EFFICIENCY, VACANCY RATES AND OVERBOOKING IN A SEQUENTIAL TRADE MODEL

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December 1999

The uncertain and sequential trading (UST) model is extended to the case in which buyers have different reservation prices. Unlike previous UST models here the probability that a market will open and the number of buyers who participate in this market are endogenous. It is shown that the UST allocation is not efficient in the relevant sense: A monopoly who faces the same informational constraints as the UST firms may improve matters by raising prices.

INTRODUCTION

Uncertain and Sequential Trading (UST) models are based on ideas in Prescott (1975) and Butters (1977). In a review article of the Phelps volume, Prescott provides a counter example to the view that precautionary unemployment is likely to be excessive because sellers of labor services have some monopoly power. Prescott considers an example in which sellers of motel rooms set prices before they know how many buyers will arrive and derive an equilibrium price distribution. He assumes that cheaper rooms are sold first and therefore in equilibrium sellers face a tradeoff between price and the probability of making a sale.

In prescott's example all motel rooms are the same and all buyers who arrive want a single room and are willing to pay up to the same reservation price. Prescott's conclusion is that "For this example, which entails monopoly power on the part of sellers facing a stochastic demand, the competitive equilibrium is efficient. If demanders were heterogeneous (in terms of preferences) and there were heterogeneity in the type of room supplied, it is possible that these conclusions would be altered. Until such an analysis is successfully performed, I see no reason to conjecture that the natural vacancy rate is either too high or too low." (page 1233).

In the UST approach taken by Eden (1990) an equilibrium distribution of prices is obtained even though sellers are allowed to change their prices during trade and have no monopoly power. The UST approach has proven useful for analyzing issues in monetary economics and business cycle. See Bental and Eden (1993, 1996), Eden (1994, forthcoming), Lucas and Woodford (1994), Williamson (1996) and Woodford (1996). But the efficiency question posed by Prescott is still open.

In Eden (1990) all buyers have the same downward sloping demand curves (rather than a demand for a single unit) and the allocation is efficient. Buyers who arrive first buy more at lower unit prices. In terms of the motel room example: Buyers who arrive first, buy larger rooms at a low price per square meter. Thus heterogeneity in the type of room supplied does not alter the efficiency result.

Recently, Dana (1998) has extended Prescott's model to an environment in which buyers have different reservation prices. He concludes that in this case the equilibrium allocation may not be efficient because of price rigidity.

Here I consider Dana's problem in a UST framework that does not assume price rigidity. Like in other UST models demand arrives sequentially in batches and firms have to make irreversible trading decisions before they know the realization of aggregate demand. But here the probability that each batch will arrive and its size are determined endogenously.[1]

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[1] In Dana (1998) the probability of arrival and the size of each batch do not depend on prices. This is because he assumes that regardless of prices he can rank aggregate states of nature by the aggregate demand so that the aggregate demand in state of nature 1 is always lower than the aggregate demand in state of nature 2. This assumption may be rather restrictive. Assume for example that there are two potential buyers and two states of nature. Buyer 1 wants to consume one unit in state 1 only and his reservation price is 10. Buyer 2 wants to consume 2 units in state 2 only and his reservation price is 5. Aggregate

The UST framework allows for a distinction between two types of efficiency criteria. I define sequential efficiency by the solution to the problem of a central planner who maximize expected surplus subject to the same informational constraints which are imposed on the UST firms: He has to make irreversible choices before he knows the realization of demand. Standard Walrasian efficiency on the other hand is defined by the solution to the unconstrained social planner problem.

Dana compares the allocation in the Prescott model to the standard Walrasian allocation and finds that the Walrasian allocation is better. This is a claim about standard Walrasian efficiency which holds in the UST model as well. The "Walrasian auctioneer" does a better job because he has more information than the UST firms: He knows aggregate demand before irreversible trade takes place.

Here I show that the UST allocation need not be sequential efficient: Even a social planner who operates under the informational constraints faced by the UST firms can improve matters. Moreover, even a monopoly who face the same informational constraints may improve matters.

Since we allow sellers to change their prices during trade, the reason for the inefficient outcome of the UST model is not in price rigidity. The reason for the standard Walrasian inefficiency is in the informational constraint implied by the sequential nature of trade. The

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demand in state 1 is thus 1 unit when $P \leq 10$ and zero otherwise. Aggregate demand in state 2 is 2 units when $P \leq 5$ and zero otherwise. Aggregate demand is larger in state 2 when $P \leq 5$ and larger in state 1 when $5 < P \leq 10$.

reason for the sequential inefficiency is in the competitive pressure to make zero expected profits.

The result about sequential inefficiency is rather surprising. The intuition for this result is roughly as follows. Under sequential trade it is possible that a low valuation buyer who arrive early will not be rationed while a high valuation buyer who arrive late will be rationed. A possible solution is to raise prices above the reservation price of the low valuation buyer. But this leads to above normal profits and therefore requires some monopoly power.

I extend the analysis to environments in which ex-ante contracts are possible and distinguish between the case in which delivery occurs sequentially and the case in which it occurs simultaneously after all buyers have arrived. In the sequential delivery case the outcome from trading in ex-ante contracts may be better than the UST outcome because the UST outcome is not sequential efficient. In the simultaneous delivery case the outcome of trading in contracts will be, under certain conditions, the Walrasian efficient outcome.

I characterize the competitive contracts in the case of simultaneous delivery. It is shown that "overbooking" is essential for achieving the Walrasian outcome. Partial refunds can be used as a screening device in cases of asymmetric information.

THE MODEL

I consider an economy with two dates (t = 0,1) and two goods (X and Y with lower case letters denoting quantities). There is a price-taking firm which has an access to a constant return to scale technology: It can produce X at the cost of λ units of Y (the numeiraire) per unit of X. Production occurs at t = 0.

There are S possible aggregate states of nature (indexed s) and J types of buyers (indexed j). The number of buyers from type j is n_j. A type j buyer demands at most one unit of X at any price less than v_j if he wants to consume and zero otherwise.

It is assumed that a fraction ϕ_{js} of type j agents want to consume in aggregate state s.

At t = 1, buyers form a line and learn their type and whether they want to consume X. After learning this information they arrive at the market-place one by one according to their place in the line. Upon arrival buyers see all price offers and choose the cheapest available offer. The sequence of events is illustrated by Figure 1.

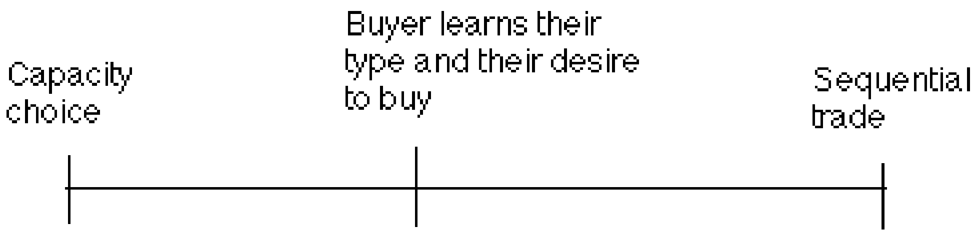

Figure 1

<u>Markets:</u> From the firm's point of view demand arrives in batches. The first batch with Δ_1 buyers arrives with certainty. They then trade and go away. After the first batch of buyers complete trade a second batch of Δ_2 buyers may arrive. In general, there can be two possible events after batch i completes trade and disappear: Either trade ends or an additional batch of Δ_{i+1} buyers arrive. The arrival process is illustrated by Figure 2.

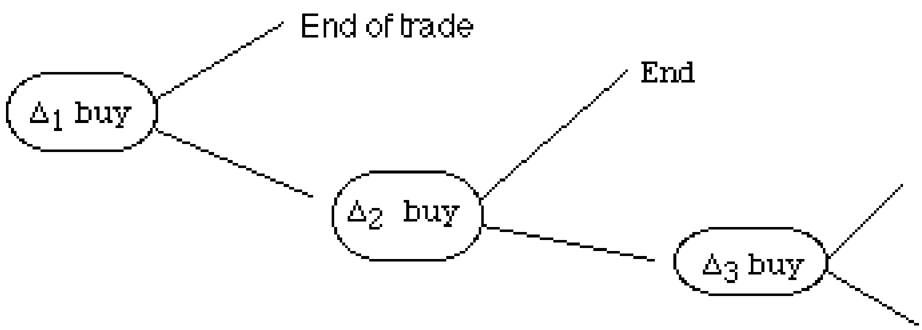

Figure 2

The firm can sell to the first batch at the price P_1. It may choose not to sell everything at this price speculating on the possibility that a second batch of buyers will arrive and buy at the higher price P_2. In general, the firm can sell at the price P_i to buyers in batch i if it arrives.

The firm makes a contingent plan to sell x_i units at the price P_i if batch i arrives, subject to:

(1) $$\Sigma_i x_i = x,$$

where x is the total amount it chooses to produce.

It helps to think in terms of markets that open sequentially. The first batch of buyers opens the first market. If the second batch of buyers arrives, it opens the second market and so on. The price in market i is P_i, the quantity supplied to market i is x_i and the quantity demanded in market i is Δ_i. In equilibrium markets which open are cleared:

(2) $$\Delta_i = x_i.$$

I now turn to describe an algorithm for computing the probability that each batch will arrive and the number of buyers it contains as functions of an arbitrarily chosen price vector: $P_1 \leq ... \leq P_S$. This is done by looking at the residual demand at each stage of trade. The next batch will arrive if there is strictly positive residual demand (buyers who wanted to buy in previous markets but could not). The number of buyers in the next batch is the minimum size of the residual demand. I now turn to a detailed description.

Market 1 will open in all states in which $\Sigma_j \phi_{js} > 0$ and there is strictly positive demand. The probability that market 1 will open is given by $q_1 = \Sigma_{s=1}^{S} \pi_s I(\Sigma_j \phi_{js})$, where π_s is the probability that state s occurs and $I(x) = 1$ when $x > 0$ and zero otherwise.

The number of type j buyers who want to buy in the first market at the price P_1 in state s is:

(3) $$N_{js}^{1}(P_1) = \phi_{js} n_j \text{ if } v_j \geq P_1 \text{ and zero otherwise.}$$

Total demand in the first market at the price P_1 is therefore:

(4) $$N_s^1(P_1) = \Sigma_j N_{js}^1(P_1).$$

The number of buyers in the first batch is the minimum demand at the price P_1 over all states:

(5) $$\Delta_1(P_1) = \min_+\{N_1^1(P_1),...,N_S^1(P_1)\},$$

where I use the operator $\min_+$ to select the smallest strictly positive number if such a number exists and zero otherwise.[2]

Market 2 will open in state of nature s if after the completion of trade in the first market there is residual demand in this state: $N_s^1(P_1) > \Delta_1(P_1)$. The probability that market 2 will open is:

(6) $$q_2(P_1) = \Sigma_{s=1}^{S} \pi_s I[N_s^1(P_1) - \Delta_1(P_1)],$$

where π_s denotes the probability that state of nature s occurs and

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[2] For example: $\min_+(1,2,0) = 1$ and $\min_+(0,0,0) = 0$.

$I(x) = 1$ if $x > 0$ and zero otherwise.

If the probability that market 2 opens is zero we are done. If not we calculate the size of the second batch in the following way. The fraction of remaining buyers out of all buyers who wanted to buy is:

(7) $$\upsilon_S^1(P_1) = 1 - [\Delta_1(P_1)/N_S^1(P_1)],$$
$$\text{if } N_S^1(P_1) > 0 \text{ and zero otherwise.}$$

It is assumed that the fraction of remaining buyers is the same for all types who wanted to buy in market 1. The number of remaining type j buyers is therefore $\upsilon_S^1(P_1)\phi_{js}n_j$. The number of remaining type j buyers who want to buy at the price P_2 is:

(8) $$N_{js}^2(P_1, P_2) = \upsilon_S^1(P_1)\phi_{js}n_j, \quad \text{if } v_j \geq P_2 \text{ and zero otherwise.}$$

And the total residual demand at the price P_2 is:

(9) $$N_S^2(P_1, P_2) = \Sigma_j\, N_{js}^2(P_1, P_2).$$

The number of buyers in batch 2 is:

(10) $$\Delta_2(P_1, P_2) = \min_+\{N_1^2(P_1, P_2),...,N_S^2(P_1, P_2)\}.$$

We proceed to calculate the probability that market 3 will open:

(11) $$q_3(P_1, P_2) = \Sigma_{S=1}^{S}\, \pi_S I[N_S^2(P_1, P_2) - \Delta_2(P_1, P_2)].$$

In general, the probability that market m will open depends only on prices in markets with indices less than m and is denoted by: $q_m(P_1, ..., P_{m-1})$. For notational convenience I use: $q_1 = q_1(P_0)$. The number of buyers in batch m depends on the prices in all the first m markets and is denoted by: $\Delta_m(P_1, ..., P_m)$.[3]

Because of constant returns to scale the supply to market m does not depend on the supply to other markets: It depends only on the probability that market m will open and the price in this market. The firm takes prices $(P_1, ..., P_m)$ as given and chooses the quantity x_m to solve:

(12) $$\max\ [q_m(P_1, ..., P_{m-1})]P_m x_m - \lambda x_m.$$

Because of constant returns to scale the firm will choose any interior solution, $0 < x_m < \infty$, if:

(13) $$[q_m(P_1, ..., P_{m-1})]P_m = \lambda.$$

Given the functions $q_m(P_1, ..., P_{m-1})$ and $\Delta_m(P_1, ..., P_m)$ I define equilibrium as follows.

<u>A UST equilibrium</u> is a vector of prices and quantities $(P_1,..., P_S; x_1,...,x_S)$ that satisfy:

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[3] Note that market m may open and $\Delta_m(P_1, ..., P_m)$ may equal zero because the price P_m is too high. Note also that by construction, the number of markets that open is less than or equal to the number of states (S).

(a) $P_1 < P_2 < \ldots < P_S$;

(b) $[q_m(P_1, \ldots, P_{m-1})]P_m = \lambda$;

(c) $\Delta_m(P_1, \ldots, P_m) = x_m$;

for all m.

Since the probability that market m opens depends only on prices in lower indexed markets, we can solve for the unique UST equilibrium in the following way. We start from $P_1 = \lambda$ and compute $N_s^1(\lambda)$ and $\Delta_1(\lambda)$. We then compute the probability that market 2 will open: $q_2(\lambda) = \Sigma_{s=1}^S \pi_s I[N_s^1(\lambda) - \Delta_1(\lambda)]$. If $q_2(\lambda) = 0$, we choose $P_m = 0$ for all $m \geq 2$. Otherwise, we choose: $P_2 = \lambda/q_2(\lambda)$ and compute $N_s^2[\lambda, \lambda/q(\lambda)]$, $\Delta_2[\lambda, \lambda/q(\lambda)]$. We then compute the probability that market 3 will open: $q_3[\lambda, \lambda/q(\lambda)] = \Sigma_{s=1}^S \pi_s I\{N_s^2[\lambda, \lambda/q(\lambda)] - \Delta_2[\lambda, \lambda/q(\lambda)]\}$. If $q_3[\lambda, \lambda/q(\lambda)] = 0$, we choose $P_m = 0$ for all $m \geq 3$. Otherwise, we choose: $P_3 = \lambda/q_3[\lambda, \lambda/q(\lambda)]$ and so on.

<u>Monopoly:</u> A monopolist will choose the prices ($P_1 \leq P_2 \ldots \leq P_S$) to maximize the expected profits:

(14) $\Sigma_{m=1}^S [q_m(P_1,\ldots,P_{m-1})][\Delta_m(P_1, \ldots,P_m)]P_m - \lambda\Sigma_{m=1}^S [\Delta_m(P_1, \ldots,P_m)]$.

Social planner:

The fraction of type j buyers in batch m in state s is:[4]

(15) $$\alpha^m_{js}(P_1,...,P_m) = N^m_{js}(P_1,...,P_m)/N^m_s(P_1,...,P_m),$$

if $N^m_s(P_1,...,P_m) > 0$ and zero otherwise.

The average valuation of the buyers in batch m, state s is:

(16) $$V^m_s(P_1,...,P_m) = \Sigma_j\, v_j\alpha^m_{js}(P_1,...,P_m).$$

The probability that $N^m_s(P_1,...,P_m) > 0$ is: $Q(P_1,..., P_m) = \Sigma^S_{s=1}\, \pi_s I[N^m_s(P_1,...,P_m)]$. The average valuation of a unit sold in market m is therefore:

(17) $$V_m(P_1,...,P_m) = \Sigma^S_{s=1}\, [\pi_s/Q(P_1,..., P_m)][V^m_s(P_1,...,P_m)],$$

if $Q(P_1,..., P_m) > 0$ and zero otherwise.

The social planner chooses prices ($P_1 \leq P_2 ... \leq P_S$) to maximize the expected consumer surplus:

(18) $$\Sigma^S_{m=1}\, [q_m(P_1,...,P_{m-1})][\Delta_m(P_1, ...,P_m)]V_m(P_1, ...,P_m) - \lambda\Sigma^S_{m=1}\, [\Delta_m(P_1, ...,P_m)].$$

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[4] It is assumed that all buyers who want to buy in market m have the same chance of making it and therefore the fraction of type j who actually buy in market m is equal to the fraction of type j in all agents that want to buy in market m.

An allocation is sequential efficient if it is the outcome of solving (18).

I now turn to compare the UST allocation with the monopoly choice (14) and the central planner's choice (18).

Example 1: There are two types of agents: Definite buyers ($j = 1$) and possible buyers ($j = 2$). The number of agents from each type is 1. Definite buyers have a reservation price of $v_1 = 10$ dollars (units of Y) and possible buyers have a reservation price of $v_2 = 7$ dollars. There are two states of nature which occurs with equal probabilities: rain ($s = 1$) and no rain ($s = 2$). Definite buyers want to consume X in both states. Possible buyers want to consume X only if there is no rain. The cost of production is $\lambda = 5$ per unit of capacity.

At $P_1 > 10$, total demand is zero. At $7 < P_1 \leq 10$ total demand is 1 and at $P_1 \leq 7$ total demand is 1 if $s = 1$ and 2 if $s = 2$. The number of buyers in the first market (= minimum demand) is thus:

$$(19) \qquad \Delta_1(P_1) = 1 \quad \text{for } P_1 \leq 10 \text{ and zero otherwise.}$$

The probability that market 2 will open is:

$$(20) \qquad q_2(P_1) = 1/2 \quad \text{for } P_1 \leq 7 \text{ and zero otherwise.}$$

This is because in state 2 there is strictly positive residual demand.

When $P_1 \leq 7$, the number of remaining buyers in state 2 is $^1/_2$ of each type. When $P_2 \leq 7$, all the remaininig buyers want to buy in market 2 and therefore:

(21) $$\Delta_2(P_1, P_2) = 1 \text{ for } P_1 < P_2 \leq 7.$$

When $7 < P_2 \leq 10$ only the definite buyers want to buy in market 2 and therefore:

(22) $$\Delta_2(P_1, P_2) = {}^1/_2 \quad \text{for } P_1 \leq 7 \text{ and } 7 < P_2 \leq 10.$$

When $P_2 > 10$ none of the remaining buyers want to buy in market 2 and therefore:

(23) $$\Delta_2(P_1, P_2) = 0 \quad \text{for } P_1 \leq 7 \text{ and } P_2 > 10.$$

Since the second market opens in this example with probability $^1/_2$, equilibrium prices are: $P_1 = \lambda = 5$ and $P_2 = 2\lambda = 10$. The number of buyers in the second batch is $^1/_2$ according with (22) and production is therefore 1.5 units at the cost of 7.5. The surplus in state 1 is: $v_1 - 7.5 = 2.5$. The surplus in state 2 is: $v_1 + (^1/_2)v_2 - 7.5 = 6$. The average surplus over the two states is 4.25.

A monopoly will choose $P_1 = 10$ and produce one unit making a profit of 5. This profit is also the surplus in this case which is greater than the competitive expected surplus.

Note that a planner in this example cannot do better than the monopoly: The planner will choose to produce one unit and will price it at $7 < P \leq 10$ so that only the high valuation buyers will get it. Thus the monopoly choice is sequential efficient.

In example 1 the UST competitive firm produces more than the monopoly but this is not efficient because the additional half a unit of capacity is being used by type 2 agents whose ex-ante valuation is less than the cost of production (3.5 per unit). The reason why in example 1 the competitive firm produces too much capacity is in the failure to allocate capacity to buyers who value it the most: Low valuation buyers who arrive early are not rationed and therefore the residual demand includes high valuation buyers who arrive late. These high valuation buyers are willing to pay enough to produce goods that will be sold with probability $\frac{1}{2}$.

<u>Example 2:</u> The same as example 1 but now $v_1 = 9$ instead of 10. In this case competitive UST prices remains the same as in the previous example: $P_1 = 5$ and $P_2 = 10$. As in the previous example, market 2 will open in state 2 but since $\Delta_2(5, 10) = 0$ it will not be active. In a UST equilibrium only one unit is produced and allocated to market 1. The surplus is: 4 in state 1 and 3 in state 2. The average surplus is: 3.5.

A monopoly will choose $P_1 = 9$ guaranteeing a profit (surplus) of 4.

In example 2 both the monopoly and the competitive firm produce the same amount but the monopoly does a better job in allocating the existing capacity to the buyers who value it the most.

<u>Example 3:</u> I now add a new type to example 1: Type 3 who wants to consume only when s = 1 (when type 2 does not want to consume) and is willing to pay only up to $v_3 = 4$.

Adding type 3 will not change the UST equilibrium and the monopoly choice. But it will change the planner's choice. Now the planner can do better by producing two units and pricing them at $P \leq 4$.

When s = 1, type 1 and type 3 will buy the good and the surplus will be 10 + 4 - 10 = 4. When s = 2, type 1 and type 2 will buy the good and the surplus will be 10 + 7 - 10 = 7. The average surplus is 5.5 which is higher than what the planner can make if he adopts the monopoly choice.

In example 3 the UST firm is producing too little relative to the sequential efficient level: 1.5 instead of 2.

The above three examples show:

<u>Proposition 1:</u> (a) The UST allocation is not necessarily sequential efficient; (b) The monopoly choice may be sequential efficient even in cases in which the UST allocation is sequential inefficient and (c) The UST output may be either too high or too low relative to the sequential efficient level of output.

As was said in the introduction improvements require the departure from zero expected profits. To improve we must either raise prices and achieve a better screening of buyers (allowing only high valuation buyers to buy) or reduce prices and allow the participation of low valuation buyers who want to consume in low demand states. This requires either positive or negative expected profits and therefore cannot occur in the UST competitive environment.

I now turn to a comparison of the UST model with the standard competitive model.

<u>The standard competitive model:</u>

The Walrasian auctioneer observes demand (the state of nature) before announcing the market clearing price. This informational advantage allows for outcomes which are better than the solution to (18). To show this let:

(24) $$n_{js}(p) = \phi_{js}n_j \text{ if } v_j \geq p \text{ and zero otherwise.}$$

Total demand in state s is given by:

(25) $$n_s(p) = \Sigma_j\, n_{js}(p).$$

Let p_s denote the price in state of nature s and define equilibrium as follows.

A standard competitive equilibrium is a vector $(p_1,...,p_S;\ V)$ such that:

(a) $\Sigma_{s=1}^{S}\ \pi_s p_s = \lambda$;

(b) $n_s(p_s) \leq V$ for all s and $P_s = 0$ when the inequality is strict.

Condition (a) insures that the firm produces the capacity V at zero expected profits. Condition (b) is a market clearing condition.

I now solve for the standard competitive equilibrium vectors in the above examples (note that equilibrium is not always unique).

Example 1: The following two vectors satisfy the conditions for standard competitive equilibrium:

$p_1 = 0$, $p_2 = 10$, $V = 1$;

$p_1 = 1$, $p_2 = 9$, $V = 1$.

In both cases the expected consumer surplus = 5.

Example 2:

$p_1 = 1$, $p_2 = 9$, $V = 1$

$p_1 = 2$, $p_2 = 8$, $V = 1$.

Expected consumer surplus = 4.

Example 3:

$p_1 = 3$, $p_2 = 7$, $V = 2$.

Expected consumer surplus = 5.5.

In these examples, the expected consumer surplus is the same as the maximum value of (18). The following example illustrates that the informational advantage of the Walrasian auctioneer permits strict improvement.

<u>Example 4:</u> The same as example 3 but now type 3 (with a reservation price of $v_3 = 4$) wants to consume in both states. In this case the standard competitive equilibrium is the same as in example 3:
$p_1 = 3$, $p_2 = 7$, $V = 2$ and expected consumer surplus = 5.5.

But here a central planner who faces the sequential constraint and solves (18) can not do better than produce $V = 1$ and charge $7 \leq P \leq 10$. The outcome of this choice in terms of consumer surplus is 5 (like the monopoly).

Thus the standard competitive allocation may be strictly better than the sequential efficient allocation.

<u>Standard Walrasian efficiency:</u>

I consider the problem of a central planner who chooses capacity at $t = 0$ but allocates it at $t = 1$ after observing the state of nature. Let $I(x) = 1$ when $x \geq 0$ and zero otherwise. The central planner's problem is to choose p_s and V which solve:

$$(26) \qquad \max \Sigma_s \pi_s \Sigma_j \phi_{js} n_j v_j I(v_j - p_s) - \lambda V$$

$$\text{s.t.} \quad \Sigma_j \phi_{js} n_j I(v_j - p_s) \leq V \text{ for all } s.$$

An allocation is Walrasian efficient if it is the outcome of solving (26). In general, the expected consumer surplus (26) is higher than the maximum value of (18). It is also well known (The first welfare Theorem) that the standard competitive allocation solves (26).

The informational difference between the sequential model and the Walrasian model is illustrated by Figure 3.

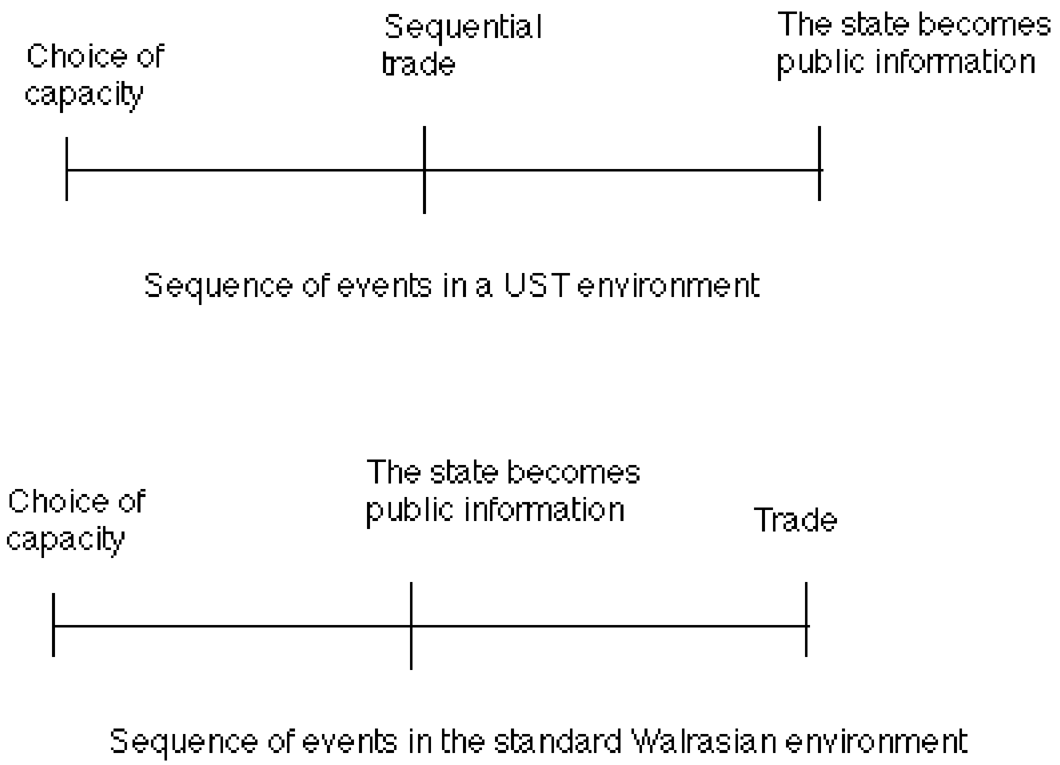

Figure 3

<u>Ex-ante contracts:</u> In many cases it is possible to buy tickets before actual delivery occurs. Is this a way of cutting on waiting time or is there another reason for advance purchase?

The answer to this question depends on whether delivery is sequential or simultaneous. When delivery is sequential the payment cannot be contingent on the realization of demand: It can only be contingent on the order of arrival. When delivery is simultaneous and occurs after all buyers have arrived the tickets may be contingent on the realization of demand. Hotels are examples for sequential delivery:

It is difficult to imagine that buyers will wait outside an hotel until there is no chance that additional buyers will arrive. This is not the case in the airline industry where technology requires that delivery occurs after all buyers have arrived.

Ex-ante contracts cannot help if buyers do not know anything about themselves at the time of capacity choice. I therefore assume that at $t = 0$, agents learn about their type where, as before, a type is characterized by the valuation v_j and the probabilities that he will want to consume: $(\phi_{j1}, ..., \phi_{jS})$. At $t = 0$ there is trade in contracts. Buyers learn about their desire to consume only at $t = 1$. Then we may have two types of deliveries: sequential delivery as in the hotel example or simultaneous delivery as in the airline example. The sequence of events is illustrated by Figure 4.

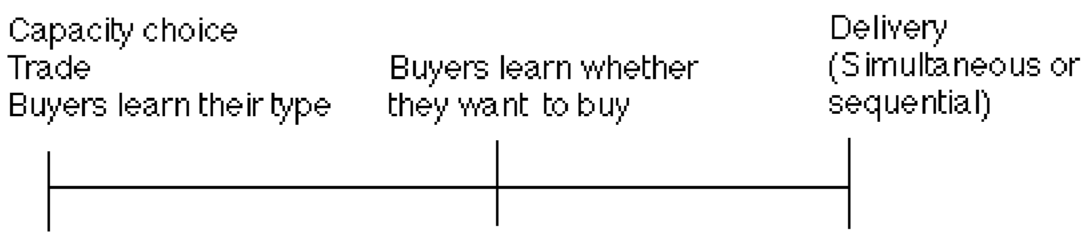

Figure 4

I start from the case of simultaneous delivery and adopt the airline example. I assume that after buyers learn about their desire to fly they may report it to the airline (by showing at the counter). The airline then chooses to whom it will deliver (in the case of "overbooking").

A contract is a triplet (μ, α, β): If at t = 1 the buyer wants to fly and the airline chooses to satisfy his demand the buyer pays the price μ. If the buyer chooses not to fly he pays α (get a refund for $\mu - \alpha$). If he wants to fly but there is "overbooking" and the airline does not deliver the buyer gets β (pays - β). It is assumed that $\mu \geq \alpha \geq 0$. This insures that a buyer who does not want to fly will report it to the airline.[5] I start from the case in which the type of the buyer can be observed by the airline at t = 0.

At t = 1 all the refund options included in the tickets are exercised. As a result the airline learns the state of nature. Let p_s denote the shadow price of a seat in state of nature s. It is assumed that the airline makes zero expected profits and therefore:

$$\Sigma_s \pi_s p_s = \lambda. \tag{27}$$

The expected revenue from selling a ticket (μ, α, β) to type j is:

$$\Sigma_s \pi_s\{\phi_{js}[\max(\mu, p_s - \beta)] + (1-\phi_{js})(p_s + \alpha)\}. \tag{28}$$

The term $\max(\mu, p_s - \beta)$ reflects the choice of the firm: Only if

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[5] A weaker condition is needed. A buyer that does not want to fly and reports that he wants to fly, will pay μ if the airline deliver and $-\beta$ if the airline does not deliver. Letting q denote the probability that the airline will deliver, he will pay on average $q\mu - (1-q)\beta$. If the buyer reports his true preference he will pay α. Thus we require: $q\mu - (1-q)\beta \geq \alpha$. This condition is satisfied if: $\mu \geq \alpha$.

$\mu \geq p_s - \beta$ it will choose to deliver. The term $(p_s + \alpha)$ is the payoff to the firm when the buyer chooses not to fly: The airline gets α from the buyer and the shadow price p_s from allocating the seat to someone else.

Using the decision rule of the firm we can calculate the expected consumer surplus from a ticket (μ, α, β). This is:

$$(29) \qquad \Sigma_s\ \pi_s\phi_{js}\{(v_j - \mu)I(\mu + \beta - p_s) + [1 - I(\mu + \beta - p_s)]\beta\} - \Sigma_s\ \pi_s(1-\phi_{js})\alpha,$$

where $I(x) = 1$ if $x \geq 0$ and zero otherwise. Note that $I(\mu + \beta - p_s) = 1$ when $\mu \geq p_s - \beta$. In this case the consumer will get the surplus $(v_j - \mu)$. When $\mu < p_s - \beta$, the firm does not deliver and the consumer gets β as compensation.

The airline allows the buyer to choose any ticket he wants subject to the zero expected profit constraint:

$$(30) \qquad \Sigma_s\ \pi_s\{\phi_{js}[\max(\mu, p_s - \beta)] + (1-\phi_{js})(p_s + \alpha)\} = \lambda.$$

The buyer therefore chooses a ticket $T_j = (\mu_j, \alpha_j, \beta_j)$ which solves the following problem:

$$(31) \qquad \max\ \Sigma_s\ \pi_s\phi_{js}\{(v_j - \mu)I(\mu + \beta - p_s) + [1 - I(\mu + \beta - p_s)]\beta\} - \Sigma_s\ \pi_s(1-\phi_{js})\alpha$$

$$\text{s.t. (30) and } \mu \geq \alpha \geq 0.$$

<u>Claim 1:</u> (a) The solution to (31) is characterized by:

$\beta_j = v_j - \mu_j$ and (b) the incentive compatibility constraint $\mu \geq \alpha \geq 0$ is not binding.

<u>Proof:</u> I start by solving (31) without the incentive compatibility constraint. Without this constraint, a solution to (31) must also maximize the total (consumer and producer) surplus:

$$(32)\ \Sigma_s\ \pi_s\{\phi_{js}[v_j I(\mu + \beta - p_s) + [1 - I(\mu + \beta - p_s)]p_s] + (1 - \phi_{js})p_s\}.$$

The choice $\beta = v_j - \mu$ maximizes (32) because it ensures that I = 1 when $v_j \geq p_s$ and zero otherwise.

To show that the incentive compatibility constraint is not binding, I substitute $\beta = v_j - \mu$ in (29). This yields:

$$(33)\quad CS_j = \Sigma_s\ \pi_s\{\phi_{js}(v_j - \mu_j) - (1 - \phi_{js})\alpha_j\} = \Phi_j v_j - \Phi_j \mu_j - (1 - \Phi_j)\alpha_j.$$

where $\Phi_j = \Sigma_s\ \pi_s\phi_{js}$ is the unconditional probability that type j will want to fly. Since it is possible to increase μ_j and reduce α_j without changing the consumer surplus, the incentive compatibility constraint is not binding. □

From (33) it follows immediately that,

<u>Corollary:</u> The solution to (31) does not determine (μ_j, α_j) uniquely. It determines only the expected payment: $\Phi_j\mu_j + (1 - \Phi_j)\alpha_j$.

I now define equilibrium as follows.

An advance purchase equilibrium is a vector

$(p_1, ..., p_S; T_1, ..., T_J; V)$ such that:

(a) $\Sigma_s \pi_s p_s = \lambda$;

(b) given the prices $(p_1, ..., p_S)$ type j's choice $T_j = (\mu_j, \alpha_j, \beta_j)$ solves (32) for all j;

(c) $\Sigma_j [I(\mu_j + \beta_j - p_s)]\phi_{js}n_j \le V$.

To find an advance purchase equilibrium we pick a standard competitive equilibrium vector: $(p_1, ..., p_S; V)$. We then let agents choose contracts by solving (31). Claim 1 implies that the buyer will get the seat only if he wants to fly and $v_j \ge p_s$. This is precisely the allocation rule used in the standard competitive model. Therefore the market clearing condition (c) is satisfied. We have thus shown,

<u>Proposition 2:</u> When delivery occurs simultaneously, the advanced purchase allocation is the same as the standard competitive allocation.

We get the Walrasian outcome because we are in a Walrasian environment: delivery takes place only after all buyers arrive and information about the state is known. The "overbooking" option is important because it allows the airline to "reverse" trading choices which were made before the resolution of uncertainty.

<u>The type is private information:</u>

I now consider the case in which the firm cannot observe the type of the buyer at the time it sells tickets. In this case, it is no longer possible to ask the buyer to solve (31) because he may have an incentive not to declare his true type.

I therefore follow the adverse selection literature and assume that the firm offers a choice of contracts and asks each buyer to choose out of the proposed menu.

An advance purchase separating equilibrium is an advance purchase equilibrium (defined above) with the added "truth telling constraint":

$$(34) \quad \Sigma_s \pi_s\phi_{js}\{(v_j - \mu_j)I(\mu_j + \beta_j - p_s) + [1 - I(\mu_j + \beta_j - p_s)]\beta_j\} - \Sigma_s \pi_s(1-\phi_{js})\alpha_j$$
$$\geq \Sigma_s \pi_s\phi_{js}\{(v_j - \mu_{j'})I(\mu_{j'} + \beta_{j'} - p_s) + [1 - I(\mu_{j'} + \beta_{j'} - p_s)]\beta_{j'}\} - \Sigma_s \pi_s(1-\phi_{js})\alpha_{j'}$$

for all j and j'.

The "truth telling" or incentive compatibility constraint says that type j will prefer ticket T_j out of all the tickets offered. Since the Corollary implies that we have a "free parameter" this constraint may not bind. The airline may issue tickets with high μ and low α and tickets with relatively low μ and high α. Buyers with low probability of "wanting to fly" will buy tickets with high μ and low α.

In examples 1 and 2, the airline can choose V = 1 and offer tickets with no refund option ($\alpha = \mu$) at the price of $\mu = 5$. These tickets will be bought only by type 1 agents. This does not always work. In example 3, trade in contracts is not sufficient to insure Walrasian efficiency. This is not surprising in view of the asymmetry in information.[6]

To illustrate the importance of partial refunds as a selection device, I add the following example.

Example 5: There are two types and two states of nature.

Type 1: $v_j = 6$ and he always want to consume;

Type 2: $v_j = 7$ and he wants to consume in state 2 only.

In this example the efficient level of production is unity but unlike the previous example, here it should be allocated to type 1 in state 1 and type 2 in state 2.

An advance purchase separating equilibrium can be achieved by choosing:

$\mu_1 = 5$; $\beta_1 = 1$; $\alpha_1 = 5$;

$\mu_2 = 6$; $\beta_2 = 1$; $\alpha_2 = 0$.

$p_1 = 4$; $p_2 = 6$.

Another possible solution:

$\mu_1 = 4$; $\beta_1 = 1$; $\alpha_1 = 4$.

$\mu_2 = 7$; $\beta_2 = 0$; $\alpha_2 = 0$.

$p_1 = 3$; $p_2 = 7$.

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[6] In example 3 we may achieve Walrasian efficiency if we allow buyers to purchase the same ticket. In this case the airline will choose V = 2 and will sell tickets with no refund option for $\mu = 5$. Type 2 and 3 will get together and buy a ticket for 5 dollars spliting the cost unevenly: 3 dollars for type 2 and 2 dollars for type 3.

Ex-ante contracts in a sequential environment: When delivery occurs sequentially, it is possible to write contracts which are contingent on the order of arrival. For example, the firm can promise the consumer to deliver one unit at the price P_m if he arrives in batch m. This will produce the same outcome as the UST equilibrium but since this outcome is not sequential efficient there may be better ways of conducting trade.

In example 1, sequential efficiency can be achieved by trading at t = 0 in non-refundable tickets. The market clearing price for these tickets is 5, only one unit will be produced and this unit will be bought by the definite buyers.

Thus in the sequential delivery case it is possible to improve on the UST allocation by trading at t = 0. This is because the UST allocation is not sequential efficient.

CONCLUDING REMARKS

The UST model was extended to the case of heterogeneous agents. Unlike other UST models here the probability that a market will open and the number of buyers participating in this market are endogenous. The next market will open if there are buyers who wanted but could not buy in the last market. The probability of this event depends on the prices in previous markets. The demand in the next market is the minimum size of the residual demand which depends on the prices in previous markets and the price in the next market.

The UST allocation is not sequential efficient. Buyers who arrive early may get the good even if they should not. A monopoly who face the same informational constraints as the firm in the UST model may improve matters by raising prices and discouraging low valuation buyers.

I compare the UST outcome with the standard Walrasian outcome. In general, the Walrasian auctioneer can do better because he does not allow for irreversible trade before the complete resolution of uncertainty.

The difference between the UST model and the standard Walrasian model is in the time of trade. The UST model allows trade before the resolution of uncertainty while the standard model does not. In an environment which allows for ex-ante contracts we can draw a similar distinction about the time of delivery. In a UST environment delivery occurs sequentially while in a Walrasian environment delivery occurs after all buyers have arrived. Trading in ex-ante contracts will produce the Walrasian outcome if delivery occurs simultaneously after all buyers have arrived and information is symmetric. When delivery is sequential (as in the motels example) trading in ex-ante contracts may improve matters because the UST outcome is not sequential efficient.

The characterization of the contracts in the case in which delivery is simultaneous is of some interest. It is shown that the "overbooking" option is crucial. Such contracts are sometimes used in practice. For example, electrical companies offer discounts to factories which agree to a cut off in the supply of electricity in cases of shortage. The option for partial refund may be used as a sorting device when information is asymmetric.

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