SEQUENTIAL INTERNATIONAL TRADE

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This paper uses the uncertain and sequential trade model to show that trade in the same good may increase or decrease welfare and output. The paper characterizes the conditions under which trade leads to an increase in welfare and output. Roughly speaking a country with a relatively stable demand may suffer from trade if as a result of trade there is more uncertainty about supply and demand conditions. The model is also used to derive an equilibrium in which there is a strong positive relationship between the nominal and the real exchange rates.

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1. INTRODUCTION

The following observations have provoked a lot of discussion in the international trade literature: (a) Countries trade in similar goods; (b) There are government made barriers to trade and (c) there is a strong correlation between changes in the real and the nominal exchange rates.

These observations are sometimes regarded as puzzles because they are at odds with the implications of the standard Walrasian model. On the basis of the standard model we expect that only the benefits from trade in goods that are not close substitutes will cover transaction and transportation costs. In the classic Heckscher-Ohlin model countries with a relative large endowment of labor export labor intensive goods and import capital intensive goods. We also expect that in the absence of transactions costs governments will reach efficient agreements among themselves and promote free trade. Finally, the law of one price suggests no correlation between changes in the real and the nominal exchange rate but Obstfeld and Rogoff (1996) argues that for countries with floating currencies and open capital markets the exchange rates are an order of magnitude more volatile than the ratio of the CPIs which hardly moves at all (page 606, see their Figure 9.2). An immediate corollary is "that the short-run volatility of real exchange rates is very similar to that of nominal exchange rates". Earlier Mussa (1986) has observed that an increase in the volatility of the nominal exchange rate leads to an increase in the volatility of the real exchange rate.

Models with increasing returns to scale and monopolistic competition are often used to account for the trade in similar goods puzzle. See for example, Helpman and Krugman (1985). In Newbery and

Stiglitz (1984) there is trade in the same good to smooth consumption over time but in a given period each country is either exporting the good or importing it. It does not do both. Newbery and Stiglitz show that when agents are risk averse and insurance markets are incomplete trade may be Pareto inferior to autarky. This may provide a partial explanation for the barriers to trade question. A different approach is taken in the political economy literature. Schattschneider (1935) and Olson (1971) argue that the benefits from a tariff is concentrated in a relatively small group while the cost is spread over a large group that cannot form an effective lobby.

Rogoff survey of the extensive literature on the purchasing power parity puzzle. He concludes that "International goods market, though becoming more integrated all the time, remain quite segmented, with large trading frictions across a broad range of goods. These frictions may be due to transportation costs, threatened or actual tariffs, nontariff barriers, information costs, or lack of labor mobility. As a consequence of various adjustment costs, there is a large buffer within which nominal exchange rates can move without producing an immediate proportional response in relative domestic prices." Rogoff (1996, page 665).

Here I analyze these questions in an uncertain and sequential trade (UST) model that assumes a competitive environment, a single good, constant or diminishing returns to scale and risk neutrality. Residents of the same country are ex-ante identical. There are no "natural barriers to trade" like transportation costs. The only so called friction is the UST friction: buyers arrive sequentially and some irreversible trade must be made before the complete resolution of uncertainty about demand and supply.

I discuss the incentives for trade from the individual agent's point of view, from the individual's country point of view and from the world's point of view. I also discuss the effect of trade on output. This is done in two models: A real model that assumes a constant per unit cost of production and a single unit demand function and a monetary cash-in-advance model that assumes increasing marginal cost of production. I start with a discussion of the efficiency properties of the underline model.

2. EFFICIENCY IN UST MODELS

Uncertain and Sequential Trading (UST) models are based on ideas in Prescott (1975) and Butters (1977). In a review article of the Phelps volume, Prescott provides a counter example to the view that precautionary unemployment is likely to be excessive because sellers of labor services have some monopoly power. Prescott considers an example in which sellers of motel rooms set prices before they know how many buyers will arrive. He assumes that cheaper rooms are sold first and therefore in equilibrium sellers face a tradeoff between price and the probability of making a sale.

In prescott's example all motel rooms are the same and all buyers who arrive want a single room and are willing to pay up to the same reservation price. Prescott's conclusion is that "For this example, which entails monopoly power on the part of sellers facing a stochastic demand, the competitive equilibrium is efficient. If demanders were heterogeneous (in terms of preferences) and there were heterogeneity in the type of room supplied, it is possible that these conclusions would be altered. Until such an analysis is successfully performed, I see no reason to conjecture that the natural vacancy rate is either too high or too low." (page 1233).

In the UST approach taken by Eden (1990) an equilibrium distribution of prices is obtained even though sellers are allowed to change their prices during trade and have no monopoly power. In this model all buyers have the same downward sloping demand curves and buyers who arrive first buy more at lower unit prices. In terms of the motel room example: Buyers who arrive early buy larger rooms at a low price per square meter. The UST outcome is efficient and may be viewed as the execution of ex-ante contingent contracts. Thus heterogeneity in the type of room supplied does not alter the efficiency result.

Recently, Dana (1998) has extended Prescott's model to the case of heterogeneous buyers. He concludes that in this case the equilibrium allocation may not be as good as the Walrasian outcome because of price rigidity.

The comparison with the Walrasian model is appropriate if the reason for the inefficiency is in rigid prices. In the UST model considered here prices are not rigid. They do not respond to the current realization of demand because of informational constraints: Sellers must make irreversible trading decisions before they know the realization of demand. The question is whether a central planner who faces the same informational constraints as the sellers in the model can improve matters.

3. A REAL UST MODEL

I consider an economy with two dates (t = 0,1) and two goods (X and Y with lower case letters denoting quantities).

There are S possible aggregate states of nature (indexed s) where state s occurs with probability Π_s. There are J types of sellers. The ability to produce depends on the aggregate state. If a seller is able to produce he can produce as many units as he wants at the price of λ units of Y per unit of X. The number of type j sellers is m_j and a fraction μ_{js} of them can produce in state of nature s. The total number of sellers who can produce in state of nature s is:

(1) $$M_s = \Sigma_j \mu_{js} m_j.$$

There are J types of buyers. A type j buyer demands one unit of X at any price less than v_j if he wants to consume and zero otherwise. There are n_j type j buyers and in state s a fraction ϕ_{js} of them want to consume. Aggregate demand per seller over all type j buyers in state s at the price p is thus:

(2) $$N_{js}(p) = \phi_{js} n_j / M_s \quad \text{if } v_j \geq p \text{ and zero otherwise.}$$

Aggregate demand per seller over all types in state s is:

(3) $$N_s(p) = \Sigma_j N_{js}(p).$$

Production occurs at t = 0. The state becomes public information only after the completion of trade at t = 1.

The $\Sigma_{j=1}^{J} \phi_{js} n_j$ buyers who want to consume form a line by a lottery that treats everyone symmetrically. After the line is formed, buyers arrive at the market place one by one according to their place in the line and choose whether to buy at the cheapest available offer. Figure 1 illustrates the sequence of events.

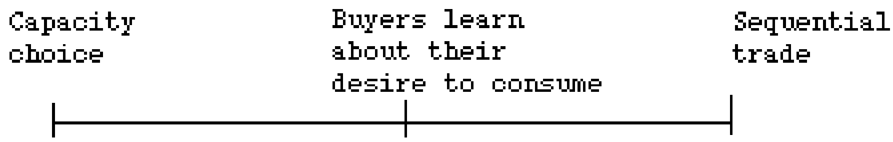

Figure 1

From the individual seller's point of view demand arrives in batches. The first batch with Δ_1 buyers per seller arrives with certainty. After the buyers in the first batch complete trade a second batch of Δ_2 buyers (per seller) may arrive. In general, there can be two possible events after buyers in batch i complete trade: Either trade ends or an additional batch of Δ_{i+1} buyers arrive. Figure 2 illustrates the sequential trade process.

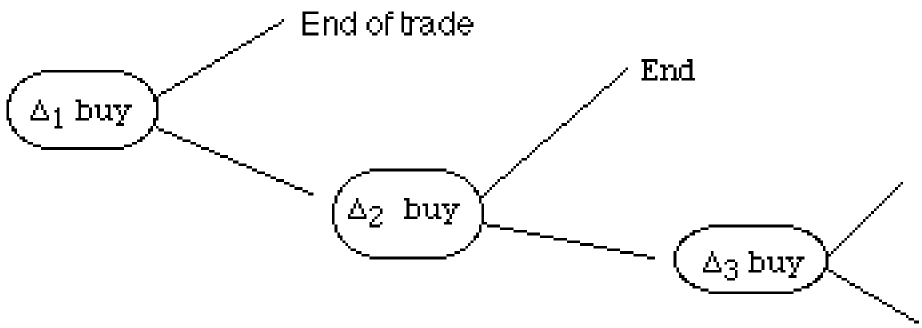

Figure 2

Sellers are price-takers. They behave as if they can sell any amount at the price P_i to buyers in batch i if it arrives. Each seller makes a contingent plan to sell x_i units to batch i if it arrives.

It helps to think in terms of markets that open sequentially. The first batch of buyers opens the first market. If the second batch of buyers arrives, it opens the second market and so on. The price in market i is P_i, the quantity supplied to market i is x_i per seller and the quantity demanded in market i is Δ_i per seller. In equilibrium markets that open are cleared:

(4) $$\Delta_i = x_i \text{ for all } i.$$

Trade occurs sequentially but does not take real time. The only information that the sellers receive during the trading process is about the number of batches that arrive: at stage i of the trading process sellers know that i batches already arrived.

The probability that market i will open, q_i, the price in market i, P_i, and the size of the batch that will trade in this market if it opens (Δ_i) are all determined endogenously. An algorithm for computing these equilibrium magnitudes is as follows. The first market opens when some buyers want to consume (regardless of their reservation price). The probability that the first market will open is the probability that a state with strictly positive demand occurs. This probability is denoted by q_1. The price in the first market is: $P_1 = \lambda/q_1$. At this price suppliers to the first market make zero expected profits ($q_1P_1 = \lambda$). Market 2 opens if there are some buyers who wanted to buy in the first market but could not. In general, an additional market opens after transactions in market i-1 are complete if there is residual demand: There are buyers who wanted to buy in market i-1 at the price P_{i-1} but could not make a buy. We compute the probability that there is residual demand after trade in market i - 1 is complete and denote it by q_i. We use this probability to compute the zero expected profit price in market i: $P_i = \lambda/q_i$. To determine Δ_i we compute the minimum (strictly positive) number of unsatisfied customers who are willing to pay the price P_i.

Appendix A provides a general treatment. It is shown there that when the reservation prices are the same across types or (and) the probability of wanting to consume is the same across types, the UST allocation is efficient in the relevant sense. We now turn to examples in which the allocation is not necessarily efficient.

Example 1: There are two types of buyers and two states of nature. The number of buyers per type is n. A type 1 buyer demands one unit if the price is less than 7 and zero otherwise ($v_1 = 7$, $\phi_{11} = \phi_{12} = 1$). A type 2 buyer demands 1 unit if the price is less than 7 and s = 2 and demands zero otherwise ($v_2 = 7$, $\phi_{21} = 0$ and $\phi_{22} = 1$). Type j buyers reside in country j. There is a known number of sellers in each country who can all produce in both states at the unit cost of $\lambda = 5$. (Thus there is no uncertainty about the supply).

Since there is no uncertainty about the demand in country 1, under autarky there will be only one market (a standard Walrasian market) in country 1. This market opens with certainty at the price of $\lambda = 5$. The quantity produced in country 1 is n and the generated surplus is $(v_1 - \lambda)n = 2n$. There is a market in country 2 that opens with probability ½ at the price of 10 but there is no demand in country 2 at this price. The quantity produced in country 2 is zero and the generated surplus is zero.

We now allow free trade. To define the market structure for this case we start by specifying the demand per seller as a function of the price and the state:

(6) $N_1(p) = n$ and $N_2(p) = 2n$ if $p \le 7$ (0 otherwise).

There are always some buyers who want to consume and therefore market 1 opens with certainty at the price: $\lambda = 5$. The minimum demand per seller at this price and hence the number of buyers in the first market is: $\Delta_1 = n$.

Market 2 will open if there are buyers who wanted to buy at the first market but could not. This will occur in state 2 with probability $q_2 = {}^1\!/_2$ and the price in the second market is therefore: $P_2 = 2\lambda = 10$. At this price there is no demand in market 2. Total production is therefore n and total surplus is 2n.

Since the lottery that determines the order of arrival of buyers treats everyone symmetrically, in state 2 $({}^1\!/_2)n$ buyers from each country make a buy in market 1. The surplus is therefore divided between the two countries: The buyers in country 1 get an average surplus of 1.5n while the buyers in country 2 get an average surplus of 0.5n. Autarky is better for the residents in country 1 and free trade is better for the residents of country 2.

Trading in ex-ante contracts: Suppose that we allow ex-ante trade (at $t = 0$) in promises to deliver X. The equilibrium price of a promise to deliver a unit of X is 5. Only type 1 buyers will choose to buy this contract and the resulting surplus is 2n.

Thus, here allowing for trade in ex-ante contracts is equivalent to autarky.

Example 2: The same as example 1 but now $v_1 = 10$.

Under autarky there will be only one market in country 1 that opens with certainty at the price of $\lambda = 5$. There is a market in country 2 that opens with probability ${}^1\!/_2$ at the price of 10 but there is no demand in country 2 at this price. The quantity produced in country 1 is n and the generated surplus is 5n.

We now allow free trade. To define the market structure for this case we start by specifying the demand per seller as a function of the price and the state:

(7) $N_s(p) = 0$ if $p > 10$; $N_s(p) = n$ if $7 < p \le 10$;

$$N_1(p) = n \text{ and } N_2(p) = 2n \quad \text{if } p \le 7.$$

The minimum number of buyers who want to consume is n. Market 1 will therefore open with certainty at the price of $P_1 = 5$. The minimum number of buyers who want to buy at this price is: $\Delta_1 = n$.

In state 2 there are buyers who wanted to buy in the first market but could not. Therefore market 2 opens with probability $\frac{1}{2}$ at the price of $P_2 = 10$.

When $s = 2$, $(\frac{1}{2})n$ buyers from each type make a buy in market 1 and the number of unsatisfied customers is $(\frac{1}{2})n$ of each type. But at the price of 10 only type 1 buyers are willing to buy. Therefore, $\Delta_2 = (\frac{1}{2})n$. Production is (1.5)n units at the cost of (7.5)n. The surplus in state 1 is: $(v_1 - 7.5)n = (2.5)n$. The surplus in state 2 is: $(v_1 + (\frac{1}{2})v_2 - 7.5)n = 6n$. The average surplus is (4.25)n.

The world surplus is less than under autarky.

A monopoly (that faces the same informational constraints as the sellers in our model) will choose $P_1 = 10$ and produce n units making a profit of 5n. This is one way of showing that when agents are heterogeneous, the UST allocation is not efficient in the relevant sense.

Example 3: The same as example 2 but now type 1 is the relatively low valuation buyer: Type 1 buyers want to consume a unit only if $p \le v_1 = 7$ and type 2 buyers want to consume a unit only if $p \le v_2 = 9$ and $s = 2$.

Under autarky, n units are produced in country 1 and none are produced in country 2. (There is one market in country 1 that opens with certainty at the price of 5 and there is one market in country 2 that opens only in state 2 at the price of 10 but there is no demand at this price). The world surplus is 2n and all of it goes to the residents of country 1.

We now allow free trade. There are always some buyers who want to buy and therefore market 1 opens with probability 1 at the price of 5. In state 2 there will be unsatisfied buyers and therefore the probability that market 2 will open is $^1/_2$ and the price in this market is 10. At this price there is no demand in market 2.

Production is n units. In state 1 these units are sold to type 1 and the surplus is: $(7 - 5)n = 2n$. In state 2, $(^1/_2)n$ units are sold to each of the two types and the surplus is: $(^1/_2)7n + (^1/_2)9n - 5n = 3n$. The expected surplus is: 2.5n. Type 1 buyers get an expected surplus of 1.5n while type 2 buyers get an expected surplus of n.

World surplus is 2.5n and is higher than under autarky.

A monopoly will set the price 7 and produce n units. The surplus in this case is:
$S = (^1/_2)7n + (^1/_2)[(^1/_2)7n + (^1/_2)9n] - 5n = 2.5n$. The monopoly will make an expected profit of 2n. Type 1 buyers will make zero expected surplus and type 2 buyers will make an expected surplus of 0.5n.

Thus in this example free trade increases world surplus but reduces the surplus of the buyers in the country with stable demand (country 1). The monopoly profits is equal to the free trade surplus.

Example 4: Type 1 wants to consume in state 1 only while type 2 wants to consume only in state 2. They are both willing to pay up to 7 units for the good in the state that they want to consume.

Under autarky there will be no trade.

Under free trade n units are supplied to a single market that opens with certainty at the price of 5. In state 1 type 1 buyers buy n units and in state 2 type 2 buyers buy n units. Surplus is 2n.

In this example trade improve welfare in both countries.

Example 5: The same as example 4 but now type 1 buyers are willing to pay only up to 4. In this case no trade will occur even when free trade is allowed.

A discriminating monopoly may improve matters by producing n units and posting a price of 4 in country 1 and a price of 7 in country 2. In this case the monopoly will sell in country 1 when $s = 1$ and in country 2 when $s = 2$, making an average profit of 0.5n.

If the discriminating monopoly is in country 2 his sales in state 1 at the price of 4 may look like "damping" to the residents of country 1 but in this case "damping" does not have an adverse effect on the welfare in country 1. See Ethier (1982) for a comprehensive discussion of "damping".

Example 6 (Supply shocks): There are two countries and there is one seller per country. In country 1 there is a (type 1) seller who can produce in both states at the cost of $\lambda = 5$ per unit. In country 2 there is a seller who can produce only in state 1 at the cost of $\lambda = 5$ per unit. Type 1 must choose production before he knows the state.

There are n buyers, living in country 1, who want to consume in both states and are willing to pay up to 10 for a unit. Buyers in country 2 do not want to consume good X (in any state).

Under autarky there is one market that opens with certainty in country 1 and the price in this market is 5. Production in country 1 is n and buyers in country 1 get a surplus of 5n. There is no production in country 2.

Under free trade, total demand per seller at the price p in state of nature s, is:

$$(8) \qquad n/M_1 = (1/2)n \text{ and } n/M_2 = n \quad \text{if } p \leq 10 \text{ and } 0 \text{ otherwise.}$$

There is some demand in both states and therefore the first market opens with probability 1 at the price of 5. The minimum demand per seller and hence the number of buyers in the first market is: $\Delta_1 = (1/2)n$.

In state 2 there will be some unsatisfied buyers. The probability that market 2 will open is $1/2$ and the price in this market is 10. The demand in the second market if it opens is $(1/2)n$.

Type 1 seller supplies $(1/2)n$ at the price 5 and $(1/2)n$ at the price of 10. Type 2 supplies in state 1 $(1/2)n$ units at the price of 5. When s = 1, n units are produced and the surplus is (10 - 5)n = 5n. When s = 2, 1.5n units are produced and the surplus is: (10 - 7.5)n = 2.5n. The average surplus is: 3.75n.

Thus here free trade is Pareto inferior to autarky.

Table 1 summarizes the results of the examples. It suggests that trade may be Pareto superior (example 4) or Pareto inferior (example 6) to autarky.[2] It also suggests that the country with a relatively stable demand and supply conditions is more likely to loose from trade while the opposite holds for the country with the relatively unstable demand and supply conditions.

These conclusions are very different from what one will get from a standard Walrasian model of trade. The reason is in the UST friction: Irreversible choices are made before the state is known. On a less abstract level there are two reasons why free trade may change the world surplus relative to autarky. It affects average capacity utilization and it affects the allocation of capacity among buyers.

In example 2, average capacity utilization (the average fraction of output sold) under autarky is 1 in country 1 and not defined in country 2. The world average capacity utilization is 1. Under free trade the world's average capacity utilization is $^5/_6$ ($^2/_3$ in state 1 and 1 in state 2). In example 3 free trade does not change average capacity utilization but it improves trade because in state 2 it achieves a better allocation of output across buyers.

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[2] Note that the adverse effect of trade on welfare occurs here under risk neutrality and is therefore different from the result in Newbery and Stiglitz (1984). In their model the strongest case against trade is in the case of perfectly negative correlation between the supply and demand conditions in the two countries. As shown by example 4, in our model this is the strongest case for (rather than against) trade.

Table 1*: Autarky, free trade and monopoly

	World surplus	surplus to buyers in country 1	surplus to buyers in country 2	output
Type 1 wants to consume in both states, type 2 wants to consume in state 2 only				
$v_1 = v_2 = 7$ (example 1):				
Autarky	2	2	0	1
Free trade	2	1.5	0.5	1
monopoly:	2	0	0	1
$v_1 = 10$, $v_2 = 7$ (example 2):				
Autarky	5	5	0	1
Free trade	4.25	3.75	0.5	1.5
monopoly:	5	0	0	1
$v_1 = 7$, $v_2 = 9$ (example 3):				
Autarky	2	2	0	1
Free trade	2.5	1.5	1	1
monopoly:	2.5	0	0.5	1
Type 1 wants to consume in state 1 and type 2 in state 2				
$v_1 = v_2 = 7$ (example 4):				
Autarky	0	0	0	0
Free trade	2	1	1	1
monopoly	2	0	0	1
$v_1 = 4$; $v_2 = 7$ (example 5):				
Autarky	0	0	0	0
Free trade	0	0	0	0
disc. monopoly	0.5	0	0	1
Supply shock: The seller in country 2 can produce only in state 1				
$v_1 = 10$; $v_2 = 0$ (example 6):				
Autarky	5	5	0	1
Free trade	3.75	3.75	0	1.5
monopoly	5	0	0	1

* This Table gives the results in examples 1-6. The first column is world surplus calculated as the expected value of the output sold minus the cost of production. Then we have the surplus of the buyers in each country and the world output. There is one type of buyer (seller) per country. v_j is the reservation price of type j buyer. In examples 1-3 type 1 buyers have stable demand. In examples 4 - 5 demand is negatively correlated. In example 6 there is supply uncertainty. For each example we compute the outcome under autarky, free trade and monopoly. In example 5 we have a discriminating monopoly.

When free trade reduces average world's capacity utilization it also leads to an increase in output (examples 2 and 6). As we shall see in the next section this result is special to the inelastic unit demand assumption.

3. A MONETARY MODEL

I now assume a cash-in-advance economy with a downward sloping demand and increasing marginal cost.

There are N infinitely lived households. As in Lucas (1980), each household consists of a seller (worker) and a buyer pair. At the beginning of the period the worker goes to work. The buyer learns about his demand (taste shock) and if he wants to consume he takes the available money and goes shopping. After trade in the goods market is complete the seller takes his revenue in the form of cash reunite with the shopper and consume whatever the shopper has bought.

The household is risk neutral and its single period utility is: $\theta c_t - v(L_t)$, where c is the quantity consumed, L is the quantity produced and θ is a random variable that takes two possible realizations: 0 if the household does not want to consume and 1 if it does. The cost function v( ) has the standard properties ($v' > 0$ and $v'' > 0$ everywhere) and the household's discount factor is $0 < \beta < 1$.

Aggregate demand is an i.i.d random variable. In each period there are S possible states of aggregate demand. In state s a fraction ϕ_s of the agents experience $\theta = 1$ and want to consume. It is assumed that: $0 < \phi_1 \le \phi_2 \le \ldots \le \phi_S$. State s occurs with probability Π_s. The probability that the state is greater than or equal to s is

$q_s = \sum_{i=s}^{S} \Pi_i$. The probability that $\theta = 1$ is the same for all agents and is equal to ϕ_s in state s. For notational convenience I set $\phi_0 = 0$.

I start by assuming a single world currency - the dollar. Household h starts the period with M_t^h dollars and gets in addition, a perfectly anticipated lump sum transfer of G_t dollars. The average per-household post transfer amount of money is: $M_t = G_t + (1/N)\sum_{h=1}^{N} M_t^h$ dollars. The deterministic rate of change in the world money supply is: $M_{t+1}/M_t = 1 + \mu$.

Buyers who want to consume spend all the money they have. As in the previous section buyers who want to consume arrive sequentially at the market-place, buy at the cheapest available price and disappear. Buyers who arrive early buy at low prices. Buyers who arrive late do not find cheap merchandise and buy at relatively high price.

From the seller's point of view nominal demand in aggregate state s is $\phi_s M_t(1 + \mu)$ dollars or ϕ_s normalized dollars, where normalized magnitudes are nominal magnitudes divided by the post transfer money supply, $M_t(1 + \mu)$.

Dollars arrive in batches. The size of the first batch is the minimum amount that will be spent: ϕ_1 normalized dollars. The size of the second batch is the minimum additional amount that will be spent if $s \geq 2$: $\min_{s \geq 2}\{\phi_s - \phi_1\} = \phi_2 - \phi_1$ normalized dollars. In general, the size of batch i is the minimum additional amount that will be spent if $s \geq i$ and i-1 batches completed trade: $\min_{s \geq i}\{\phi_s - \phi_{i-1}\} = \phi_i - \phi_{i-1}$ normalized dollars. Batch i buys in market i at the price of p_i normalized dollars per unit if it arrives.

Workers/sellers choose how much to produce and make a contingent plan which specifies the amount that they will sell to each batch if it arrives. This contingent plan is described as an allocation of output across the potential markets: k_i to market i.

The sequence of events within the period is as follows. Prices are announced for each of the S markets and the workers choose production and the allocation of output across the potential markets.[3] Buyers then learn about their desire to consume and those who want to consume arrive sequentially at the market-place. Figure 3 illustrates the sequence of events.

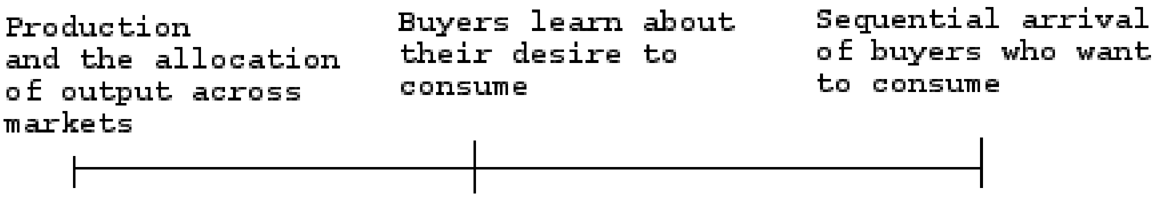

Figure 3

The expected purchasing power of a dollar when exactly s markets open is:

(9) $$z_s = \Sigma_{i=1}^{s}(\upsilon_i^s/p_i),$$

where υ_i^s is the probability that the dollar will buy in market i: $\upsilon_i^s = (\phi_i - \phi_{i-1})/\phi_s$.

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[3] It is possible to show tha during trade there are no incentive to change the allocation of goods across markets or to change prices.

The household starts with m normalized dollars and receives a transfer worth g normalized dollars.[4] It solves the following Bellman's equation:

$$(10) \qquad V(m) = \Sigma_{s=1}^{S} \Pi_s \phi_s (m + g) z_s + \max_{k_s} - v(\Sigma_{s=1}^{S} k_s)$$

$$+ \beta \Sigma_{s=1}^{S} \Pi_s \{\phi_s V[(\Sigma_{i=1}^{s} p_i k_i)\omega] + (1 - \phi_s) V[(m + g + \Sigma_{i=1}^{s} p_i k_i)\omega]\},$$

where $\omega = (1 + \mu)^{-1}$ is used to convert current normalized dollars into next period's normalized dollars. Here V(m) is the maximum expected utility that the household can achieve; $\Sigma_{s=1}^{S} \Pi_s \phi_s (m + g) z_s$ is the expected current consumption; $v(\Sigma_{s=1}^{S} k_s)$ is the utility cost of producing $L = \Sigma_{s=1}^{S} k_s$ units. To understand the future utility terms note that the worker's revenue is $\Sigma_{i=1}^{s} p_i k_i$ if exactly s markets open this period. When $\theta = 1$ the buyer spends everything he has and the household will have at the end of the period $\Sigma_{i=1}^{s} p_i k_i$ normalized dollars which will be worth, in the next period, $(\Sigma_{i=1}^{s} p_i k_i)\omega$ normalized dollars. When $\theta = 0$ the buyer saves everything and the household will start next period with $(m + g + \Sigma_{i=1}^{s} p_i k_i)\omega$ normalized dollars.

In aggregate state s, a normalized dollar will buy on average z_s units if the buyer want to consume (with probability ϕ_s) and will be saved and become ω normalized dollars in the next period if the buyer does not want to consume. The expected utility from a normalized dollar is thus:

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[4] Since we normalize by the post transfer money supply: $g = \mu/(1 + \mu)$.

(11) $$z = \Sigma_{s=1}^{S}\Pi_s[\phi_s z_s + (1-\phi_s)\beta\omega z].$$

Using $\zeta = \Sigma_{s=1}^{S}\Pi_s\phi_s$ to denote the unconditional probability of wanting to consume, we can solve (11) and get:

(12) $$z = (1 - \beta\omega + \zeta\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\phi_s z_s.$$

The expected benefits from supplying a unit to market s is $q_s p_s \omega z$ utils: If the market opens (with probability q_s) the seller gets p_s current normalized dollars which will become $p_s\omega$ next period's normalized dollars and bring on average $p_s\omega z$ utils. The first order conditions for an interior solution to (10) can therefore be written as:

(13) $$q_s\beta p_s\omega z = \beta p_1\omega z = v'(\Sigma_{s=1}^{S} k_s).$$

This says that the discounted expected benefit from supplying a unit is the same for all markets and is equal to the marginal cost. I will refer to $\beta p_1\omega z$ as the expected discounted real wage or the real wage for short.

Equilibrium requires that the first order conditions (13) are satisfied and markets which open are cleared:

(14) $$p_s k_s = \phi_s - \phi_{s-1} \quad \text{for all s.}$$

Welfare in equilibrium is an increasing function of the real wage, $w = \beta p_1\omega z$, and can be measured by:

(15) $$A(w) = \max_L \; wL - v(L),$$

where wL - v(L) is the expected utility that the worker gets for a given choice of L (as a function of L and w) and A(w) is the maximum expected utility as a function of w. It is therefore useful to characterize the equilibrium real wage as a function of the probability distribution of ϕ.

In equilibrium average capacity utilization must equal average consumption per unit of labor supplied and is therefore related to the real wage. The capacity utilization when exactly s markets open is: $CU_s = \Sigma_{i=1}^{s} k_i / \Sigma_{s=1}^{S} k_s$. The expected capacity utilization is:

$$CU = \Sigma_{s=1}^{S} \Pi_s CU_s . \tag{16}$$

I now show the following Claim.

Claim 1: The equilibrium expected capacity utilization is:
$CU = 1 - [\Sigma_{s=1}^{S}\Pi_s\Sigma_{i=s+1}^{S} \Pi_i(\phi_i - \phi_s)/\zeta]$.

The proof of this and all other Claims is in Appendix B. We now turn to show that the real wage is related to average capacity utilization.

Claim 2: The equilibrium real wage is:
$w = \beta\omega p_1 z = CU\{\beta\omega\zeta/(1 - \beta\omega + \zeta\beta\omega)\}$.

Note that when $\beta\omega = 1$, w = CU. This is because when $\beta\omega = 1$ (a policy known as the Friedman rule) workers are fully compensated for the delay in spending their money income. In general the real wage is increasing in $\beta\omega$, average capacity utilization (CU) and the unconditional probability that a buyer will want to consume (ζ). The probability of wanting to consume plays a role only when $\beta\omega < 1$ because a higher probability of wanting to consume means, on average, less delay in spending the money.

To study the effect of changes in the distribution of ϕ on capacity utilization and the real wage, I now show the following Claim.

<u>Claim 3:</u> $CU = 1 - \Sigma_{s=1}^{S} \phi_s \Pi_s \lambda(s)/\zeta$, where $\lambda(s) = \Sigma_{i=1}^{s-1} \Pi_i - \Sigma_{i=s+1}^{S} \Pi_i = 1 - q_s - q_{s+1}$.

Note that $\lambda(s)$ is an increasing function, $\lambda(1) < 0$ and $\lambda(S) > 0$. Since $0 < \Sigma_{i=1}^{S} \phi_i \Pi_i \lambda(i)/\Sigma_{i=1}^{S} \Pi_i \phi_i < 1$, there exists a cut-off point s^* such that
$\lambda(s) \leq \Sigma_{i=1}^{S} \phi_i \Pi_i \lambda(i)/\Sigma_{i=1}^{S} \Pi_i \phi_i$ for $s \leq s^*$ and
$\lambda(s) > \Sigma_{i=1}^{S} \phi_i \Pi_i \lambda(i)/\Sigma_{i=1}^{S} \Pi_i \phi_i$ otherwise. Figure 4 illustrates the cut-off point.

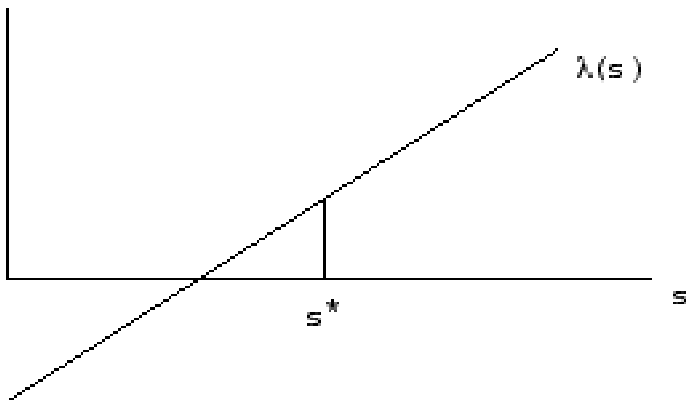

Figure 4

An immediate Corollary is as follows.

Corollary: A small increase in ϕ_s (holding other ϕ_i constant) increases CU if $s \le s^*$ and reduces CU otherwise.

The intuition is as follows. Increasing the fraction of buyers who want to consume in states that this fraction is low or reducing it in states that it is high reduce the uncertainty about demand and therefore increase average capacity utilization.

3.1 INCENTIVES TO TRADE FROM THE INDIVIDUAL AND THE SOCIAL POINT OF VIEW

When the probability distribution of ϕ is country specific there are incentive to trade in the same good, at least from the individual's point of view. This is because equilibrium prices (the solution to [14] and [15]) depend on the probability distribution of ϕ and are country specific under autarky. When prices are country specific, a seller in country j will have, for example, an incentive to sell in country j' if there is a market s for which: $p_{j's} > p_{js}$.

An individual that takes prices as given can only benefit from trade. But a country as a whole that recognize the effect of trade on prices and capacity utilization may actually suffer from trade.

I now turn to characterize the condition under which free trade can improve the welfare for an individual country j. For this purpose let ϕ_{js} denote the fraction of buyers who want to consume in country j in state s and for simplicity let us assume:

$0 = \phi_{j0} \le \phi_{j1} \le \phi_{j2} \le ... \le \phi_{jS}$ for all j. The real wage under autarky can be derived by adding the country index j to the expression in Claim 2. It is:

$$\beta\omega p_{j1}z_j = CU_j[\beta\omega\zeta_j/(1 - \beta\omega + \zeta_j\beta\omega)], \quad (17)$$

where $CU_j = 1 - [\Sigma_{i=s}^{S}\Pi_i(\phi_{ji} - \phi_{js})/\Sigma_{s=1}^{S}\Pi_s\phi_{js}]$ and $\zeta_j = \Sigma_{s=1}^{S}\Pi_s\phi_{js}$.

Free trade equilibrium: We now assume n countries of equal size and calculate the real wage under free trade. Although prices are the same for all buyers, the real wage is country specific because the probability of wanting to consume is country specific.

I use:

$$\phi_s = (1/n)\Sigma_{j=1}^{n} \phi_{js}, \quad (18)$$

to denote the worldwide fraction of buyers who want to consume. The probability that a dollar will buy in market i when s markets open is not type specific and is given by $\upsilon_i^s = (\phi_i - \phi_{i-1})/\phi_s$. Since prices are also common to all buyers the expected purchasing power of a dollar when s markets open is: $z_s = \Sigma_{i=1}^{s}(\upsilon_i^s/p_i)$ and is not type specific. We can now modify (10) to formulate type j's Bellman's equation:

$$V_j(m) = \Sigma_{s=1}^{S} \Pi_s\phi_{js}(m + g)z_s + \max_{k_{js}} - v(\Sigma_{s=1}^{S} k_{js}) \quad (19)$$

$$+ \beta\Sigma_{s=1}^{S} \Pi_s\{\phi_{js}V_j[(\Sigma_{i=1}^{s} p_i k_{ji})\omega] + (1 - \phi_{js})V_j[(m + g + \Sigma_{i=1}^{s} p_i k_{ji})\omega]\}.$$

To state the first order conditions for this problem we modify (12) and define the expected utility from a normalized dollar by:

$$z_j = \Sigma_{s=1}^{S}\Pi_s[\phi_{js}z_s + (1-\phi_{js})\beta\omega z_j] = (1 - \beta\omega + \zeta_j\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\phi_{js}z_s, \quad (20)$$

where $\zeta_j = \Sigma_{s=1}^{S}\Pi_s\phi_{js}$ is the probability that type j will want to consume. We can now state the first order condition to (19) by:

(21) $$\beta\omega q_s p_s z_j = \beta\omega p_1 z_j = v'(\Sigma_{s=1}^{S} k_{js}).$$

The average per country supply to market s is:

(22) $$k_s = (1/n)\Sigma_{j=1}^{n} k_{js}.$$

The market clearing conditions can now be stated as:

(23) $$p_s k_s = \phi_s - \phi_{s-1}.$$

Claim 4: Under free trade, $CU = 1 - \Sigma_{s=1}^{S}\Pi_s\Sigma_{i=s+1}^{S} \Pi_i(\phi_i - \phi_s)/\zeta$ and the real wage for type j sellers is:

$\beta\omega p_1 z_j = CU\{\beta\omega E(\phi_j/\phi)\zeta/(1 - \beta\omega + \zeta_j\beta\omega)\}$,

where $E(\phi_j/\phi) = \Sigma_{s=1}^{S}\Pi_s(\phi_{js}/\phi_s)$.

We can now compare the expected wage under free trade and autarky. $CU\{\beta\omega E(\phi_j/\phi)\zeta/(1 - \beta\omega + \zeta_j\beta\omega) > CU_j\{\beta\omega\zeta_j/(1 - \beta\omega + \zeta_j\beta\omega)\}$ implies: $CU > CU_j/\delta_j$ and vise versa, where

$\delta_j = \zeta_j/\zeta E(\phi_j/\phi) = E(\phi_j)/E(\phi)E(\phi_j/\phi)$. Thus,

Claim 5: Free trade increases welfare in country j if: $CU > CU_j/\delta_j$.

The magnitude $\delta_j = E(\phi_j)/E(\phi)E(\phi_j/\phi)$ may be either larger or smaller than unity.[5] Therefore the condition $CU > CU_j$ is neither sufficient nor necessary.

Example 8: There are two countries with the same population. The residents of country 1 always want to consume. In country 2 either half of the residents want to consume or all of the residents want to consume; with equal probabilities of occurrence.

For this example, with $\phi_{11} = \phi_{12} = 1$, $\phi_{21} = 1/2$; $\phi_{22} = 1$. We compute: $CU_1 = 1$, $CU_2 = 2/3$, $CU = 6/7$, $\delta_1 = 48/49$ and $\delta_2 = 36/35$. Since $CU_2/\delta_2 = 70/108 = 0.648 < CU = 6/7 = 0.857$ country 2 gains from trade. Since $CU_1/\delta_1 = 49/48 = 1.02 > CU = 0.857$ country 1 looses from trade.

Example 9: There are two countries with the same population and the same independent distribution of demand: Either half of the residents want to consume or all of the residents want to consume; with equal probabilities of occurrence.

There are four states of the world which occur with equal probabilities. In state 1 demand is low in both countries; in state 2 and 3 it is low in one country and high in the other and in state 4 it is high in both countries. In this symmetric example:

$\phi_{11} = \phi_{21} = 1/2$; $\phi_{12} = 1$ and $\phi_{22} = 1/2$; $\phi_{13} = 1/2$ and $\phi_{23} = 1$;

$\phi_{14} = \phi_{24} = 1$.

We compute average capacity utilization under autarky:

$CU_1 = CU_2 = = 2/3$.

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[5] $E(\phi_j/\phi) = E(\phi_j)E(1/\phi) + Cov(\phi_j, 1/\phi) \leq E(\phi_j)E(1/\phi) \geq E(\phi_j)/E(\phi)$

The first inequality is because $Cov(\phi_j, 1/\phi) < 0$. The second is because of Jensen's inequality.

To compute average capacity utilization under free trade, note that the average fraction of buyers who want to consume is: $\phi_1 = {}^1/_2$; $\phi_2 = {}^3/_4$; $\phi_3 = {}^3/_4$ and and $\phi_4 = 1$. Average capacity utilization under free trade is: $CU = 1 - {}^7/_{48} = {}^{41}/_{48} = 0.854$. In this symmetric example, $\delta_j = 1$. Since $CU > CU_j$ both countries benefit from trade.

3.2 EXCHANGE RATES

I now assume J currencies (indexed j). Let M^h_{jt} denotes the amount of currency j held by househld h at time t and let E_{jt} denotes the value of currency j in terms of dollars. The dollar value of the portfolio of currencies held by household h is:

$$M^h_t = \Sigma^J_{j=1} E_{jt}M^h_{jt}. \tag{24}$$

In addition, all households get a perfectly anticipated lump sum transfer. The dollar value of the transfer payment is the same for all households and is equal to G_t. The average per-household post transfer world money supply is: $M_t = G_t + (1/N)\Sigma^N_{h=1}M^h_t$ dollars. As in the single currency case the rate of change in the world money supply is non-random and is given by: $M_{t+1}/M_t = 1 + \mu$.

Exchange rates are determined at the beginning of the period in a random manner but, as in Karaken and Wallace (1981), on average they are expected to remain constant:

$$E(E_{jt+1}|E_{jt}) = E_{jt}, \tag{25}$$

where E denotes the expectations operator.

Since the world money supply in terms of dollars grow at a constant rate, equilibrium prices in terms of dollars grow at a constant rate. In our formulation we divide all dollar magnitudes by the post transfer money supply ($M_t[1 + \mu]$) and assume that prices in terms of normalized dollars do not change over time. This allow us to use the amount of normalized dollar held at the beginning of the period as the only state variable in our dynamic programming formulation and write the Bellman equation (10), where m is now interpreted as the value of the beginning of period portfolio in terms of normalized dollars.

Sellers may be viewed as maximizing revenues in terms of next period's normalized dollars and because of (25) they are willing to accept any currency. If the seller sells a unit at a price of 1 dollar he will get $1/(1 + \mu)$ in terms of next period's normalized dollars. If instead he sells the unit for $1/E_{jt}$ units of currency j he will have next period E_{jt+1}/E_{jt} dollars, which under (25) will equal on average to 1 dollar or $1/(1 + \mu)$ normalized dollars. Thus under (25) a seller who maximizes the expected value of revenues in terms of next period's normalized dollars will accept payment in any currency.

Since dollar prices increases at a constant rate sellers also behave as if they maximize revenues in terms of dollars (rather than next period's normalized dollars). Is this true also for say marks? A seller who maximizes revenues in terms of next period's expected marks will prefer 1 dollar (which is worth on average $E[1/E_{jt+1}]$ marks in the next period) to $1/E_{jt}$ marks. This is because Jensen's inequality and (25) imply: $E(1/E_{jt+1}) \geq 1/E(E_{jt+1}) = 1/E_{jt}$. But our sellers are indifferent between marks and dollars.

To resolve this apparent paradox (sometimes called the Siegel's paradox) we formulated the dynamic programming problem in terms of normalized marks, where a normalized mark is the per household mark money supply. This revealed that the beginning of period normalized marks can be used as the only state variable and our seller may be viewed as maximizing the expected value of revenues in terms of next period normalized marks. But this is different from maximizing the expected revenues in terms of next period's regular marks. To see this point, let $DM_t = M_t/E_{jt}$ denote the money supply in terms of marks. Since the dollar money supply grows at the rate of μ, $E_{jt+1}DM_{t+1}/E_{jt}DM_t = 1 + \mu$ and therefore $DM_{t+1}/DM_t = E_{jt}(1 + \mu)/E_{jt+1}$. Since E_{jt+1} is a random variable the rate of growth of the world money supply in terms of marks is a random variable. It follows that a normalized mark will become $\omega_j = E_{jt+1}/E_{jt}(1 + \mu)$ next period's normalized marks, where now ω_j is a random variable. Now 1 dollar will become on average $E(1/E_{jt+1})\omega_j = 1/E_{jt}(1 + \mu)$ next period's normalized marks. And $1/E_{jt}$ marks will also become $1/E_{jt}(1 + \mu)$ next period's normalized marks. Therefore a seller who maximize revenues in terms of next period's normalized marks will be indifferent between accepting 1 dollar to accepting $1/E_{jt}$ marks.

3.3 ASYMMETRIC EQUILIBRIA AND THE CORRELATION BETWEEN THE NOMINAL AND THE REAL EXCHANGE RATE

The above symmetric formulation assumes that the representative household allocates capacity to all S markets. We may think of this allocation as a price setting choice: The seller post many prices and puts the price tag p_s on k_s units.

I now assume that each seller quotes one price only and a fraction k_s/L of the sellers choose to participate in market s and offer to sell at the price p_s. The expected utility of the seller does not depend on his choice of market.

I also assume that sellers in country j (type j) quote their price in terms of currency j (but nevertheless are willing to accept payment in any other currency). I assume that a seller changes his price in terms of the local currency only if there are strictly positive benefits from doing it. This assumption may be motivated by the existence of small menu type costs for changing nominal price quotations.

To get the observed correlation between the nominal exchange rate and the real exchange rate I assume a continuum of countries and two potential markets. Equilibrium prices are 1 dollar worth in market 1 and 2 dollars worth in market 2.

Suppose now that at time t the exchange rate of currency j was 1 and sellers in country j supplied to market 2. Currency j depreciated at t + 1 and is now worth 0.5 dollars. Sellers in country j do not change their nominal price quotations (in terms of their currency) and as a result the dollar value of their price goes down by the percentage of the devaluation. After the devaluation, country j supplies to market 1 instead of market 2 but because the country is small all the equilibrium conditions are satisfied and country j sellers have no incentive to change their price quotation.

CONCLUSIONS

The sequential trade model abandons the law of one price in favor of an equilibrium price distribution. Therefore we may get deviations from the purchasing power parity (PPP) and a high correlation between the real and the nominal exchange rates.

Under autarky the distribution of equilibrium prices are in general country specific and therefore individual agents have an incentive to trade, in the same good, with agents in other countries. However, the country (or the world) as a whole may be better off under autarky. This occurs when trade increases the uncertainty about supply and demand conditions.

An individual country benefits from trade if as a result of trade average capacity utilization increases by a sufficient amount. The exact condition is: $CU > CU_j/\delta_j$, where CU is average capacity utilization under free trade, CU_j is average capacity utilization under autarky and δ_j is a country specific constant. This constant is close to unity in the examples we have worked out.

We mentioned "damping" in the context of example 5. A more complete analysis of this issue is a possible topic for future research.

The analysis of the great depression is another possible application. Recently Cole and Ohanian (2001) describe the new deal policies which were designed to limit competition. They think of these policies as a huge "mistake" that contributed to the persistent of the depression. But it is possible that in 1933 there was a lot of uncertainty about supply and demand conditions and limiting competition may have worked in the direction of reducing this uncertainty.

APPENDIX A: THE REAL UST MODEL

In this Appendix I define the UST equilibrium, the monopoly problem and the social planner's problem and study the efficiency properties of the UST allocation.

I start from the case in which the reservation price is the same across buyers: $v_j = v$ for all j. I use $N_s = N_s(v)$ for the number of buyers per seller and assume: $N_1 < N_2 < ... < N_S$.

The minimum demand per seller (at a price $p \leq v$) is: $\Delta_1 = N_1$. After the first Δ_1 units are sold by each seller there may be two possibilities: either s = 1 and there are no more buyers or s > 1. If s > 1, then there are $N_s - N_1$ unsatisfied buyers per seller. The minimum residual demand per seller is:
$\Delta_2 = \min_s\{N_s - N_1\} = N_2 - N_1$. The probability that s > 1 is
$q_2 = 1 - \Pi_1$ and this is the probability that market 2 will open. After transactions in market 1 are completed, there may be again two possibilities: either no additional buyers arrive or, if s > 2, some additional buyers do arrive. The probability that s > 2, is
$q_3 = 1 - \Pi_1 - \Pi_2$ and this is the probability that market 3 will open. The minimum residual demand per seller if s > 2 is:
$\Delta_3 = \min_s\{N_s - N_2\} = N_3 - N_2$. Proceeding in this way we define q_s and Δ_s for all s = 1, ...,S.

Because of constant returns to scale equilibrium prices are determined by supply considerations only. The expected revenue from supplying a unit to market i is $q_i P_i$. When $q_i P_i = \lambda$ the expected revenue is equal to the expected cost and sellers are indifferent about the quantity supplied and are willing to satisfy demand.

A UST spot markets equilibrium is a vector of prices $(P_1,...,P_S)$ and a vector of supplies per seller $(x_1,...,x_S)$ such that: (a) $P_i = \lambda/q_i = \lambda/\Sigma^S_{s=i}\Pi_s$ and (b) $x_i = \Delta_i = N_i - N_{i-1}$ if $P_i \le v$ and zero otherwise.

To solve for the equilibrium quantities we substitute the equilibrium condition (a) in (b) to get: $x_i = \Delta_i$ if $\lambda \le vq_i$ and zero otherwise. This says that capacity will be created to satisfy the demand of batches that arrive with a high enough probability so that the cost per unit λ is less than the expected value vq_i.

We now define sequential efficiency as the maximum surplus per seller that can be obtained by a central planner who does not violate the informational constraints of the environment. In particular the central planner must allocate goods to buyers as they arrive without knowing the state. More specifically, the social planner's problem in this environment is to choose capacity per seller and allocate it across the potential markets to maximize expected surplus per seller:

$$\text{(A1)} \qquad \max_{x_i} \; v\Sigma_i\, q_i x_i - \lambda\Sigma_i\, x_i \;;\; \text{s.t. } x_i \le \Delta_i.$$

The solution to (A1) is: $x_i = \Delta_i$ if $\lambda \le vq_i$ and zero otherwise. This coincides with the equilibrium outcome and therefore the UST spot markets allocation is sequential efficient in our special case.

Note that prices may appear rigid in the sequential environment because they do not respond to the realization of demand (the state s). Nevertheless sellers do not have an incentive to change prices during trade.[6]

I now turn to another special case in which v_j may vary across types but the consumption probabilities are the same across types.

I assume that $\phi_{js} = \phi_s$ for all j and we can rank the number of buyers per seller: $N_1 < N_2 < ... < N_S$. To simplify I also assume that this is also the ranking of the fraction of buyers who want to consume: $0 = \phi_0 \leq \phi_1 \leq \phi_2 \leq ... \leq \phi_S = 1$. The reservation price v_j may vary across types. I use

(A2) $$d_{ji} = 1 \text{ if } v_j \geq p_i \text{ and zero otherwise,}$$

to denote the demand per type j buyer in market i. Since $p_i \leq p_{i+1}$, $d_{ji} = 0$ implies $d_{ji+1} = 0$.

I now calculate the demand per seller in each market given the prices p_i. The minimum number of buyers who want to consume is $\phi_1 \Sigma_{j=1}^{J} n_j$ and the minimum demand per seller is:

[6] To show this claim we apply Bayes' rule and compute the probability that market i > s will open given that market s opens. This is: Π_i/q_s. The probability that market i will open given that market s opens is: $\Sigma_{k=i}^{S}\Pi_k/q_s$. In equilibrium the unconditional expected revenue (from a unit supplied to market i) is $P_i\Sigma_{k=i}^{S}\Pi_k = \lambda$ and the conditional expected revenue (from a unit supplied to market i given that market s opens) is: $P_i\Sigma_{k=i}^{S}\Pi_k/q_s = \lambda/q_s$. Since in equilibrium $P_s = \lambda/q_s$ the opening of market s does not provide an incentive for the firm to move units from market s to market i or vice versa. Since the conditional expected revenue is λ/q_s for all i > s, the firm does not have an incentive to move units allocated to markets s+1,...,S. Thus, not surprisingly the initial plan is time consistent.

$\Delta_1 = \phi_1\Sigma_{j=1}^{J} n_j d_{j1}/M_1$. After the first batch has completed trade a second batch will arrive if $(\phi_S - \phi_1)\Sigma_{j=1}^{J} n_j d_{j1}/M_S > \Delta_1$ and not all buyers who wanted to buy in market 1 made it. This event occurs with probability $q_2 = 1 - \Pi_1$. The minimum residual demand per seller at the price p_2 is: $\Delta_2 = (\phi_2 - \phi_1)\Sigma_{j=1}^{J} n_j d_{j2}/M_2$ units and this is the demand in the second market if it opens. In general, market i opens with probability $q_i = \Sigma_{s=i}^{S} \Pi_s$ and the demand per seller in market i if it opens is:

(A3) $$\Delta_i = (\phi_i - \phi_{i-1})\Sigma_{j=1}^{J} n_j d_{ji}/M_i.$$

UST spot markets equilibrium is a vector $\{d_{ji}, p_i, \Delta_i\}$ that satisfies:

(a) $p_i = \lambda/q_i$;

(b) $d_{ji} = 1$ if $v_j \geq p_i$ and zero otherwise;

(c) $\Delta_i = (\phi_i - \phi_{i-1})\Sigma_{j=1}^{J} n_j d_{ji}/M_i$.

Substituting equilibrium condition (a) in (b) and using (c) leads to the equilibrium allocation rule:

(A4) $$\Delta_i = (\phi_i - \phi_{i-1})\Sigma_{j=1}^{J} n_j I(q_i v_j \geq \lambda)/M_i,$$

where I(statement) = 1 if the statement is true and zero otherwise.

To study efficiency it is useful to imagine an hypothetical central planner who may dictate the quantity per seller that will be sold to each type according to his order of arrival. Let Δ_{ji} denote the amount per seller that will be supplied to type j buyers who arrive in batch i. The hypothetical planner solves the following problem:

(A5) $$\max_{\Delta_{ji}} \Sigma_{j=1}^{J}\Sigma_{i=1}^{S} q_i\Delta_{ji}v_j - \lambda\Delta_{ji}$$

$$\text{s.t. } 0 \le \Delta_{ji} \le (\phi_i - \phi_{i-1})n_j/M_i.$$

If the planner chooses $\Delta_{ji} = (\phi_i - \phi_{i-1})n_j/M_i$ he will get $q_i\Delta_{ji}v_j$ at the cost of $\lambda\Delta_{ji}$. He will therefore make this choice only if $q_iv_j \ge \lambda$. The equilibrium allocation (A4) also says that a type j that arrive in batch i will get a unit if $q_iv_j \ge \lambda$. We have thus shown that the equilibrium allocation (A4) solves the planner's problem (A5) and is therefore sequential efficient. We can now combine the results of the two special cases as follows.

Proposition 1: When the reservation price is the same across types (v_j = v for all j) or the consumption probabilities are the same across types ($\phi_{js} = \phi_s$ for all j and s) the UST spot markets allocation is efficient in the sequential sense.

I now turn to the more general case. I start by an algorithm that compute the probability that each market will open and the demand in each market as a function of the prices in the S potential markets: $(P_1,..., P_S)$.

Market 1 will open when $\Sigma_j \phi_{js} > 0$, with probability $q_1 = \Sigma_{s=1}^{S} \Pi_s I(\Sigma_j \phi_{js} > 0)$. To simplify, I assume $q_1 = 1$.

The number of type j buyers per seller who want to buy in the first market at the price P_1 in state s is:

(A6) $$N_{js}^{1} = \phi_{js}n_j/M_s \quad \text{if } v_j \ge P_1 \text{ and zero otherwise.}$$

Total demand per seller in the first market at the price P_1 is therefore:

(A7) $$N_s^1(P_1) = \Sigma_j\ N_{js}^1(P_1).$$

The number of buyers in the first batch is the minimum demand at the price P_1 over all states:

(A8) $$\Delta_1(P_1) = \min_+\{N_1^1(P_1),...,N_S^1(P_1)\},$$

where I use the operator $\min_+$ to select the smallest strictly positive number if such a number exists and zero otherwise.[7]

Market 2 will open in state s if after the completion of trade in the first market there is residual demand in this state: $N_s^1(P_1) > \Delta_1(P_1)$. The probability that market 2 will open is thus:

(A9) $$q_2(P_1) = \Sigma_{s=1}^{S}\ \Pi_s I[N_s^1(P_1) > \Delta_1(P_1)].$$

If $q_2(P_1) = 0$, we set $q_i = 0$ and $\Delta_i = 0$ for $i > 1$. If $q_2(P_1) > 0$, we calculate the size of the second batch. The fraction of satisfied buyers out of all buyers who wanted to buy is:

(A10) $\psi_s^1(P_1) = \Delta_1(P_1)/N_s^1(P_1)$ if $N_s^1(P_1) > 0$ and zero otherwise.

It is assumed that the fraction of satisfied buyers is the same for all types who wanted to buy in market 1. The number of unsatisfied

[7] For example: $\min_+(1,2,0) = 1$.

type j buyers is $[1 - \psi_S^1(P_1)]\phi_{js}n_j$ and the number of unsatisfied type j buyers who want to buy at the price P_2 is:

(A11) $N_{js}^2(P_1, P_2) = [1 - \psi_S^1(P_1)]\phi_{js}n_j$ if $v_j \geq P_2$ and zero otherwise.

The total residual demand at the price P_2 is:

(A12) $$N_S^2(P_1, P_2) = \Sigma_j\ N_{js}^2(P_1, P_2).$$

We can now compute the number of buyers in batch 2 by taking the minimum residual demand at the price P_1 over all states:

(A13) $$\Delta_2(P_1, P_2) = \min_+\{N_1^2(P_1, P_2),...,N_S^2(P_1, P_2)\}.$$

Market 3 will open if after completion of trade in market 2 there is strictly positive residual demand. This occurs with probability:

(A14) $$q_3(P_1, P_2) = \Sigma_{s=1}^{S}\ \Pi_s I[N_s^2(P_1, P_2) > \Delta_2(P_1, P_2)].$$

In general, the probability that market i will open depends only on prices in markets with indices less than i and is denoted by: $q_i(P_1, ..., P_{i-1})$. The probability $q_i(P_1, ..., P_{i-1})$ is zero if $q_{i-1}(P_1, ..., P_{i-2})$ is zero. When $q_i(P_1, ..., P_{i-1}) > 0$, the number of buyers in batch i, $\Delta_i(P_1, ..., P_i)$, depends on the prices in all the first i markets. If $q_i(P_1, ..., P_{i-1}) = 0$ then $\Delta_i(P_1, ..., P_i) = 0$.[8]

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[8] Note that market i may open and $\Delta_i(P_1, ..., P_i)$ may equal zero because the price P_i is too high. Note also that by construction,

Because of constant returns to scale the supply to market i does not depend on the supply to other markets: It depends only on the probability that market i will open and the price in this market. The firm takes prices (P_1, ..., P_i) as given and chooses the quantity x_i to solve:

(A15) $$\max\ [q_i(P_1, \ldots, P_{i-1})]P_i x_i - \lambda x_i.$$

The firm will choose any interior solution, $0 < x_i < \infty$, if:

(A16) $$[q_i(P_1, \ldots, P_{i-1})]P_i = \lambda.$$

Given the functions $q_i(P_1, \ldots, P_{i-1})$ and $\Delta_i(P_1, \ldots, P_i)$ I define equilibrium as follows.

<u>UST spot markets equilibrium</u> is a vector of prices and quantities (P_1,..., P_S; x_1,...,x_S) that satisfies:

(a) $P_i q_i(P_1, \ldots, P_{i-1}) = \lambda$ if $q_i(P_1, \ldots, P_{i-1}) > 0$ and P_i is large and finite otherwise;

(b) $x_i = \Delta_i(P_1, \ldots, P_i)$.

Since the probability that market i opens depends only on prices in lower indexed markets, we can solve for the unique UST equilibrium in the following way. Since $q_1 = 1$, we set $P_1 = \lambda$ and compute $N_s^1(\lambda)$ and $\Delta_1(\lambda)$. We then compute the probability that market 2 will open: $q_2(\lambda) = \Sigma_{s=1}^{S} \Pi_s I[N_s^1(\lambda) > \Delta_1(\lambda)]$. If $q_2(\lambda) = 0$ then

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the number of markets that open is less than or equal to the number of states (S).

$q_i = \Delta_i = 0$ for all $i > 1$ and we are done. Otherwise, we choose: $P_2 = \lambda/q_2(\lambda)$ and compute $N_s^2[\lambda, \lambda/q(\lambda)]$, $\Delta_2[\lambda, \lambda/q(\lambda)]$. We then compute the probability that market 3 will open:

$q_3[\lambda, \lambda/q(\lambda)] = \Sigma_{s=1}^{S} \Pi_s I\{N_s^2[\lambda, \lambda/q(\lambda)] > \Delta_2[\lambda, \lambda/q(\lambda)]\}$. If $q_3[\lambda, \lambda/q(\lambda)] = 0$, we are done. Otherwise, we choose:

$P_3 = \lambda/q_3[\lambda, \lambda/q(\lambda)]$ and so on. Thus,

Proposition 2: There exists a unique UST spot markets equilibrium.

I now use the functions $q_i(P_1,...,P_{i-1})$ and $\Delta_i(P_1, ...,P_i)$ to discuss the choice of a monopoly and the choice of a social planner.

Monopoly: A monopolist who must satisfy demand sequentially, will choose the prices ($P_1 \leq P_2 ... \leq P_S$) to maximize the expected profits:

$$\text{(A17)} \quad \Sigma_{i=1}^{S} P_i[q_i(P_1,...,P_{i-1})]\Delta_i(P_1, ...,P_i) - \lambda\Sigma_{i=1}^{S} \Delta_i(P_1, ...,P_i).$$

Social planner: We start by computing the average valuations of buyers in batch i.

The fraction of type j buyers in all the buyers who want to buy in market i is:

$$\text{(A18)} \qquad \alpha_{js}^{i}(P_1,...,P_i) = N_{js}^{i}(P_1,...,P_i)/N_s^i(P_1,...,P_i),$$

if $N_s^i(P_1,...,P_i) > 0$ and zero otherwise.

It is assumed that this is also the fraction of type j buyers who actually buy in market i. The average valuation of the buyers in batch i is therefore:

(A19) $$V_i(P_1,...,P_i) = \Sigma_s \Pi_s \Sigma_j v_j \alpha^i_{js}(P_1,...,P_i).$$

The social planner chooses prices ($P_1 \le P_2 ... \le P_S$) to maximize the expected consumer surplus:

(A20) $$\Sigma^S_{i=1} q_i(P_1,...,P_{i-1})[\Delta_i(P_1, ...,P_i)]V_i(P_1, ...,P_i)$$

$$- \lambda\Sigma^S_{i=1} \Delta_i(P_1, ...,P_i).$$

An allocation is sequential efficient if it solves (A20). We already showed by example 1 that the UST allocation may not be efficient in the sequential sense and a monopoly may improve matters. In example 1 the UST competitive firm produces more than the monopoly. I now show by example that excess production is not the only reason for the inefficiency.

APPENDIX B: PROOFS OF THE CLAIMS IN THE MONETARY MODEL

<u>Proof of Claim 1:</u> Using the market clearing condition (14) we get:

(B1) $$k_i = k_1(\phi_i - \phi_{i-1})p_1/p_i\phi_1 = k_1(\phi_i - \phi_{i-1})q_i/\phi_1.$$

Average waste if exactly s markets open is:

(B2) $$\Sigma^S_{i=s+1} k_i / \Sigma^S_{i=1} k_i = \Sigma^S_{i=s+1} (\phi_i - \phi_{i-1})q_i / \Sigma^S_{i=1} (\phi_i - \phi_{i-1})q_i$$

We now use the definition $q_s = \Sigma_{i=s}^{S} \Pi_i$ to get:

(B3) $$\Sigma_{i=s+1}^{S} (\phi_i - \phi_{i-1})q_i =$$

$$= (\phi_{s+1} - \phi_s)q_{s+1} + (\phi_{s+2} - \phi_{s+1})q_{s+2} +...+ q_S(\phi_S - \phi_{S-1})$$

$$= -\phi_s q_{s+1} + \phi_{s+1}(q_{s+1} - q_{s+2}) + ... + q_S\phi_S$$

$$= - \phi_s q_{s+1} + \phi_{s+1}\Pi_{s+1} +...+ q_S\phi_S =$$

$$= \Sigma_{i=s+1}^{S} \Pi_i\phi_i - \phi_s\Sigma_{i=s+1}^{S} \Pi_i = \Sigma_{i=s+1}^{S}\Pi_i(\phi_i - \phi_s).$$

We can now get the expected waste by substituting (B3) in (B2) and computing the expected value of this expression. This leads to: $\Sigma_{s=1}^{S}\Pi_s\Sigma_{i=s+1}^{S} \Pi_i(\phi_i - \phi_s)/\Sigma_{s=1}^{S}\Pi_s\phi_s$. Since expected capacity utilization is one minus the expected waste we get:

$CU = 1 - \Sigma_{s=1}^{S}\Pi_s\Sigma_{i=s+1}^{S} \Pi_i(\phi_i - \phi_s)/\Sigma_{s=1}^{S}\Pi_s\phi_s$. □

Proof of Claim 2:

Substituting (12) in the real wage (13) leads to:

(B4) $$\beta\omega p_1 z = \beta\omega p_1(1 - \beta\omega + \zeta\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\phi_s z_s.$$

Substituting (9), $q_i p_i = p_1$, $\upsilon_i^s = (\phi_i - \phi_{i-1})/\phi_s$ and $\zeta = \Sigma_{s=1}^{S}\Pi_s\phi_s$ in (B4) leads to:

(B5) $$\beta\omega p_1 z = \beta\omega p_1(1 - \beta\omega + \zeta\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\phi_s\Sigma_{i=1}^{S}(\upsilon_i^S/p_i)$$

$$= \beta\omega(1 - \beta\omega + \zeta\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\phi_s\Sigma_{i=1}^{s} q_i\upsilon_i^s$$

$$= \beta\omega(1 - \beta\omega + \zeta\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\Sigma_{i=1}^{s} q_i(\phi_i - \phi_{i-1})$$

$$= \beta\omega\Sigma_{s=1}^{S}\Pi_s\Sigma_{i=1}^{s} q_i(\phi_i - \phi_{i-1})/[1 - \beta\omega + \beta\omega\Sigma_{s=1}^{S}\Pi_s\phi_s].$$

We now use $q_s = \Sigma_{i=s}^{S} \Pi_i$ to get:

(B6) $$\Sigma_{i=1}^{S} q_i(\phi_i - \phi_{i-1}) =$$

$$= \phi_1 + q_2(\phi_2 - \phi_1) + q_3(\phi_3 - \phi_2) + ... + q_S(\phi_S - \phi_{S-1})$$

$$= (1 - q_2)\phi_1 + (q_2 - q_3)\phi_2 + + q_S\phi_S = \Pi_1\phi_1 + \Pi_2\phi_2 + \Pi_3\phi_3 + + q_S\phi_S$$

$$= \Sigma_{i=1}^{S-1}\Pi_i\phi_i + q_S\phi_S = \Sigma_{i=1}^{S-1}\Pi_i\phi_i + \phi_S\Sigma_{i=S}^{S}\Pi_i$$

$$= \Sigma_{s=1}^{S}\Pi_s\phi_s - \Sigma_{i=s}^{S}\Pi_i(\phi_i - \phi_s).$$

Substituting (B6) in (B5) and using Claim 1 leads to:

$\beta\omega p_1 z = \beta\omega[\Sigma_{s=1}^{S}\Pi_s\phi_s - \Sigma_{s=1}^{S}\Pi_s\Sigma_{i=s}^{S}\Pi_i(\phi_i - \phi_s)]/[1 - \beta\omega + \beta\omega\Sigma_{s=1}^{S}\Pi_s\phi_s].$

$= \beta\omega CU\Sigma_{s=1}^{S}\Pi_s\phi_s/[1 - \beta\omega + \beta\omega\Sigma_{s=1}^{S}\Pi_s\phi_s].$ □

Proof of Claim 3:

$\Sigma_{s=1}^{S}\Pi_s\Sigma_{i=s+1}^{S} \Pi_i(\phi_i - \phi_s) =$

$= \Pi_1\Pi_2(\phi_2 - \phi_1) + \Pi_1\Pi_3(\phi_3 - \phi_1) + \Pi_1\Pi_4(\phi_4 - \phi_1) + ... + \Pi_1\Pi_S(\phi_S - \phi_1)$

$+ \Pi_2\Pi_3(\phi_3 - \phi_2) + \Pi_2\Pi_4(\phi_4 - \phi_2) + ... + \Pi_2\Pi_S(\phi_S - \phi_2)$

$+ \Pi_3\Pi_4(\phi_4 - \phi_3) + \Pi_3\Pi_5(\phi_5 - \phi_3) + ... + \Pi_3\Pi_S(\phi_S - \phi_3)$

$+...+ \phi_S\Pi_S(\Sigma_{i=1}^{S-1} \Pi_i) =$

$= \phi_1\Pi_1(-\Pi_2 - \Pi_3 - \Pi_4 -... - \Pi_S)$

$+ \phi_2\Pi_2(\Pi_1 - \Pi_3 - \Pi_4 -... - \Pi_S)$

$+ \phi_3\Pi_3(\Pi_1 + \Pi_2 - \Pi_4 -... - \Pi_S)$

$+...+ \phi_S\Pi_S(\Sigma_{i=1}^{S-1} \Pi_i)$

$= \Sigma_{s=1}^{S} \phi_s\Pi_s\{\Sigma_{i=1}^{s-1} \Pi_i - \Sigma_{i=s+1}^{S} \Pi_i\} = \Sigma_{s=1}^{S} \phi_s\Pi_s\{(1 - q_s) - q_{s+1}\}.$

Substituting this in Claim 1 leads to the Claim. □

Proof of Claim 4: Substituting (20) in the real wage (21) leads to:

(B7) $$\beta\omega p_1 z_j = \beta\omega p_1(1 - \beta\omega + \zeta_j\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\phi_{js}z_s.$$

Substituting (9), $q_i p_i = p_1$, $\upsilon_i^s = (\phi_i - \phi_{i-1})/\phi_s$ and $\zeta_j = \Sigma_{s=1}^{S}\Pi_s\phi_{js}$ in (B7) leads to:

(B8) $$\beta\omega p_1 z_j = \beta\omega p_1(1 - \beta\omega + \zeta_j\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\phi_{js}\Sigma_{i=1}^{s}(\upsilon_i^s/p_i)$$

$$= \beta\omega(1 - \beta\omega + \zeta_j\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s\phi_{js}\Sigma_{i=1}^{s}\ q_i\upsilon_i^s$$

$$= \beta\omega(1 - \beta\omega + \zeta_j\beta\omega)^{-1}\Sigma_{s=1}^{S}\Pi_s(\phi_{js}/\phi_s)\Sigma_{i=1}^{s}\ q_i(\phi_i - \phi_{i-1})$$

$$= \beta\omega\Sigma_{s=1}^{S}\Pi_s(\phi_{js}/\phi_s)\Sigma_{i=1}^{s}\ q_i(\phi_i - \phi_{i-1})/[1 - \beta\omega + \beta\omega\Sigma_{s=1}^{S}\Pi_s\phi_{js}]$$

Substituting (B6) in (B5) and using Claim 1 leads to:

$$\beta\omega p_1 z_j =$$
$$\beta\omega\Sigma_{s=1}^{S}\Pi_s(\phi_{js}/\phi_s)[\Sigma_{s=1}^{S}\Pi_s\phi_s - \Sigma_{i=s}^{S}\Pi_i(\phi_i - \phi_s)]/[1 - \beta\omega + \beta\omega\Sigma_{s=1}^{S}\Pi_s\phi_{js}]$$
$$= \beta\omega CU\Sigma_{s=1}^{S}\Pi_s(\phi_{js}/\phi_s)\Sigma_{s=1}^{S}\Pi_s\phi_s/[1 - \beta\omega + \beta\omega\Sigma_{s=1}^{S}\Pi_s\phi_{js}]. \quad \square$$

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