This chapter is taken from the book:

"A Course in Monetary Economics: Sequential Trade, Money and Uncertainty"

By Benjamin Eden, Published by Blackwell, 2005.

CHAPTER 15

A Monetary Model

In chapters 15–18 we use monetary versions of the UST model to account for the money/output relationship. We start with an overlapping generations model and then use a cash-in-advance model. The overlapping generations model is simpler and does not require dynamic programming. The cash-in-advance model is useful for studying the role of inventories in propagating monetary shocks. A reader who is interested in the role of inventories may go directly to chapter 17.

In monetary UST models sellers may know the number of buyers that will arrive but do not know the amount that they will spend. This may depend on the realization of the random monetary transfer. Thus in monetary UST models uncertainty about the number of dollars that will arrive plays the role of uncertainty about the number of buyers in real UST models.

Unlike Lucas' confusion model (reviewed in chapter 10), here everyone observes the money supply process and there is no confusion. The monetary transfer process is like rain. Everyone observes the amount of monetary "rain" accumulated from the beginning of the period but no one knows when it will stop. Sellers do not sell everything they have at the beginning of the process because they speculate on the event that the monetary rain will continue and they will be able to sell at a higher price.

Changes in the money supply affect capacity utilization and output first and quoted prices later (with a one period lag). But this Keynesian feature of the model does not imply an exploitable trade-off between inflation and output.

The first generation of UST monetary models are the papers by Eden (1994) and Lucas and Woodford (1994). In the Lucas and Woodford model, trade is sequential but the transfer of money is not: Trade starts only after buyers know the total amount of money transferred. This asymmetry in information about the money supply may lead to rationing. In Eden (1994) both the money transfer and trade are perfectly synchronized processes: Each batch of dollars transferred triggers more trade. Lucas and Woodford use an infinite horizon model, a Nash equilibrium concept, and exogenous capacity. Eden uses an overlapping generations model, a competitive equilibrium concept and allows for production. Here we present the model in Eden (1994).

15.1 AN EXAMPLE

We consider an overlapping generations model. Two identical individuals are born at the beginning of each period. Each lives for two periods. In the first period of his life the representative agent produces and sells his output for money. He then uses, in the second period, the proceeds of first period sales plus a transfer that he may receives from the government to buy goods. There is a single consumption good and fiat money is the only asset.

The buyer (an old agent) shops in two locations. In the first location he spends the proceeds of period $t-1$ sales (M_t). He then goes to the second location. If he receives a transfer at this second location he uses it to buy goods. Otherwise, he goes home. The amount of transfer is one dollar per dollar held at the beginning of the period and the probability of receiving it is 1/2. Figure 15.1 illustrates the sequence of events from the buyer's point of view.

There is a single seller (young agent) in each location and at each round of trade a single buyer may appear in his location. (Thus buyers move but sellers do not.) From the representative seller's point of view demand arrives in batches: The first buyer spends M_t dollars with certainty and disappears. Then we have two possibilities: Either trade for the period ends or a second buyer arrives (with probability 1/2) and spends an additional M_t dollars. Figure 15.2 illustrates the seller's point of view.

The seller is a price-taker and knows that he can sell to the first buyer at the dollar price P_{1t}. He can sell to the second buyer, if he gets the transfer payment, at the dollar price $P_{2t} > P_{1t}$. He chooses total capacity before the beginning of trade and makes a contingent plan as to how to utilize it. He may plan for example to sell 80% of his capacity to the first buyer and 20% to the second buyer if he arrives.

As in the real models, it is useful to think of Walrasian markets that open sequentially in response to the arrival of purchasing power. The first batch of M_t dollars buys in market 1 at the price P_{1t}. If the second batch of M_t dollars arrives it buys in market 2 at the price P_{2t}. Thus market 1 opens with certainty and market 2 opens with probability 1/2.

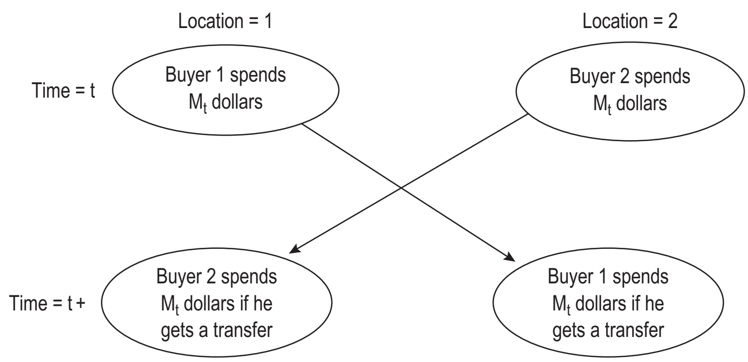

Figure 15.1 Events over time and locations

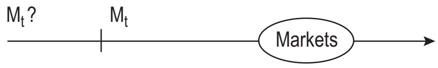

Figure 15.2 The seller's point of view

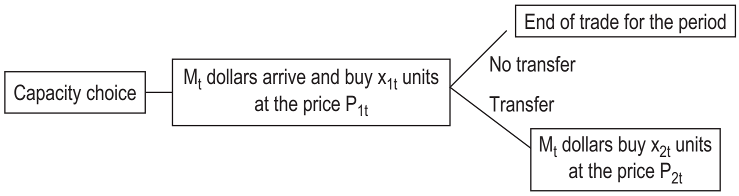

Figure 15.3 Sequence of events

It takes one unit of labor to produce one unit of capacity. The representative young agent at time t, has a utility function which is linear in consumption:

$$c_{t+1} - (x_t)^2, \tag{15.1}$$

where c_{t+1} is the expectation at time t of consumption at time $t + 1$ and x_t is the total amount of work (capacity).

The representative young agent chooses total capacity, x_t, before the beginning of trade and allocates x_{1t} to market 1 and x_{2t} to market 2 ($x_{1t} + x_{2t} = x_t$). It is costless to convert capacity into output and storage is not possible. Capacity allocated to market 2 is utilized (converted to output) only if market 2 opens. Figure 15.3 describes the sequence of events from the representative seller's point of view.

Since money received today will be spent in the next period the seller must form expectations about next period's prices in order to choose capacity (effort) and to allocate it between the two markets. For this purpose it is useful to compute the expected purchasing power of a dollar earned in each market.

Let P^s_{it+1} denote the expected next period price in market i when exactly s markets open in the current period. A dollar earned this period, allows its owner to spend one dollar with certainty in the next period's first market. Since the transfer payment is proportional (a dollar per dollar held at the beginning of period) it also promises a 50% chance of spending an additional dollar in next period's market 2. The expected purchasing power of a dollar earned at time t in market 2 is therefore:

$$Z_{2t} = 1/P^2_{1t+1} + \tfrac{1}{2}\left(1/P^2_{2t+1}\right). \tag{15.2}$$

There is no uncertainty in (15.2) about the superscript (the number of markets open in the current period) because a sale is made in market 2 only if exactly 2 markets open this period. This is not the case when the dollar is earned in market 1. The expected purchasing power of a dollar earned at time t in market 1 is:

$$Z_{1t} = \tfrac{1}{2}\left[1/P^1_{1t+1} + \tfrac{1}{2}\left(1/P^1_{2t+1}\right)\right] + \tfrac{1}{2}\left[1/P^2_{1t+1} + \tfrac{1}{2}\left(1/P^2_{2t+1}\right)\right]. \tag{15.3}$$

The expected real revenue from a unit of capacity allocated to market 1 is $P_{1t}Z_{1t}$ and the expected revenue from a unit of capacity allocated to market 2 is: $(1/2)P_{2t}Z_{2t}$. The representative young agent's problem is to choose the capacities x_{st} that solve:

$$\max_{x_{st}} P_{1t}Z_{1t}x_{1t} + \tfrac{1}{2}P_{2t}Z_{2t}x_{2t} - (x_{1t} + x_{2t})^2. \tag{15.4}$$

The first order conditions for an interior solution to (15.4) require that the expected real revenue per unit of capacity is the same for both markets and is equal marginal cost:

$$P_{1t}Z_{1t} = \tfrac{1}{2}P_{2t}Z_{2t} = 2(x_{1t} + x_{2t}). \tag{15.5}$$

A partial equilibrium for time t is defined for given expectations (P^s_{it+1}). It is a vector $(P_{1t}, P_{2t}, x_{1t}, x_{2t})$ that satisfies (15.5) and the market-clearing conditions:

$$M_t/P_{1t} = x_{1t} \quad \text{and} \quad M_t/P_{2t} = x_{2t}. \tag{15.6}$$

A full equilibrium endogenizes expectations. To do that, we assume that agents use a quantity theory type model and expect prices proportional to the beginning of period money supply. They thus assume the existence of normalized prices p_s such that:

$$P_{st} = p_s M_t \quad \text{for all t}. \tag{15.7}$$

Using (15.7), sellers form the following rational expectations:

$$P^1_{st+1} = p_s M_t \quad \text{and} \quad P^2_{st+1} = p_s 2M_t. \tag{15.8}$$

A full equilibrium for time t is a vector $(p_1, p_2, x_{1t}, x_{2t}, P_{1t}, P_{2t}, P^1_{1t+1}, P^1_{2t+1}, P^2_{1t+1}, P^2_{2t+1}, Z_{1t}, Z_{2t})$ satisfying (15.2), (15.3) and (15.5)–(15.8).

Note that once we know normalized prices (p_1, p_2), we can easily compute (P_{1t}, P_{2t}) by using (15.7), the period t expectations $(P^1_{1t+1}, P^1_{2t+1}, P^2_{1t+1}, P^2_{2t+1})$ by using (15.8) and the expected purchasing power expression by using (15.2) and (15.3). Therefore, rather than solving for full equilibrium, we turn now to a more direct way of solving for quantities and normalized prices.

15.2 WORKING WITH THE MONEY SUPPLY AS THE UNIT OF ACCOUNT

Rather than formulating the model in terms of dollar magnitudes, we may do it directly in terms of normalized magnitudes.

To build intuition let us start with the concept of an indexed dollar. The number of cents in an indexed dollar is proportional to the money supply: If, in the current period there are 100 cents in an indexed dollar and the money supply went up by 10% then in the next period we require 110 cents per indexed dollar. A seller who posts his prices in terms of indexed dollars does not change posted prices in response to neutral changes in the money supply.

Alternatively, we may use the money supply (per household) itself as a unit of account and call it a normalized dollar. For example, if the price of a unit is one normalized dollar it means that you must pay the whole money supply (average per household) to get it. Like with an indexed dollar, a seller that posts prices in terms of normalized dollars does not change posted prices in response to neutral money supply changes.

A normalized dollar this period is worth $\omega^s = M_t/M^s_{t+1}$ in terms of next period's normalized dollars, where M^s_{t+1} is the next period money supply if s markets open today. In our example:

$$\omega^1 = 1 \quad \text{and} \quad \omega^2 = \tfrac{1}{2}. \tag{15.9}$$

A normalized dollar earned in market 2 is thus worth ω^2 next period's normalized dollars because making a sale in market 2 implies that exactly 2 markets open this period. A normalized dollar earned in the first market is worth $\omega = (1/2)\omega^1 + (1/2)\omega^2$ in terms of next period's normalized dollars because making a sale in market 1 does not tell us anything new about the number of markets that will open.

The purchasing power of a normalized dollar held at the beginning of the period is:

$$z = (1/p_1) + \tfrac{1}{2}(1/p_2), \tag{15.10}$$

because it will be spent in the first market for sure and it promises (with probability 1/2) a transfer of a normalized dollar that will be spent in market 2.

The young agent's problem can now be written as:

$$\max\, (p_1\omega z)x_1 + \tfrac{1}{2}(p_2\omega^2 z)x_2 - (x_1 + x_2)^2. \tag{15.11}$$

The product $p_1\omega$ is the expected number of next period's normalized dollars that a young agent can get by allocating a unit of capacity to market 1 and similarly, $(1/2)p_2\omega^2$ is the revenue from allocating a unit to market 2. Multiplying these magnitudes by the expected purchasing power of a normalized dollar (z) yields the relevant real price in each market.

The first order conditions for the problem (15.11) are:

$$p_1\omega z = \tfrac{1}{2}p_2\omega^2 z = 2(x_1 + x_2). \tag{15.12}$$

To interpret (15.12) note that a unit supplied to market 1 promises, on average, $p_1\omega z$ units of consumption. This must equal the expected amount of consumption that one can get from supplying a unit to the second market, $(1/2)p_2\omega^2 z$, and the marginal cost, $2(x_1 + x_2)$.

Since one normalized dollar will arrive in each market that is opened the market-clearing conditions can be written as:

$$1/p_1 = x_1 \quad \text{and} \quad 1/p_2 = x_2. \tag{15.13}$$

We can now define *equilibrium* as a solution (p_1, p_2, x_1, x_2) to (15.12) and (15.13). The solution is: $p_1 = 64/21; p_2 = 64/7; x_1 = 21/64; x_2 = 7/64$.

The equilibrium normalized prices and (15.7) and (15.8) can be used to compute actual and expected prices. For example, if $M_t = 1$, actual prices are equal to normalized prices and expected prices are: $P^1_{1t+1} = 64/21; P^2_{1t+1} = 128/21; P^1_{2t+1} = 64/7; P^2_{2t+1} = 128/7$.

Note that (15.12) implies:

$$p_1/p_2 = \tfrac{1}{2}\omega^2/\omega < \tfrac{1}{2}. \tag{15.14}$$

Thus unlike the real UST model, here the ratio of prices is less than the probability that market 2 will open. The intuition is that a normalized dollar earned in market 1 is worth more in terms of next period's normalized dollars and therefore $p_1 < (1/2)p_2$.

This result is analogous to the winner-curse effect in the auction literature (see for example the discussion in Milgrom and Weber [1982]). Here there is an adverse effect of a success in selling in market 2 on the value of the dollar.

15.3 ANTICIPATED AND UNANTICIPATED MONEY

We now consider a more general case that allows for a distinction between anticipated and unanticipated money. We assume $M_{t+1} = \theta\lambda M_t$, where λ is the average (anticipated) gross rate of change in the money supply and θ is the unanticipated component.

At the first location the buyer receives a transfer of $(\theta_1\lambda - 1)$ per dollar and spends a total of $\theta_1\lambda M_t$ dollars or $\theta_1\lambda$ normalized dollars. With probability $1/2$ the buyer may get a transfer of $(\theta_2 - \theta_1)\lambda$ normalized dollars in the second location. We assume $E\theta = 1$. In the above numerical example: $\lambda = 3/2$, $\theta_1 = 2/3$ and $\theta_2 = 4/3$.

A normalized dollar this period will become:

$$\omega^s = M_t/M^s_{t+1} = 1/\theta_s\lambda, \tag{15.15}$$

next period's normalized dollars, if s markets open this period. The expected purchasing power of a normalized dollar held at the beginning of the period is now

$$z = (\theta_1\lambda/p_1) + \tfrac{1}{2}(\theta_2 - \theta_1)\lambda/p_2. \tag{15.16}$$

The purchasing power of a normalized dollar earned in market 2 is: $\omega^2 z = (1/\theta_2)[\theta_1/p_1 + (1/2)(\theta_2 - \theta_1)/p_2]$. The expected consumption that a unit of output supplied to market 2 will bring is:

$$\tfrac{1}{2}p_2\omega^2 z = \tfrac{1}{2}(1/\theta_2)\left[\theta_1 p + \tfrac{1}{2}(\theta_2 - \theta_1)\right], \tag{15.17}$$

where $p = p_2/p_1$. Similarly, $\omega z = [E(1/\theta)][(\theta_1/p_1) + \frac{1}{2}(\theta_2 - \theta_1)/p_2]$ and

$$p_1\omega z = [E(1/\theta)]\left[\theta_1 + \tfrac{1}{2}(\theta_2 - \theta_1)(1/p)\right], \tag{15.18}$$

where $E(1/\theta) = (1/2)(1/\theta_1) + (1/2)(1/\theta_2)$. The first order condition (15.12) can now be written as:

$$\begin{aligned}&[E(1/\theta)]\left[\theta_1 + \tfrac{1}{2}(\theta_2 - \theta_1)(1/p)\right]\\ &\quad = \tfrac{1}{2}(1/\theta_2)\left[\theta_1 p + \tfrac{1}{2}(\theta_2 - \theta_1)\right]\\ &\quad = 2(x_1 + x_2).\end{aligned} \tag{15.19}$$

Market-clearing conditions are now:

$$\theta_1\lambda/p_1 = x_1 \quad \text{and} \quad (\theta_2 - \theta_1)\lambda/p_2 = x_2. \tag{15.20}$$

Dividing the first equation in (15.20) by the second yields:

$$p\theta_1/(\theta_2 - \theta_1) = x_1/x_2. \tag{15.21}$$

Equilibrium is a solution (p, x_1, x_2) to (15.19) and (15.21). Note that this is a system of three equations in three unknowns: (p, x_1, x_2) which do not depend on the parameter λ.

Using the price in the first market as a numeraire, the distribution of relative prices is: A fraction $x_1/(x_1 + x_2)$ of quoted prices is unity and a fraction $x_2/(x_1 + x_2)$ of quoted prices is $p = p_2/p_1$. This distribution of relative prices depends only on the distribution of θ and not on the expected rate of change λ. Thus,

Claim: Anticipated money is neutral: It does not affect the allocation of resources and the distribution of relative prices.

15.4 LABOR CHOICE, AVERAGE CAPACITY UTILIZATION AND WELFARE

The first order condition (15.12) implies $p_1\omega z = (1/2)p_2\omega^2 z$ and $1/p = p_1/p_2 = (1/2)(\omega^2/\omega)$. Substituting this and $\omega^2/\omega = (1/\theta_2)/[E(1/\theta)]$ in (15.18) leads to:

$$w = p_1\omega z = \tfrac{3}{4} + \tfrac{1}{4}(\theta_1/\theta_2). \tag{15.22}$$

Since by construction $\theta_2 \geq \theta_1$, the real wage is maximized at the level of unity when $\theta_2 = \theta_1$ and there is no uncertainty about the money supply.

Average capacity utilization: Using the market-clearing conditions (15.20) and $1/p = (1/2)(\omega^2/\omega) = (1/2)(1/\theta_2)/[E(1/\theta)]$ leads (after substantial amount of algebra) to:

$$CU = \tfrac{1}{2}x_1/(x_1 + x_2) + \tfrac{1}{2} = \tfrac{3}{4} + \tfrac{1}{4}(\theta_1/\theta_2), \tag{15.23}$$

where CU is expected capacity utilization. When $\theta_1 = \theta_2$ there is no uncertainty about the money supply, only one market is active (the demand in the second market $(\theta_2 - \theta_1)\lambda = 0$) and average capacity utilization is maximized.

Comparing (15.23) to (15.22) reveals:

$$w = CU. \tag{15.24}$$

This is because in equilibrium a supply of one unit of labor leads to an increase in CU units of consumption on average.

Welfare: The welfare of the representative consumer can be measured by:

$$V(w) = \max_x wx - (x)^2. \tag{15.25}$$

Since V(w) is an increasing function, welfare is maximized when monetary uncertainty is eliminated and $w = CU$ is maximized.

15.5 A GENERALIZATION TO MANY POTENTIAL MARKETS

We now generalize the transfer process. The variable θ is an i.i.d. random variable with S possible realizations: $\theta_1 < \theta_2 < \cdots < \theta_S$. The realization θ_i occurs with probability Π_i. For convenience we set $\theta_0 = 0$.

At the first location the buyer receives a transfer of $(\theta_1\lambda - 1)$ dollars per dollar, where $\lambda - 1$ denotes the anticipated rate of change in the money supply. At this stage the buyer has

$(\theta_1 - \theta_0)\lambda M_t$ dollars which he spends in the first market. If there are no additional transfers, trade for period t ends and the buyer consumes whatever was bought in the first location. But, with probability $q_2 = 1 - \Pi_1$, he gets an additional transfer of $(\theta_2 - \theta_1)\lambda M_t$ dollars. If he gets it he spends it. In general, the transfer $(\theta_s - \theta_{s-1})\lambda M_t$ will be realized with probability $q_s = \sum_{j=s}^{S} \Pi_j$ and the buyer spends it immediately after getting it. There are S different locations and each transfer is spent in a different location. The end of period money supply is: $M_{t+1} = \theta\lambda M_t$.

From the representative seller's point of view demand arrives in batches. The first batch of $(\theta_1 - \theta_0)\lambda$ normalized dollars arrives with certainty. The second batch of $(\theta_2 - \theta_1)\lambda$ normalized dollars arrives with probability q_2 and so on. Each batch of dollars that arrives opens a new market.

A normalized dollar this period is worth $\omega^s = 1/\lambda\theta_s$ normalized dollars next period if exactly s markets open this period. A normalized dollar earned in market s is worth

$$\omega_S = \sum_{j=s}^{S} (\Pi_j/q_s)\omega^j, \tag{15.26}$$

in terms of next period's normalized dollars. To build (15.26) recall that Π_j is the unconditional probability that exactly j markets open today. When market 1 opens the seller does not learn anything and does not revise these probabilities. But when market 2 opens, the seller learns that at least 2 markets will open today. He therefore applies Bayes rule and assigns a probability of zero to the event that exactly one market will open today and Π_j/q_2 to the event that exactly $j \geq 2$ markets will open today. In general, when market s opens the revised probability that exactly j markets will open is: Π_j/q_s if $j \geq s$ and zero otherwise.[1]

There is a "winner curse" here. A seller who succeeds in selling must take into account the adverse implications of his success on the money supply. Specifically, an increase in the number of markets open today reduces the value of a normalized dollar in terms of next period's normalized dollars ($\omega^j > \omega^{j+1}$) and therefore:

$$\omega_s > \omega_{s+1}. \tag{15.27}$$

This says that a normalized dollar earned in market s is worth on average more than a normalized dollar earned in market s + 1.

Since a normalized dollar held at the beginning of the period promises a transfer of $(\theta_s - \theta_{s-1})\lambda$ normalized dollars if market s opens, the expected purchasing power of a normalized dollar held at the beginning of the period is:

$$z = \sum_{s=1}^{S} q_s(\theta_s - \theta_{s-1})\lambda/p_s, \tag{15.28}$$

where p_s is the normalized price in market s. The expected purchasing power of a normalized dollar earned in market s is $\omega_s z$ and the expected purchasing power that one can get by supplying a unit to market s is: $q_s p_s \omega_s z$. The representative young agent's problem is to choose the capacities x_s which solve:

$$\max \sum_s q_s(p_s \omega_s z)x_s - v\left(x = \sum_s x_s\right), \tag{15.29}$$

where v(x) is the utility cost of producing x units of capacity. The first order conditions for an interior solution to (15.29) require that the expected real revenue per unit of capacity is equal to the marginal cost:

$$q_s(p_s \omega_s z) = v'(x) \quad \text{for all s.} \tag{15.30}$$

Since $(\theta_s - \theta_{s-1})\lambda$ normalized dollars will buy in market s if it opens, the market-clearing conditions are:

$$(\theta_s - \theta_{s-1})\lambda / p_s = x_s \quad \text{for all s.} \tag{15.31}$$

A solution $(p_1, p_2, \ldots, p_S, x_1, x_2, \ldots, x_S)$ to (15.30) and (15.31) is a symmetric steady-state equilibrium.

Note that from (15.27) and (15.30) it follows that

$$q_s p_s = q_{s+1} p_{s+1}(\omega_{s+1}/\omega_s) < q_{s+1} p_{s+1}. \tag{15.32}$$

The expected normalized price in market s is less than the expected normalized price in market s + 1 because a normalized dollar earned in market s worth more in terms of next period's normalized dollars.

Equation (15.30) implies: $p_s/p_1 = (q_1/q_s)(\omega_1/\omega_s)$. Since $\omega_1/\omega_s > 1$, the "winner curse" effect contributes to price dispersion.

15.6 ASYMMETRIC EQUILIBRIA: A PERFECTLY FLEXIBLE PRICE DISTRIBUTION IS CONSISTENT WITH INDIVIDUAL PRICES THAT APPEAR TO BE "RIGID"

In the symmetric steady-state equilibrium each seller allocates a fraction $\mu_s = x_s/x$ of his capacity to market s. Since our risk neutral sellers are indifferent about the way they allocate capacity across markets, there exists an asymmetric equilibrium in which a fraction μ_s of the sellers supply all their output to market s.[2] In this asymmetric equilibrium it is enough that only some sellers will change their posted dollar prices in response to an increase in the money supply, because most sellers are compensated for the reduction in the normalized price by the increased probability of making a sale.

To illustrate, we assume that the range of the equilibrium distribution of normalized prices $p = P/M$ is between $p = 1$ to $p = 4$. Assume that in the previous period the money supply was $M = 10$ and in the current period it doubled to $M' = 20$. A seller whose dollar price in the previous period was $P = 30$ (and his normalized price was 3) will have a normalized price of 1.5 in the current period if he does not change his dollar price. This seller will not care about the decline in his normalized price because he is compensated by the increase in the probability of making a sale. But a seller whose dollar price was $P = 15$ (and normalized price was 1.5) will have a current normalized price of 0.75 if he does not change his dollar price. Since this seller can increase his dollar price to 20 (and normalized price to 1) without affecting the probability of making a sale, he will definitely choose to increase his dollar price. Thus, the distribution of normalized prices may be always in equilibrium even when some sellers do not change their dollar prices.

15.7 SUMMARY OF THE IMPLICATIONS OF THE MODEL

The main implications of the model are as follows.

1 There is a positive correlation between capacity utilization and unanticipated money. If we measure output as capacity which is utilized (a meal prepared but not sold is not counted as output) then the model predicts a positive correlation between output and unanticipated money.
2 Money surprises affect output first and quoted prices only in the following period.
3 The distribution of relative prices (p_s/p_1) depends only on the distribution of the unanticipated rate of change in the money supply θ and not on the expected rate of change λ.
4 The expected real wage is equal to average capacity utilization. Both are maximized in the absence of monetary uncertainty (when θ is a degenerate random variable). Welfare is a monotonic function of the expected real wage and it is also maximized in the absence of monetary uncertainty.
5 Monetary uncertainty reduces the level of average output (relative to the case of no uncertainty) because it reduces average capacity utilization and the expected real wage.
6 Changes in nominal prices need not be synchronized across sellers. Sellers may let inflation erode their relative price as long as it stays in the equilibrium range. This is because the reduction in real price is compensated by the increase in the probability of making a sale. Therefore, although the distribution of dollar prices adjusts to changes in the money supply with a one period lag, the dollar price of a particular seller may appear "rigid."

PROBLEMS WITH ANSWERS

1 Consider the following variation of the example in section 15.2. We assume now that the buyer may get, with probability 1/2, a transfer payment of two (rather than one) dollars per dollar held at the beginning of the period.

(a) Write the equilibrium conditions;
(b) What is the ratio of p_1/p_2?

Answer

(a) $\omega^1 = 1,\ \omega^2 = 1/3,\ \omega = 2/3;$
$z = 1/p_1 + 1/p_2$
First order conditions:
$p_1\omega z = (1/2)p_2\omega^2 z = 2(x_1 + x_2)$
Market-clearing conditions:
$1/p_1 = x_1$ and $2/p_2 = x_2$.
(b) $p_1/p_2 = (1/2)\omega^2/\omega = 1/4$

2 Answer 1 under the assumption that the transfer payment is in a lump sum form.

Answer

(a) $\omega^1 = 1,\ \omega^2 = 1/3,\ \omega = 2/3;$
$z = 1/p_1$
First order conditions:
$p_1 \omega z = (1/2) p_2 \omega^2 z = 2(x_1 + x_2)$
Market-clearing conditions:
$1/p_1 = x_1$ and $2/p_2 = x_2$.
(b) $p_1/p_2 = (1/2)\omega^2/\omega = 1/4$

NOTES

1 To apply Bayes rule recall that Prob $(A|B)$ = Prob(A and B)/Prob(B). Here the event A is "exactly j markets open"; the event B is "at least s markets open"; Prob(A and B) = Prob(A) = Π_j if $j \geq s$ and zero otherwise (because A implies B but not vice versa) and Prob(B) = q_s. Thus Prob$(A|B) = \Pi_j/q_s$ if $j \geq s$ and zero otherwise.

2 There are many other asymmetric equilibria arising from the observation that equilibrium conditions (15.36) and (15.37) determine the total capacity allocated to each market and not the number of sellers in each market.