INVENTORIES AND THE BUSINESS CYCLE: TESTING THE IMPLICATIONS OF A UST MODEL

Benjamin Eden*

The University of Haifa

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I test the implications of a monetary version of the Uncertain and Sequential Trading (UST) model using post war US data. The data support the hypothesis about the effect of demand shocks: Low demand has a persistent positive effect on inventories and a persistent negative effect on output, prices and labor supply (employment, hours per employee and effort).

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Changes in inventories are volatile and small. Christiano (1988) reports that quarterly changes in inventory investment are on average 0.6% of GDP but about half the size of changes in GDP. This type of observation led Blinder (1981, page 500) to conclude that "to a great extent, business cycle are inventory fluctuations". We can therefore hope to use inventories behavior for testing competing business cycles theories. Here I focus on the implications of an Uncertain and Sequential Trading (UST) model.

UST models are based on ideas in Prescott (1975) and Butters (1977). Prescott considers an environment in which sellers set prices before they know how many buyers will arrive at the market-place and derive an equilibrium price distribution. He assumes that cheaper goods are sold first and therefore in equilibrium sellers face a tradeoff between price and the probability of making a sale. In the UST approach taken by Eden (1990) an equilibrium distribution of prices is obtained even though sellers are allowed to change their prices during trade. While Prescott describes his model as a model in which sellers have monopoly power and prices are rigid, in my version of the model sellers are price-takers and prices are flexible. Recently the UST approach has been used in monetary economics to study the real effects of money and other issues. See Eden (1994), Lucas and Woodford (1994), Williamson (1996) and Woodford (1996).

Bental and Eden (1993, 1996; hereafter BE) introduced storage to the UST framework. In the 1993 paper i.i.d. demand shocks arise as a result of taste shocks. In the 1996 paper i.i.d. demand shocks arise as a result of money supply shocks. The predictions of the model about the behavior of real variables do not depend on the source of the demand shock. Here I focus on the 1996 monetary version.

BE (1996) use a cash-in-advance economy populated by infinitely lived households. Each household consists of two people: a seller (producer) and a buyer. At the beginning of each period the household has money and inventories. The seller takes the inventories and goes to work. He produces some additional output and tries to sell the accumulated stock. The buyer takes the money and goes shopping. On the way to the shopping location, the buyer may receive a monetary transfer. Once the buyer arrives at the shopping location he spends part or all of the money he has and returns home. The household consumes whatever the buyer managed to buy. The only uncertainty in the model is about the number of buyers that will receive the transfer payment.

The seller stays in one location. He knows that a certain minimal amount of money will arrive. We say that this minimal amount buys in the first market. With some probability, more buyers will get a transfer and more money will arrive. The additional money, if it arrives, opens the second market and so on. The seller, after having produced, allocates the available supply (output + beginning of period inventories) among all potential markets. If a particular market opens the seller sells the supply allocated to that market for cash. If that market does not open, the supply is carried over to the following period as inventories. Inventories may also be held for purely speculative reasons.

The model developed here is aimed at simplifying BE (1996) and making it user friendly while adding empirically relevant features. Unlike BE (1996) here demand can take only two possible realizations, the cash in advance is alwasy binding and pure speculation is not allowed. But here I add serially correlated supply shocks and a richer menu of labor supply choices.

The "undesired inventories" hypothesis

Before turning to the model, I would like to build some intuition about the main result in BE (1993, 1996): The negative relationship between the beginning of period inventories and output. In Figure 1 the price in the second market (p_2) is on the vertical axis while total supply (k = inventories + output) is on the horizontal axis. Equilibrium prices move together and therefore we can think of p_2 as representing the average price. An increase in the beginning of period inventories (which occurs say as a result of a negative demand shock in the previous period) shifts the supply curve to the right without affecting the demand curve. As a result, prices go down. From the diagram we can see that a unit increase in inventories is associated with less than a unit increase in k. Therefore, output goes down in response to the increase in inventories.

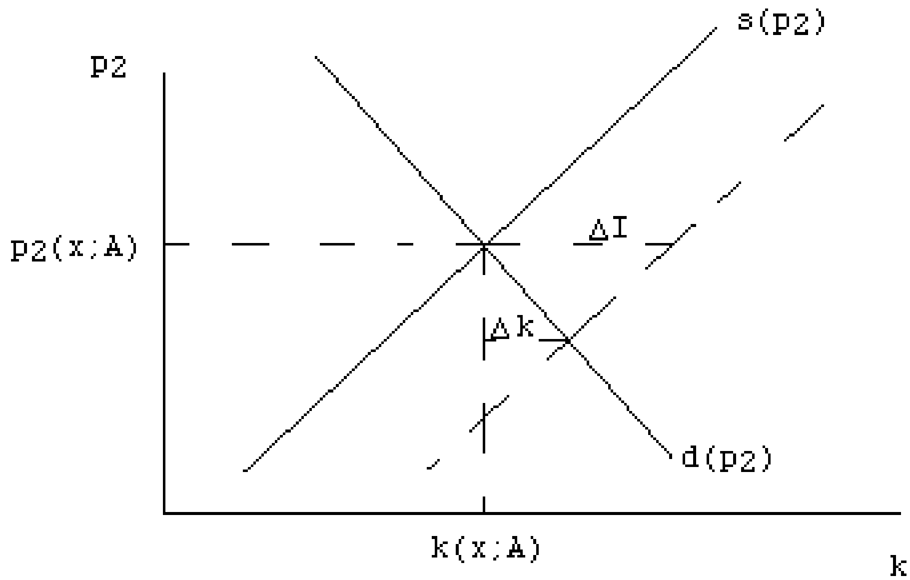

Figure 1

This is different from the real business cycle model in Kydland and Prescott (1982) and Cooley and Prescott (1995) who treat inventories as part of capital. This input view of inventories suggests a positive correlation between the beginning of period level of inventories (input) and output.

The production smoothing model suggests that on average the beginning of period inventories will be large in a period of high demand and low in periods of low demand. Production is expected to behave in the same way leading to a positive correlation between the beginning of period inventories and output.

Blinder and Fischer (1981) build on Lucas' confusion hypothesis and write down a modified Lucas-type supply curve where production depends not only on the price level and trend output but also on the difference between desired and final goods inventories. This should lead to a negative relationship between the beginning of period inventories and output. The major difference between the implications of the Blinder-Fischer model and the implications of the UST model is about the effect of the initial monetary shock. The Blinder-Fischer model predicts a change in the price level in response to a money supply shock while in the UST model current prices do not move in response to a monetary shock.

The model

There are n households. Each household consists of many people: a seller (manager) a buyer and a large number of workers. The manager of household h takes the beginning of period inventories (I^h_{t-1}) and the workers and go to work. He chooses the number of workers which will be employed (N^h_t), the number of hours per employee (H^h_t) and effort (F^h_t = the fraction of productive time out of total time spent at the work-place). Total productive hours or labor input is given by:

$$L^h_t = (N^h_t)(H^h_t)(F^h_t). \tag{1}$$

The household is risk neutral and dislikes all three components of labor. Its single period utility is:

$$c^h_t - \upsilon(N^h_t,\ H^h_t,\ F^h_t), \tag{2}$$

where c is consumption and the function $\upsilon(\ )$ is the disutility from labor supply components. The household does not treat all components of labor supply as perfect substitutes. He may prefer for example, to supply N = H = F = 1 rather than N = 1, H = 2 and F = $^1/_2$. To simplify the exposition, I assume that υ is symmetric. For example: $\upsilon = (^1/_2)(N^{1/3} + H^{1/3} + F^{1/3})^6$.

The minimum cost of supplying L units of labor is given by:

(3) $$v(L) = \min \upsilon(N, H, F) \quad \text{s.t. } (N)(H)(F) = L.$$

It is assumed that there exists a unique solution to (3) and because of symmetry:

(4) $$N = H = F = L^{1/3}.$$

It is further assume that v(L) has the standard properties of a cost function: $v'(0) = 0$, $v' > 0$ and $v'' > 0$.

Labor is the only input and the production function takes the Cobb-Douglas form:

(5) $$y_t^h = \varepsilon_t(L_t^h)^\alpha,$$

where ε_t is an i.i.d supply shock. Total supply is:

(6) $$k_t^h = y_t^h + I_{t-1}^h.$$

Demand: The buyer takes the beginning of period balances (M_t^h dollars) and goes shopping. On the way to the market the buyer may receive a transfer of T_t dollars. The number of buyers that will receive a transfer is unknown. For simplicity it is assumed that either a fraction

γ or a fraction 2γ of the buyers will get a transfer. These events occur with equal probabilities and the identity of the buyers that will receive the transfer is determined by an i.i.d. lottery.[1]

The buyer spends all the available cash: M_t^h if he did not get a transfer and $M_t^h + T_t$ if he did get a transfer. Buyers arrive in the market sequentially. The order of arrival is determined each period by an i.i.d lottery. Upon arrival, each buyer sees all the available offers and buys at the lowest price. Since cheaper goods are bought first, buyers that arrive late may face a higher price.

The average beginning of period balances is: $M_t = (1/n)\Sigma_{h=1}^{n} M_t^h$. For simplicity I assume: $T_t = M_t$. The amount spent per seller is therefore: $M_{t+1} = M_t + \gamma T_t = (1 + \gamma)M_t$ if a fraction γ got a transfer and $M_{t+1} = (1 + 2\gamma)M_t$ if a fraction 2γ got a transfer.

Markets: From the sellers' point of view, purchasing power arrives in batches. The first batch of $\Delta_{1t} = (1 + \gamma)M_t$ dollars arrives with certainty. The second batch of $\Delta_{2t} = \gamma M_t$ dollars arrives with probability $^1/_2$. I say that the first batch of dollars buys in the first market at the price P_{1t}. The second batch of dollars buys, if it arrives, in the second market at the price P_{2t}.

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[1] The following story may help. The government is committed to a welfare program which pays T_t dollars to households which qualifies. The criteria for qualification is not well understood by the public and therefore ex-ante all households assign the same (random) probability to winning the welfare lottery.

<u>The household's maximization problem:</u> The household's current consumption depends on the beginning of period balances (M_t^h), on whether it gets a transfer payment and on the market (price offer) it draws. The expected current consumption is denoted by:

$$(7) \qquad E\{(M_t^h + \tilde{i}_t T_t)/P_{\tilde{s}t}\},$$

where tilde denotes a random variable; $\tilde{i} = 1$ if the buyer gets a transfer and $\tilde{i} = 0$ otherwise; and expectations are taken with respect to $\tilde{i}$ (i = 0,1) and $\tilde{s}$ (s = 1,2).[2]

Since in the UST model (and most other models) observed changes in the beginning of period money supply are neutral, I use the beginning of period money supply per household as the unit of account and call it a

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[2] To compute (7), note that if only one market opens, the buyer will buy at the price P_{1t} with probability one. If he gets a transfer (with probability γ) he will have $(M_t^h + T_t)/P_{1t}$ units of consumption. Otherwise, he will have M_t^h/P_{1t} units. The expected consumption if only the first market opens is therefore: $[\gamma(M_t^h + T_t) + (1 - \gamma)M_t^h]/P_{1t}$. If two markets open in the current period, then the buyer will participate in the first market with probability $\Delta_{1t}/(\Delta_{1t} + \Delta_{2t})$, which is the fraction of dollars in batch 1 out of the post-transfer money supply. The probability of a transfer in this case is: 2γ. The expected consumption given that two markets open is therefore:
$\{[\Delta_{1t}/(\Delta_{1t} + \Delta_{2t})][2\gamma(M_t^h + T_t) + (1 - 2\gamma)M_t^h]/P_{1t}$
$+ [\Delta_{2t}/(\Delta_{1t} + \Delta_{2t})][2\gamma(M_t^h + T_t) + (1 - 2\gamma)M_t^h]/P_{2t}\}$.
The unconditional expected consumption is:
$E_{i,s}\{(M_t^h + i_t T_t)/P_s\} = (1/2)[\gamma(M_t^h + T_t) + (1 - \gamma)M_t^h]/P_{1t}$
$+ (1/2)\{[\Delta_{1t}/(\Delta_{1t} + \Delta_{2t})][2\gamma(M_t^h + T_t) + [1 - 2\gamma]M_t^h]/P_{1t}$
$+ [\Delta_{2t}/(\Delta_{1t} + \Delta_{2t})][2\gamma(M_t^h + T_t) + [1 - 2\gamma]M_t^h]/P_{2t}\}$.

normalized dollar. For example, if the beginning of period money supply is M_t, a price of 1 normalized dollar means that you have to pay M_t regular dollars to get a unit of the good. Note also that the assumption T = M means that the transfer per buyer is one normalized dollar. I use lower case letters to denote normalized prices and balances and write (7) as:

$$(7') \qquad E\{(m_t^h + \tilde{\imath}_t)/p_{\tilde{s}t}\}.$$

When exactly s markets open this period $M_{t+1}/M_t = 1 + s\gamma$ and a normalized dollar this period will become:

$$(8) \qquad \omega^s = 1/(1 + s\gamma),$$

normalized dollars in the next period, if held as a non-interest bearing asset.

Seller (manager) h allocates the available supply across the two markets:

$$(9) \qquad k_{1t}^h + k_{2t}^h = k_t^h,$$

where k_{jt}^h is the supply to market j.[3]

The average per household beginning of period inventories is:

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[3] Thus, pure speculation is not allowed. For the more general case in which $k_{1t}^h + k_{2t}^h \leq k_t^h$, see BE (1996).

$I_{-1} = (1/n)\Sigma_{h=1}^{n} I_{-1}^{h}$. I use $x = (I_{-1}, \varepsilon)$ to denote the current aggregate state and assume an equilibrium in which all magnitudes are functions of x. I drop the superscript to denote average per household magnitudes and use

$$k(x) = \varepsilon[L(x)]^{\alpha} + I_{-1}, \tag{10}$$

to denote total supply per household and

$$I^{s}(x) = k(x) - \Sigma_{j=1}^{s} k_j(x), \tag{11}$$

to denote the average per household level of next period inventories if exactly s markets open today.

In equilibrium, household h chooses (N^h, H^h, F^h) to solve the following Bellman equation:

$$V(m^h, I_{-1}^{h}; x) = \max E\{(m^h + \tilde{I})/p_{\tilde{s}}(x)\} - \upsilon(N^h, H^h, F^h) + \tag{12}$$

$$\beta EV\{\Sigma_{j=1}^{\tilde{s}} p_j k_j^h)/(1+\tilde{s}\gamma),\ k^h - \Sigma_{j=1}^{\tilde{s}} k_j^h;\ [I^{\tilde{s}}(x), \tilde{\varepsilon}]\}$$

s.t.

$$L^h = (N^h)(H^h)(F^h),$$

$$k_1^h + k_2^h = k^h = \varepsilon(L^h)^{\alpha} + I_{-1}^{h} \text{ and non negativity constraints,}$$

where $V(m^h, I_{-1}^{h}; x)$ is the maximum expected utility when the aggregate state is x and the household starts the period with the predetermined variables (m^h, I^h_{-1}). Expectations are taken with respect to this period number of markets ($\tilde{s}$) and next period supply shock ($\tilde{\varepsilon}$).

Equilibrium conditions: To state the first order conditions for an interior solution to (12), I compute the expected purchasing power of a normalized dollar held by the buyer at the beginning of the period:

$$(13) \qquad z(x) = (1/2)[1/p_1(x)] + (1/2)[\theta_1/p_1(x) + \theta_2/p_2(x)],$$

where θ_s is the probability that a dollar is in batch s given that two markets open: $\theta_1 = (1+\gamma)/(1+2\gamma)$ and $\theta_2 = \gamma/(1+2\gamma)$. The first term on the right hand side of (12) is the purchasing power of a normalized dollar if only one market opens. The second term is the expected purchasing power if two markets open.

The expected purchasing power of a normalized dollar next period given that s markets open this period is:

$$(14) \qquad Z[I^s(x)] = E\{z[I^s(x), \tilde{\varepsilon}]\}.$$

At the optimum producing an additional unit and supplying it to the first market will not change the expected utility. The marginal cost must therefore equal the expected discounted real price in the first market:

$$(15) \qquad mc(x) = v'[L(x)]/\varepsilon\alpha[L(x)]^{\alpha-1} = \beta p_1(x)E\{\omega^{\tilde{s}}Z[I^{\tilde{s}}(x)]\}.$$

The right hand side of (15) is the expected discounted real price: p_1 is the price in terms of current normalized dollar, $p_1\omega^s$ is the price in terms of next period normalized dollars and $p_1\omega^s Z$ is the price in terms of next period expected consumption.

The expected marginal cost next period given that s markets open today is:

(16) $$MC[I^s(x)] = E\{mc[I^s(x), \tilde{\varepsilon}]\}.$$

Since at an interior optimum the seller must be indifferent between supplying to the first and to the second market we have:

(17) $$p_1(x)E\{\omega^{\tilde{s}}Z(I^{\tilde{s}})\}$$
$$= (1/2)p_2(x)\omega^2 Z(I^2) + (1/2)MC(I^1).$$

The left hand side of (17) is the expected consumption from supplying a unit to the first market. The right hand side is the expected consumption from supplying a unit to the second market. The right hand side has two elements. The first is the expected consumption given that market 2 opens and the unit is sold. The second is the value of the unit if market 2 does not open and it is carried as inventories to the next period. Since in this case, the unit may be used to substitute for next period production, the value of inventories is equal to next period expected marginal cost.

A full (steady state) equilibrium is a vector of functions

$[p_1(x), p_2(x), L(x), N(x), H(x), F(x), k(x), k_1(x), k_2(x), z(x), mc(x),$ $I^1(x), I^2(x), Z[I^s(x)], MC[I^s(x)], V(m,I;x)]$ such that: (9) - (17) are satisfied and markets which open are cleared:

(18) $$1 + \gamma = p_1(x)k_1(x) \; ; \; \gamma = p_2(x)k_2(x).$$

<u>Solving for a partial equilibrium:</u> A partial equilibrium is defined for a given current state x and given expectation functions:
$A = \{Z(\bullet), MC(\bullet)\}$.

A partial equilibrium for given (x, A) is a vector $[p_1(x; A), p_2(x; A), L(x; A), N(x; A), H(x; A), F(x; A), k(x; A),$ $k_1(x; A), k_2(x; A), z(x; A), mc(x; A), I^1(x; A), I^2(x; A)]$ that satisfies (9) - (13), (15), (17), (18).

I now solve for the current period magnitudes $k_s(x; A)$ and $p_s(x; A)$ under the assumption that $Z(\bullet)$ is an increasing function and $MC(\bullet)$ is a decreasing function. I start by choosing p_2 arbitrarily. The price in the first market must satisfy (17) which can now be written as:

(19) $$p_1 E[\omega^{\tilde{s}} Z(I^{\tilde{s}})] = (1/2)p_2[\omega^2 Z(I^2)] + (1/2)MC(I^1).$$

Substituting $I^2 = 0$ and $I^1 = \gamma/p_2$ leads to:

(20) $$p_1[(1/2)\omega^1 Z(0) + (1/2)\omega^2 Z(\gamma/p_2)]$$
$$= (1/2)p_2[\omega^2 Z(0)] + (1/2)MC(\gamma/p_2).$$

Denote the solution to (20) by: $p_1(p_2)$. Since Z( ) is an increasing function and MC( ) is a decreasing function, $p_1(p_2)$ is an increasing function.

Total demand at the prices p_2 and $p_1(p_2)$ is:

(21) $$d(p_2) = \gamma/p_2 + (1 + \gamma)/p_1(p_2),$$

which is a decreasing function of p_2 as in Figure 1.

To find labor supply we substitute (20) in (15) to get:

(22) $$mc = v'(L)L^{1-\alpha}/\varepsilon\alpha = \beta\{(^1/_2)p_2[\omega^2 Z(0)] + (^1/_2)MC(\gamma/p_2)\}.$$

Let $mc(p_2)$ and $L(p_2)$ denote the solution to this equation. Since MC( ) is a decreasing function, mc( ) is an increasing function of p_2. Since $v'' > 0$, $L(p_2)$ is also an increasing function. Total supply is given by:

(23) $$s(p_2) = \varepsilon[L(p_2)]^{\alpha} + I_{-1},$$

which is an increasing function. A solution can be obtained by equating supply and demand: $s(p_2) = d(p_2)$, as illustrated by Figure 1.

<u>Solving for a full equilibrium:</u> The above partial equilibrium solution was computed for a given x. We now vary x to get the partial equilibrium functions and use (14) and (16) to compute the functions $\{Z(\bullet, A), MC(\bullet, A)\}$. We then check whether the assumed functions $A = \{Z(\bullet), MC(\bullet)\}$ are the same as the partial equilibrium functions:

$A' = \{Z(\bullet; A), MC(\bullet; A)\}$. If they are the same, we are done. If not we compute a partial equilibrium for the new vector A' and so on with the hope that this iteration procedure will converge.[4]

<u>Properties of the equilibrium functions:</u> Since a full equilibrium is also a partial equilibrium, we can derive the properties of the equilibrium functions by using the algorithm for computing partial equilibrium. Changes in I_{-1} and ε affect the supply schedule $s(p_2)$ but not the demand schedule $d(p_2)$. An increase in I_{-1} will shift the supply curve to the right, reduce prices and increase total supply by less than the increase in inventories: $\Delta k < \Delta I$. It follows that an increase in the beginning of period inventories reduces output and labor supply. An increase in ε will not change the right hand side of (22) and will therefore increase $L(p_2)$. It follows that the supply curve (23) will shift to the right and as a result prices, output and labor supply will go down. This type of analysis leads to:

<u>Claim 1:</u> The equilibrium functions L(x), N(x), H(x), F(x) are decreasing in I and increasing in ε. The equilibrium functions k(x), $k_1(x)$, $k_2(x)$, z(x), $I^1(x)$, $I^2(x)$ are increasing in I and in ε. The equilibrium functions $p_1(x)$, $p_2(x)$ are decreasing in I and in ε.

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[4] A formal existence proof for the Bental and Eden (1996) model can be obtained from the authors. For a published existence proof in a similar model see Bental and Eden (1993). Both existence proofs use Schauder's fixed point theorem.

Serially correlated supply shocks

I now turn to the case in which the supply shock follows an AR(1) process and is given by:

(24) $$\varepsilon_t = \rho\varepsilon_{t-1} + u_t,$$

where u_t are i.i.d error terms. To analyze this case I redefine the expected purchasing power and the expected marginal cost in (14) and (16) as follows:

(14') $$Z[I^S(x), \varepsilon] = E\{z[I^S(x), \tilde{\varepsilon}_{+1} = \rho\varepsilon + \tilde{u}]\},$$

(16') $$MC[I^S(x), \varepsilon] = E\{mc[I^S(x), \tilde{\varepsilon}_{+1} = \rho\varepsilon + \tilde{u}]\},$$

where the expectations operator is taken with respect to $\tilde{u}$.

Solving for a partial equilibrium: As before a partial equilibrium is defined for a given current state x and given expectation functions: $A = \{Z(\bullet,\bullet), MC(\bullet,\bullet)\}$. I now solve for the current period magnitudes $k_S(x; A)$ and $p_S(x; A)$ assuming that $Z(\bullet,\bullet)$ is increasing in its first argument (inventories) and $MC(\bullet,\bullet)$ is decreasing in its first argument.

As before, I start by choosing p_2 arbitrarily. The price in the first market must satisfy:

(20') $$p_1[(1/2)\omega^1 Z(0, \varepsilon) + (1/2)\omega^2 Z(\gamma/p_2, \varepsilon)] = (1/2)p_2[\omega^2 Z(0, \varepsilon)] + (1/2)MC(\gamma/p_2, \varepsilon).$$

Denote the solution to (20') by: $p_1(\varepsilon, p_2)$. Since Z( ) is an increasing function of next period inventories and MC( ) is a decreasing function of next period inventories, $p_1(\varepsilon, p_2)$ is an increasing function of p_2.

Total demand at the prices p_2 and $p_1(\varepsilon, p_2)$ is:

$$(21') \qquad d(\varepsilon, p_2) = \gamma/p_2 + (1 + \gamma)/p_1(\varepsilon, p_2),$$

which is a decreasing function of p_2 as in Figure 1.

To find labor supply we substitute (20') in (15) to get:

$$(22') \quad mc = v'(L)L^{1-\alpha}/\varepsilon\alpha = \beta\{(1/2)p_2[\omega^2 Z(0, \varepsilon)] + (1/2)MC(\gamma/p_2, \varepsilon)\}.$$

Let $mc(\varepsilon, p_2)$ and $L(\varepsilon, p_2)$ denote the solution to this equation. Since MC( ) is a decreasing function of next period inventories, mc( ) is an increasing function of p_2. Since $v'' > 0$, $L(\varepsilon, p_2)$ is also an increasing function of p_2. Total supply is:

$$(23') \qquad s(\varepsilon, p_2) = \varepsilon[L(\varepsilon, p_2)]^\alpha + I_{-1},$$

which is an increasing function of p_2. Thus, also in this more general case a solution can be obtained by equating supply and demand, $s(\varepsilon, p_2) = d(\varepsilon, p_2)$, as illustrated by Figure 1.

Properties of the equilibrium functions: As in the i.i.d. case changes in I_{-1} shifts the supply curve only and therefore:

Claim 2: When the supply shocks are AR(1), the equilibrium functions $L(x)$, $N(x)$, $H(x)$, $F(x)$, $p_1(x)$, $p_2(x)$ are decreasing in I_{-1} and the equilibrium functions $k(x)$, $k_1(x)$, $k_2(x)$, $z(x)$, $I^1(x)$, $I^2(x)$ are increasing in I_{-1}.

Unlike the i.i.d case changes in ε will in general affect both supply and demand and therefore the sign of the partial derivatives with respect to ε cannot be determined without some additional assumptions. As a working hypothesis, I assume that changes in ε have the effects described in Claim 1.

Implementation

I start with the analysis of Hodrick-Prescott (H-P) log detrended variables.[5] The same symbols is used now to denote the detrended log of the variables. I use the following model to interpret vector auto regression (VAR) of output hours and inventories.

(25) $$Y_t = \alpha L_t + \varepsilon_t$$

(26) $$TH_t = N_t + H_t$$

(27) $$L_t = TH_t + F_t$$

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[5] The logs of the variables were detrended by the Hodrick-Prescott (H-P) filter, using the standard choice of $\lambda = 1600$.

(28) $$TH_t = - \beta_1 I_{t-1} + \beta_2 \varepsilon_t$$

(29) $$F_t = - \gamma_1 I_{t-1} + \gamma_2 \varepsilon_t$$

(30) $$I_t = \delta_1 I_{t-1} + \delta_2 \varepsilon_t - \delta_3 s_t,$$

where TH = (N)(H) denotes total hours, F denotes effort and L = F + TH is labor input. To save notation the intercept term was omitted from all equations (but is present in all regressions).

Equation (25) is a Cobb-Douglas production function. (It is linear because the detrended variables are in log forms).

Equation (26) defines total hours (TH) as the sum of employment and average hour per employee. Equation (27) defines labor supply as the sum of total hours and effort. Equations (28) - (29) specify total hours and effort as a function of the beginning of period inventories and the supply shock. Equation (30) specifies inventories as a function of the beginning of period inventories, the supply shock ε and the number of markets open (s). We expect: α, β_i, γ_i, $\delta_i > 0$.

Substituting (28) and (29) in (25) leads to:

(31) $$Y_t = - \alpha(\beta_1 + \gamma_1) I_{t-1} + \alpha(\beta_2 + \gamma_2 + 1)\varepsilon_t = -dI_{t-1} + e\varepsilon_t.$$

In the serially correlated supply shocks case we get:

(32) $$\begin{aligned} Y_t &= -dI_{t-1} + e\varepsilon_t = -dI_{t-1} + e(\rho\varepsilon_{t-1} + u_t) \\ &= -dI_{t-1} + \rho(Y_{t-1} + dI_{t-2}) + eu_t \\ &= \rho Y_{t-1} - dI_{t-1} + \rho dI_{t-2} + eu_t. \end{aligned}$$

In a similar way we get:

(33) $$TH_t = \rho TH_{t-1} - \beta_1 I_{t-1} + \rho\beta_1 I_{t-2} + \beta_2 u_t.$$

The specification (32) and (33) says that lag variables matter. The number of lag depends of course on the number of lags in the AR process (24).[6] I therefore experiment with various lags starting from VAR with one lag only. The main hypothesis is that the beginning of period inventories have a negative effect on all components of labor supply (employment, hours per employee and effort) and therefore on output and a positive effect on the end of period inventories.

Disaggregation by stage-of-fabrication: In the version of the UST model considered here (chapter 7.5) there is no production lag and no distinction among inventories at different stages of the production process (work in progress and final goods) and no special category for materials and supplies.[7] I therefore looked at the change in the aggregate level of inventories. Blinder (1986) argues that the concept of inventories that satisfies the equation:

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[6] When the supply shock follows an AR(p) process we should modify the theoretical section and define the aggregate state by:
$x = (\varepsilon, \varepsilon_{-1},...,\varepsilon_p, I_{-1})$. This should not change the result in Claim 2.

[7] See for example, Abramovitz (1950) for an early study and Reagan and Sheehan (1985) for a more recent study that distinguishes among these types of inventories.

output = sales + change in inventories is finished goods + work in progress. Since this equation is used in the model, the aggregate concept of inventories is appropriate. Here I follow Blinder in using the aggregate concept of inventories.

Data: I use NIPA data from 1959:1 to 1997:4.[8] These data have three concepts of outputs: Goods, Durable Goods and Non-Durable Goods. (GDP = Services + Goods; Goods = Durables + Non-Durables). In 1997:4 the goods producing sector was about 40% of GDP and was equally divided between durables (20%) and non-durables (20%).

The standard deviation of the cyclical variables in the goods sector are 0.026 for output, 0.011 for sales, 0.014 for inventories and 0.035 for total hours.

The correlation matrix for the detrended (cyclical) variables is in Table 1. The correlation between the beginning of period inventories and output (levels of detrended variables) is positive for the durable sector (0.2) and negative for the non-durable sector (-0.1). The correlation for the goods sector as a whole is positive (0.2). The correlation between hours and lag inventories are all positive and somewhat higher: 0.4 for the goods and durable sectors and 0.1 for the non-durable sector.

[8] The data is on the internet at: http://www.stat-usa.gov/

Table 1: The correlation matrix for the detrended (ln) variables.

	G	I_{-1}	TH	D	I_{-1}	TH	ND	I_{-1}	TH
G	1								
I_{-1}	0.2	1							
TH	0.9	0.39	1						
D	0.94	0.19	0.89	1					
I_{-1}	0.29	0.9	0.39	0.2	1				
TH	0.91	0.39	0.98	0.92	0.4	1			
ND	0.64	0.17	0.47	0.35	0.26	0.43	1		
I_{-1}	-0.05	0.59	0.16	0	0.19	0.14	-0.1	1	
TH	0.82	0.1	0.89	0.8	0.07	0.85	0.46	0.1	1

* All variables were logged before the H-P filter was applied. We have three concepts of outputs: goods (G), durable goods (D) and non-durable goods (ND). For each output concept we have the lag stock of inventories (I_{-1}) and hours (TH). The correlations between output and lag inventories are in large font.

The vector auto regressions are in Table 2. The sign of the coefficients of lag inventories are as predicted by theory. The lags of output and total hours are statistically significant (t statistics in parentheses). This suggests the rejection of the i.i.d supply shocks hypothesis in favor of the serially correlated supply shocks.

Table 2*: VAR using H-P detrended log variables

	Y_{-1}	TH_{-1}	I_{-1}	Adj.R^2
Goods measures				
dependent var: Y	0.59	0.33	-0.69	0.72
	(6.0)	(4.2)	(-6.8)	
dependent var: TH	0.22	0.92	-0.70	0.84
	(2.2)	(11.2)	(-6.6)	
dependent var: I	0.09	0.09	0.68	0.90
	(2.6)	(3.3)	(20.0)	
Durables measures				
dependent var: Y	0.49	0.61	-0.90	0.68
	(4.2)	(4.2)	(-6.8)	
dependent var: TH	0.14	0.96	-0.64	0.85
	(1.9)	(10.5)	(-7.7)	
dependent var: I	0.05	0.15	0.65	0.90
	(1.6)	(3.9)	(18.8)	
Non-dur. measures				
dependent var: Y	0.37	0.21	-0.38	0.33
	(5.0)	(4.2)	(-4.3)	
dependent var: TH	0.03	0.84	-0.16	0.69
	(0.4)	(16.6)	(-1.8)	
dependent var: I	0.06	0.05	0.81	0.73
	(1.7)	(2.0)	(18.4)	

* t statistics in parentheses.

We can examine the joint hypothesis about the effect of inventories on effort by running Two Stage Least Squares regression of output on total hours and lag inventories, using lag variables as instruments. The results in Table 3 are consistent with the prediction of the theory.

Table 3: Dependent variable = Y; 2SLS using H-P detrended log variables

	Y_{-1}	I_{-1}	TH	Adj.R^2
Goods measures		-0.27	0.69	0.83
		(-4.1)	(23.3)	
	0.51	-0.44	0.36	0.83
	(5.6)	(-6.1)	(5.4)	
Durables measures		-0.31	1.04	0.87
		(-4.4)	(26.3)	
	0.40	-0.49	0.64	0.84
	(4.1)	(-5.6)	(5.9)	
Non-dur. measures		-0.25	0.38	0.23
		(-2.7)	(6.5)	
	0.37	-0.34	0.26	0.35
	(4.9)	(-3.9)	(4.2)	

In Tables 4 and 5 I test the hypothesis about the effect of the beginning of period inventories on employment and hours per employee. The results are consistent with the prediction of a negative effect.

Table 4: Dependent variable = number of employees (N); using H-P detrended log variables

	Y_{-1}	I_{-1}	N_{-1}	Adj.R^2
Goods measures	0.25	-0.43	0.88	0.89
	(4.9)	(-5.5)	(12.7)	
Durables measures	0.15	-0.41	0.95	0.89
	(4.6)	(-5.9)	(12.6)	
Non-dur. measures	0.09	-0.09	0.86	0.78
	(2.2)	(-1.9)	(19.7)	

Table 5: Dependent variable = hours per employee (H); using H-P detrended log variables

	Y_{-1}	I_{-1}	H_{-1}	Adj.R^2
Goods measures	0.04	-0.26	0.82	0.72
	(0.9)	(-5.4)	(10.4)	
Durables measures	0.06	-0.27	0.78	0.79
	(1.6)	(-6.8)	(8.8)	
Non-dur. measures	-0.05	-0.07	0.79	0.58
	(-1.2)	(-1.4)	(14.1)	

Impulse response functions

The model has sharp predictions about the effect of demand shocks. A negative shock to demand leads to the accumulation of inventories and has a persistent negative effect on output, prices and end of period inventories. The persistence in this model arises because of inventories: an increase in the beginning of period inventories by one unit leads to a less than a unit increase in the beginning of next period inventories and this leads to even smaller effect on the beginning of two periods ahead inventories and so on until the effect dies out. Since a shock to inventories leads to a persistent effect on inventories, it leads also to a persistent effect on output.

Since output is determined before the beginning of trade, demand shocks affect the end of period inventories only. This allows for the identification of demand and supply shocks in an impulse response analysis. A shock to output is a supply shock. Holding contemporaneous output constant, a shock to inventories is a demand shock. Here I study the effect of these shocks on contemporaneous and future variables.

For the sake of comparision, I use the sample period in Christiano Eichenbaum and Evans (1998) which is 1964:3 - 1995:2 and allows for four lags. All variables are not detrended and except for the interest rate they are in log form.

I start with a simple VAR system of output and inventories:

(34) $$Y_t = d^1_{11}Y_{t-1} + d^1_{12}I_{t-1} + \ldots + d^4_{11}Y_{t-4} + d^4_{12}I_{t-4} + e_{11}\varepsilon_t$$

(35) $$I_t = d^1_{21}Y_{t-1} + d^1_{22}I_{t-1} + \ldots + d^4_{21}Y_{t-4} + d^4_{22}I_{t-4} + e_{21}\varepsilon_t + e_{22}s_t,$$

where d^q_{ij} are coefficients of the lag variables and e_{ij} are the coefficients of the supply and demand shocks (ε and s). I start by analyzing the effects of a demand shock. This is done by an impulse response to a shock in I, ordering Y first.

To understand the nature of the experiment we can imagine the following procedure. We first run:

(36) $$I_t = b_1 Y_t + \text{lagged variables} + \theta_t.$$

Since in this regression Y_t serves as a variable which is perfectly correlated with the supply shock, the residual θ is (perfectly correlated with) the demand shock. We can now imagine that we keep all the variables in the regression (36) constant and change only the realization of θ by σ units where σ is the estimated standard deviation of θ.

By construction,

(37) $$\Delta Y_t = 0 \; ; \; \Delta I_t = \sigma,$$

where Δ denotes the change from the initial path. The changes (37) imply the following changes at t+1:

(38) $$\Delta Y_{t+1} = d^1_{12}\sigma \; ; \; \Delta I_{t+1} = d^1_{22}\sigma.$$

The changes (37) and (38) imply the following changes at t+2:

(39) $\Delta Y_{t+2} = d^1_{11}d^1_{12}\sigma + d^1_{12}d^1_{22}\sigma + d^2_{12}\sigma$; $\Delta I_{t+2} = d^1_{21}d^1_{12}\sigma + d^1_{22}d^1_{22}\sigma + d^2_{22}\sigma$

and so on.

I ran this impulse response procedure for four different sectors (GDP, GOODS, DURABLES AND NONDURABLES). Figure 2 uses GDP for Y and inventories in the goods producing sector for I. An increase in inventories by one standard deviation increases inventories initially by about 0.4 percent. The effect on inventories diminishes gradually until it dies out. The shock to inventories reduces output by about 0.5 percent after one quarter and this effect persists over time.

Figure 3 uses output and inventories in the goods sector. The impulse response functions look qualitatively similar but the effect on output is stronger (about 1 percent). Figure 4 uses output and inventories in the durables sector. Figure 5 uses the magnitudes from the non-durable sector. The effect of a shock to inventories on the output in the durable sector is much larger than in the non-durable sector: In the durable sector the maximal decline occurs after 3 to 6 quarters and is about 2%. In the non-durable sector the maximal decline occurs after about four quarters and is roughly 0.5%.

These impulse responses are consistent with the hypothesis that an increase in the beginning of period inventories (due to a negative demand shock in the previous period) has a persistent negative effect on output and a persistent positive effect on inventories.

Adding money and prices:

The model predicts a negative persistent effect of a negative shock to demand (positive shock to inventories) on prices. To test this hypothesis we need to control for the money supply. As a proxy for the money supply, I used the Federal funds rate (FF) and the money supply (M = M1). The results are robust to the inclusion of additional monetary aggregates like total reserves and non-borrowed reserves as in Christiano, Eichenbaum and Evans (1998). For prices I use the implicit price deflator (P) or the producer price index (PPIALL). In addition, I follow Christiano, Eichenbaum and Evans (1998) in using an index of prices for sensitive commodities (PCOM) and running a four lag VAR.

Figure 6 uses GDP and inventories in the goods producing sector. A shock to inventories is followed by about 0.2% immediate decline in output which persist for half a year. Output then go back to normal in about 1 year and then "over shoot". The implicit price deflator P goes down gradually and then return gradually to normal. The decline in prices is slow reaching a maximum effect of about 0.2% after about two years. Prices of sensitive commodities (PCOM) react much faster to the change in inventories. The reduction is about 0.5% after one quarter. These prices go back to normal after about a year and then "over shoot".

The impulse response functions are consistent with the hypothesis that the Fed react to the increase in inventories by lowering the federal fund (FF) rate. This leads to an increase in the money supply. The effect of the shock to inventories on future inventories is positive and diminishes initially. But unlike the simple VAR it "over shoot" after about a year and a half.

On the whole the impulse response functions for the first four quarters behave according to the predictions of the theory. Output falls and prices fall in spite of the fact that the money supply (M1) does not decline.

Figure 7 uses output and inventories in the goods producing sector and the producer price index (PPIALL) for prices. The results are qualitatively the same. The effect of the shock of inventories on output is stronger: Output declines by about 0.5% after two quarters.

Figures 8 and 9 are for the durables and non-durables sectors. The effect of inventories on output is about the same in both sectors: A maximal effect of about 0.5%. In the nondurable sector prices tend to decline after an inventories shock (especially PCOM) in spite of the fact that the money supply tends to go up. In the durables sector prices and the money supply do not move much. The effect on inventories is positive in both cases but in the nondurables sector inventories decline after an initial rise. On the whole it seems that the behavior of the nondurables sector is more in line with the predictions of the model.

The effect of supply shocks

I have not been able to derive sharp predictions about the effect of serially correlated supply shocks. However, as a working hypothesis, I adopt the implications of i.i.d. shocks (Claim 1) which says that output and the end of period inventories are both increasing functions of the supply shock (ε).

The impulse response functions in Figures 10 - 17 do reveal a positive effect of a shock to output on the end of period inventories.

There is a difference between the durables and nondurables sectors. This difference can be illustrated by the full VAR in Figures 16 and 17. In the durables sector the maximal effect on inventories occurs after about a year and is close to 1%. In the nondurable sector the maximal effect occurs after one quarter and is about 0.5%.

The difference is even larger when looking at the effect of a supply shock on prices. In the durables sector prices tend to go up in spite of the reduction in the money supply. In the nondurables sector prices tend to fall in spite of the increase in the money supply.

On the whole, the responses of inventories and prices to a supply shock in the nondurables sector is in line with my intuition about the UST model.

Using employment instead of output

The identifying assumption in (34) and (35) is that demand shocks affect inventories only. This assumption is questionable because selling goods requires real resources. Even if selling goods does not require real resources there is a measurement problem that stems for the fact that our model has many prices for the same good while the NIPA measurements assumes a single price.

To see the measurement problem, note that since inventories are valued at replacement cost (p_1) nominal income depends on the number of markets open (s) and is given by: $p_1k_1 + p_1k_2$ if s = 1 and $p_1k_1 + p_2k_2$ if s = 2. Since $p_1 \le p_2$, nominal income is an increasing function of s. Since the price index which is used for deflating nominal income is based on quoted price and is not a function of s also real income depends on s.

Employment (N) suffers less from the first problem because it is a quasi fixed factor: Changes in demand during the quarter are likely to cause changes in hours per employee rather than the number of employees (see Eden and Griliches [1993], for example). Employment (number of employees) does not suffer from the second (measurement) problem because it does not use prices. I therefore ran the same VARs using employment (N) instead of output. The impulse response functions did not change much as a result of this substitution: See Figures 18 - 21 for the goods producing sector.

CONCLUSIONS

The negative relationship between the beginning of period inventories and output is an important prediction of the UST model. The raw correlations between these variables is positive (and small) when using H-P detrended variables. But when lag variables are held constant, the effect of inventories on output is negative and highly significant. In this case, inventories have a negative effect on all three components of labor supply: employment, hours per employee and effort.

The negative effect of the beginning of period inventories on output is rather robust. It holds when using first differences (not reported here) and when using the actual log variables in VAR analysis.

The model assumes that production decisions are made before the realization of demand. Under this identifying assumptions shocks to output are supply shocks and shocks to inventories (holding contemporaneous output constant) are (negative) demand shocks.

The impulse responses (Figures 2 - 9) are consistent with the predictions of the theory about the effects of demand shocks when looking at the first four quarters after the shock. Both output and prices decline in response to a negative demand shock (positive shock to inventories).

I was not able to derive the implications of the model about the effects of serially correlated supply shocks. I expected that similar to the case of serially independent supply shocks, end of period inventories will rise in response to a supply shock. This occurs in both the durables and nondurables sectors. I also expected that the effect on inventories will die out gradually. This occurs in the nondurables sector but not in the durables sector. Similarly I expected that prices will decline in response to a supply shock. This does not occur in the durables sector.

The finding of a negative effect of the beginning of period inventories on output and a positive effect of output on the end of period inventories is consistent with a positive correlation between the change in inventories and the change in output.[9] In the UST model a unit increase in inventories leads, on average, to a reduction in output and to a less than a unit increase in end of period inventories. The change in inventories like the change in output is therefore negative.

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[9] See Abramowitz (1950) for early empirical work and Ramey and West (1997) for the study of recent experience in the G7 countries.

Furthermore, a supply shock moves output and end of period inventories in the same direction.[10]

The results here are different from the findings in Cooley and Prescott (1995) for two reasons. First, Cooley and Prescott look at the correlation between the detrended <u>change</u> in inventories and detrended output and report high positive correlations. I look at the correlation between the detrended level of beginning of period inventories and detrended output. This correlation is positive but small. Second, I focus on the partial correlation rather than the simple correlation, and find a strong negative partial correlation.

PLEASE ADD HERE THE IMPULSE RESPONSE FUNCTIONS (FIGURES 2 - 21)

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[10] Technically,

$Cov(Y_t - Y_{t-1}, I_t - I_{t-1})$

$= Cov(Y_t, I_t) - Cov(Y_t, I_{t-1}) - Cov(Y_{t-1}, I_t) + Cov(Y_{t-1}, I_{t-1})$.

Therefore, the finding $Cov(Y_t, I_t) = Cov(Y_{t-1}, I_{t-1}) > 0$ and $Cov(Y_t, I_{t-1}) < 0$ is consistent with the finding $Cov(Y_t - Y_{t-1}, I_t - I_{t-1}) > 0$.

Figure 2
Response to One S.D. Innovations ± 2 S.E.
GDP sector

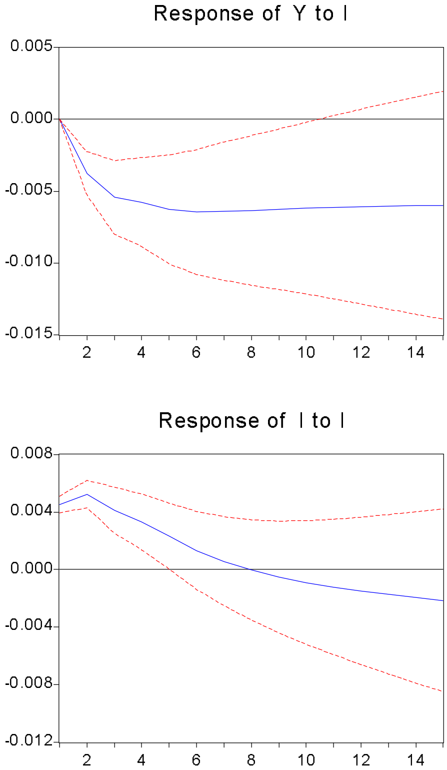


Figure 3
Response to One S.D. Innovations ± 2 S.E.
GOODS sector

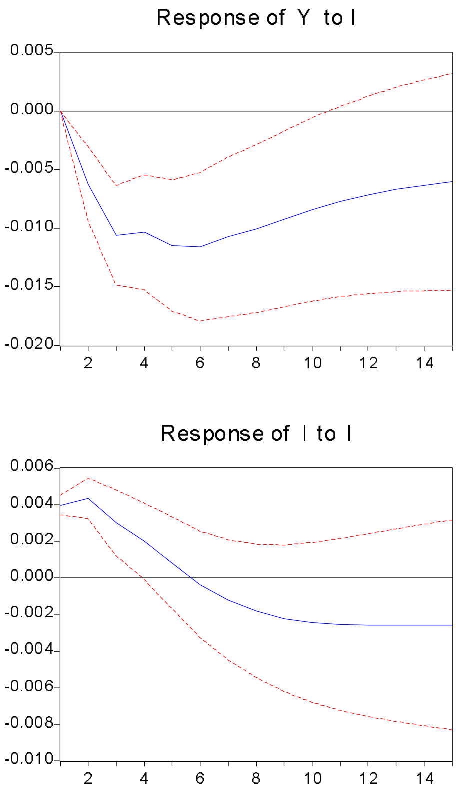


Figure 4
Response to One S.D. Innovations ± 2 S.E.
DURABLES sector

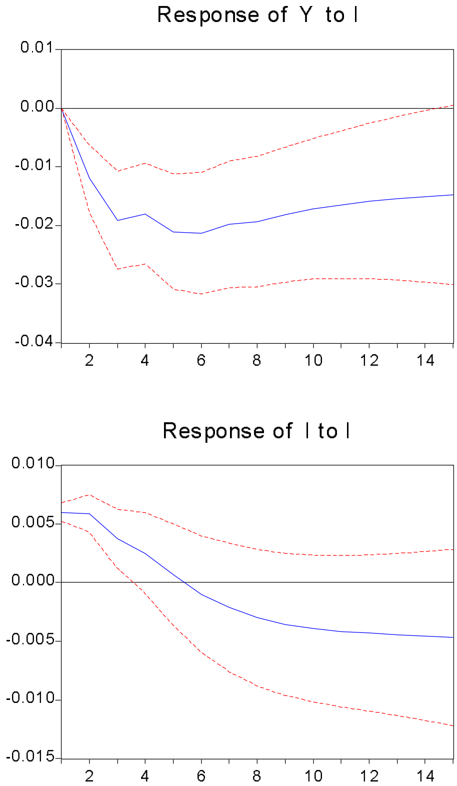


Figure 5
Response to One S.D. Innovations ± 2 S.E.
NONDURABLES sector

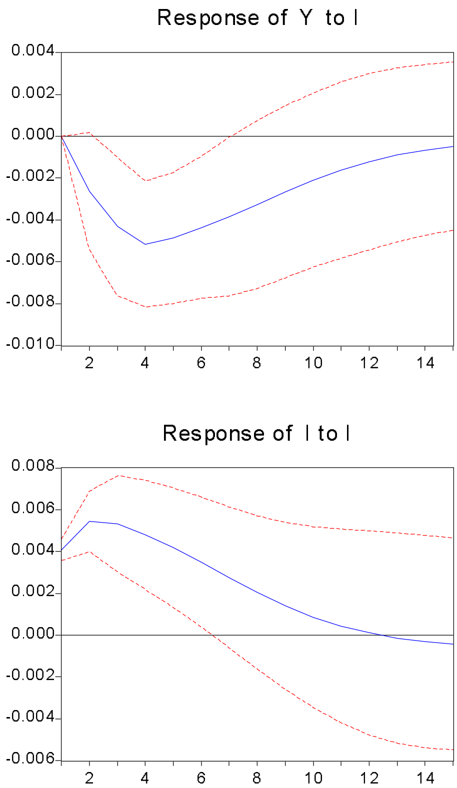


Figure 6
Response to One S.D. Innovations ± 2 S.E.
GDP sector

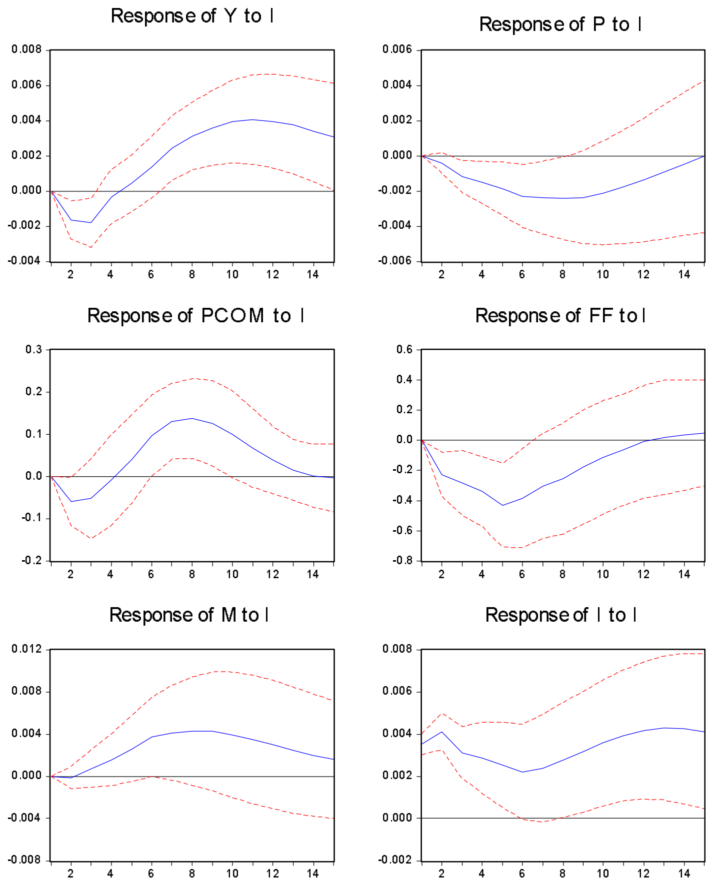


Figure 7
Response to One S.D. Innovations ± 2 S.E.
GOODS sector

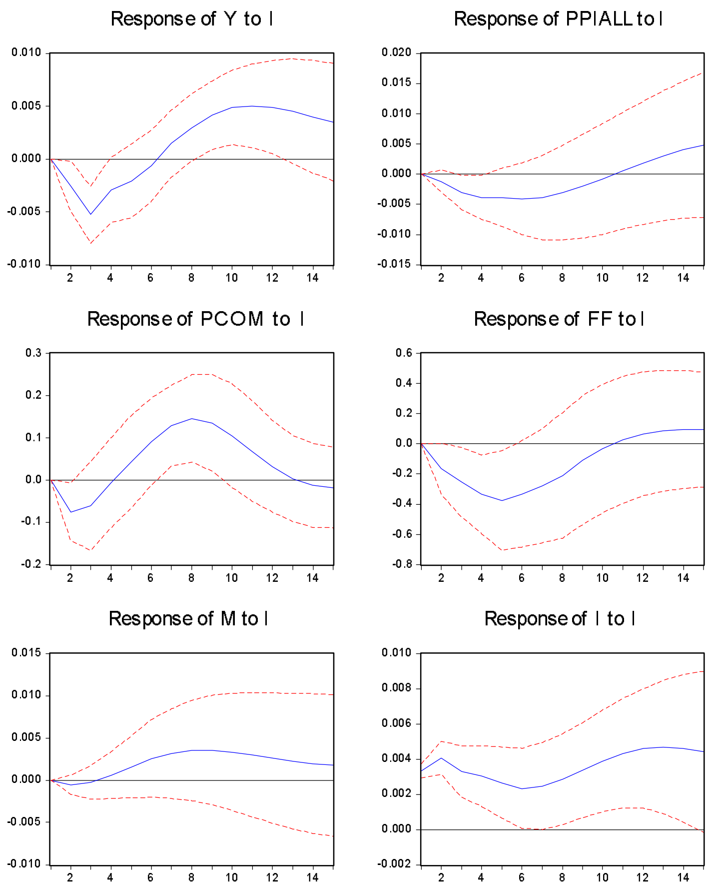


Figure 8
Response to One S.D. Innovations ± 2 S.E.
DURABLES sector

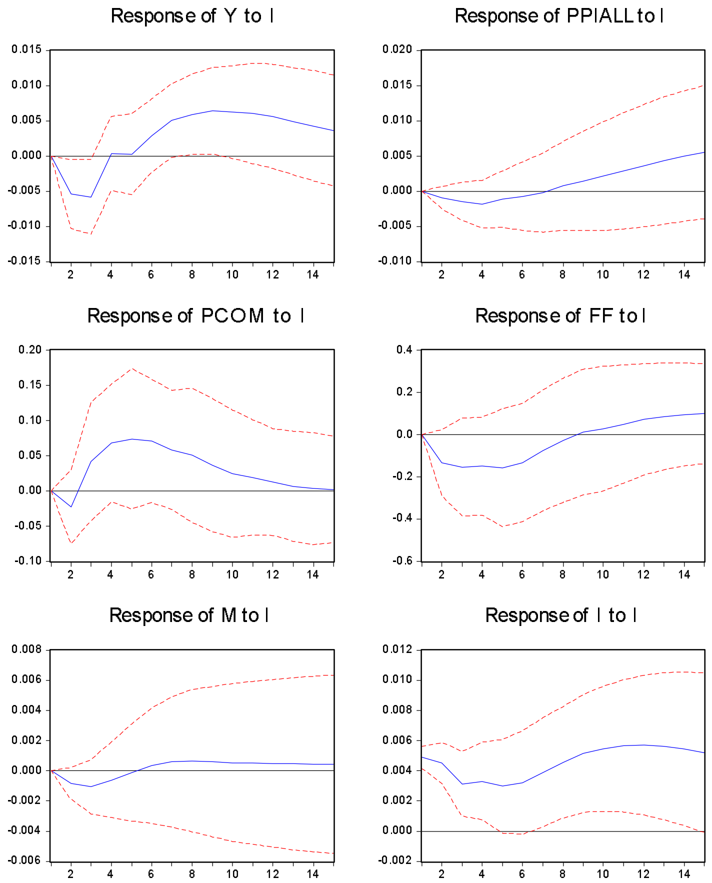


Figure 9
Response to One S.D. Innovations ± 2 S.E.
NONDURABLES sector

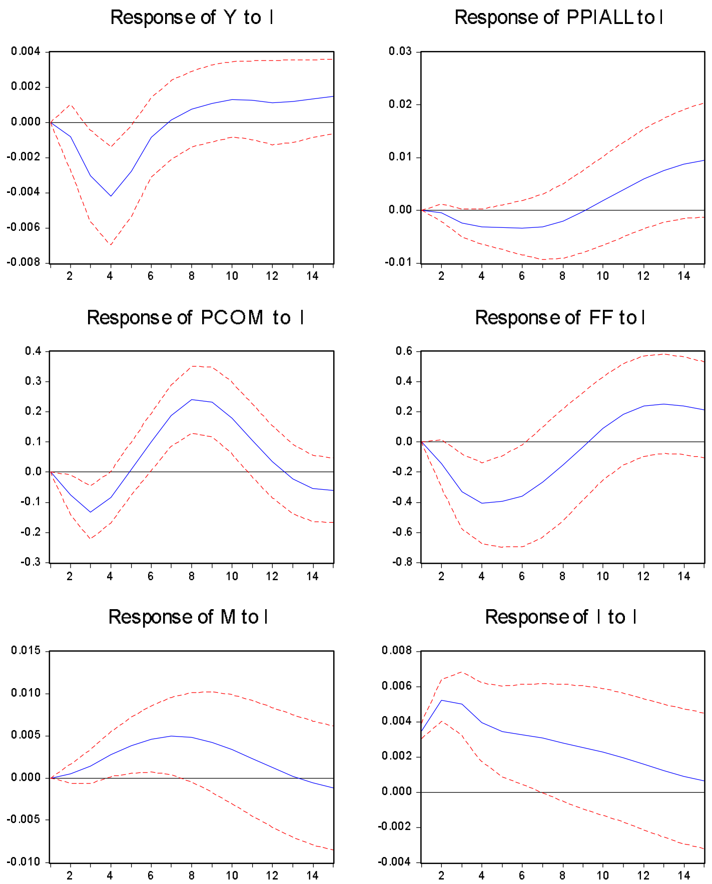


Figure 10
Response to One S.D. Innovations ± 2 S.E.
GDP sector

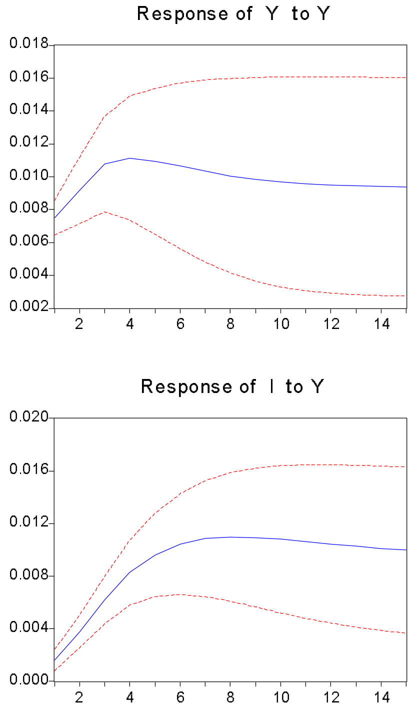


Figure 11
Response to One S.D. Innovations± 2 S.E.
GOODS sector

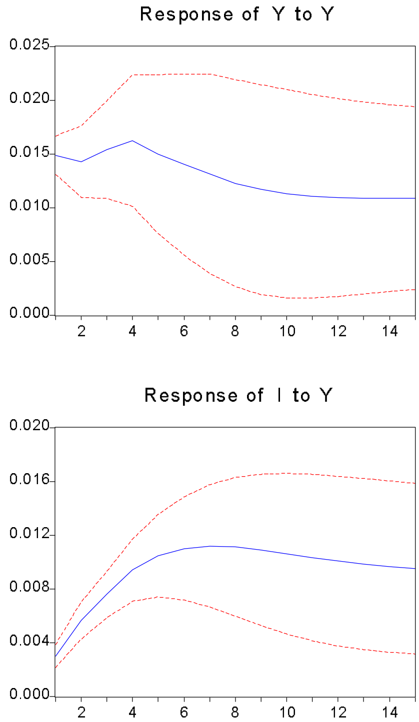


Figure 12
Response to One S.D. Innovations± 2 S.E.
DURABLES sector

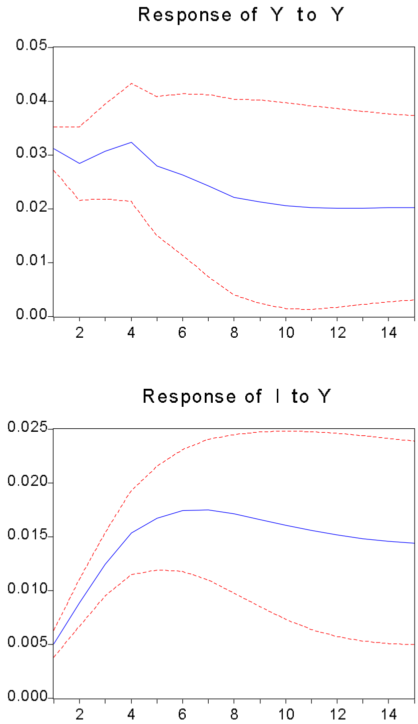


Figure 13
Response to One S.D. Innovations ± 2 S.E.
NONDURABLES sector

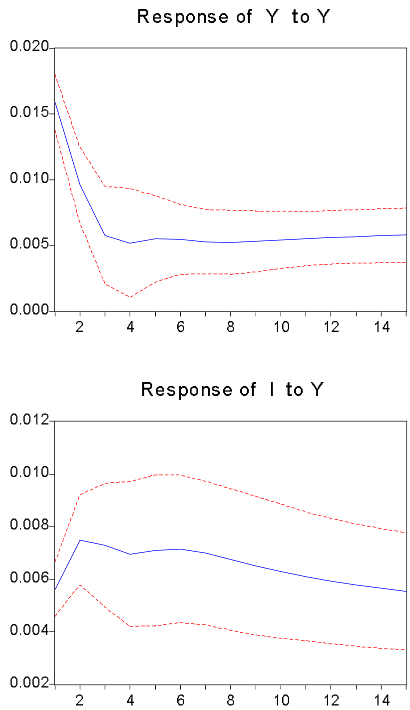


Figure 14
Response to One S.D. Innovations ± 2 S.E.
GDP sector

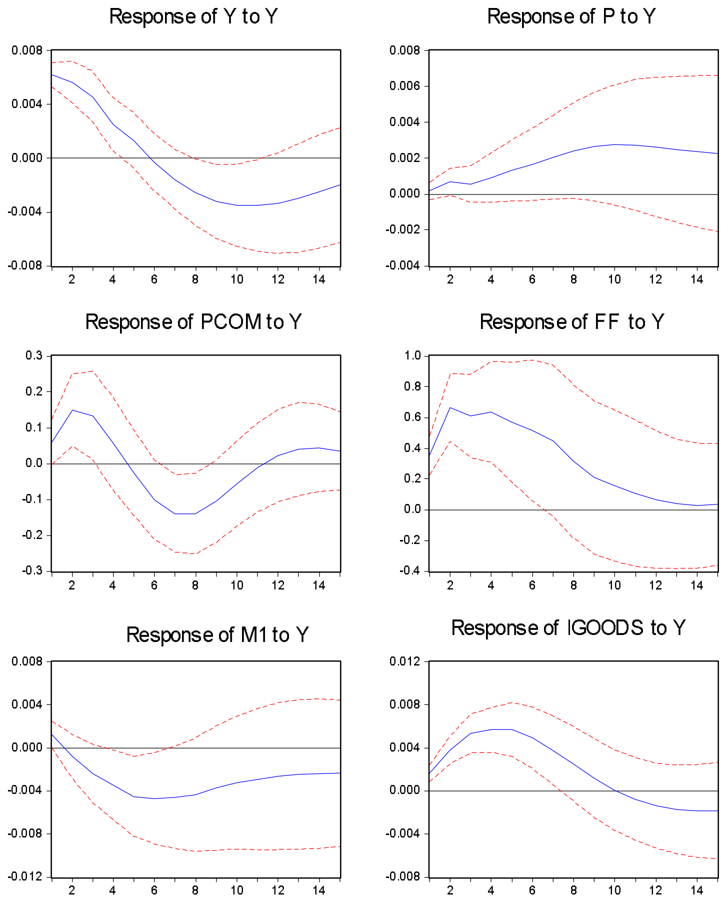


Figure 15
Response to One S.D. Innovations ± 2 S.E.
GOODS sector

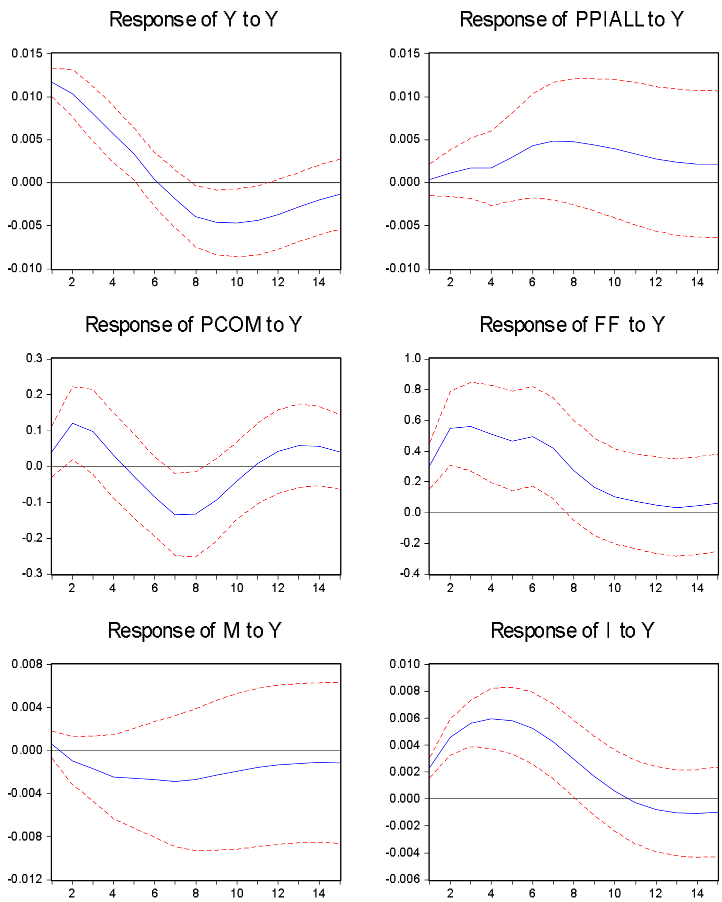


Figure 16
Response to One S.D. Innovations ± 2 S.E.
DURABLES sector

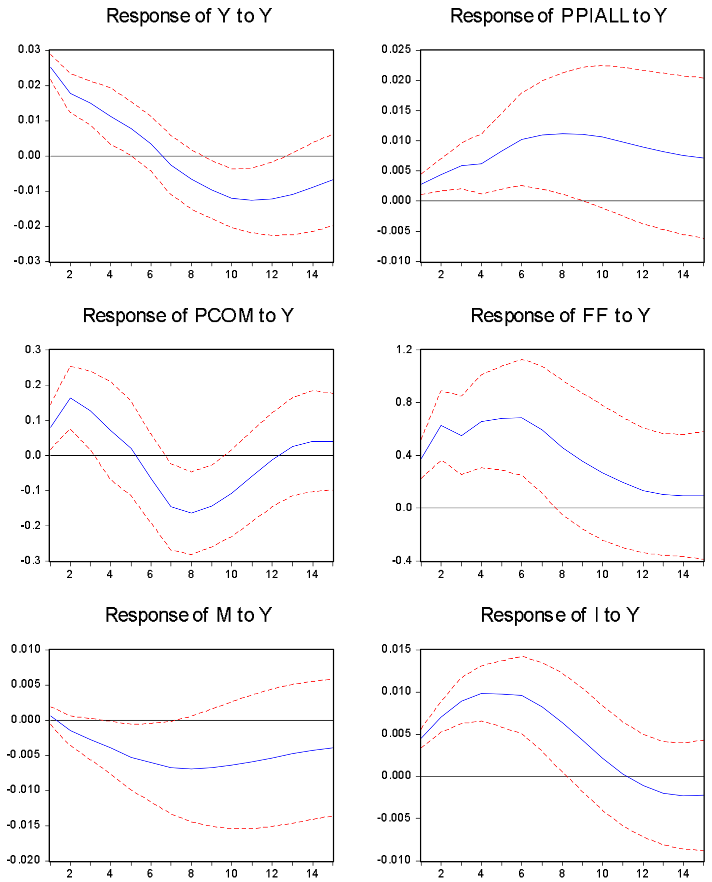


Figure 17
Response to One S.D. Innovations ± 2 S.E.
NONDURABLES sector

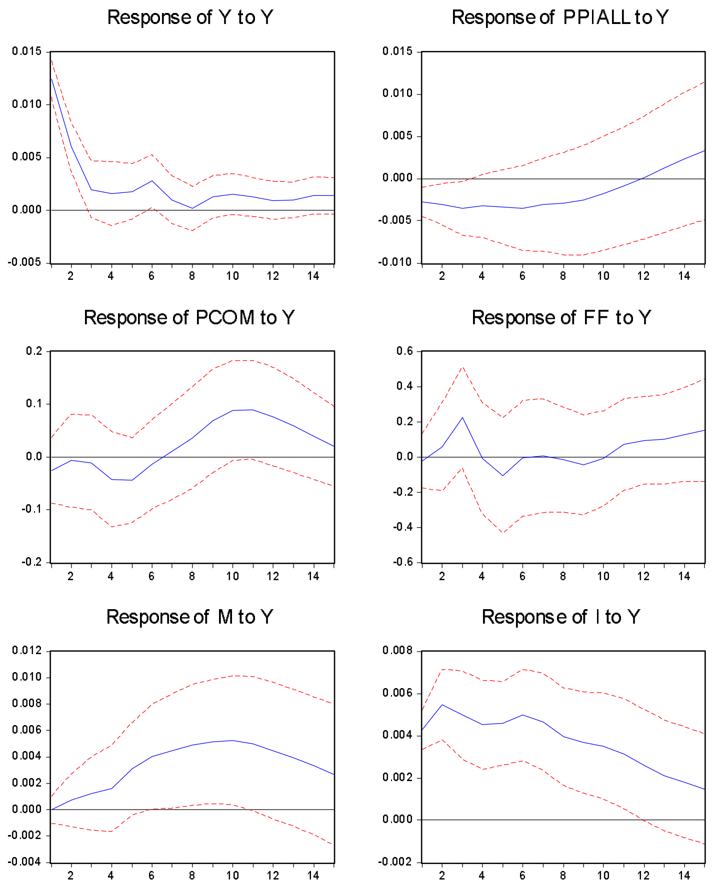


figure 18
Response to One S.D. Innovations ± 2 S.E.
GOODS sector

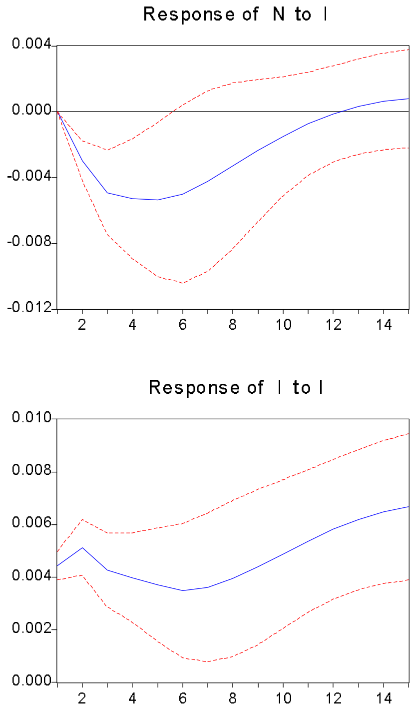


Figure 19
Response to One S.D. Innovations ± 2 S.E.
GOODS sector

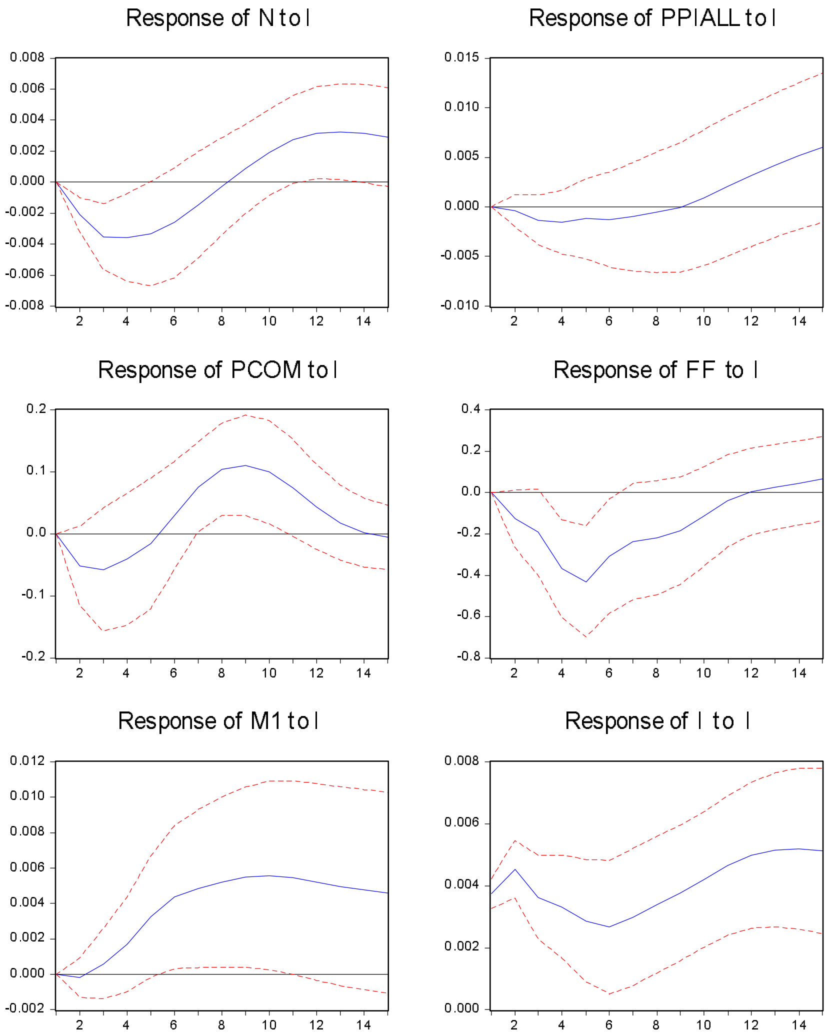


Figure 20
Response to One S.D. Innovations ± 2 S.E.
GOODS sector

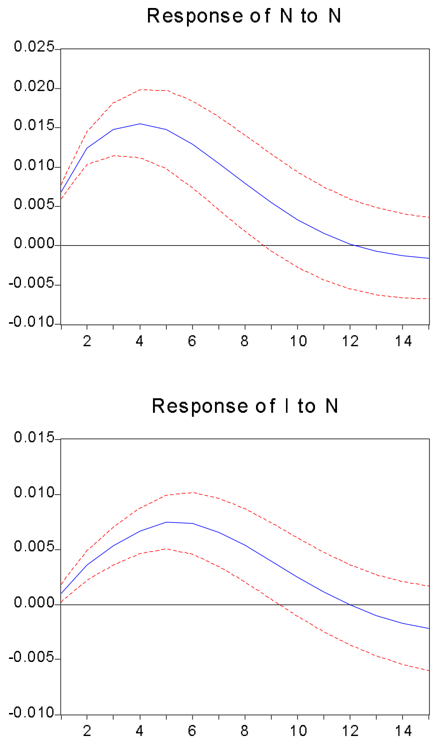
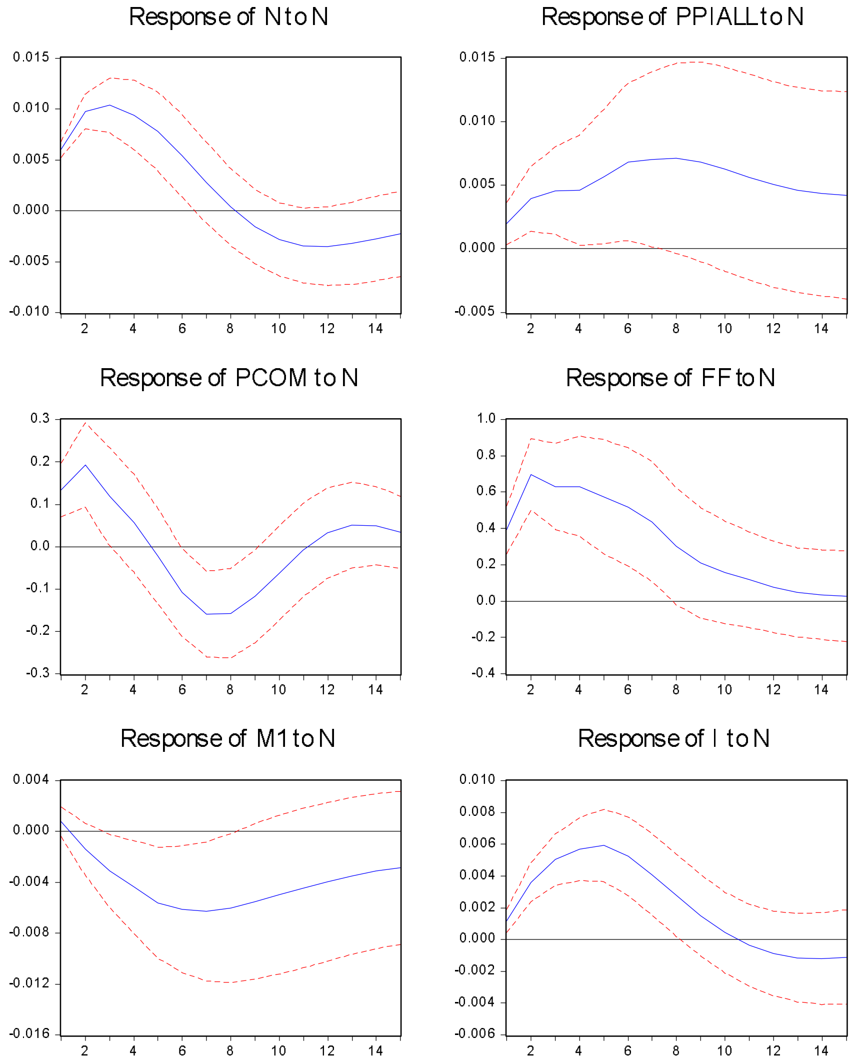


Figure 21
Response to One S.D. Innovations ± 2 S.E.
GOODS sector



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