EFFICIENT BARRIERS TO TRADE: A SEQUENTIAL TRADE MODEL WITH HETEROGENEOUS AGENTS

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ABSTRACT

This paper studies the choice of tariffs and other type of consumption taxes and subsidies in a flexible price version of the Prescott (1975) hotels model. It is shown that a country with unstable demand may benefit from a tariff on imports. More surprisingly, the exporting country may also benefit from the tariff. In general, I consider the problem of a world planner who chooses country specific consumption taxes and subsidies. I show that buyers in countries that tend to consume relatively more in the high demand state should be taxed and buyers in countries that tend to consume relatively more in the low demand state should be subsidized.

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KEY WORDS: Optimal Tariffs, Sequential Trade, Demand Uncertainty, Terms of Trade

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1. INTRODUCTION

The predictability of demand is different across countries. Recently Stock and Watson (2003, Table 2) estimated the predictability of GDP for the G-7 countries. Their estimates imply that since 1984 the one-step ahead forecast RMSE for Japan was higher than the RMSE for the US by 90%. Aguiar and Gopinath (2007) found that emerging markets experience substantial volatility in trend growth and that the average standard deviation of change in output in emerging markets is about twice as that of developed markets. Here I study the implications of differences in predictability for the gains from trade question and the effect of barriers to trade.

The paper complements the analysis in Eden (2007). In this earlier paper I show that a country with a stable demand may suffer from trade with a country with unstable demand, and describe the trade patterns that may emerge. Here I focus on the choice of subsidies, taxes and tariffs. This leads to some surprising results. I show that a tariff, for example, may lead to a Pareto improvement over the free trade outcome.

I use a version of the Prescott (1975) hotels model. In Prescott's model, sellers of motel rooms set prices before they know how many buyers will arrive. Prescott assumes that cheaper rooms are sold first and therefore in equilibrium sellers face a tradeoff between price and the probability of making a sale.

In Prescott's example all motel rooms are the same and all buyers who arrive want a single room and are willing to pay up to the same reservation price. Dana (1998) has extended the rigid price version of the Prescott's model to the case of heterogeneous potential buyers who demand at most one unit and have different valuation and different probabilities of becoming active. He shows that firms in the Prescott model have incentives to offer advance-purchase discounts and in equilibrium advance purchase sales are made to low valuation customers. He concludes that because of price rigidity the equilibrium allocation may not be as good as the Walrasian outcome (first best) even when buyers have a demand for one unit only. Deneckere and Peck (2005) show that once we allow for heterogeneity, the allocation is not first best. But we can achieve the first best if we allow buyers to return for a second round of trading.

A flexible price version of the Prescott model is in Eden (1990, 2005) and Lucas and Woodford (1993). This approach assumes that buyers arrive sequentially, see all available offers and after buying at the cheapest available offer they consume and go elsewhere. I refer to this version of the Prescott model as the Uncertain and Sequential Trade (UST) model.

From a positive economics point of view it does not matter whether prices in the model are flexible or rigid. But for the question of efficiency, which is the focus of this paper, it does matter. I show that the UST outcome is efficient if the probability of becoming active does not depend on the buyer's type, even when buyers have different downward sloping demand functions. I now show by example that the UST allocation may not be efficient when the probability of becoming active depends on the buyer's type. In the example, countries benefit from barriers to trade.

2. AN EXAMPLE

I consider a single period economy with two goods (X and Y where lower case letters denote quantities) and two states of nature (1 and 2) that occur with equal probability. There are three types of agents. All types are risk neutral and get a large endowment of Y. Type 0 agents (sellers) can produce X at the per unit cost of λ units of Y, where in this example I assume: $\lambda=1$. Sellers derive utility only from Y and their utility function is: $u^0(x,y)=y$. Only sellers can produce X. The other two types are the potential buyers of X. Type 1 agents derive utility from both goods and their utility function is: $u^1(x,y)=U(x)+y$. Type 2 agents derive utility from X only in state 2. Their random utility function is: $u^2(x,y)=y$ in state 1 and $u^2(x,y)=V(x)+y$ in state 2. I assume that $U(x)=\alpha\ln(x)$ and $V(x)=\beta\ln(x)$, where $\alpha=20$ and $\beta=10$.

The number of buyers from each type is the same and is normalized to unity. I use $d(p)=\arg\max U(x)-px={}^{\alpha}\!/_{p}$ to denote the demand function of a type 1 agent at the price of p and $d^*(p)=\arg\max V(x)-px={}^{\beta}\!/_{p}$ to denote the demand of a type 2 agent in state 2. Type 1 buyers reside in country 1 (the home country) and type 2 buyers reside in country 2. There are sellers in both countries and the number of sellers in each country is known.

Production choices are made before the state is known.

<u>Autarky</u>: In country 1 there is no uncertainty about demand and therefore the standard Walrasian model can be applied. The price of X is $\lambda=1$ and the supply of X is $\alpha=20$. The buyer's surplus is: $20\ln(20)-20=39.915$.

In country 2 the good is sold only in state 2. In state 1 there is no demand and the output is wasted. The seller in country 2 chooses x to maximize $(1/2)px-x$. An interior solution to this problem requires: $p=2$. The price $p=2$ compensates the seller for the risk of not making a sale and at this price he is willing to produce any amount. Since in state 2 the demand at the price of 2 is 5, sellers will produce 5 units that will be sold and used only in state 2. Welfare is: $(1/2)10\ln(5)-5=3.047$.

<u>Free trade</u>: I now assume a fully integrated world. The cost of transporting goods across countries is small and will be treated as zero in most of the analysis. Trade is done on the internet in a sequential manner. Buyers who want to consume place orders. Those who get first on line buy at a low price. Those who go on line relatively "late" may have to pay a higher price for the good.

In state 1 only country 1 buyers want to consume and they will all be able to buy at the low price. In state 2 not all buyers will be able to buy at the low price and those who are "late" in placing their order will buy at the high price. Thus as in the autarkic case we have two prices but now the identity of those who buy at the low price (in state 2) is determined by a lottery that treats all active buyers symmetrically.

It is convenient to assume two hypothetical markets: The first market opens with probability 1 at the price of 1 and the second market opens with probability $1/2$ at the price of 2. The supplies to the two markets are perfectly elastic and sellers satisfy the demand in markets that open. The minimum demand at the price of 1 is $d(1) = \alpha$ units. To satisfy the minimum demand sellers supply $x_1 = d(1) = \alpha$ units to the first market.

In state 1, only type 1 buyers are active and they all buy in market 1. In state 2 both types are active. Since buyers are treated symmetrically, the average demand per buyer is: $A = (1/2)[d(1) + d^*(1)] = (1/2)(\alpha/1) + (1/2)(\beta/1) = (1/2)(\alpha + \beta) = 15$ and $\Delta = x_1/A = 2\alpha/(\alpha+\beta) = 4/3$ buyers will be serviced in market 1. The fraction of buyers serviced in market 1 is: $\theta = \Delta/2 = \alpha/(\alpha+\beta) = 2/3$. The remaining $1-\theta = 1/3$ buyers from each type will buy at the price 2. The demand in the second market is: $(1/3)(\alpha/2) + (1/3)(\beta/2) = (1/6)(\alpha+\beta) = 5$. To satisfy the demand of all buyers who could not make a buy in the first market, sellers supply to the second market $x_2 = 5$ units.

Equilibrium is thus as a vector $(p_1, p_2, x_1, x_2, A, \Delta, \theta)$ such that:

$p_1 = \lambda = 1$ = the price in the first market;

$p_2 = 2\lambda = 2$ = the price in the second market;

$x_1 = d(p_1) = \alpha/p_1$ = first market clearing condition;

$A = (1/2)[d(p_1) + d^*(p_1)]$ = average demand in state 2 at the low price;

$\Delta = x_1/A$ = the number of buyers who buy at the low price in the high demand state;

$\theta = \Delta/2$ = the fraction of buyers who buy at the low price in the high demand state;

$x_2 = (1-\theta)[d(p_2) + d^*(p_2)] = (1/6)(\alpha+\beta)$ = second market clearing condition.

This is equilibrium in the sense that at the equilibrium prices sellers cannot make profits and markets that open are cleared.

When transportation costs are literally zero, the identity of the sellers in each market is not determined. But if the buyer has to pay small transportation costs for shipping goods from the foreign country, he will prefer to buy from a local seller unless a foreign seller offers the good at a lower price. I therefore assume that only sellers from country 1 supply to the first market.[1]

In our numerical example, the quantities supplied to each market are the same as under autarky but now the identity of the buyers in each market is different and therefore the distribution of surpluses is different. The surplus for buyers in country 1 is: $(1/2)\left(20\ln(20)-20\right)+(1/2)\left\{(2/3)\left(20\ln(20)-20\right)+(1/3)\left(20\ln(10)-20\right)\right\}=37.604$. The surplus in country 2 is: $(1/2)\left\{(2/3)\left(10\ln(10)-10\right)+(1/3)\left(10\ln(5)-10\right)\right\}=5.358$. Thus,

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[1] When the buyer must pay ε units of Y per unit of X that is transported, there is no equilibrium in which sellers in country 2 supply to market 1. To see this, assume that sellers from country 2 sell in the first market at the producer price of 1 (otherwise, they make non zero profits). In this case, at the low demand state buyers pay $1+\varepsilon$ per unit and therefore sellers from country 1 can make profits by say selling at $1+\varepsilon/2$.

The identity of the sellers in market 2 is less important to our analysis. In the presence of transportation costs, local sellers will satisfy the demand of local buyers who arrived late and could not make a buy in market 1. To see that this must be the case, note that if a seller sells to a foreign buyer at the price of 2 then a foreign seller can make profits by selling at the price of say, $2+\varepsilon/2$. In our numerical example, sellers in country 1 supply 20 units to market 1 and $10/3$ units to market 2. Sellers in country 2 supply $5/3$ units to market 2 (and nothing to market 1). In Eden (2007) transportation costs are explicit.

relative to autarky, country 2 gains from trade and country 1 looses from trade, but total surplus is the same: $37.604 + 5.358 = 42.962$.

<u>Tariffs, Export Taxes and Subsidies</u>: Is it possible to improve on the free trade outcome? To answer this question I assume that there are governments in both countries that can affect prices. I limit the choice of policy instruments. The government in country 1 may choose a subsidy of $\sigma \geq 0$ per unit of the good bought by local buyers from local producers at the cheaper price of 1. It can also charge an export tax of $\eta \geq 0$ per unit. The government in country 2 may choose a tariff of $\tau^* \geq 0$ per unit, where $\tau^* + \eta \leq 1$.

The producer prices are 1 in the first market and 2 in the second market. Country 1 buyers pay $1-\sigma$ in the first market and 2 in the second market (the subsidy is given only if you buy from a local seller at the price of 1). Country 2 buyers pay $1+\tau^* + \eta \leq 2$ in the first market and 2 in the second market. Since country 1 buyers pay $1-\sigma$ per unit in the first market, sellers in country 2 cannot guarantee the making of a sale at a producer price of 1. Therefore when $\sigma > 0$, only country 1 sellers supply to the first market. Because of small transportation costs, I assume that this is also the case when $\sigma = 0$.

The clearing of the first market requires:

$$x_1 = d(1-\sigma) = \frac{\alpha}{1-\sigma} \tag{1}$$

In the high demand state (state 2) buyers from country 2 who make it to the first market will buy the good at the price of $1+\tau^* + \eta$. The demand per country 2's buyer in the first market is therefore:

$$d^*(1+\tau^* + \eta) = \frac{\beta}{1+\tau^* + \eta}. \tag{2}$$

The average demand per buyer in market 1 (state 2) is: $A = (½)\left(d^*(1+\tau^*+\eta)+d(1-\sigma)\right)$. The number of buyers serviced in market 1 is: $\Delta = \frac{x_1}{A}$. The fraction of buyers who buy in market 1 in the high demand state is:

$$\theta(\sigma,\eta,\tau^*) = \frac{\Delta}{2} = \frac{d_1(1-\sigma)}{d_1(1-\sigma)+d_2(1+\tau^*+\eta)} = \frac{\alpha(1+\tau^*+\eta)}{\alpha(1+\tau^*+\eta)+\beta(1-\sigma)} \tag{3}$$

The participation function $\theta(\sigma,\eta,\tau^*)$ plays an important role in determining the effect of a change in policy on welfare. It is increasing in all its arguments. An increase in τ^* reduces the demand of type 2 buyers and the average demand per buyer (A) in the first market. Since the supply to the first market is not affected by a change in τ^* this leads to an increase in the number of buyers (Δ) that make a buy in the first market and increase in θ. An increase in η has the same effect. An increase in σ increases both the average demand per buyer (A) and the supply to the first market (x_1). But since the effect on the supply is larger, the ratio Δ goes up and θ goes up.

I now turn to specify welfare in each country as a function of the buyers' surplus in each market and government revenues. The revenues of government 1 are: $-\sigma d(1-\sigma)$ in state 1 and $\theta\left(\eta d^*(1+\tau^*+\eta)-\sigma d(1-\sigma)\right)$ in state 2. Summing over both states we get:

$$g(\sigma,\eta,\tau^*) = \theta\eta d^*(1+\tau^*+\eta)-\sigma(1+\theta)d(1-\sigma) \tag{4}$$

Summing the surplus of country 1 over both states we get:

$$W(\sigma,\eta,\tau^*) = [U(x_1)-(1-\sigma)x_1] \tag{5}$$
$$+\theta\left\{U\left(d(1-\sigma)\right)-(1-\sigma)d(1-\sigma)\right\}+(1-\theta)\left\{U\left(d(2)\right)-2d(2)\right\}+g$$

The first term on the right hand side (RHS) of (5) is the surplus that country 1 buyers make in state 1. The second two terms are the surplus that country 1 buyers make in state 2 and the last term is government's revenues.

The tariff revenues of government 2 are given by:

$$g^*(\sigma,\eta,\tau^*) = \theta\tau^* d^*(1+\tau^*+\eta) = \frac{\alpha\beta\tau^*}{\alpha(1+\tau^*+\eta)+\beta(1-\sigma)} \qquad (6)$$

The surplus of country 2 in state 2 is:

$$\begin{aligned} W^*(\sigma,\eta,\tau^*) = \theta\Big\{V\big(d^*(1+\tau^*+\eta)\big) - (1+\tau^*+\eta)d^*(1+\tau^*+\eta)\Big\} \\ +(1-\theta)\Big\{V\big(d^*(2)\big) - 2d^*(2)\Big\} + g^* \end{aligned} \qquad (7)$$

The first term on the RHS of (7) is the surplus earned by buyers who make a buy in market 1. The second term is the surplus earned by buyers in market 2. The last term is government revenues.

When country 2 is passive and chooses $\tau^* = 0$, the choice that maximizes (5) is: $\eta = 1, \sigma = 0.073$. Relative to free trade, this choice increases surplus in country 1 (from 37.604 to 40.587), and reduces the surplus in country 2 (from 5.358 to the autarkic level 3.047). It increases aggregate world surplus by roughly 1.5 percent: from 42.962 to 43.634.

When country 1 is passive and chooses $\eta = \sigma = 0$, country 2 will choose: $\tau^* = 0.24$. Relative to free trade this will improve welfare in both countries.

To better understand these surprising results I now turn to analyze the effects of changes in (σ,η,τ^*). For this purpose I define the per buyer surpluses:

$$\begin{aligned} S^* = V\big(d^*(1+\tau^*+\eta)\big) - (1+\eta)d^*(1+\tau^*+\eta)\ ;\ s^* = V\big(d^*(2)\big) - 2d^*(2); \\ S = U\big(d(1-\sigma)\big) - d(1-\sigma);\ s = U\big(d(2)\big) - 2d(2) \end{aligned} \qquad (8)$$

Thus, S^* is the surplus in market 1 per type 2 buyer (including government revenues); s^* is the surplus in market 2 per type 2 buyer; S is the surplus in market 1 (minus government spending on the subsidy) per type 1 buyer and s is the surplus in market 2 per type 1 buyer. Figure 1 illustrates the definition of S^*. From the point of view of country 2 the cost of a unit exported from country 1 is $1+\eta$. The tariff creates a distortion measured by the area D. The per type 2 buyer surplus in market 1 is $S^* = A + B$.

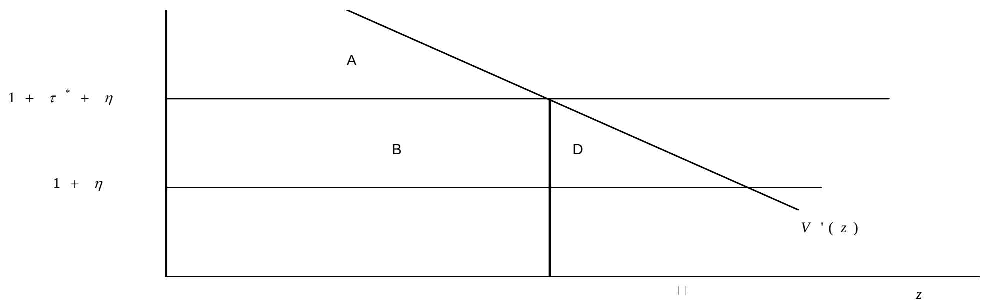

Figure 1: The surplus in market 1 per type 2 buyer: $S^* = A + B$

Figure 2 illustrates the definition of S. The cost of a unit bought in market 1 is $\lambda = 1$. The subsidy creates a distortion measured by the area B and $S = A - B$.

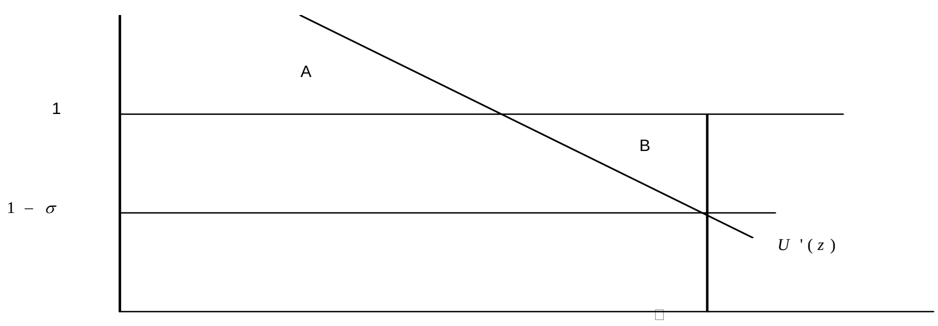

Figure 2: The surplus in market 1 per type 1 buyer: $S = A - B$

Using (8), welfare in country 2 can be written as $W^* = \theta S^* + (1-\theta)s^*$ and welfare in country 1 can be written as $W = S + \theta S + (1-\theta)s + \theta\eta d^*(1+\tau^*+\eta)$. A change in policy will affect the per buyer surplus in the first market (S and S^*) and the participation in the first market θ. I now turn to use these definitions to analyze changes in the policy parameters.

An increase in η: Figure 3 computes welfare in both countries as a function of η, where welfare is measured relative to the free trade level. As can be seen, the relationship between welfare in country 1 and η is increasing for country 1 and decreasing for country 2. An increase in η from zero to 0.1 decreases welfare in country 2 by close to 5% and increases welfare in country 1 by 1.2%.

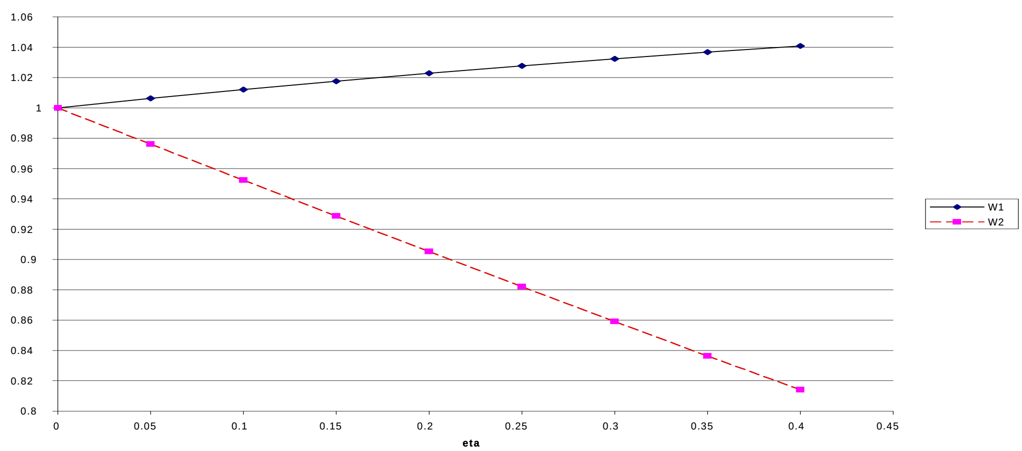

Figure 3: Welfare in country 1 (W1) and in country 2 (W2) as a function of η (assuming $\sigma = \tau^* = 0$)

The surplus in country 1 increases with η, because an increase in η has a positive effect on participation in the first market (θ) and on the revenues of the government. The surplus in country 2 decreases. The total surplus that type 2 buyers make in the second market, $(1-\theta)s^*$, decreases because θ increases. The per buyer surplus that type 2 make in the first market, S^*, decreases with η. It turns out that in our example this effect dominates and the total surplus in market 1, θS^*, is decreasing in η. Thus the surplus that type 2 buyers make decreases in both markets and therefore welfare in country 2 decreases with η.

<u>An increase in τ^*</u>: Figure 4 computes the effect of τ^* on normalized welfare. Welfare in country 1 is increasing in τ^* because an increase in τ^* increases the participation in the first market, θ. The relationship between welfare in country 2 and τ^* is "hump shaped".

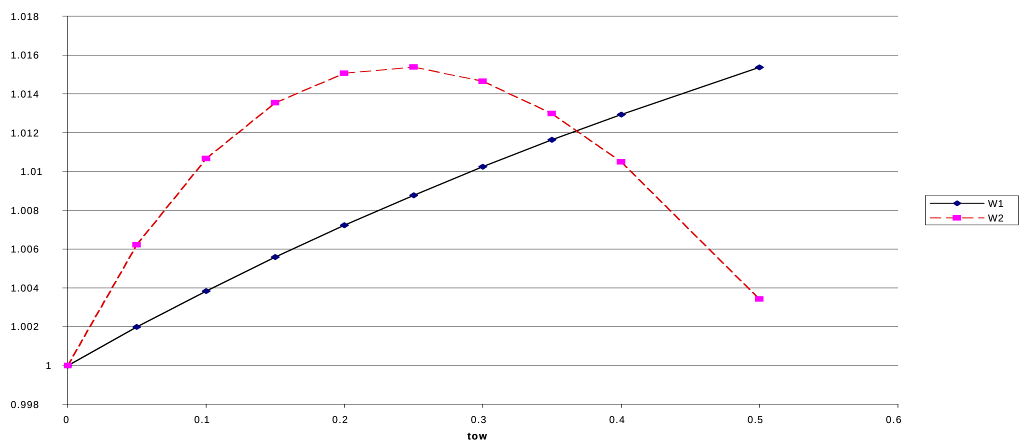

Figure 4: The effect of changes in τ^* on welfare (assuming $\sigma = \eta = 0$)

To understand the effect on welfare in country 2, note that the derivative of $W^* = \theta S^* + (1-\theta)s^* = \theta(S^* - s^*) + s^*$ with respect to τ^* can be written as:

$$\left(\partial S^* \big/ \partial \tau^*\right) \frac{1}{S^* - s^*} + \left(\partial \theta \big/ \partial \tau^*\right) \frac{1}{\theta} \tag{9}$$

This is the sum of two elasticities: The surplus per buyer elasticity and the participation elasticity. An increase in τ^* has a negative effect on the per buyer surplus and a positive effect on the participation in the first market.

Figure 5 illustrates the negative effect of an increase in τ^* on the per buyer surplus. An increase in τ^* by ε units leads to a reduction in S^* by the area d in Figure 5. When V' is close to linear, the area d is larger the larger τ^* is and therefore $\left| \partial S^* / \partial \tau^* \right|$ is increasing in τ^*. Since the difference $S^* - s^*$ declines with τ^* the absolute value of the first term in (9) increases with τ^*.

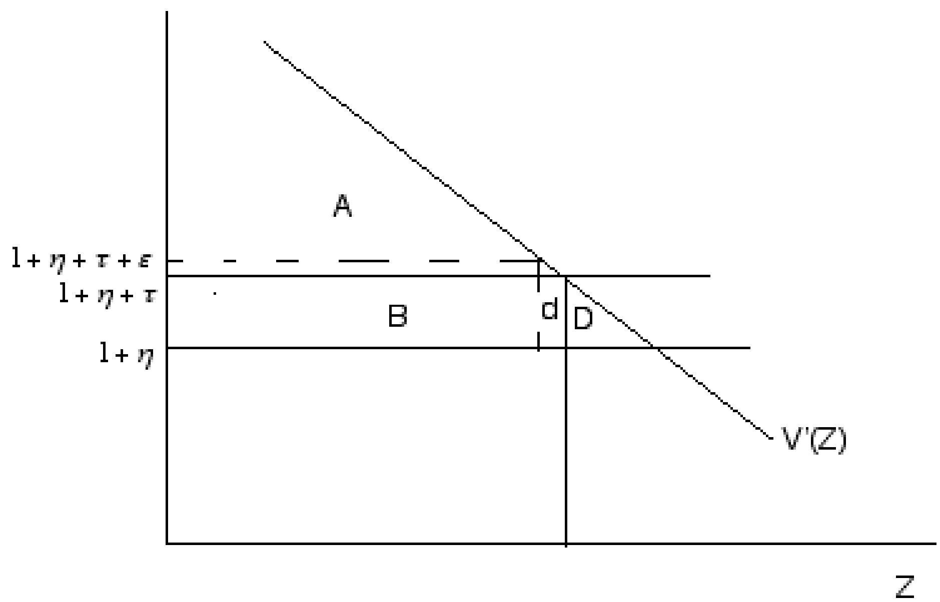

Figure 5: S^* declines with τ^* and $\left| \partial S^* / \partial \tau^* \right|$ is increasing in τ^*.

In our example, $\partial S^* / \partial \tau^* = 0$ when $\tau^* = 0$ and therefore the first term in (9) is zero when $\tau^* = 0$. The elasticity $\left(\partial \theta / \partial \tau^* \right) \frac{1}{\theta} = \frac{\beta\alpha(1-\sigma)}{\left(\alpha(1+\tau^*+\eta)+\beta(1-\sigma)\right)\alpha(1+\tau^*+\eta)}$ is strictly positive when $\tau^* = 0$. Therefore initially the welfare in country 2 is increasing in τ^*. When τ^* is large the first term in (9) dominates and welfare is decreasing in τ^*. This leads to the “hump shape” relationship in Figure 4.

An increase in σ: Figure 6 computes normalized welfare as a function of σ. As can be seen, the relationship between welfare in country 1 and σ is "hump shaped". Welfare in country 2 is increasing in σ because of the positive effect of σ on θ. In our numerical example, an increase in σ from zero to 0.1 increases welfare in country 2 by 1.5% and welfare in country 1 by only 0.17%.

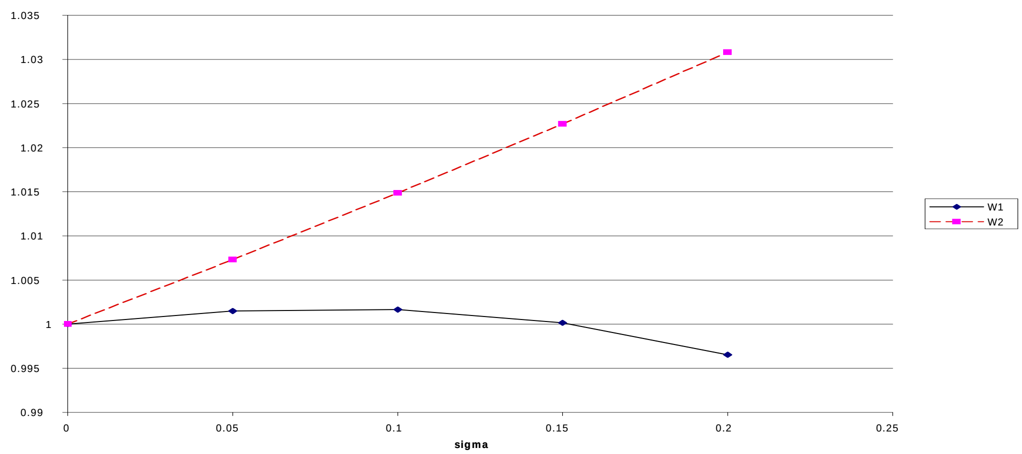

Figure 6: Welfare as a function of σ (normalized; assuming $\tau^* = \eta = 0$)

To understand the relationship between σ and the welfare in country 1, I take the derivative of W with respect to σ:

$$\partial W / \partial \sigma = \left(\partial S / \partial \sigma \right)(1+\theta) + \left(\partial \theta / \partial \sigma \right)\left(S - s + \eta d^*(1+\eta+\tau^*) \right) \quad (10)$$

When σ is small, $\partial W / \partial \sigma > 0$ and when σ is large, $\partial W / \partial \sigma < 0$. To see this I start with the surplus in market 1 per type 1 buyer: $S = A - B$ in Figure 7. Since an increase in σ leads to an increase in the area B, the derivative $\partial S / \partial \sigma$ is negative. Increasing σ by ε units reduces welfare by the area d. When U' is close to linear this area is larger the larger σ is. Therefore the absolute value of the derivative $\partial S / \partial \sigma$ increases with σ. The second term on the RHS of (10) is positive because the supply to the first market and therefore the participation in the first market (in the high demand state) increases with σ. When σ is low the second term dominates. When σ is large the first term dominates and therefore the relationship between σ and welfare in country 1 is "hump shaped".

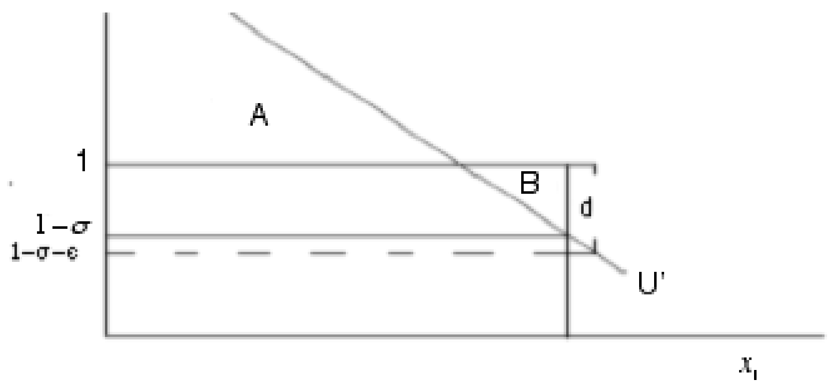

Figure 7: The surplus per type 1 buyer in market 1 (S) is decreasing in σ and the effect of an increase in σ is larger when σ is large.

Overall, in our example, the effect of changing the policy instruments on welfare in country 2 is much larger than the effect on welfare in country 1 and the effect of changing η is much larger (in absolute value) than the effect of changing τ^* and σ.

Figure 8 plots indifference curves in the (σ, τ^*) plane. The broken curves are for country 2. An increase in σ improves welfare in country 2 because of its positive effect on θ. An increase in τ^* improves welfare in country 2 when τ^* is small and reduces welfare when τ^* is large. The slope of the indifference curves for country 2 is therefore negative for small τ^* and positive for large τ^*. The slope is zero when τ^* is about 0.24. The solid curves are for country 1. Welfare in country 1 increases with τ^* because of its positive effect on θ. Welfare in country 1 increases with σ when σ is small and decreases with σ when σ is large. Here I plot only the indifference curves in the relevant range when σ is large ($\sigma \geq 0.073$). Note that welfare for country 1 increases in the South East direction. A Pareto efficient choice of the policy instruments must have τ^* at the range in which the indifference curves of country 2 are upward sloping ($\tau^* > 0.24$).

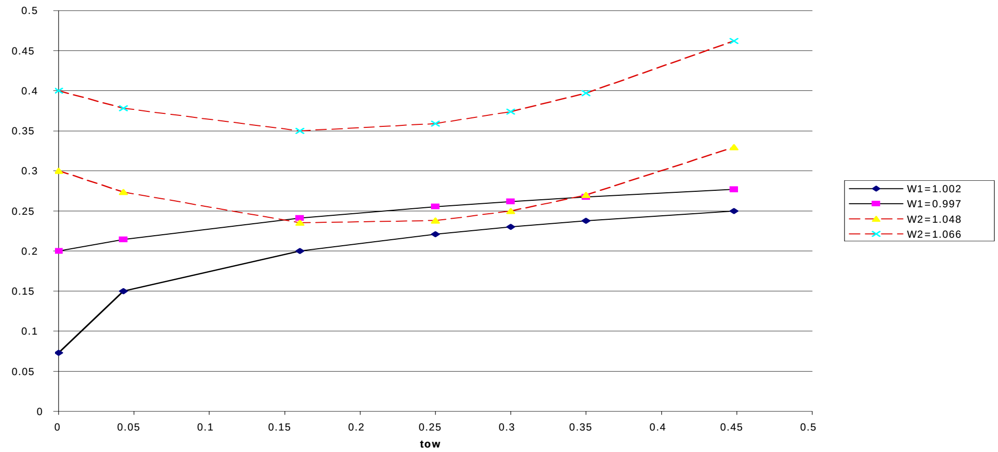

Figure 8: Indifference curves in the (τ^*,σ) plane

<u>A planner's problem</u>: If the two countries enter negotiations they may choose to set country specific consumer prices to maximize total surplus and then divide the surplus between them. Here I consider the problem of choosing country specific prices in the first market only assuming that the second market consumer price is 2.

The optimal first market prices may be achieved by an appropriate choice of the policy parameters (σ,η,τ^*). Since (η,τ^*) have a symmetric effect on the price paid by country 2 buyers, it is enough to use only two policy parameters: (σ,η) or (σ,τ^*).

Figure 9 plots aggregate normalized welfare ($W + W^*$ divided by the free trade level) as a function of τ^* for various σ. The best policy is in the neighborhood of $\tau^* = 0.7$ and $\sigma = 0.1$. The maximum aggregate welfare is obtained for $\tau^* = 0.71$ and $\sigma = 0.08$ and in this case aggregate world welfare increases by 1.7%.

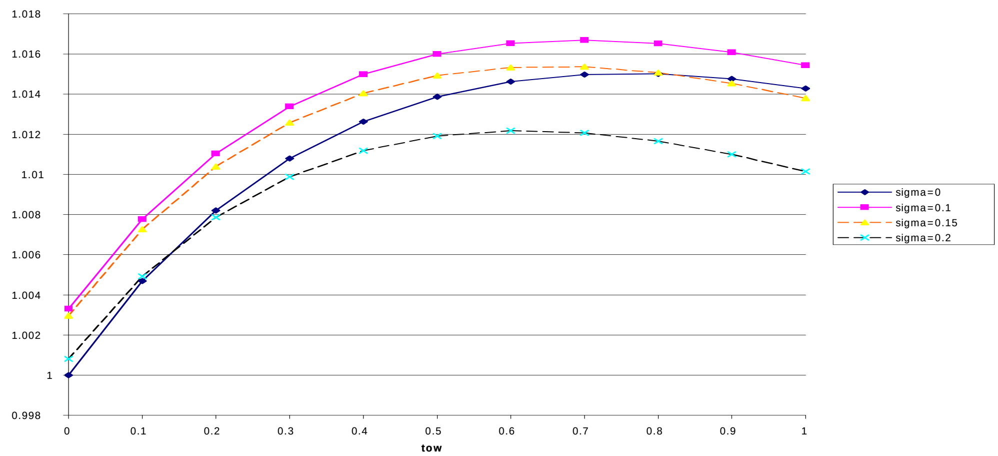

Figure 9: Aggregate welfare as a function of τ^* for different levels of σ

Table 1 summarizes some of the results obtained for the above example where all the surpluses are divided by the free trade level.

Table 1: Normalized surpluses

	Country 1	Country 2	Aggregate
Autarky	1.06	0.57	1
Free Trade	1	1	1
Planner: $\sigma = 0.08$ $\tau^* = 0.71,$ $\eta = 0$			1.017
Gov. 1 active: $\sigma = 0.073,$ $\eta = 1,$ $\tau^* = 0$	1.079	0.57	1.016
Gov. 2 active: $\tau^* = 0.24,$ $\eta = \sigma = 0$	1.008	1.015	1.009

The Table shows that a Pareto improvement over the free trade equilibrium is possible and the free trade equilibrium is not a Nash equilibrium: if one country is passive the other country can improve matters by some intervention. To get intuition I now turn discuss two variations on the UST environment.

<u>A Walrasian environment:</u> I consider now the problem of a Walrasian planner who unlike the sellers in our model can distribute output after he knows the state (but capacity choice must still be made before the state is known). This is the peak-load pricing model analyzed by Williamson (1966).

Let x denotes total capacity and k denotes the amount that the Walrasian planner allocates to buyers from country 2 in the high demand state. The planner maximizes the world's expected surplus by solving: $\max_{x,k} (\frac{1}{2})\left(U(x)+U(x-k)+V(k)\right)-\lambda x$. The first order conditions for this problem are: $(\frac{1}{2})\left(U'(x)+U'(x-k)\right)=\lambda$ and $V'(k)=U'(x-k)$. The solution is: $x=3k=24.9$, $k=8.3$, $V'(k)=U'(x-k)=1.2$ and $U'(x)=0.8$. Figure 10 illustrates. The curves are the marginal utilities of the buyer from country 1 (in the two states and the expected marginal utility) as a function of x taking $k=8.3$ as given.

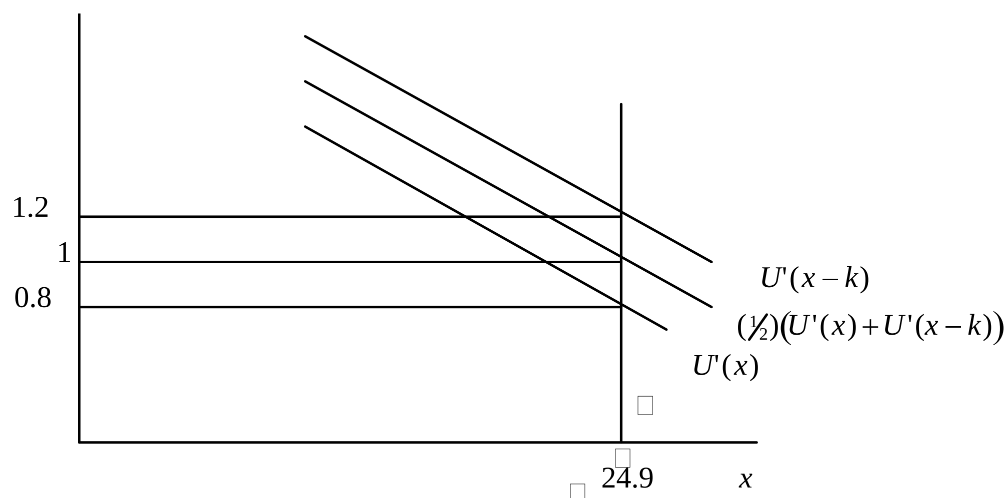

Figure 10: The solution to the problem of a Walrasian planner

The Walrasian planner charges all active buyers in the high demand state the same price of 1.2 and all active buyers in the low demand state the price of 0.8. By doing that he achieves a total surplus of 52.459 which is higher by 22% relative to the free trade case in our UST environment. This implies that in our example removing the constraint that the allocation to the first batch of buyers must be made before the state of demand in known will increase welfare by about 20%.

The Walrasian planner chooses prices that depend on the state. Prices in the UST environment cannot depend on the state but may depend on the country of residence and the transaction price. The UST planner has less information than the Walrasian planner but he tries to do the same thing: Charge a higher price in the high demand state. The UST planner charges a higher price from type 2 buyers that arrive in the first market because he knows that type 2 buyers arrive only in the state of high demand.

A related sequential environment: In the UST model the fraction of buyers who make a buy in the first market (θ) is endogenous. To gain additional intuition, I now consider an environment in which this fraction is exogenous.

I assume that production occurs at $t=0$ at the cost of $\lambda=1$ per unit. Trade occurs in real time: It occurs at $t=1$ and may occur at $t=2$ (with probability ½). The number of buyers that will arrive at $t=1$ is known and is normalized to one. The composition of buyers at $t=1$ is not known. In state 1 all buyers are from country 1. In state 2 half are from country 1 and half are from country 2. The number of buyers who arrive at date 2 is zero in state 1 and one in state 2. If buyers arrive at date 2 they consist of an equal number from both countries.

Sellers announce prices at $t=0$ and must satisfy demand at the announced prices. The producer price at $t=1$ is 1 and the producer price at $t=2$ is 2.

There is a world government (a planner) that can levy date and country specific consumption taxes (which may be negative as in the case of subsidy) and collect lump sum taxes. The planner announces the tax rates at $t=0$. Similar to prices, tax rates are sticky and cannot be changed after $t=0$.

Buyers from country 1 (country 2) pay at $t=1$ the price T (T^*), where T and T^* are the gross consumption tax rates imposed by the planner. There are no taxes at $t=2$ and at this date buyers pay the price of 2 if they want to consume.

Sales at each date depend on the state. I use x_{ts} to denote sales at date t in state s, where: $x_{11}=d(T)$, $x_{12}=(1/2)d(T)+(1/2)d^*(T^*)$, $x_{21}=0$, $x_{22}=(1/2)d(2)+(1/2)d^*(2)$. Assuming that the maximum demand occurs in state 2 the cost of production is: $(1/2)d(T)+(1/2)d^*(T^*)+(1/2)d(2)+(1/2)d^*(2)$. Since the sellers cannot change prices, I assume that the planner cannot change the tax rates that he announces at $t=0$. The planner chooses the gross tax rates T and T^* by solving the following problem.

$$\max_{T,T^*}(1/2)U(d(T))+(1/2)\left\{(1/2)U(d(T))+(1/2)V\left(d^*(T^*)\right)+(1/2)U(d(2))+(1/2)V\left(d^*(2)\right)\right\}$$
$$-\ (1/2)\left\{d(T)+d^*(T^*)+d(2)+d^*(2)\right\}$$

Note that by choosing the tax rates, the planner can choose the per buyer quantities at $t=1$. I use $y_1=d(T)$ and $y_1^*=d^*(T^*)$. Using these notation we can write the planner's problem as:

$$\max_{y_t,y_t^*}(1/2)U(y_1)+(1/2)\left\{(1/2)U(y_1)+(1/2)V\left(y_1^*\right)\right\}-\left\{(1/2)y_1+(1/2)y_1^*\right\}.$$

The first order conditions for this problem are: $U'(y_1)=2/3$ and $V'\left(y_1^*\right)=2$. The first order conditions for the buyers' problem are: $U'(y_1)=T$ and $V'\left(y_1^*\right)=T^*$. Therefore the optimal solution to the planner's problem can be implemented by imposing a tax (subsidy) of $T=2/3$ on buyers from country 1 and a tax of $T^*=2$ on buyers from country 2.

Relative to the free trade outcome (with $T=T^*=1$), the planner can increase the surplus from the $t=1$ market by 5.2%. I will use this related environment to get intuition also for the more general case that follows.

3. THE MODEL

I consider an economy with two dates $(t = 0,1)$ and two goods (X and Y with lower case letters denoting quantities). There are Z possible aggregate states of nature. State s occurs with probability Π_s.

There are many ex-ante identical sellers in each country. I start with the case in which the number of sellers in each country is known. Sellers are risk neutral and derive utility from Y only. Sellers can produce X at the per unit cost of λ units of Y. Unlike sellers, buyers are heterogeneous. There are J types of buyers. The number of type j (potential) buyers is n_j. All buyers are endowed with a large quantity of Y. In aggregate state s the utility function that a fraction ϕ_{js} of type j buyers realize is: $u_{js}(x,y) = U_j(x) + y$, where $U_j(x)$ is strictly monotone, strictly concave and differentiable. The remaining $(1-\phi_{js})n_j$ buyers who realize the utility function $u_{js}(x,y) = y$ are not active. The random utility of a type j buyer in aggregate state s is thus:

$$u_{js}(x,y) = U_j(x) + y \text{ with probability } \phi_{js} \text{ and } u_{js}(x,y) = y \text{ otherwise.} \tag{11}$$

A type j buyer demands $d_j(p)$ units of X at the price p if he wants to consume, where the individual demand function is defined by:

$$d_j(p) = \arg\max_{x \ge 0} U_j(x) - px. \tag{12}$$

The first order condition for the problem in (12) is:

$$U_j'(x) \le p \text{ with equality if } x > 0. \tag{13}$$

Production (capacity choice) occurs at $t=0$. After production choice is made, buyers realize a utility function and those who want to consume form a line. I treat all buyers symmetrically and assume that any segment taken from this line accurately represents the type composition of buyers who want to consume: In state s, $\sum_i \phi_{is} n_i$ buyers want to consume and the fraction of type j buyers in any segment of the line is: $\vartheta_{js} = \frac{\phi_{js} n_j}{\sum_i \phi_{is} n_i}$. After the line is formed, buyers arrive at the market place one by one according to their place in the line and buy at the cheapest available offer. The sequential trade does not take real time (it occurs in meta time).

I start with the relatively simple case in which the type composition of active buyers do not depend on the state.

3.1 THE PROBABILITY OF BECOMING ACTIVE DOES NOT DEPEND ON THE BUYER'S TYPE

I assume that the probability of becoming active depends only on the aggregate state and not on the buyer's type: $\phi_{js} = \phi_{1s} = \phi_s$ for all j. I choose indices so that demand is increasing in the state: $0 = \phi_0 \le \phi_1 \le ... \le \phi_Z = 1$.

In state s, the number of active buyers is $N_s = \phi_s N$ where $N = \sum_j n_j$ is the number of potential buyers. The fraction of type j buyers in any segment of the line, $\vartheta_j = \frac{\phi_s n_j}{\sum_i \phi_s n_i} = \frac{n_j}{N}$ is independent of s.

The minimum number of buyers that will arrive is $\phi_1 N = \min_s\{\phi_s N\}$ and the demand of this first batch (at the price p) is: $D_1(p) = \phi_1 \sum_j n_j d_j(p)$ units. If $s > 1$, there are $N_s - N_1$ buyers who could not make a buy in the first market. The minimum number of unsatisfied buyers if $s > 1$, is $(\phi_2 - \phi_1)N = \min_{s>1}\{(\phi_s - \phi_1)N\}$ and this is the number of buyers who will buy in the second market. Since the composition of buyers in any segment of the line $(\vartheta_1, ..., \vartheta_J)$ does not depend on the state s, we can figure out the demand of this second batch of buyers. This is: $D_2(p) = (\phi_2 - \phi_1)\sum_j n_j d_j(p)$ units. In general, the demand of batch i at the price p is: $D_i(p) = (\phi_i - \phi_{i-1})\sum_j n_j d_j(p)$. I use $q_i = \sum_{s=i}^{Z} \Pi_s$ to denote the probability that batch i will arrive. Since each batch will buy at a different price it is convenient to think of a sequence of Walrasian markets and assume that batch i buys in market i and the probability that market i opens is q_i.

The seller is a price-taker and behaves as if he can sell any amount at the price P_i in market i if it opens. The expected revenue from supplying a unit to market i is $q_i P_i$. In equilibrium expected profits are zero and prices satisfy: $q_i P_i = \lambda$.[2]

Free trade equilibrium is a vector of prices $(P_1,...,P_Z)$ and a vector of supplies $(x_1,...,x_Z)$ such that: (a) $P_i = \lambda / q_i$ and (b) $x_i = D_i(P_i)$.

In free trade equilibrium a type j buyer who arrives in batch i consumes $d_j(\lambda / q_i)$ units. To evaluate this outcome I now assume a planner (a World government) that can collect taxes. Taxes may be negative as in the case of subsidies. The planner can collect country specific lump sum taxes and country and batch specific consumption taxes. The consumption tax rate for a type j buyer who bought in market i is τ_{ji} and this buyer will therefore pay $P_i \mathrm{T}_{ji}$ and consume $d_j(P_i \mathrm{T}_{ji})$, where $\mathrm{T}_{ji} = 1 + \tau_{ji}$ is the gross tax rate. Here the consumption tax rate may depend on the country of residence and the transaction price. I now define equilibrium as follows. Equilibrium with taxes is a vector of producer prices $(P_1,...,P_Z)$, a vector of country and batch specific gross consumption tax rates $(\mathrm{T}_{11},...,\mathrm{T}_{1Z};\mathrm{T}_{21},...,\mathrm{T}_{2Z};...;\mathrm{T}_{J1},...,\mathrm{T}_{JZ})$ and a vector of supplies $(x_1,...,x_Z)$ such that:
(a) $P_i = \lambda / q_i$ and (b) $x_i = \sum_{j=1}^{J} (\phi_i - \phi_{i-1}) n_j d_j(P_i \mathrm{T}_{ji})$.

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[2] Note that (posted) prices may appear rigid because they do not respond to the realization of demand (the state). Nevertheless, it can be shown that sellers' contingent selling plans are time consistent and they do not have an incentive to change prices during trade.

From the planner's point of view the total utility from X derived in market i is: $(\phi_i - \phi_{i-1})\sum_{j=1}^{J} n_j U_j\left(d_j(P_i \mathrm{T}_{ji})\right)$ and the expected utility from X is: $\sum_{i=1}^{Z} q_i(\phi_i - \phi_{i-1})\sum_{j=1}^{J} n_j U_j\left(d_j(P_i \mathrm{T}_{ji})\right)$. Since by construction demand is highest in state Z and the planner satisfies demand in all markets that open, the capacity cost is: $\lambda\sum_{i=1}^{Z}\sum_{j=1}^{J}(\phi_i - \phi_{i-1}) n_j d_j(P_i \mathrm{T}_{ji})$. The planer will choose the tax rates by solving the following problem:

$$\max_{T_{ji}} \sum_{i=1}^{Z} q_i(\phi_i - \phi_{i-1})\sum_{j=1}^{J} n_j U_j\left(d_j(P_i \mathrm{T}_{ji})\right) - \lambda\sum_{i=1}^{Z}\sum_{j=1}^{J}(\phi_i - \phi_{i-1}) n_j d_j(P_i \mathrm{T}_{ji}). \tag{14}$$

Claim 1: The solution to the planner's problem (14) is $\mathrm{T}_{ji} = 1$ (and $\tau_{ji} = 0$) for all i, j.

This and all other claims are proved in the Appendix. The Claim says that the free trade allocation is efficient when the probabilities of becoming active do not depend on the buyer's type. Note that this is efficiency in the second best (constrained) sense. If the planner had the information about the state and could choose state dependent tax rates (τ_{jis}), he would have administered the Walrasian (Peak-Load-Pricing) allocation.

The intuition is in the comparison with the Walrasian planner who distributes output after the complete resolution of uncertainty. As was said before the UST planner tries to approximate the first best by charging a higher price from buyers who are more likely to buy in the high demand states. Here all buyers are equally likely to buy in the high demand state and therefore the UST planner cannot improve matters.

I now turn to consider the case of autarky.

Autarky in country j: I assume that there is one seller in country j. The minimum number of country j buyers that want to consume is: $\phi_1 n_j$. If $s > 1$ a second batch of

$(\phi_2-\phi_1)n_j$ buyers will arrive and demand $(\phi_2-\phi_1)n_j d_j(p)$ units. In general, the demand of batch i at the price p is $(\phi_i-\phi_{i-1})n_j d_j(p)$ and we can define equilibrium under autarky as follows.

Equilibrium in country j under autarky is a vector of prices $(P_1,...,P_Z)$ and a vector of supplies $(x_{j1},...,x_{jZ})$ such that: (a) $P_i = \lambda/q_i$ and (b) $x_{ji}=(\phi_i-\phi_{i-1})n_j d_j(P_i)$.

I now show the following Claim.

Claim 2: The allocation under autarky is the same as the allocation under free trade.

Thus here the individual country does not gain from trade and does not suffer from trade. I now turn to the case in which the probability of becoming active is type-dependent. I start with the case in which potential buyers have the same demand functions.

3.2 BUYERS HAVE THE SAME DEMAND FUNCTIONS

I assume $U_j(x)=U(x)$, $d_j(p)=d(p)$ and $\phi_{j1}\le\phi_{j2}\le...\le\phi_{jZ}$ for all j. I use $N_s=\sum_j \phi_{js}n_j$ for the number of active buyers. Thus, $N_1<N_2<...<N_S$. Here we do not know the type composition of the buyers who arrive in each batch (ϑ_{js}). Because all types have the same demand function, the value of ϑ_{js} is not relevant for computing the demand of each batch and for defining free trade equilibrium. But as we shall see it is relevant for the social planner.

The algorithm for computing the number of buyers in each batch is similar to what we had in the previous case. The minimum number of (active) buyers is: $\Delta_1=N_1$. The first batch of Δ_1 buyers arrives with certainty. After buyers in this first batch complete trade and go away there are two possibilities. If $s=1$ trade ends. If $s>1$, there are N_s-N_1 unsatisfied buyers. The minimum number of unsatisfied buyers if $s > 1$ is: $\Delta_2 = \min_s\{N_s - N_1\} = N_2 - N_1$ and this is the number of buyers in batch 2. The probability that $s > 1$ is $q_2 = 1 - \Pi_1$ and this is the probability that batch 2 will arrive. Proceeding in this way we define q_s and Δ_s for all $s = 1,...,Z$. As before, it is convenient to think of a sequence of Walrasian markets, where batch i buys in market i and the seller supplies x_i units to market i.

Free trade equilibrium is a vector of prices $(P_1,...,P_Z)$ and a vector of supplies $(x_1,...,x_Z)$ such that: (a) $P_i = \lambda/q_i$ and (b) $x_i = (N_i - N_{i-1})d(P_i) = \Delta_i d(P_i)$.

To solve for the equilibrium quantities we substitute the equilibrium condition (a) in (b) to get:

$$x_i = \Delta_i d(\lambda/q_i). \tag{15}$$

The consumption of type j buyers who buy in market i in state s is:

$$x_{jis}^{FT} = \vartheta_{js} x_i = \frac{\phi_{js} n_j d(P_i) \sum_m (\phi_{mi} - \phi_{mi-1}) n_m}{\sum_m \phi_{ms} n_m} \tag{16}$$

<u>Autarky in country j</u>: I use $N_{js} = \phi_{js} n_j$ to denote the number of active buyers in state s. Under autarky, the number of buyers in batch i is $(N_{ji} - N_{ji-1})$ and equilibrium is defined as follows.

Equilibrium under autarky is a vector of prices $(P_1,...,P_Z)$ and a vector of supplies $(x_{j1},...,x_{jZ})$ such that: (a) $P_i = \lambda/q_i$ and (b) $x_{ji} = (N_{ji} - N_{ji-1})d(P_i)$.

Thus prices are the same under autarky but the allocation may be different. The consumption of type j buyers who buy in market i in state s is now:

$$x_{jis}^{A} = (\phi_{ji} - \phi_{ji-1}) n_j d(P_i) \tag{17}$$

Note that (17) is equal to (16) if $\phi_{js} = \phi_s$ for all s. Otherwise the two allocations are different.

Claim 3: Country j will do better under autarky if: $N_{ji} \geq \vartheta_{ji} N_i$ for all i.

The Claim says that autarky is better if the number of type j buyers served under autarky is higher than the number of type j buyers served under free trade in all stages of trade. Under this condition more buyers will be able to buy at cheaper prices under autarky. In our example, in the high demand state all of country 1 buyers could buy at the first market price under autarky but only a fraction θ could buy in the first market under free trade. Therefore autarky was better than free trade for country 1 (and the opposite is true for country 2). Figure 11 illustrates the sufficient condition under which autarky is better.

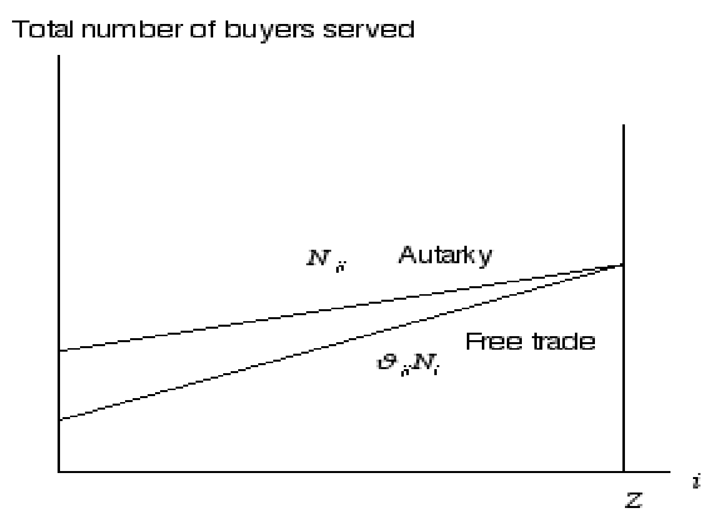

Figure 11: Autarky is better if the number of buyers served in markets $s \leq i$ is higher under autarky for all i.

The related environment: I now assume the related environment in which the fraction of buyers who make a buy in each market is exogenous. This related environment allows for a simple analytical solution to the optimal tax problem. In the next section I provide an algorithm for computing equilibrium in a UST environment.

I assume that at $t=0$ there is an auctioneer who announces the producer prices $(P_1,...,P_Z)$ and there is a World government that announces (country and batch specific) tax rates $(\mathrm{T}_{11},...,\mathrm{T}_{1Z};...;\mathrm{T}_{J1},...,\mathrm{T}_{JZ})$. Prices and tax rates cannot be changed after $t=0$ and sellers must satisfy demand.

Given the producer prices, sellers choose total capacity (production) at $t=0$. Trade occurs in real time during the next day (period). I divide the trading day into Z sub-periods: $t=1,...,Z$. At $t=1$ a batch of N_1 buyers arrive. In state s the demand of the first batch is $\sum_{j=1}^{J}\vartheta_{js}N_1 d(P_1\mathrm{T}_{j1})$ units and sellers sell $x_{1s}=\sum_{j=1}^{J}\vartheta_{js}N_1 d(P_1\mathrm{T}_{j1})$ units to satisfy the demand. After the first batch complete their transactions and disappear, we may have two possibilities. Either no additional buyers arrive or at $t=2$, a second batch of N_2-N_1 buyers arrives. In general, N_i-N_{i-1} buyers may arrive at sub-period $t=i$ and demand $x_{is}=\sum_{j=1}^{J}\vartheta_{js}(N_i-N_{i-1})d(P_i\mathrm{T}_{ji})$ units. Sellers satisfy the demand of the buyers who arrived in batch i at the producer price P_i.

Since sellers always satisfy demand, the total supply (capacity) x must be

$$x=\max_s\left\{\sum_{j=1}^{J}\sum_{i=1}^{s}\vartheta_{js}(N_i-N_{i-1})d(P_iT_{ji})\right\} \qquad (18)$$

I now define equilibrium as follows. Equilibrium in the related environment is a vector of producer prices $(P_1,...,P_Z)$, a vector of country and batch specific gross consumption taxes $(\mathrm{T}_{11},...,\mathrm{T}_{1Z};...;\mathrm{T}_{J1},...,\mathrm{T}_{JZ})$, total capacity x and a vector of quantities sold $(x_{11},...,x_{Z1};x_{12},...,x_{Z2};...;x_{1Z},...,x_{ZZ})$ such that $P_i=\lambda/q_i$, (18) is satisfied and the quantity sold in market i in state s is: $x_{is}=\sum_{j=1}^{J}\vartheta_{js}(N_i-N_{i-1})d(P_iT_{ji})$.

I now turn to the planner's problem. In state s, the utility that country j derives from buying X in market i is: $\vartheta_{js}(N_i - N_{i-1})U\left(d(P_i \mathrm{T}_{ji})\right)$. The utility they derive from buying X in all the markets that open in state s ($i \le s$) is: $\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U\left(d(P_i \mathrm{T}_{ji})\right)$. The expected utility that country j derives from the consumption of X is therefore: $\sum_{s=1}^{Z}\Pi_s\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U\left(d(P_i \mathrm{T}_{ji})\right)$. The expected utility that all countries derive from the consumption of X is: $\sum_{j=1}^{J}\sum_{s=1}^{Z}\Pi_s\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U\left(d(P_i \mathrm{T}_{ji})\right)$. Since the planner always satisfies demand he must satisfy (18). The planner chooses the country and batch specific taxes T_{ji} to maximize expected world surplus. He thus solves:

$$\max_{\mathrm{T}_{ji}} \sum_{j=1}^{J}\sum_{s=1}^{Z}\Pi_s\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U\left(d(P_i \mathrm{T}_{ji})\right) - \lambda \max_s\left\{\sum_{j=1}^{J}\sum_{i=1}^{s}\vartheta_{js}(N_i - N_{i-1})d(P_i \mathrm{T}_{ji})\right\} \qquad (19)$$

To simplify, I assume that at the solution to (19), the maximum total demand occurs when all Z markets open: $Z = \arg\max_s\left\{\sum_{j=1}^{J}\sum_{i=1}^{s}\vartheta_{js}(N_i - N_{i-1})d(P_i \mathrm{T}_{ji})\right\}$. We may now define the quantity consumed by a type j buyer that arrives in batch i by $y_{ji} = d(P_i \mathrm{T}_{ji})$ and set the planner's problem (19) in terms of quantities:

$$\max_{y_{ji}} \sum_{j=1}^{J}\sum_{s=1}^{Z}\Pi_s\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U(y_{ji}) - \lambda\sum_{j=1}^{J}\sum_{i=1}^{Z}\vartheta_{jZ}(N_i - N_{i-1})y_{ji} \qquad (20)$$

Note that y_{ji} matters only when $s \ge i$ and market i opens. We can therefore find the first order condition to the problem in (20) by taking the derivative of $(N_i - N_{i-1})U(y_{ji})\sum_{s=i}^{Z}\Pi_s\vartheta_{js} - \lambda\vartheta_{jZ}(N_i - N_{i-1})y_{ji}$. Equating this derivative to zero and using the equilibrium conditions $P_i = \lambda/q_i$ and $P_i \mathrm{T}_{ji} = U'(y_{ji})$ leads to the equilibrium conditions:

$$P_i \mathrm{T}_{ji} = U'(y_{ji}) = \frac{\lambda \vartheta_{jZ}}{\sum_{s=i}^{S} \Pi_s \vartheta_{js}} = \frac{\lambda \vartheta_{jZ}}{q_i \sum_{s=i}^{S} (\Pi_s / q_i) \vartheta_{js}} = P_i \frac{\vartheta_{jZ}}{\sum_{s=i}^{S} (\Pi_s / q_i) \vartheta_{js}} = P_i \left(\vartheta_{jZ} / \overline{\vartheta}_{ji} \right) \qquad (21)$$

where $\overline{\vartheta}_{ji} = \sum_{s=i}^{Z} (\Pi_s / q_i) \vartheta_{js}$ is the expected value of ϑ_j given that market i opens. This first order conditions imply:

$$\mathrm{T}_{ji} = \vartheta_{jZ} / \overline{\vartheta}_{ji} . \qquad (22)$$

The price paid by type j buyer who arrives in batch i depends on the representation of country j in the high demand state (ϑ_{jZ}) relative to the conditional expected representation of country j. Thus types that are represented in the high demand state more than their average representation will pay a tax. Those who are represented in the high demand state less than their average representation will get a subsidy. The intuition is that the planner tries to mimic the Walrasian planner's first best solution. As was said before, a Walrasian planner would charge the same price from all agents: a low price in the low demand state and a high price in the high demand state. Our planner must make irreversible choices before he knows the state. He may therefore want to charge a relatively low price from buyers that are more likely to buy in the low demand state and a relatively high price from buyers who are more likely to buy in the high demand states.

As was said before the demand-satisfying environment allows for the supply to each market to depend on the state. I now go back to the UST environment in which the supply to each market cannot depend on the state.

3.3 THE GENERAL UST CASE

I relax the assumption $\phi_{js} \le \phi_{js+1}$ for all j and extend the analysis to the case in which the probability of wanting to consume is not restricted. I also introduce uncertainty about the number of active sellers. Uncertainty about the number of active sellers may be important for application of this model. For example we may have a weak housing market because of over supply of new homes that was caused by the failure of builders to coordinate their effort.

I assume many potential ex-ante identical sellers. Active sellers are chosen randomly out of the group of potential sellers. An active seller (a seller for short) can produce as many units as he wants at the price of λ units of Y per unit of X. In state s there are M_s active sellers. The probability of state s from an active seller's point of view (conditional on being chosen as an active seller) is Π_s.

As before buyers arrive in batches but here the size of each batch is endogenous. I now turn to describe an algorithm for computing the size of each batch for an arbitrarily chosen price vector ($P_1 \le P_2 \le ... \le P_Z$).

Roughly speaking, the size of the first batch is the minimum demand per active seller at the price P_1. Market 2 opens if there are some buyers who wanted to buy in the first market but could not. In general, market i opens if there is residual demand after transactions in market $i-1$ are complete. The size of batch i is the minimum residual demand per active seller. I now turn to a detailed description of this algorithm.

Demand per seller in state s at the price P_1 is:

$\frac{\sum_j \phi_{js} n_j d_j(P_1)}{M_s}$. I choose indices such that state 1 is the state of minimum demand,

$1 = \arg\min_s \left\{ \frac{\sum_j \phi_{js} n_j d_j(P_1)}{M_s} \right\}$. The per seller demand of buyers in the first batch is:

$D_1(p) = \frac{\sum_j \phi_{j1} n_j d_j(P_1)}{M_1}$ units. It is assumed that each active seller supplies that many units at the price P_1.

If $s = 1$ then all buyers are served in the first market and trade ends. Otherwise, if $s > 1$, a demand for $\sum_j \phi_{js} n_j d_j(P_1) - M_s D_1(P_1) \geq 0$ units was not satisfied. The fraction of demand satisfied in market 1 is: $1 - \chi_s^1(P_1) = \frac{M_s D_1(P_1)}{\sum_j \phi_{js} n_j d_j(P_1)}$. The residual demand per seller at the price P_2 is $\frac{\chi_s^1(P_1) \sum_j \phi_{js} n_j d_j(P_2)}{M_s}$. We now choose the indices $s > 1$ so that

$2 = \arg\min_{s>1} \left\{ \frac{\chi_s^1(P_1) \sum_j \phi_{js} n_j d_j(P_2)}{M_s} \right\}$ and the minimum residual demand per seller is in state 2. The demand of buyers in batch 2 is:

D2(P1, P2) = $D_2(P_1, P_2) = \frac{\chi_2^1(P_1) \sum_j \phi_{j2} n_j d_j(P_2)}{M_2}$ units.

In general, we start iteration i having already computed the fraction Ω_s^{i-2} of demand that was not satisfied in markets $1, ..., i-2$. For example, if $i = 3$ we already know the fraction of demand that was not satisfied in market 1: $\Omega_s^1 = \chi_s^1(P_1)$. We have also computed the amount per active seller supplied to market $i-1$: $D_{i-1}(P_1, ..., P_{i-1})$. If $s > i-1$, the demand in market $i-1$ is: $\Omega_s^{i-2} \sum_j \phi_{js} n_j d_j(P_{i-1})$. The supply to this market is: $M_s D_{i-1}(P_1, ..., P_{i-1})$. The fraction of the residual demand satisfied in market $i-1$ is:

$$1 - \chi_s^{i-1}(P_1, ..., P_{i-1}) = \frac{M_s D_{i-1}(P_1, ..., P_{i-1})}{\Omega_s^{i-2} \sum_j \phi_{js} n_j d_j(P_{i-1})}. \tag{23}$$

The fraction of buyers who could not buy in markets $1, ..., i-1$ is: $\Omega_s^{i-1} = \Omega_s^{i-2}\ \chi_s^{i-1}$. When $s > i-1$, the residual demand per seller at the price P_i is:

$\frac{\Omega_s^{i-1}\sum_j \phi_{js} n_j d_j(P_i)}{M_s}$. We choose indices $s > i-1$ such that:

$i = \arg\min_{s>i-1}\left\{\frac{\Omega_s^{i-1}\sum_j \phi_{js} n_j d_j(P_i)}{M_s}\right\}$. The demand per seller in batch i is:

$$D_i(P_1,...,P_i) = \frac{\Omega_i^{i-1}\sum_j \phi_{ji} n_j d_j(P_i)}{M_i}, \tag{24}$$

units.

Given the construction of the demand functions $D_i(P_1,...,P_i)$ we can now define a symmetric equilibrium as follows.

A free trade equilibrium is a vector of prices $(P_1,...,P_Z)$ and a vector of per seller supplies $(x_1,...,x_Z)$ such that: (a) $P_i = \frac{\lambda}{q_i}$ and (b) $x_i = D_i(P_1,...,P_i)$.

In equilibrium the seller must know prices and all sellers must coordinate on the supply to each of the Z potential market. We may describe equilibrium in the following way. Each seller puts a price tag of P_i on x_i units and then remains passive. He knows that the lowest priced x_1 units will be sold first with certainty. Then if there is additional demand (with probability q_2) the x_2 units with the price tag P_2 will be sold and so on. The seller does not use the type composition of batch i to update the probabilities of the states. But the amount that is sold to each buyer is type dependent. We may therefore think of the seller as having many outlets and since trade does not take real time he cannot get aggregate statistics on the type composition during trade.

To find out whether country j can benefit from trade, the above algorithm should be applied to solve for equilibrium in country j. Then we can compare the allocations to country j under autarky and free trade. Claim 3 still hold if the ordering assumption holds and $\phi_{js} \le \phi_{js-1}$ where now the indices s are determined in the above algorithm applied to the free trade case. To solve for the optimal tax problem, we must modify the above algorithm to the case in which prices may depend on the type and the batch. The example in the first part of this paper demonstrates that a planner can improve matters.

4. CONCLUSIONS

Barriers to trade are viewed in the profession as a way of protecting an industry or improving the terms of trade of one country at the expense of another country. Here I studied a competitive model in which barriers to trade are motivated by efficiency considerations.

We showed by example that a planner who can charge type (and batch) specific prices may improve on the UST allocation. Tariffs, export taxes and subsidies may be viewed as a way of implementing type specific prices. In our example, a tariff leads to an increase in welfare in both countries.

A more general model was used to show that the UST free trade outcome is efficient when there is no uncertainty on the supply side and the probability of becoming active does not depend on the buyer's type. In this case the autarkic allocation is the same as the free trade allocation.

The case in which the probability of becoming active is type dependent is more interesting. Under the assumption that the probability of becoming active is increasing with the state for all types ($\phi_{js} \geq \phi_{js-1}$) we can show that a country can do better under autarky if the total number of buyers served in markets $s \leq i$ (at the prices: $P_1,...,P_i$) is greater under autarky for all i. Roughly speaking this condition holds if country j has a relatively stable demand.

In general the number of buyers and their composition that participate in each of the UST markets are endogenous. In states in which the average demand per buyer is high the number of buyers serviced in each market will be relatively low. This complicates the planner's optimal tax problem even when all buyers have the same demand function. To allow for analytical results to the palnner's problem we considered a simplified sequential trade environment in which the number of buyers in each batch is exogenous. In this environment sellers must satisfy demand at sticky prices and therefore the quantity sold in each market depends on the state but the price in each market does not depend on the state. The planner's country and batch specific consumption tax rates are also sticky and cannot be changed during trade. For this simplified sequential trading environment we showed that the planner will tax buyers whose representation in the high demand state (ϑ_{jZ}) is higher than their representation in lower demand states (ϑ_{js} for $s < Z$). The planner will subsidize buyers who are represented relatively more in the low demand states.

The paper also contributes to the discussion of efficiency in Prescott (1975) type models. We distinguish between versions that allow for the distribution of output after the state is known to versions in which some irreversible distribution decisions are made before the state is known. A planner who distributes output after he learns the state can easily improve on the Prescott allocation. This occurs even in the case of homogeneous buyers who have a downward sloping demand function (as opposed to a single unit inelastic demand function). In this case, a planner who observes the state will distribute more to buyers who arrive early if he knows that this is a state of low demand.

In a UST type environment the distribution of output is made before the state is known. A planner that operates in the same environment as the sellers in the UST model cannot improve matters if the probability of becoming active is type independent.

The positive implications of the theory should be further developed. In our model, the efficient level of tariffs and other barriers to trade is endogenous and an agreement to reduce tariffs may not be followed by an increase in trade. For example, efficiency requires no barriers to trade when the probabilities of wanting to consume is the same across countries. But in this case there are no gains from trade. This suggests that when the business cycles of two countries become similar, they may have incentives to reduce barriers to trade but this does not mean that an increase in trade will follow. This is consistent with Rose’s finding of no relationship between membership in the WTO and the volume of trade (Rose [2004]).

APPENDIX: PROOFS

Proof of Claim 1: Since the social planner can choose type and batch specific prices ($P_i \mathrm{T}_{ji}$) he can choose the amount $y_{ji} = d_j(P_i \mathrm{T}_{ji})$ that a type j buyer who arrives in batch i will get. We can therefore write the problem (14) as:

$$\max_{y_{ji}} \sum_{i=1}^{S} q_i(\phi_i - \phi_{i-1}) \sum_{j=1}^{J} n_j U_j(y_{ji}) - \lambda \sum_{i=1}^{S} \sum_{j=1}^{J} y_{ji}(\phi_i - \phi_{i-1}) n_j . \tag{A1}$$

The first order conditions for this problem are:

$$q_i U_j'(y_{ji}) \le \lambda \text{ with equality if } y_{ji} > 0. \tag{A2}$$

The first order condition for the consumer's problem (13) and $P_i = \lambda / q_i$ ensure that in equilibrium (A2) is satisfied. Thus the UST free trade allocation solves the planner's problem (A1) and therefore zero taxes and subsidies are optimal. □

Proof of Claim 2: Note that prices under autarky are the same as under free trade. We need to show that also the probability of buying in any given market is the same as under free trade. In state s there are $\phi_s n_j$ type j active buyers. The number of buyers who buy in market $i \le s$ under autarky is $(\phi_i - \phi_{i-1})n_j$ and the probability that an active buyer will buy in market $i \le s$ is: $\frac{(\phi_i - \phi_{i-1})n_j}{\phi_s n_j} = \frac{\phi_i - \phi_{i-1}}{\phi_s}$. Under free trade, there are $\phi_s N$ active buyers (from all types) and $\vartheta_j \phi_s N$ type j buyers. The number of type j buyers in market $i \le s$ is $\vartheta_j(\phi_i - \phi_{i-1})N$ and the probability that a type j buyer will buy in market $i \le s$ is: $\frac{\vartheta_j(\phi_i - \phi_{i-1})N}{\vartheta_j \phi_s N} = \frac{\phi_i - \phi_{i-1}}{\phi_s}$. Thus the probability of buying in market $i \le s$ under free trade is the same as under autarky and therefore the allocation under autarky is identical to the allocation under free trade.

the allocation under autarky is identical to the allocation under free trade. Thus, when the probability of becoming active does not depend on the buyer's type there are no gains from trade and an individual country is indifferent between autarky and free trade. □

<u>Proof of Claim 3</u>: We write the consumer surplus in market i: $CS_i = U(d(P_i)) - P_i d(P_i)$. Total surplus to country j if exactly i markets open:

$$W_{ji}^{A} = \sum_{s=1}^{i}(N_{js} - N_{js-1})CS_s = \sum_{s=1}^{i} N_{js}CS_s - \sum_{s=1}^{i} N_{js-1}CS_s \qquad \text{(A3)}$$
$$= N_{j1}(CS_1 - CS_2) + N_{j2}(CS_2 - CS_3) + \ldots + N_{ji}(CS_{i-1} - CS_i)$$

Total surplus under free trade if exactly i markets open is:

$$W_{ji}^{FT} = \vartheta_{ji}N_1(CS_1 - CS_2) + \vartheta_{ji}N_2(CS_2 - CS_3) + \ldots + \vartheta_{ji}N_i(CS_{i-1} - CS_i). \qquad \text{(A4)}$$

Since $CS_i \ge CS_{i+1}$, $W_{ji}^{A} \ge W_{ji}^{FT}$ if $N_{ji} \ge \vartheta_{ji}N_i$. Thus we have shown that welfare under autarky is higher if exactly i markets open. Expected welfare is higher under autarky when $N_{ji} \ge \vartheta_{ji}N_i$ holds for all i. □

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