terms of the producer currency (PCP) and pricing in terms of the consumer currency (LCP). They show that the implications of risk for foreign trade are highly sensitive to the choice of currency at which prices are set. Here we let the seller choose the pricing currency.

It is convenient to assume four hypothetical markets: a market (indexed 1) for exchanging goods for dollars at the price $p_1(y)$ that opens if $M^1 > 0$; a market (indexed 2) for exchanging goods for shekels at the price $p_1^*(y)$ that opens if $S^1 > 0$; a market (indexed 3) for exchanging goods for dollars at the price $p_2(y)$ that opens if $\theta = 1$ and $M^2 > 0$ and a market (indexed 4) for exchanging goods for shekels at the price $p_2^*(y)$ that opens if $\theta = 1$ and $S^2 > 0$. There are thus dollar markets (indexed by odd numbers) and shekel markets (indexed by even numbers). A market opens if buyers with its payment currency do arrive. To simplify, I assume that buyers hold dollars and $M^1, M^2 > 0$. Under this assumption the first dollar market opens with certainty and the second dollar market opens with probability π. The seller knows that he can make a sale in any market that opens. Seller h supplies x_i^h units to market i. Figure 1 describes the sequence of events within the period.

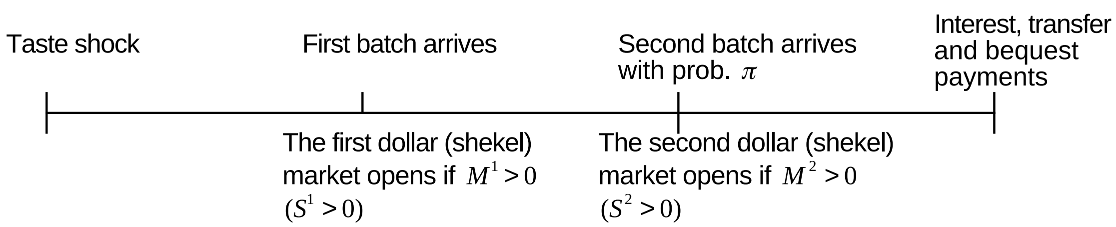

Figure 1

Sellers form expectations about the probability that each asset will be accepted in the next period as payment for goods. They assume that their own actions cannot affect these probabilities. (The state y is the average portfolio held by each buyer type and an individual agent is small and cannot affect the average portfolio). They expect that if they will not be able to buy in the first markets they will be able buy in the second markets.

Their expectations with respect to the probability of buying in the first markets are determined by the fraction of the dollar supply (shekel supply) that will be held in the next period by type 1 buyers. These fractions are: $m(y)$ and $n(y)$ respectively. In the state of high demand ($s=2$) the probability of buying in the first dollar (shekel) market is m (n). Agents expect that in the state of low demand ($s=1$) the first dollar (shekel) market will open if $m>0$ ($n>0$). Agents expect that in the low demand state they will be able to buy in any market that opens.

In state of demand s, exactly s dollar markets open and a dollar will buy on average $z_s(y)$ units of consumption where

(1) $$z_1(y) = \frac{1}{p_1(y)} \text{ if } m(y)>0 \text{ and } \frac{1}{p_2(y)} \text{ otherwise;}$$

$$z_2(y) = \frac{m(y)}{p_1(y)} + \frac{1-m(y)}{p_2(y)}.$$

Similarly, the expected purchasing power of a shekel is:

(2) $$z_1^*(y) = \frac{1}{p_1^*(y)} \text{ if } n(y)>0 \text{ and } \frac{1}{p_2^*(y)} \text{ otherwise;}$$

$$z_2^*(y) = \frac{n(y)}{p_1^*(y)} + \frac{1-n(y)}{p_2^*(y)}.$$

Sellers also form expectations about the average portfolios in the next period, $y_s(y)$, if the current demand state is s. I use

(3) $$\bar{z}_2 = \pi z_2(y_2) + (1-\pi) z_2(y_1)\ ;\ \bar{z}_2^* = \pi z_2^*(y_2) + (1-\pi) z_2^*(y_1)$$

to denote the expected value of next period purchasing power ($z_2,\ z_2^*$) before the state of current demand is known.

<u>Seller 2</u> can get p_2R dollars per unit in the second dollar market. If the second markets open the relevant deflator is $z_2(y_2)$ and therefore the expected real dollar price in market 3 is $p_2Rz_2(y_2)$ units of consumption. Seller 2 can also get p_1R dollars per unit in the first dollar market. Since in the first market he does not know the state of demand, the relevant deflator is $\bar{z}_2$ and the expected real dollar price in market 1 is $p_1R\bar{z}_2$. The expected real shekel prices can be calculated in a similar way.

Seller 2 will thus choose his supplies to the four markets by solving the following problem.

(4) $$\max_{x_i^2}\ p_1x_1^2R\bar{z}_2 + p_1^*x_2^2R^*\bar{z}_2^* + \pi\left\{p_2x_3^2Rz_2(y_2) + p_2^*x_4^2R^*z_2^*(y_2)\right\}$$

s.t. $x_1^2 + x_2^2 + x_3^2 + x_4^2 = \lambda^2$ and non-negativity constraints.

The first two terms in (4) are the expected consumption from supplying to the first markets. The rest is the expected consumption from supplying to the second markets. The constraint says that the total supplies to the four markets must equal the endowment.

I assume a solution in which the supplies to the dollar markets are strictly positive: ($x_1^2, x_3^2 > 0$). The first order conditions for a solution of this type are:

(5) $$p_1R\bar{z}_2 \geq p_1^*R^*\bar{z}_2^* \quad \text{with equality if } x_2^2 > 0;$$

(6) $$p_2Rz_2(y_2) \geq p_2^*R^*z_2^*(y_2) \quad \text{with equality if } x_4^2 > 0;$$

(7) $$p_1\bar{z}_2 = \pi p_2 z_2(y_2)$$

The first two conditions say that the expected real dollar price must be greater than the expected real shekel price. The last equality says that the expected real price in the first dollar market must equal the expected real price in the second dollar market.

<u>Seller 1</u> will always want to consume. He therefore uses the unconditional expected purchasing power of a dollar (Z) and a shekel (Z^*):

(8) $$Z(y) = \pi z_2(y) + (1-\pi)z_1(y)\ ;\ Z^*(y) = \pi\, z_2^*(y) + (1-\pi)\, z_1^*(y)$$

I use

(9) $$\bar{Z} = \pi Z(y_2) + (1-\pi)Z(y_1)\ ;\ \bar{Z}^* = \pi Z^*(y_2) + (1-\pi)Z^*(y_1)$$

to denote the expected value of next period Z (Z^*) before the state of current demand is known. Seller 1 solves:

(10) $$\max_{x_i^1}\ p_1 x_1^1 R\bar{Z} + p_1^* x_2^1 R^* \bar{Z}^* + \pi\left\{ p_2 x_3^1 R Z(y_2) + p_2^* x_4^1 R^* Z^*(y_2)\right\}$$
$$\text{s.t. } x_1^1 + x_2^1 + x_3^1 + x_4^1 = \lambda^1 \text{ and non-negativity constraints.}$$

The first order condition for a solution in which the supply is strictly positive for the dollar markets ($x_1^1, x_3^1 > 0$) are:

(11) $$p_1 R\bar{Z} \geq p_1^* R^* \bar{Z}^* \text{ with equality if } x_2^1 > 0;$$

(12) $$p_2 R Z(y_2) \geq p_2^* R^* Z^*(y_2) \text{ with equality if } x_4^1 > 0;$$

(13) $$p_1\bar{Z} = \pi p_2 Z(y_2)$$

The market clearing conditions are:

(14) $$p_1(x_1^1 + x_1^2) = M^1 \; ; \; p_1^*(x_2^1 + x_2^2) = S^1; \; p_2(x_3^1 + x_3^2) = M^2; \; p_2^*(x_4^1 + x_4^2) = S^2$$

Note that the supplies to the first markets must equal the minimum demand. Since only type 1 agents buy in the low demand state, I require that the value of the goods offered in the first dollar (shekel) market is equal to the amount of dollars (shekels) held by type 1 buyers. When demand is high some buyers from both types do not make a buy in the first market. These buyers hold a total of M^2 dollars and S^2 shekels. The purchasing power that could not make a buy in the first market buys in the second market. Thus, markets that open are cleared.

The first M^1 (S^1) dollars (shekels) that arrive will buy in the first dollar (shekel) market. Therefore, the probabilities m (n) are given by the fraction of dollars (shekels) that make a buy in the first dollar (shekel) market:

(15) $$m = \frac{M^1}{M^1 + M^2} \; ; \; n = \frac{S^1}{S^1 + S^2}.$$

This says that the probability of buying with dollars (shekels) in the first market in the high demand state is equal to the value of first market goods offered for dollars (shekels) relative to the dollar (shekel) supply. Note that it is possible to have $M^1 < S^1$ and $m > n$. What is important for liquidity is the use of the asset in its first market relative to its supply and not relative to the other asset.

The next period state is $y_s = (M_s^1, M_s^2, S_s^1, S_s^2)$ if the state of current demand is s where:

(16) $$M_1^h = Rp_1x_1^h + g_1^h \; ; \; M_2^h = Rp_1x_1^h + Rp_2x_3^h + g_2^h ;$$
$$S_1^h = R^* p_1^* x_2^h + g_1^{*h} \; ; \; S_2^h = R^* p_1^* x_2^h + R^* p_2^* x_4^h + g_2^{*h}$$

The first two equations in (16) say that the beginning of next period dollar balances held by type 1 sellers is equal to the government transfer plus the revenues in the first dollar market when only one market opens and the revenues in the two dollar markets when both open. The holding of shekel balances is calculated in a similar way.

Equilibrium is a policy choice $(g_1^1, g_2^1, g_1^2, g_2^2, R, R^*)$ and a vector of functions $(p_1, p_2, p_1^*, p_2^*, m, n, y_1, y_2, z_1, z_2, z_1^*, z_2^*, x_1^1, x_2^1, x_3^1, x_4^1, x_1^2, x_2^2, x_3^2, x_4^2)$ such that all functions are from y to the real line and satisfy the conditions in (1)-(16).

We may use the following classification of equilibrium solutions.
(a) No trade equilibrium (none of the four markets ever open); (b) Only one currency is used (shekel markets never open) and (c) Both currencies are used (at least one shekel market may open). I focus on the third alternative.

I focus on a steady state equilibrium in which the portfolio held by the old agents remains constant over time $y_1 = y_2 = y$ and equilibrium is a vector of scalars rather than a vector of functions. I now show the following Lemmas.

Lemma 1: If both currencies are used then in the steady state $p_1 = \pi p_2$ and $p_1^* = \pi p_2^*$.

Lemma 2: If a seller is willing to supply a strictly positive amount to one shekel market, then he is also willing to supply a strictly positive amount to the other shekel market.

The proofs of these and all other claims are in the Appendix.

I assume that in a steady state with $S^h > 0$ seller h supplies a strictly positive amount to at least one of the shekel markets:

(17) $$sign(S^1) = sign\left(\max(x_2^1, x_4^1)\right) \; ; \; sign(S^2) = sign\left(\max(x_2^2, x_4^2)\right)$$

where $sign(x) = 1$ if $x > 0$ and $sign(x) = 0$ if $x = 0$. This assumption rules out steady states in which the government gives shekel transfers to sellers who never sell for shekels.

Claim 1: A steady state equilibrium with $S^1 > 0$ and $S^2 > 0$ requires $m = n$.

Claim 1 says that agents must hold a symmetric portfolio if market 2 (the first shekel market) opens and market 4 (the second shekel market) opens in the high demand state. This is different from Karaken and Wallace (1981) who argued that in equilibrium in which the two currencies are perfect substitutes the agents' portfolios (and hence the exchange rate) are indeterminate.

The knife-edge property $m = n$ will not hold in equilibrium in which the portfolios y fluctuate around the steady state level and I therefore focus on asymmetric equilibrium in which agents may hold different portfolios. I start by ruling out the possibility that type 2 specializes in dollars.

Claim 2: It is not possible to have a steady state with $S^1 > 0$ and $S^2 = 0$.

Claim 2 says that it is not possible to have a steady state in which market 2 opens and market 4 never opens.

Claim 3: There exists a steady state in which $S^1 = 0$ and $S^2 > 0$.

Claim 3 says that in the asymmetric steady state, market 2 (the first shekel market) does not open and market 4 opens with probability π. Since in the asymmetric equilibrium the first shekel market does not open, dollars are more liquid than shekels. Type 1 agents specialize in dollars because the advantage of the dollar is larger in the low demand state and therefore they are willing to pay a higher liquidity premium for holding dollars.

In the asymmetric equilibrium we have: $1 > m > n = 0$. Using Lemma 1 and the definitions (1) and (2) leads to:

$$\text{(18)} \qquad \frac{R}{R^*} = \frac{p_2^* z_2^*}{p_2 z_2} = \frac{1}{p_2 z_2} = \frac{1}{m(p_2 / p_1) + (1-m)} = \frac{\pi}{m + (1-m)\pi} < 1.$$

Thus $R < R^*$ reflecting the need of type 2 agents to get a premium for holding the less liquid currency. Note that the interest rates levels are not determined in equilibrium, only their ratio (the illiquidity premium) is determined by (18).

I now turn to the description of the government transfer policy. It was assumed in (17) that the government does not give shekel transfers to sellers who never sell for shekels. This is all that is required for the above claims. But for the sense of concreteness I limit the government transfer so that the buyers' portfolio is equal to their first period revenues in the high demand state. This is done by lump sum taxes equal to the interest payments and a bequest tax: $g_1^1 = g_2^1 = -rM^1$; $g_1^2 = -rM^2 - Rp_1 x_1^2$; $g_2^2 = -rM^2$; $g_1^{*2} = g_2^{*2} = -r^* S^2$. To simplify, I assume that seller 1 supplies to the first market only and $x_2^1 = x_3^1 = x_4^1 = 0$. Under this assumption, the steady state portfolios $y = (M^1, S^1 = 0, M^2, S^2)$ are:

$$\text{(19)} \qquad M^1 = p_1 x_1^1;\ S^2 = p_2^* x_4^2\ ;\ M^2 = p_2 x_3^2 + p_1 x_1^2$$

Thus in the steady state the agents' portfolios are equal to their revenues in the high demand state. I now turn to the analysis of a productive economy with a privately backed asset.

4. EQUITY PREMIUM IN A PRODUCTION ECONOMY

I assume that the consumption preferences of the agents are the same as before but now agents are endowed with labor inputs rather than final goods: a type h young agent is endowed with λ^h units of labor. I add firms that last forever. These firms buy labor inputs from young agents and sell their output to old agents who want to consume.

There is one type of government bonds and there are stocks. Stocks are claims on the after tax profits of the firms. Stocks here play the role of shekels in the previous section. We may think of bonds as interest bearing money and stocks as dividend bearing money.

As before a unit of bond (a dollar) promises $R = 1 + r$ units of bonds at the end of the period. The aggregate state of the economy is the buyers' portfolios $y = (M^1, M^2, S^1, S^2)$, where here M^h is the amount of bonds and S^h is the amount of stocks per type h old agent.

The representative firm uses L units of labor to produce L^α units of consumption where $0 < \alpha < 1$ is the return to scale parameter. Production takes place before the realization of the taste shock θ. The price of labor is W dollars.

From the firm's point of view, purchasing power arrives sequentially in batches. A first batch arrives with probability 1 and a second batch arrives if $\theta = 1$ with probability π. Figure 2 describes the sequence of events within the period.

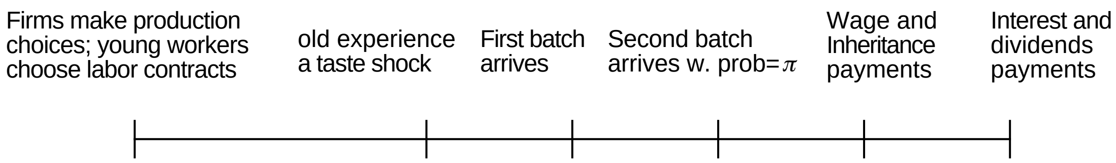

Figure 2

The firm is a price-taker. It knows that it can sell at the low price of $p_1(y)$ dollars if $M^1 > 0$. It can sell at the higher price of $p_2(y)$ dollars if $\theta = 1$ and $M^2 > 0$. Similarly, it can sell for $p_1^*(y)$ stocks per unit if $S^1 > 0$ and for $p_2^*(y)$ stocks per unit if $\theta = 1$ and $S^2 > 0$.

As before, it may be helpful to think of sellers that put a price tag on each unit that they offer for sale. Here a price tag may specify the cost of the unit in terms of bonds or in terms of stocks. It may also be useful to think of an alternative “cash-in-advance” scenario. We may assume that the firm issues chips (firm backed money) that are similar to chips issued by casinos. The firm promises to sell the good for chips at a constant price of 1 chip per unit. Buyers first go to the firm's "bank" and change their assets for chips. Buyers in the first batch get $1/p_1$ chips per unit of bonds and $1/p_1^*$ chips per stock. After the firm has observed that a certain amount of bonds and stocks were exchanged at these prices it changes the assets chip prices and starts exchanging the assets for the second batch prices: $1/p_2$ chips per unit of bonds and $1/p_2^*$ chips per stock. Thus asset chip prices fall when the second batch arrives. Although this cash-in-advance scenario makes the connection between the dictionary definitions of liquidity in the introduction and the definition here, it is nevertheless easier to write the model in terms of price tags on goods and I shall stick to that.

As before, it is convenient to assume four hypothetical markets. The price in the first bonds market (market 1) and the first stock market (market 2) are p_1(y) and p_1^*(y). The prices in the second bonds market (market 3) and the second stock market (market 4) are p_2(y) and p_2^*(y). Market 1 (market 2) opens if $M^1 > 0$ ($S^1 > 0$).

Market 3 (market 4) opens if $\theta = 1$ and $M^2 > 0$ ($S^2 > 0$). The firm supplies x_i units to market i.

To simplify, I assume that $M^1 > 0$ and $M^2 > 0$. Thus the first bonds (dollar) market always open and the second bonds market open if $\theta = 1$. I use the number of dollar markets that open to indicate the current demand state.

At the end of the period the government gives the firm a lump sum transfer of g_s units of bonds if exactly s bonds market open (where $s = 1$ if $\theta = 0$ and $s = 2$ if $\theta = 1$). There is also a lump sum tax on bequest of τ units of bonds.

There are incomplete markets for contingent claims. It is assumed that young agents and the firm can bet in these markets on the realization of the taste shock of the current old. The environment does not permit the old people to bet on their own shock: After the realization of their own taste shock they meet one firm (out of many identical firms) buy goods and disappear before the realization of the shock becomes public knowledge.

The price of a claim on a dollar that will be delivered (after the end of trade and before interest payments) if exactly s dollar markets open is Π_s. The price of a claim on the ownership of the firm that will be delivered (after the end of trade and before dividends payments) if exactly s dollar markets open is: Ω_s. The firm maximizes the present value of its profits by solving the following problem:

$$(20) \qquad \max_{x_i, L} \; \Pi_1 p_1 x_1 + \Pi_2 (p_1 x_1 + p_2 x_3) + \Omega_1 p_1^* x_2 + \Omega_2 (p_1^* x_2 + p_2^* x_4)$$

$$- WL + \Pi_1 g_1 + \Pi_2 g_2$$

$$\text{s.t. } x_1 + x_2 + x_3 + x_4 = L^\alpha, \text{ and } x_i \geq 0.$$

The first two terms in (20) are the present value of the revenues from goods offered for bonds: $p_1 x_1$ is the revenue from goods supplied to the first dollar market and $p_2 x_3$ is the contingent revenue from goods supplied to the second dollar market.

The revenues in state s are multiplied by the price of a dollar in state s, Π_s, so that the total revenues are in terms of "current" dollars (dollars delivered regardless of the state). The next two terms are the value of the revenues from goods offered for stocks: $p_1^* x_2$ is the stock revenue in the first stock market and $p_2^* x_4$ is the (contingent) stocks revenue in the second stock market. The next term is labor cost and the last two terms are the value of the contingent transfer payments. The constraint says that the amount allocated to the four markets must equal total output.

<u>The workers' problems</u>: Workers expect that they will buy in market i if they arrive when this market is open. They expect that they will arrive when market 1 (market 2) is open with probability $m(y)$ $(n[y])$ in (15). They also expect that if market 1 (market 2) is closed then market 2 (market 4) is open. The expected purchasing power of a unit of bonds ($z_s(y)$) and a unit of stocks ($z_s^*(y)$) are given by (1) and (2).

As in the previous section, workers take prices and the probabilities m and n as given. It is useful to think of dollars and stocks (rather than buyers) as arriving sequentially in batches: M^1 (S^1) dollars (stocks) arrives with certainty and M^2 (S^2) arrives with probability π. The first M^1 (S^1) dollars (stocks) open market 1 (market 2) if $M^1 > 0$ ($S^1 > 0$). The second M^2 (S^2) dollars (stocks) open market 3 (market 4) if $\theta = 1$ and $M^2 > 0$ ($S^2 > 0$). In the low demand state workers expect that their assets will arrive in the first batch and will be able to buy if the appropriate market opens. (Note again that a worker cannot open a market by his own action: If for example we are in a steady state with $S^1 = 0$, the first stock market does not open and an individual type 1 worker cannot open it by choosing to hold some stocks). In the high demand state the fraction of dollars that arrive in market 1 (market 2) is m (n). Workers therefore assume that this is the probability that their dollars (stocks) will arrive in the first batch of dollars (stocks).

A labor contract is a vector (a_1, a_2, b_1, b_2) where b_s is the amount of (before interest payment) dollars that will be delivered if exactly s dollar markets open and a_s is the amount of (before dividends payment) stocks that will be delivered if exactly s dollar markets open.

The firm let worker h choose a labor contract out of the following budget constraint:

(21) $$\Omega_1 a_1^h + \Omega_2 a_2^h + \Pi_1 b_1^h + \Pi_2 b_2^h = W\lambda^h .$$

Workers expect that the firm will distribute at the end of the period $D_s(y)$ units of bonds as dividends if exactly s dollar markets open. They also expect that the aggregate state in the next period will be $y_s = y_s(y)$ if exactly s dollar markets open.

Worker 2: A contract $(a_1^2, a_2^2, b_1^2, b_2^2)$ owned by a type 2 worker born at t promises: $a_s^2\left(z_2^*(y_s) + D_s(y)z_2(y_s)\right) + b_s^2 R(y) z_2(y_s)$ units of consumption (at $t+1$) if in the current period exactly s dollar markets open and $\theta_{t+1} = 1$. A type 2 worker chooses $(a_1^2, a_2^2, b_1^2, b_2^2)$ to maximize his expected consumption given that he wants to consume (θ_{t+1} = 1) by solving the following problem.

(22) $$\max_{a_s^2, b_s^2} \ \pi a_2^2\left[z_2^*(y_2) + D_2(y)z_2(y_2)\right] + (1-\pi)a_1^2\left[z_2^*(y_1) + D_1(y)z_2(y_1)\right] + \ \pi b_2^2 R z_2(y_2) \ + \ (1-\pi) b_1^2 R z_2(y_1) \quad \text{s.t. (21)} .$$

Worker 1: A type 1 agent always wants to consume. He therefore uses the unconditional expected purchasing power of a unit of bonds (Z) and a unit of stocks (Z^*) in (8). A type 1 worker will get on average: $a_s^1[Z^*(y_s) + D_s(y)Z(y_s)] + b_s^1 R(y) Z(y_s)$ units of consumption if exactly s dollar markets open in the current period. He will thus solve:

(23) $$\max_{a_s^1, b_s^1} \; \pi a_2^1[Z^*(y_2) + D_2(y)Z(y_2)] + (1-\pi)a_1^1[Z^*(y_1) + D_1(y)Z(y_1)] + \pi b_2^1 RZ(y_2) + (1-\pi)b_1^1 RZ(y_1) \quad \text{s.t. (21)} .$$

As before I focus on equilibrium in which both assets are valued. Since the firm is willing to accept bonds as payment for goods, I assume that stockholders are willing to hold bonds and they actually hold bonds in some form:

(24) $$\text{If } sign\left(\max(a_1^h, a_2^h)\right) = 1 \text{ then } sign\left(\max(b_1^h, b_2^h)\right) = 1$$

Note that this constraint introduces asymmetry between the two assets: It is possible to specialize in bonds but it is not possible to specialize in stocks.

Market clearing conditions: Equity market clearing requires that the claims on the ownership of the firm plus bequest of stocks must sum to 1:

(25) $$a_2^1 + a_2^2 = 1; \quad a_1^1 + a_1^2 + S^2 = 1.$$

The labor market clearing condition is:

(26) $$\text{L} = \lambda^1 + \lambda^2.$$

The goods market clearing conditions are:

(27) $$p_1 x_1 = M^1 \; ; \; p_1^* x_2 = S^1 \; ; \; p_2 x_3 = M^2 \; ; \; p_2^* x_4 = S^2 \; .$$

As in the previous section, the supplies to the first markets must equal the minimum demands. Since only type 1 agents buy in the low demand state, I require that the value of the goods offered in the first dollar market is equal to the amount of dollars held by type 1 buyers. Similarly, the stock value of the goods offered in the first stock market must be equal to the amount of stocks held by type 1 buyers. When demand is high some buyers from both types cannot make a buy in the first markets and the buyers who were rationed hold a total of S^2 stocks and M^2 bonds. The purchasing power that could not make a buy in the first markets will buy in the second markets.

The bonds market clearing conditions are:

(28) $$D_1(y) + R(y)(b_1^1 + b_1^2) = R(y)M^1 + g_1 ;$$
$$D_2(y) + R(y)(b_2^1 + b_2^2) = R(y)(M^1 + M^2) + g_2$$

The left hand side is the demand: The amount of bonds promised by the firm to stockholders and workers. The supply on the right hand side, is the bonds revenues and the transfer payment. Equation (28) pins down the expectations about dividends. In the low demand state the firm's bonds revenue is: M^1. The firm must pay the workers a total of $b_1^1 + b_1^2$ units of bonds. The amount of bonds the firm has after the end of trade is: M^1 - $(b_1^1 + b_1^2)$ units. These bonds earn interest and there is a transfer from the government. Therefore at the beginning of next period the firm will distribute $D_1(y) = R(y)\left[M^1 - (b_1^1 + b_1^2)\right] + g_1$ dollars if only one dollar market opens. When two dollar markets open the firm's bond revenues are $M^1 + M^2$ and it will distribute: $D_2(y) = R(y)\left[(M^1 + M^2) - (b_2^1 + b_2^2)\right] + g_2$.

I now turn to describe the next period state if exactly s dollar markets open in the current period: $y_s(y) = [M_s^1(y), M_s^2(y), S_s^1(y), S_s^2(y)]$. The next period holding of bonds is:

(29) $$M_1^1 = R\,b_1^1 + a_1^1 D_1;\; M_1^2 = R\,b_1^2 + (a_1^2 + S^2)D_1 + R\,M^2 - \tau$$
$$M_2^1 = Rb_2^1 + a_2^1 D_2;\; M_2^2 = Rb_2^2 + a_2^2 D_2$$

The next period holding of stocks is:

(30) $$S_1^1 = a_1^1;\; S_1^2 = a_1^2 + S^2;\; S_2^1 = a_2^1;\; S_2^2 = a_2^2.$$

Equilibrium is a policy choice (g1, g2, τ) and a vector of <u>functions</u> $(R, W, \Pi_1, \Pi_2, \Omega_1, \Omega_2, p_1, p_2, p_1^*, p_2^*, L, x_1, x_2, \psi_1, \psi_2, z_1, z_2, z_1^*, z_2^*, Z, Z^*, D_1, D_2, a_1^1, a_2^1, b_1^1, b_2^1, a_1^2, a_2^2, b_1^2, b_2^2, n, m, y_1, y_2)$ such that (a) all the functions are from the state $y = (M^1, M^2, S^1, S^2)$ to the real line; (b) given $(R, W, \Pi_1, \Pi_2, \Omega_1, \Omega_2, p_1, p_2, p_1^*, p_2^*)$, (L, x_i) solves the firm's problem (20); (c) given $(R, W, \Pi_1, \Pi_2, \Omega_1, \Omega_2, z_1, z_2, z_1^*, z_2^*, Z, Z^*, D_1, D_2)$, $(a_1^2, a_2^2, b_1^2, b_2^2)$ is a solution to (22) and $(a_1^1, a_2^1, b_1^1, b_2^1)$ is a solution to (23); (d) the conditions in (1), (2), (15), (20)-(30) are satisfied.

<u>Steady state</u>: To simplify, I focus on the case in which prices and the portfolios of buyers do not change over time and assume the following additional properties:

(31) $$D_1 = D_2 = D;\; y_1 = y_2 = y;\; \Pi_1 = 1 - \pi;\; \Pi_2 = \pi;$$
$$q = \Omega_1 + \Omega_2;\; \Omega_1 = (1-\pi)q;\; \Omega_2 = \pi q;\; p_s = q p_s^*$$

Note that in the steady state contingent claims are priced in an actuarially fair manner and the price of the firm is q. Under (31) stocks promise non-random return and in this sense they are similar to shekels discussed in the previous section. But unlike shekels (24) does not allow specialization in stocks. As with shekels agents will demand an illiquidity premium for holding stocks. The steady state assumption makes it easier to compute the illiquidity premium but is not necessary for the qualitative results because agents are risk-neutral in our model.

In the steady state, the firm's problem (20) can be written as:

(32) $$\max_{x_i,L}\ (1-\pi)p_1x_1 + \pi(p_1x_1 + p_2x_3) + (1-\pi)qp_1^*x_2 + \pi q(p_1^*x_2 + p_2^*x_4)$$
$$- WL + \Pi_1 g_1 + \Pi_2 g_2$$
$$\text{s.t. } x_1 + x_2 + x_3 + x_4 = L^{\alpha}, \text{ and } x_i \geq 0.$$

A solution in which the firm supplies a strictly positive amounts to the dollar markets ($x_1, x_3 > 0$) must satisfy the following first order conditions:

(33) $$p_1 = \pi p_2 = \frac{W}{\alpha L^{\alpha-1}}$$

where $W/\alpha L^{\alpha-1}$ is the marginal cost.

I now turn to the workers' problems in the steady state. I start with a claim that is similar to Lemma 2 in the previous section.

Claim 4: In the steady state: (a) workers who buy contingent claims on stocks are indifferent between alternative combinations of contingent claims on stocks (if the worker buys a_1^h>0 or a_2^h>0 then any choice of a_1^h and a_2^h that satisfies the budget constraint is a solution to the worker's problem); (b) workers who buy contingent claims on bonds are indifferent between alternative combinations of contingent claims on bonds.

The Claim does not rule out the possibility that workers may strictly prefer bonds to stocks but a worker that chooses to hold claims on the ownership of the firm, may own claims that will be delivered in one state only or in both states. Similarly, a worker may hold claims on bonds in one state only or in both states.

To simplify the description of the steady state, I assume that type 2 workers buy claims for delivery in state 2 only and type 1 workers buy the same amount of claims in both state of demand. I also assume that workers choose to hold a strictly positive amount of a claim on bonds. Thus,

(34) $$b_2^2 = b > 0;\ a_2^2 = a;\ b_1^2 = a_1^2 = 0;\ b_1^1 = b_2^1 = B > 0,\ a_1^1 = a_2^1 = A$$

Using this simplified notation, we can write the budget constraints (21) as:

(35) $$Aq + B = W\lambda^1 \text{ for type 1 and } aq + b = W\lambda^2/\pi \text{ for type 2.}$$

I now turn to specify the type specific rates of return in the steady state. It is easier to work with the purchasing power of a dollar worth of stocks: $v_s = z_s^* / q$. Using $p_s = qp_s^*$ yields:

(36) $$v_1 = \frac{1}{p_1} \text{ if } n > 0 \text{ and } \frac{1}{p_2} \text{ otherwise; } v_2 = \frac{n}{p_1} + \frac{1-n}{p_2};\ V = \pi v_2 + (1-\pi)v_1$$

The expected rates of return depends on the type because the deflators are type dependent. I use R_e^h (R_b^h) to denote the expected real return per dollar invested in equity (bonds) to a type h worker:

(37) $$R_e^2 = \pi \frac{qv_2 + Dz_2}{q};\ R_e^1 = \frac{qV + DZ}{q};\ R_b^2 = \pi R z_2;\ R_b^1 = RZ$$

To derive (37) note that the ownership of the firm promises on average $qv_2 + Dz_2$ units of consumption to a type 2 agent who wants to consume and $qV + DZ$ units of consumption to a type 1 agent.

Since (24) rules out specialization in stocks, we must have:

(38) $$\frac{R_e^1}{R_b^1} \le \frac{R_e^2}{R_b^2} = 1; \text{ or } 1 = \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2}$$

Under the first alternative type 1 prefers bonds and type 2 is indifferent between the two assets. Under the second alternative type 1 is indifferent between the two assets and type 2 prefers bonds.

Using (37) and (38) leads to:

(39) $$q = \frac{Dz_2}{Rz_2 - v_2} \text{ if } \frac{R_e^1}{R_b^1} \le \frac{R_e^2}{R_b^2} = 1 \text{ and } q = \frac{DZ}{RZ - V} \text{ if } 1 = \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2}.$$

Thus, the value of the firm is determined from the point of view of the stockholders. In the special case $\frac{R_e^1}{R_b^1} = \frac{R_e^2}{R_b^2} = 1$, $q = \frac{Dz_2}{Rz_2 - v_2} = \frac{DZ}{RZ - V}$. As will be shown later, this special case requires: v_2 / z_2 = V/Z and $m = n$.

To rule out some equilibria with a knife-edge property, I assume that workers specialize in bonds only when bonds earn on average a higher rate of return:

(40) $$A = 0 \text{ if } \frac{R_e^1}{R_b^1} < \frac{R_e^2}{R_b^2} = 1 \text{ and } A > 0 \text{ otherwise;}$$
$$a = 0 \text{ if } 1 = \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2} \text{ and } a > 0 \text{ otherwise.}$$

We can now define steady-state equilibrium as a policy choice (g_1, g_2) and a vector of scalars $(R, W, q, p_1, p_2, L, x_1, x_2, \psi_1, \psi_2, z_1, z_2, Z, v_1, v_2, V, D, R_e^1, R_b^1, R_e^2, R_b^2, a, A, b, B, y, n, m)$ that satisfies: $x_1 + x_2 = L^{\alpha}$, (1), (8), (15), (26), (27), (33)-(40) and the (equity, goods and bonds) market clearing conditions:

(41) $$a + A = 1;$$

(42) $$p_1 x_1 = M^1 \; ; \; p_1 x_2 = qS^1 \; ; \; p_2 x_3 = M^2; \; p_2 x_4 = qS^2$$

(43) $$D = R(M^1 - B) + g_1 = R(M^1 + M^2 - B - b) + g_2$$

I assume that the portfolios in the steady state are equal to what is promised by the labor contract in the high demand state:

(44) $$M^1 = AD + BR; \; M^2 = aD + bR; \; S^1 = A; \; S^2 = a.$$

Note that type 2 old agent holds S^2 stocks and the young type 2 worker chooses $a = S^2$. Therefore, the fraction of the firm owned by type 2 agents does not change over time: They may get it as a wage payment when $\theta = 1$ or as a bequest when $\theta = 0$. Government transfers and a bequest tax are used to finance the interest payments. Conditions (41), (43) and (44) imply $g_2 = -(R-1)M = -rM$ and $g_1 = -rM^1 + aD = -rM + rM^2 + aD$, where $M = M^1 + M^2$. We may define $\tau = rM^2 + aD$ as a bequest tax and write the government's "budget constraints" as:

(45) $$g_2 = -rM; \; g_1 = -rM + \tau$$

Thus in the high demand state the government finances the interest payments by a lump sum tax on the firm. When demand is low, interest payments are financed by both a tax on the firm and a tax on bequest.

I now show that Claim 1 and Claim 2 hold also in the productive economy. I repeat these Claims for convenience.

Proposition 1: (a) A steady state with $S^1 > 0$ and $S^2 > 0$ requires $m = n$, (b) It is not possible to have a steady state with $S^1 > 0$ and $S^2 = 0$.

I now turn to solve for a steady state equilibrium in which type 1 specializes in bonds.

Solution: It is easier to think of D as the policy choice variable and assume that (g_1, g_2) are chosen to satisfy (43).

I normalize by assuming $\lambda^1 + \lambda^2 = 1$ and W = 1. Substituting L = 1 in (33) yields:

(46) $$p_1 = \frac{1}{\alpha}\ ;\ p_2 = \frac{1}{\pi\alpha}$$

I use $n = 0$, (1), (36), (39) and (46) to get:

(47) $$q = \frac{Dz_2}{Rz_2 - v_2} = \frac{D(m + (1 - m)\pi)}{R[(m + (1 - m)\pi)] - \pi}$$

I use (15), $A = 0$, $a = 1$, (35) and (43) to get:

(48) $$m = \frac{BR}{BR + D + bR} = \frac{R\lambda^1}{R\lambda^1 + D + R[(R\lambda^2/\pi) - q]}$$

I use (40) and (42) to get:

$$(49) \qquad x_1 + x_2 = \frac{BR}{p_1} + \frac{q + D + bR}{p_2} = \alpha R + \pi\alpha(D - qr) = 1$$

We now have three equations, (47)-(49), and three unknowns: q, m, R. I assume that D is not too large[2] and show the following Proposition.

Proposition 2: (a) There exists a unique solution to (47)-(49) with the added constraints: $q > 0$, $R > 0$ and $0 < m < 1$; (b) An increase in D leads to an increase in the fraction of bonds held by type 1 agents (m); (c) In the proposed steady state, stocks earn a premium: $\frac{D}{q} > R - 1 = r$.

The intuition for (c) is as follows. Since type 1 specializes in bonds, market 2 never opens and stocks can buy only at a relatively high price. Therefore stocks will be held only if they promise a higher rate of return on average. The premium on stocks is enough to compensate type 2 agents for their relative illiquidity but it is too small for compensating type 1 agents who choose to specialize in bonds.

To get a sense about the working of the model, I now turn to numerical examples.

The special case $D = 0$: In this case (47) implies $q = 0$ and only government bonds are valued. Substituting $D = q = 0$ in (50) leads to $R = 1/\alpha$. To build intuition it may be useful to consider the case in which there is no demand uncertainty, $\pi = 1$ and only one market opens. The price of goods is the first market price in (46): $1/\alpha \geq 1$. But the

---

[2] The exact condition is: $D < \min\left\{\frac{(1/\alpha) - 1}{\pi}, \frac{\lambda^2}{\pi^2}\right\}$.

total wage payment is 1. To clear the good market the government must transfer money (the profits that it has taken as taxes) to the buyers. Here the transfer is in the form of interest payments. When $\alpha = 0.96$ the interest rate should be about 4%.

<u>The case of $R = 1$</u>: This is a case of minimal government intervention. Substituting $R = 1$ in (43) and (45) leads to: $g_2 = 0$ and $g_1 = \tau = D$. Thus the government intervenes only in the low demand state: It transfers the bequest tax revenues to the firm that is owned by the same people that pay the bequest tax. Substituting $R = 1$ in (49) leads to $D = \frac{1-\alpha}{\pi\alpha}$. When $\alpha = 0.96$ and $\pi = 0.9$, we get: $D = 0.046$.

I now turn to discuss changes in the "policy variable" D.

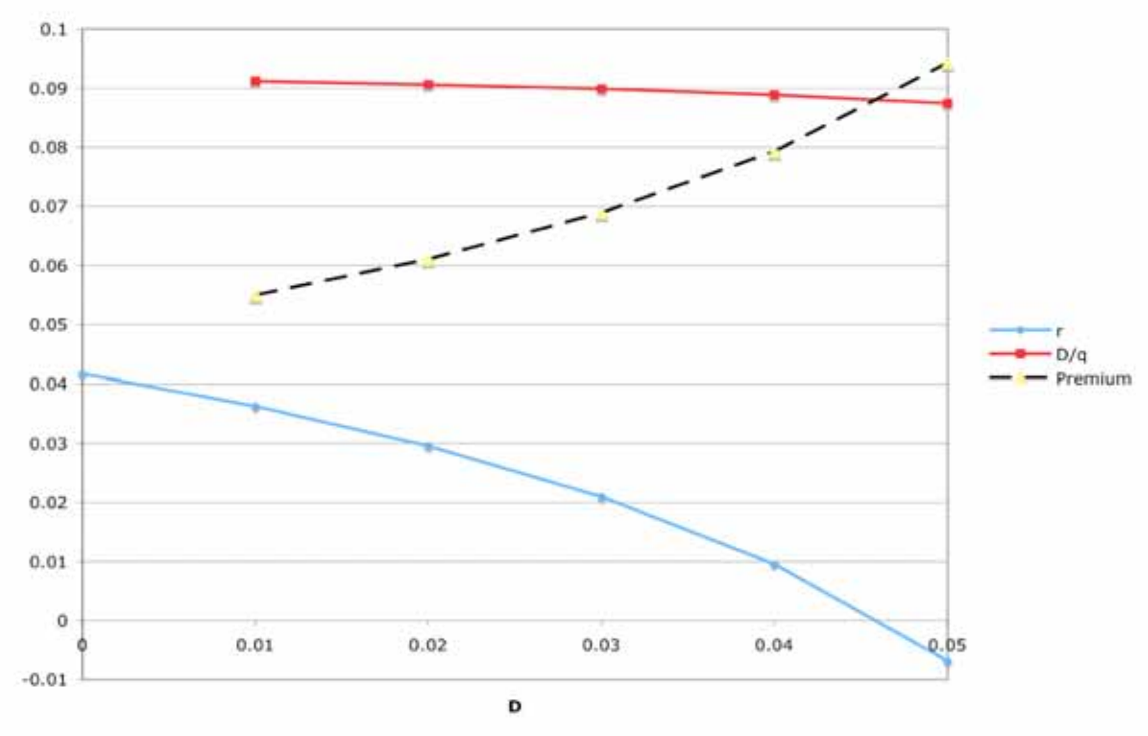

Figure 3: r, D/q and $D/q - r$ as a function of D

$(\alpha = 0.96, \pi = 0.9,\ \lambda^1 = \lambda^2 = 1/2)$

<u>Changing D</u>: Figure 3 describes the equilibrium levels of the interest rate (r), the rate of return on equity (D/q) and the difference between them (the equity premium) as a function of D. I use the parameters: $\lambda^1 = \lambda^2 = 1/2$, $\pi = 0.9$ and $\alpha = 0.96$. As we can see the interest rate decreases with D but the return on equity hardly changes with D. As a result the equity premium increases with D. The negative relationship between the interest on bonds and D can be seen with the help of the "government budget constraint", (45). Starting from $D = 0$, an increase in D implies less taxes on the firm and as a result the interest payment transfer is lower.

Figure 4 describes total bond holdings before the beginning of trade ($M = M^1 + M^2$), the amount of bonds held by type 1 buyer (M^1), the amount of bonds held by type 2 buyer (M^2) and the fraction held by type 1 ($m = M^1/M$). As was said in Proposition 2, an increase in D leads to an increase in m. Here this occurs mainly because of the reduction in M^2.

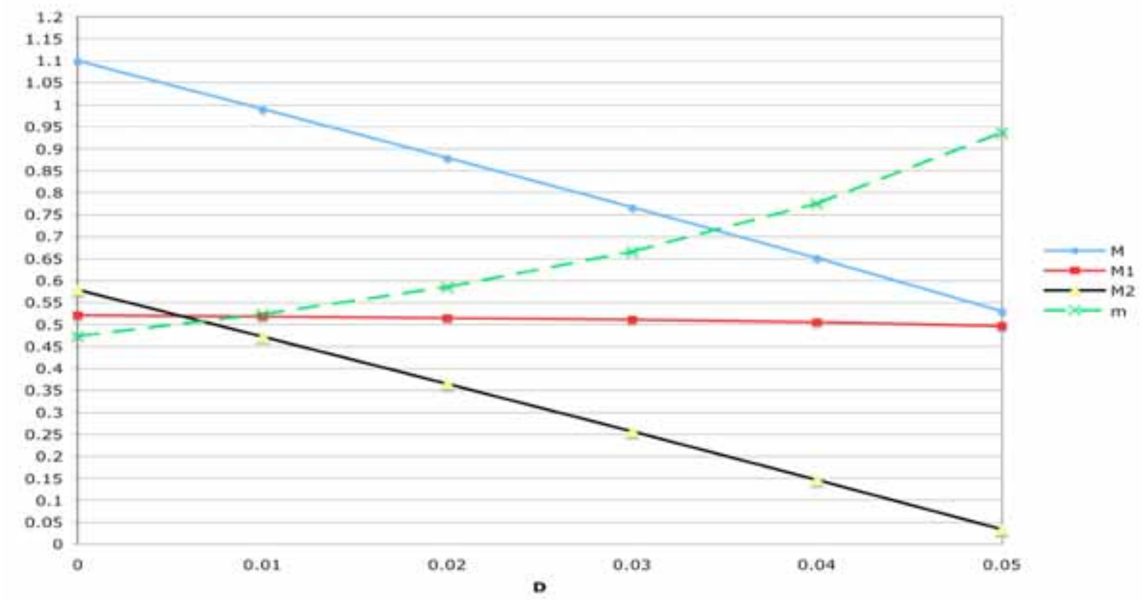

Figure 4: M, M^1, M^2, m as a function of D ($\alpha = 0.96, \pi = 0.9$, $\lambda^1 = \lambda^2 = 1/2$)

Figure 5 describes M, q and $M + q$ as a function of D. An increase in D leads to an increase in q. It also leads to a decrease in M by almost the same amount and as a result $M+q$ does not change. This is analogous to the case of currency substitution discussed in the literature. Note that since prices do not depend on D in our model, the measure of money that is consistent with the quantity theory is $M+q$.

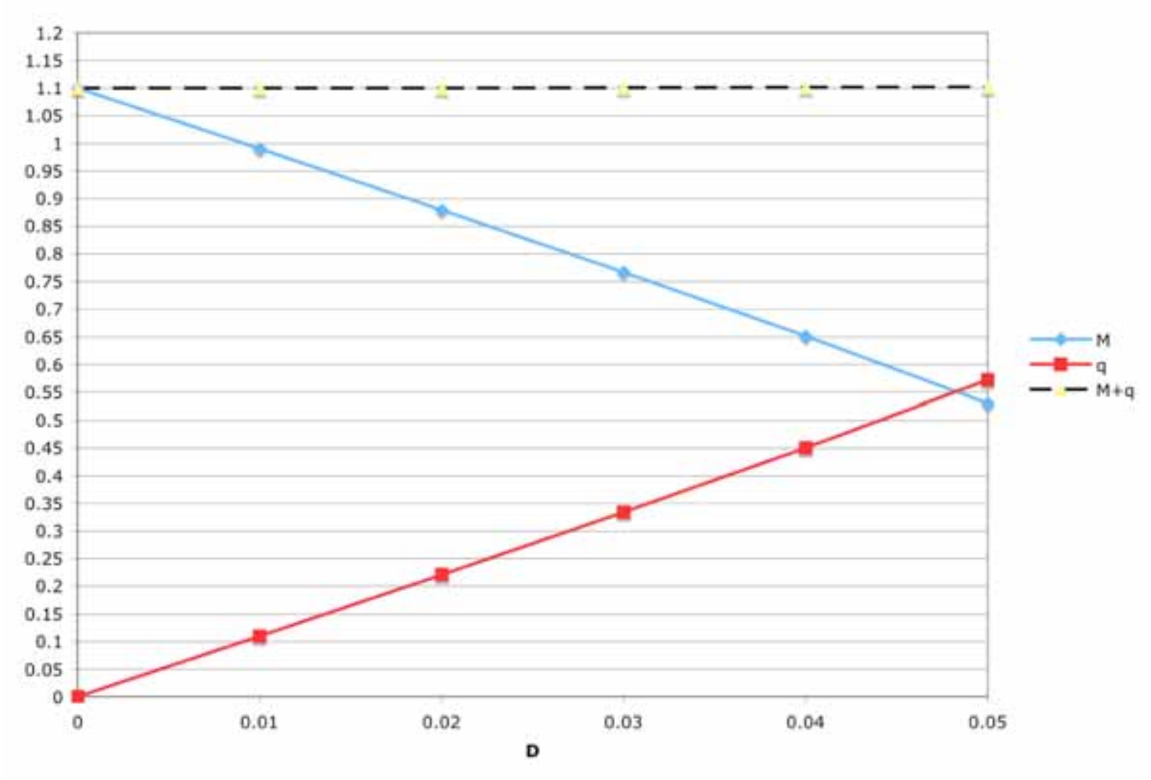

Figure 5: M,q and $M+q$ as a function of D ($\alpha=0.96, \pi=0.9$, $\lambda^1=\lambda^2=1/2$)

Can the model accounts for the observed equity premium? Mehra and Prescott (1985) observed an average rate of return on stocks of about 7% and an average real interest on short-term government bonds of less than 1%. They used data from 1890-1979 and their findings imply an equity premium of about 6%. Mankiw and Zeldes (1991) calculated an 8% equity premium for the period 1948-1988.

As can be seen from Figure 3, our model can account for these findings under the assumptions: $\alpha=0.96, \pi=0.9$, $\lambda^1=\lambda^2=1/2$ and $D=0.04$. Is this a reasonable choice of parameters?

At the end of their paper Eeckhout and Jovanovic (1992, page 1299) provide a mini survey of the empirical estimates of the elasticity of output with respect to inputs. They cite estimates of α in the range 0.95-0.99. Their own estimate is in the range: 0.94 - 0.99. Our choice of α = 0.96 is in this range. As can be seen from Figure 6, the equity premium does not change much with changes in α, but the rates of return are highly sensitive to the choice of α. The interest rate declines with α because an increase in α reduces the amount of taxes that is required to maintain constant dividends. This and the "government budget constraint" (45) lead to a decline in interest payments.

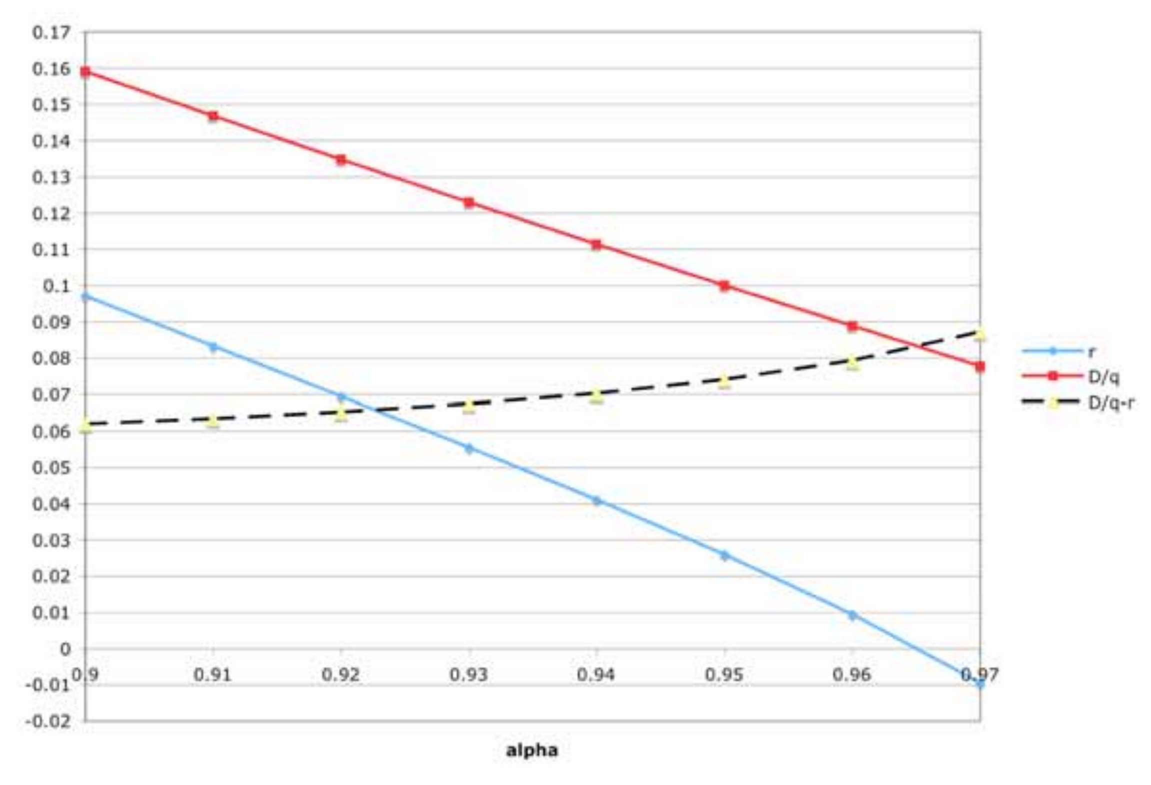

Figure 6: r, D/q and $D/q - r$ as a function of α $(D = 0.04, \pi = 0.9,\ \lambda^1 = \lambda^2 = 1/2)$

Corporate profits after tax were somewhat less than 6% of GDP during the period 1947-2007. In our model the firm represents large corporations that are publicly traded and their stocks are relatively liquid. The after tax profits of large corporations that are publicly traded is less than 6%. We chose 4% ($D = 0.04$).

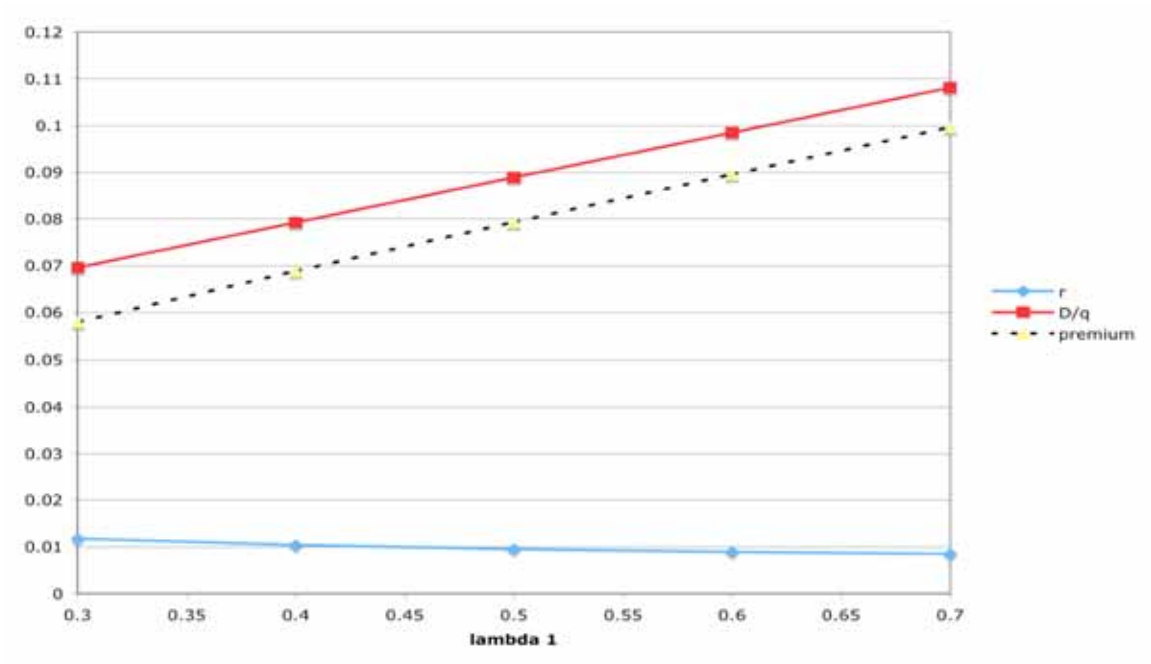

Figure 7: r, D/q and $D/q - r$ as a function of λ^1 ($\alpha = 0.96, \pi = 0.9$, D = 0.04)

Mankiw and Zeldes (1991) used a survey of 2998 US families in 1984 Panel Study of Income Dynamics (PSID). They found that only 27.6% of households hold stocks. Some stockholders own small amounts of stock. Only 23.2% of the sample holds equity in excess of 1000 dollars and only 11.9% holds equity in excess of 10,000 dollars. The fraction of stockholders increases with labor income and education. Mankiw and Zeldes report that in their sample, stockholders earn 38% of disposable income.

Since in our equilibrium type 2 workers are indifferent between stocks and bonds the fraction that actually hold stocks is not determined by the model. It is possible that all type 2 workers choose to hold stocks and it is also possible that only a fraction of them hold stocks provided that the total amount held is one. But the fraction of type 2 workers must be greater than the fraction observed in the data. This suggests $\lambda^2 > 0.38$ ($\lambda^1 < 0.62$). Our baseline specification of $\lambda^1 = \lambda^1 = \frac{1}{2}$ is consistent with this restriction.

Figures 7 computes the equilibrium rates of returns for different λ^h, assuming $\alpha = 0.96$, π = 0.9, D = 0.04. An increase in λ^1 has almost no effect on r but increases the rate of return on equity and the equity premium.

Figure 8 describes the rates of return on equity and bonds as a function of π. A close fit for the observed rates of return is obtained when π = 0.92. In this case, $r = 0.9\%$, $D/q = 0.79\%$ and the difference between the two is: 7%. Note that a decrease in π leads to an increase in D/q and a decrease in q. This may be viewed as "flight for quality" in response to an increase in uncertainty.

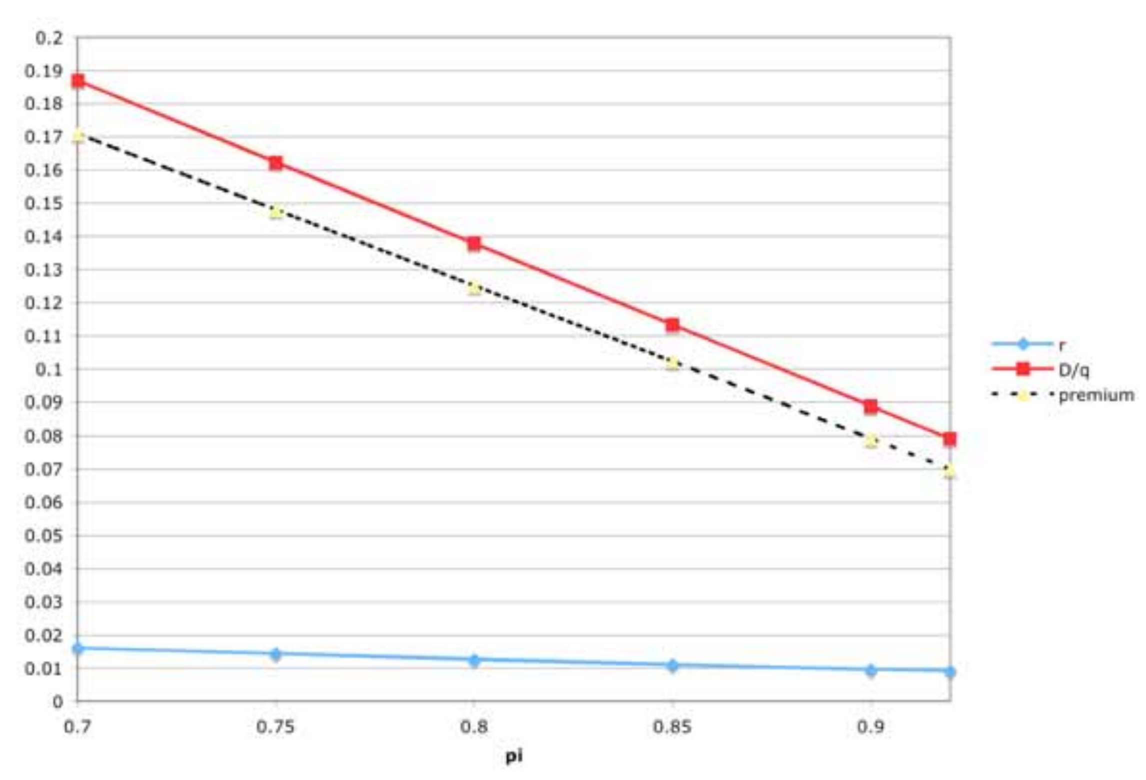

Figure 8: r, D/q and $D/q - r$ as a function of π ($\alpha = 0.96, \lambda^1 = \lambda^2 = \frac{1}{2}, D = 0.04$)

Figure 9 describes the share of stocks in total wealth $\frac{q}{M+q}$ and the share of bonds in total wealth $\frac{M}{M+q}$, where $M = M^1 + M^2$ is total bonds holdings. Our model does not distinguish between cash, short-term bonds and long-term bonds. This is a problem when trying to compare Figure 9 to data. However if we define the liquid asset in our model as cash and short-term bonds we may get a lower bound on π. In the 1994 wealth supplement to PSID the ratio of the value of stocks to the value of (cash + bonds + stocks) is 30%.[3] The ratio of the value of stocks to the value of (liquid assets + stocks) should be higher because not all bonds are short term. This implies: $\frac{q}{M+q} > 0.3$. As Figure 9 shows, this constraint is satisfied when $\pi > 0.85$.

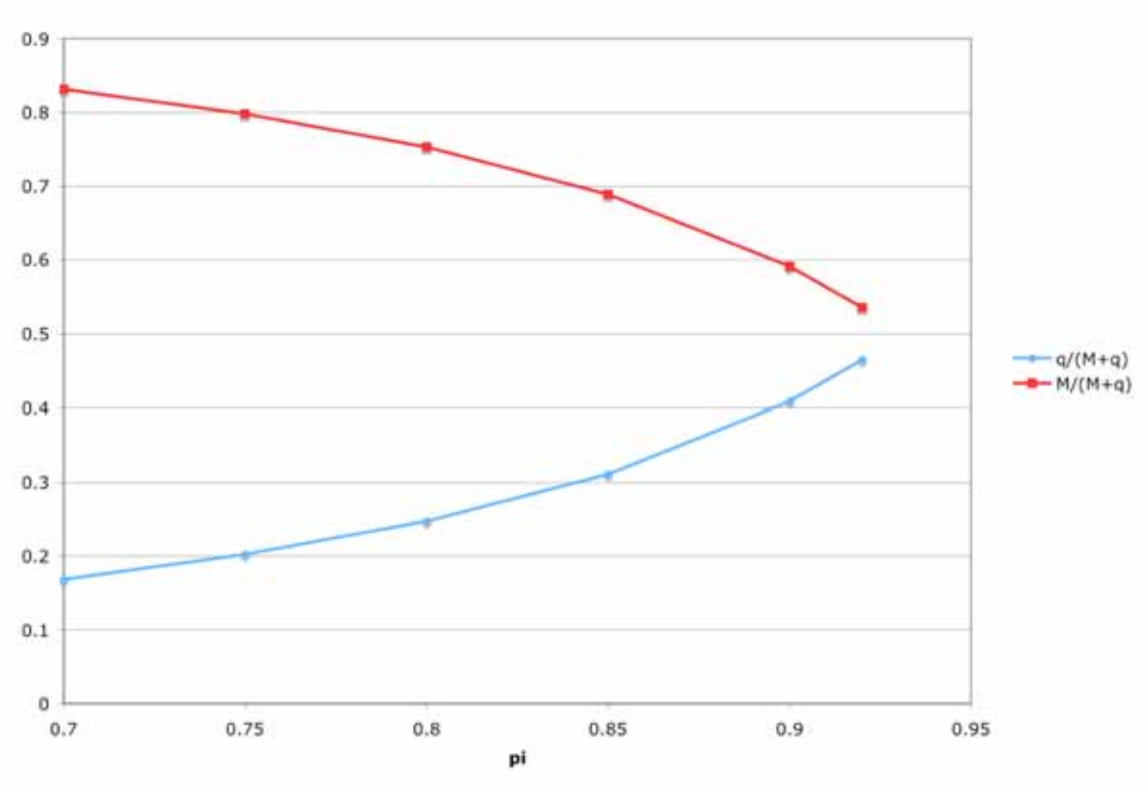

Figure 9: The average portfolio as a function of π

$(\alpha = 0.96, \lambda^1 = \lambda^2 = \frac{1}{2}, D = 0.04)$

[3] I am indebted to Matt Chambers and Don Schlagenhauf for the data used in these computations.

I now turn to the connection between the model and the dictionary definitions of liquidity cited in the introduction.

Velocity is often used as a measure of liquidity. Stocks are held by type 2 and they are exchanged in a market transaction with probability π. The average time between transactions is $1/\pi$. Bonds held by type 1 are exchanged every period while bonds held by type 2 are exchanged on average every $1/\pi$ periods. On average bonds are exchanged every $m + (1-m)\frac{1}{\pi} < 1/\pi$. Thus on average the time between transactions is shorter for bonds and bonds' velocity is higher.

The ratio of buying to selling price is also used to define liquidity. To make the connection with this definition, I now go back to the case in which there are chips backed by the firm's promise to sell one unit of the good for one chip. Buyers exchange their assets for chips before they enter the goods market and then use them to buy goods. The firm's bank exchanges the first M^1 units of bonds for $1/p_1$ chips per unit. If additional M^2 bonds arrive the bank exchanges them for $1/p_2$ chips per unit. Similarly, the bank exchanges the first S^1 stocks that arrive for $1/p_1^*$ chips per unit and it exchanges the second S^2 stocks for $1/p_2^*$ chips per unit. I use chips as the unit of account.

In the equilibrium of interest, the firm sells stocks to type 2 workers. The selling price in terms of chips is $1/p_1^* = q/p_1$. Whenever the firm buys stocks it pays $1/p_2^* = q/p_2$ chips. The ratio of the buying to the selling price is: $s^e = \frac{p_1}{p_2} < 1$. The firm sells bonds to type 1 worker at the chips price of $1/p_1$ per unit. In the low demand state it also buys bonds at this price. In the high demand state it buys a fraction m of the bonds for $1/p_1$ chips per unit and a fraction $1-m$ of the bonds for $1/p_2$ chips per unit. The average ratio of the buying to the selling price in the high demand state is:

$s^b = m + (1-m)\frac{p_1}{p_2} > s^e$. Thus in the state of high demand the buying to the selling price ratio is higher for bonds. In the low demand state the firm in our equilibrium does not buy stocks for chips and this ratio is not defined.

On the whole we may say that once we adopt chips as the unit of account, bonds appear to be more liquid both when using the velocity definition and when using the ratio of buying to selling price definition.

5. CONCLUDING REMARKS

We used price dispersion to model liquidity. Excluding steady states with knife-edge properties and assuming that both assets are valued, leads to asymmetric steady state in which assets earn different nominal rates of return. Since we assume risk neutrality, the difference in the rates of return may be called illiquidity premium. In the steady state, the stable demand type specializes in the more liquid asset. This is a result not an assumption: There is no steady state equilibrium in which the unstable demand type specializes in the more liquid asset.

The calculation of the actual real rates of return is different for the two types because they use different deflators. The stable demand type strictly prefers bonds (the more liquid asset) because they buy also in the low demand state and therefore the liquidity advantage is relatively large for them.

Our calibration exercise suggests that the model can account for the rates of returns estimated by Mehra and Prescott (1985). It can also account for the observation that only a fraction of the population holds stocks.

There are many possible extensions. We may consider a financial crisis scenario in which there is a change in expectations about the ability of the firms to pay dividends and as a result firms stop accepting other firms stocks as payment for goods. If this occurs after prices are already set and if prices cannot be changed, then expectations may be self-fulfilling and may lead to bankruptcies. In this case there may be a reason for the central bank to intervene and increase the dollar supply.

Another related extension is the analysis of non-steady state equilibria. I expect that the correlation between consumption and the return on stocks will be positive once we allow for a non-steady-state equilibrium in which prices and dividends fluctuate.

APPENDIX

Proof of Lemma 1: In the steady state, $\bar{z}_2 = z_2$, $\bar{z}_2^* = z_2^*$, $\bar{Z}^* = \pi z_1^* + (1-\pi)z_2^*$ and $\bar{Z} = \pi z_1 + (1-\pi)z_2$. The equality $p_1 = \pi p_2$ follows directly from (7) and (13) when using the steady state assumption.

To show that $p_1^* = \pi p_2^*$, assume that seller 1 is willing to accept both currencies in both markets. Then (11) and (12) hold with equality and imply: $\frac{R}{R^*} = \frac{p_1^*}{p_1} = \frac{p_2^*}{p_2}$. This and $p_1 = \pi p_2$ leads to: $p_1^* = \pi p_2^*$. A similar argument can be made for the case in which seller 2 is willing to accept both currencies.

Proof of Lemma 2: Suppose that seller 2 is willing to supply a strictly positive amount to the first shekel market. Then (5) hold with equality and the steady state assumption implies: $p_1 R z_2 = p_1^* R^* z_2^*$. Using this and Lemma 1, leads to: $p_2 R z_2 = p_2^* R^* z_2^*$ and therefore he must also be willing to supply a strictly positive amount to the second shekel market. A similar argument can be used to show that if he is willing to supply a strictly positive amount to the second shekel market he is also willing to supply a strictly positive amount to the first shekel market. The argument with respect to seller 1 is symmetric.

<u>Proof of Claim 1</u>: When $S^1 > 0$ and $S^2 > 0$, Lemma 2 and (17) imply that both types of sellers are willing to accept shekels and (11) and (5) hold with equality. This leads to:

(A1) $$\frac{R}{R^*} = \frac{p_1^* \bar{Z}^*}{p_1 \bar{Z}}$$

(A2) $$\frac{R}{R^*} = \frac{p_1^* \bar{z}_2^*}{p_1 \bar{z}_2}$$

Therefore both types will accept shekels only if:

(A3) $$\frac{\bar{z}_2^*}{\bar{z}_2} = \frac{\bar{Z}^*}{\bar{Z}}$$

In the steady state, $\bar{z}_2 = z_2$, $\bar{z}_2^* = z_2^*$, $\bar{Z}^* = \pi z_1^* + (1-\pi) z_2^*$ and $\bar{Z} = \pi z_1 + (1-\pi) z_2$.

Therefore (A3) implies:

(A4) $$\frac{z_2^*}{z_2} = \frac{\pi z_1^* + (1-\pi) z_2^*}{\pi z_1 + (1-\pi) z_2}$$

This condition will be satisfied for $0 < \pi < 1$ only if $\frac{z_2^*}{z_1^*} = \frac{z_2}{z_1}$. To see this we write

$z_1^* = k^* z_2^*$ and $z_1 = k z_2$, where k^*, k are constants. Then we can write (A4) as:

(A5) $$1 = \frac{\pi k^* + 1 - \pi}{\pi k + 1 - \pi}$$

This equality holds only if $k = k^*$. To show that (A4) requires $m = n$, I use Lemma 1 and the definitions (1) and (2) we get:

(A6) $$\frac{z_2^*}{z_1^*} = n + \frac{(1-n) p_1^*}{p_2^*} = n + \frac{1-n}{\pi}\ ;\ \frac{z_2}{z_1} = m + \frac{(1-m) p_1}{p_2} = m + \frac{1-m}{\pi}$$

Therefore $\frac{z_2^*}{z_1^*} = \frac{z_2}{z_1}$ only if $m = n$.

<u>Proof of Claim 2</u>: When $S^1 > 0$, seller 1 is willing to accept both currencies and (A1) holds. From (5) we get:

(A2') $$\frac{R}{R^*} \geq \frac{p_1^* \bar{z}_2^*}{p_1 \bar{z}_2}$$

Therefore (A1) and (A2') imply:

(A3') $$\frac{\bar{z}_2^*}{\bar{z}_2} \leq \frac{\bar{Z}^*}{\bar{Z}}$$

In the steady state, $\bar{z}_2 = z_2$, $\bar{z}_2^* = z_2^*$, $\bar{Z}^* = \pi z_1^* + (1-\pi)z_2^*$ and $\bar{Z} = \pi z_1 + (1-\pi)z_2$.

Therefore (A3') implies:

(A4') $$\frac{z_2^*}{z_2} \le \frac{\pi z_1^* + (1-\pi)z_2^*}{\pi z_1 + (1-\pi)z_2}$$

We write $z_1^* = k^* z_2^*$ and $z_1 = kz_2$, where k^*, k are constants. Then we can write (A4') as:

(A5') $$1 \le \frac{\pi k^* + 1 - \pi}{\pi k + 1 - \pi}$$

This inequality requires $k^* \ge k$. (A6) implies that $k^* \ge k$ and $\frac{z_2^*}{z_1^*} \le \frac{z_2}{z_1}$ only if $m \ge n$.

But when seller 2 specializes in dollars and seller 1 holds all the shekels, $n = 1$. Therefore we cannot have $m \ge n$ and it is not possible to have equilibrium in which seller 2 specializes in dollars and seller 1 accepts both currencies.

<u>Proof of Claim 3</u>: I now use (19) and the sellers budget constraints to solve for the steady state magnitudes $(p_1, p_2, p_2^*, x_1^1, x_1^2, x_3^2, x_4^2, M^1, M^2, S^2)$. Under the assumption that seller 1 supplies only to the first market, the budget constraint of seller 1 implies:

(A7) $$x_1^1 = \lambda^1$$

Substituting (A7) in the first equation of (18) leads to:

(A8) $$p_1 = \frac{M^1}{\lambda^1}$$

Using Lemma 1 leads to:

(A9) $$p_2 = \frac{M^1}{\pi\lambda^1}$$

Substituting (A8) and (A9) in the third equation of (19), $M^2 = p_2 x_3^2 + p_1 x_1^2$, leads to:

(A10) $$\frac{x_3^2}{\pi} + x_1^2 = \frac{M^2 \lambda^1}{M^1}$$

In addition to (A10) the second equation in (19) and the budget constraint of seller 2 must be satisfied. These are:

(A11) $$S^2 = p_2^* x_4^2,$$

(A12) $$x_1^2 + x_3^2 + x_4^2 = \lambda^2$$

We now have a system of 3 equations, (A10)-(A12), in 7 unknowns: $(p_2^*, x_1^2, x_3^2, x_4^2, M^1, M^2, S^2)$. In general there are many solutions.

To reduce the number of unknowns I assume $x_1^2 = 0$ so that seller 2 supplies to markets 3 and 4 only. In this case (A10) implies $x_3^2 = \frac{\pi M^2 \lambda^1}{M^1}$. Substituting this in (A12) leads to $x_4^2 = \lambda^2 - \frac{\pi M^2 \lambda^1}{M^1}$. Therefore, a steady state with $x_1^2 = 0$ exists if: $M^1\lambda^2 - M^2\pi\lambda^1 > 0$.

Proof of Claim 4: The first order conditions for the problem (23) are:

(A13) $\pi\left[z_2^*(y_2) + D_2(y)z_2(y_2)\right] - \lambda\Omega_2 \le 0$ with equality if $a_2^2>0$;

(A14) $(1-\pi)\left[z_2^*(y_1) + D_1(y)z_2(y_1)\right] - \lambda\Omega_1 \le 0$ with equality if $a_1^2>0$;

(A15) $\pi R z_2(y_2) - \lambda\Pi_2 \le 0$ with equality if $b_2^2>0$;

(A16) $(1-\pi)Rz_2(y_1) - \lambda\Pi_1 \le 0$ with equality if $b_1^2>0$;

where λ is the Lagrangian multiplier.

In the steady state (A13) and (A14) can be written as:

(A17) $z_2^* + Dz_2 - \lambda q \le 0$ with equality if $a_2^2>0$ or $a_1^2>0$.

Therefore if $a_2^2>0$ then the first order condition for any $a_1^2>0$ is satisfied and vice versa. This implies (a) for type 2. The argument for type 1 is the same.

To show (b) note that in the steady state (A15) and (A16) can be written as:

(A18) $Rz_2 - \lambda \le 0$ with equality if $b_2^2>0$ or $b_1^2>0$.

This implies (b) for type 2. The argument for type 1 is the same.

Proof of Proposition 1: Note that (37) implies:

(A19) $$\frac{R_e^1}{R_b^1} = \frac{1}{R}\left(\frac{V}{Z} + \frac{D}{q}\right);\ \frac{R_e^2}{R_b^2} = \frac{1}{R}\left(\frac{v_2}{z_2} + \frac{D}{q}\right)$$

I now use (A19) to characterize the relationship between these type specific ratios and (m, n).

Lemma 3: The definitions (1), (8), (36) and condition (33) imply:
(A) $\frac{R_e^1}{R_b^1} < \frac{R_e^2}{R_b^2} = 1$ if $m > n = 0$; (B) $\frac{R_e^1}{R_b^1} = \frac{R_e^2}{R_b^2} = 1$ if $m = n > 0$;
(C) $1 = \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2}$ if $n > m = 0$; (D) $1 = \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2}$ if $m > n > 0$;
(E) $\frac{R_e^1}{R_b^1} < \frac{R_e^2}{R_b^2} = 1$ if $n > m > 0$.

Proof: Note that (33) implies $p_2 > p_1$. To show (A) note that when $m > n = 0$ we must have: $v_2 = V = v_1 < z_2 < Z < z_1$. It follows that $\frac{V}{Z} < \frac{V}{z_2} = \frac{v_2}{z_2}$ and therefore (A19) leads to (A). To show (B) note that when $n = m$, we must have: $\frac{V}{Z} = \frac{v_2}{z_2} = 1$.

To show (C) note that when $n > 0$ and $m = 0$, we must have: $z_2 = Z = z_1 < v_2 < V < v_1$. In this case $\frac{V}{Z} > \frac{v_2}{Z} = \frac{v_2}{z_2}$ and (A13) leads to (C).

To show (D) note that when $0 < n < m$, $v_1 = z_1$ and $v_2 < z_2$. In this case:

(A20) $$\frac{V}{Z} = \frac{\pi v_2 + (1-\pi)v_1}{\pi z_2 + (1-\pi)z_1} = \frac{\pi v_2 + (1-\pi)z_1}{\pi z_2 + (1-\pi)z_1} > \frac{v_2}{z_2}$$

and this leads to (D). When $0 < m < n$, $v_1 = z_1$ and $v_2 > z_2$. In this case the inequality in (A20) is reversed and (A19) leads to (E).

We now turn to see which of the alternatives (A) - (E) in the Lemma satisfy (15), (40) and (44). Under (A) in the Lemma $m > n = 0$ and (44) implies $A = 0$ which is consistent with (40). Under (B), (44) implies $A > 0$ and $a > 0$ and this is also consistent with (40). I now argue that (C)-(E) are not consistent with (40) and (44).

Under (C) type 1 worker strictly prefers stocks but specialization in stocks is ruled out in (40). Under (D) type 2 specializes in bonds and $n = 1$. Therefore, $m < n = 1$. Under (E) type 1 specializes in bonds and therefore $n = 0$. We have thus shown that only alternatives (A) and (B) in the Lemma are consistent with (40) and (44). These two alternatives correspond to alternatives (a) and (b) in Proposition 1.

Proof of Proposition 2: We can write (47) as:

(A21) $$q = \frac{Dk}{Rk - \pi},$$

where $k = m + (1-m)\pi$. Substituting (A21) in (48) leads to:

(A22) $$F(k) = m + (1-m)\pi = \frac{x + \pi\left(y - z(k)\right)}{x + y - z(k)}$$

where $x = R\lambda^1, y = D + R(R\lambda^2/\pi)$ and $z(k) = \frac{RDk}{Rk - \pi}$.

Lemma: When $D < \lambda^2/\pi^2$, there exists $\pi < k < 1$ such that $k = F(k)$.

Proof: Assuming $k > \pi$, it can be shown that $z(k) > 0$, $z'(k) < 0$ and $F'(k) < 0$. I now show that $F(k = \pi) > \pi$ and $F(1) < 1$ as in Figure A1.

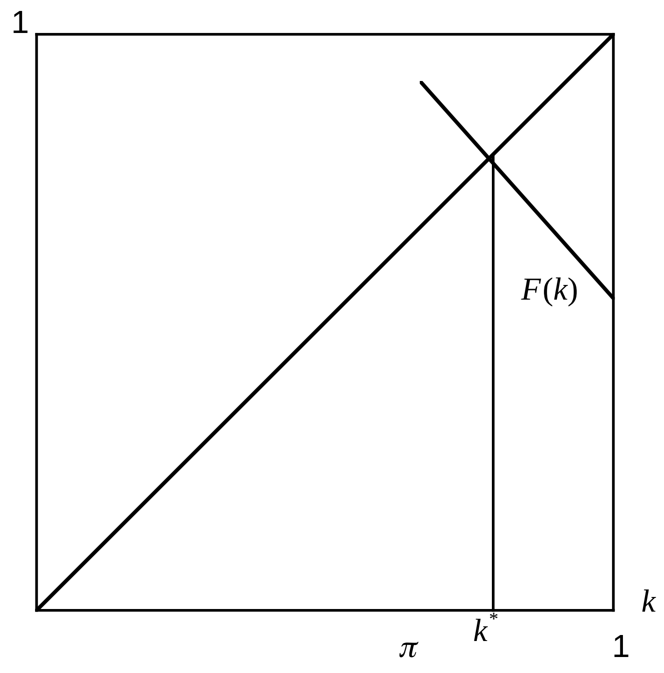

Figure A1

To show $F(k = \pi) > \pi$ note that $z(\pi) = RD/r$, where $r = R - 1$. Substituting this in (A2) leads to: $F(\pi) > \pi$ because $x + \pi\left(y - RD/r\right) - \pi\left(x + y - RD/r\right) = x(1-\pi) > 0$. To show $F(1) < 1$, note that under the above condition:

$F(1)=\dfrac{R\lambda^1+\pi\left(D+R\left((R\lambda^2/\pi)-\dfrac{D}{R-\pi}\right)\right)}{R\lambda^1+D+R\left((R\lambda^2/\pi)-\dfrac{D}{R-\pi}\right)}<1$. Thus there exists a fixed point $\pi<k^*<1$ as in Figure A1.

We can now solve for $0<m=\dfrac{k^*-\pi}{1-\pi}<1$. I now turn to solve for R and q. Using (49) we get:

(A23) $$R=\frac{(1/\alpha)-\pi D-\pi q}{1-\pi q}$$

Substituting (A21) in (A23) leads to:

(A24) $$R=\frac{(1/\alpha)-\pi D-\dfrac{\pi Dk}{Rk-\pi}}{1-\dfrac{\pi Dk}{Rk-\pi}}$$

In order for $R\geq 1$ we must require:

(A25) $$D\leq\frac{(1/\alpha)-1}{\pi}$$

Combining (A25) with the condition in the Lemma leads to: $D<\min\left\{\dfrac{(1/\alpha)-1}{\pi},\dfrac{\lambda^2}{\pi^2}\right\}$.

Thus when D is not too large in this sense, there exists a solution to (47)-(49).

To show the comparative static in part (b) of the Proposition, I define $G(D)=\dfrac{x+\pi\left(Y-Z(D)\right)}{x+Y-Z(D)}$, where $Y=R(R\lambda^2/\pi)$ and $Z(D)=D+\dfrac{RDk}{Rk-\pi}$. Note that $G(D)$ is (A22) expressed as a function of D. Since $G'=Z'(1-\pi)>0$, an increase in D leads to an upward shift in the $F(k)$ curve in Figure A1 and therefore the solution k^* goes up. This implies that the solution m^* also goes up with D.

To show (c) note that when type 1 specializes in bonds, (39) implies: $\dfrac{D}{q}=R-\dfrac{v_2}{z_2}$.

Since in this case, $0=n<m$ and $\dfrac{v_2}{z_2}<1$, we get: $\dfrac{D}{q}>r$.

REFERENCES

Abel Andrew B. "Precautionary Saving and Accidental Bequest" The American Economic Review, Vol. 75, No. 4 (Sep.,1985),pp.777-791.

Ait-Sahalia Yancine, Jonathan A. Parker, and Motohiro Yogo., "Luxury Goods and the Equity Premium" The Journal of Finance, Vol. LIX, No. 6, Dec. 2004, pp. 2959-3004.

Amihud Yakov, "Illiquidity and stock returns: cross-section and time-series effects" Journal of Financial Markets 5 (2002) 31-56.

Amihud Yakov, Haim Mendelson, Lasse Heje Pederson., "Liquidity and Asset Prices" Foundation and Trends in Finance, 1:4 (2005)

Attanasio Ozario P., James Banks and Sarah Tanner. "Asset Holding and Consumption Volatility" Journal of Political Economy, 2002, Vol. 110, no. 4, pp: 771-792.

Barro Robert J. "Are Government Bonds Net Wealth?" Journal of Political Economy, November/December 1974, 82, 1095-1117.

______ "Rare Disasters and Asset Markets in the Twentieth Century" The Quarterly Journal of Economics, August 2006, pages: 823-866.

Cochrane John H. "Stock as Money: Convenience Yield and the Tech-Stock Bubble" in William C. Hunter, George G. Kaufman and Michael Pomerleano, Eds. Asset Price Bubbles, Cambridge: MIT Press 2003 (previously NBER working paper 8987).

Dana James D. Jr. "Advance-Purchase Discounts and Price Discrimination in Competitive Markets“ Journal of Political Economy, Vol.106, Number 2, April 1998, 395-422.

Devereux Michael B. and Charles Engel., "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility" Review of Economic Studies, 70 (2003), 765-783.

Diamond, D. and P. Dybvig (1983). "Bank Runs, Deposit Insurance, and Liquidity" Journal of Political Economy 91, 401-419.

Eeckhout Jan and Boyan Jovanovic. "Knowledge Spillovers and Inequality" American Economic Review, Dec. 2002, Vol. 92, No.5, 1290-1307.

Eden, Benjamin. “Marginal Cost Pricing When Spot Markets are Complete” Journal of Political Economy, Dec.1990. Vol. 98, No.6,1293-1306.

_______ “The Adjustment of Prices to Monetary Shocks When Trade is Uncertain and Sequential” Journal of Political Economy, Vol. 102, No.3, 493-509, June 1994.

______ "International Seigniorage Payments", 2006, Vanderbilt University WP # 06-W22.

Kiyotaki, N. and R. Wright, "A Search-theoretic Approach to Monetary Economics" American Economic Review, 83 (1993), 63 - 77.

Kocherlakota, Narayana R. "The Equity Premium: It's Still a Puzzle" Journal of Economic Literature 34 (March 1996): 42-71.

Karekan, J. and N. Wallace, "On the Indeterminacy of Equilibrium Exchange Rates" Quarterly Journal of Economics, 96 (1981), pp. 207-222.

Lucas, Robert E., Jr. and M. Woodford., "Real Effect of Monetary Shocks in an Economy with Sequential Purchases" NBER Working Paper No. 4250, January 1993.

Mankiw, N. Gregory, and Stephen P. Zeldes "The Consumption of Stockholders and Nonstockholders" Journal of Financial Economics, 29 (March 1991): 97-112.

Matsuyama, M., Kiyotaki, N. and A. Matsui, "Towards a Theory of International Currency" Review of Economic Studies, 60 (1993), 283-307.

McGrattan Ellen, R. and Prescott Edward C. ""Average Debt and Equity Returns: Puzzling?" American Economic Review, Papers and Proceedings, May 2003, Vol. 93 No.2, pp. 392-397.

Mehra, Rajnish and Prescott, Edward C. "The Equity Premium: A Puzzle." Journal of Monetary Economics, 15 (March 1985): 145-61.

Prescott, Edward. C., “Efficiency of the Natural Rate” Journal of Political Economy, 83 (Dec. 1975): 1229-1236.

Rietz, Thomas. A., "The Equity Risk Premium: A Solution" Journal of Monetary Economics, XXII (1988), 117-131.

Shi, S. "Money and Prices: A Model of Search and Bargaining" Journal of Economic Theory, 27 (1995), 467-496.

Trejos, A. and R. Wright "Search, Bargaining, Money and Prices" Journal of Political Economy, 103 (1995), 118-41.

Vissing-Jorgensen Annette and Ozario P. Attanasio. "Stock-Market Participation, Intertemporal Substitution and Risk-Aversion" American Economic Review Papers and Proceedings, May 2003, p. 383-391.

Woodford, Michael. Interest and Prices, Princeton University Press, 2003.