

## INTERNATIONAL SEIGNIORAGE PAYMENTS

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What are the "liquidity services" provided by "over-priced" assets? Will competition drive international seigniorage payments to zero? Does a country gain when other hold its "over-priced" assets? These questions are analyzed here in a model with demand uncertainty (taste shocks) and sequential trade. It is shown that a country with a relatively stable demand may issue "over priced" debt and get seigniorage payments from countries with unstable demand. But this does not necessarily improve welfare in the stable demand country.

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## 1. INTRODUCTION

Will seigniorage payments disappear in a technological advanced society? In Woodford (2003, Ch. 2) cashless economy, money does not enter as an argument in the utility function, markets are complete, money earns interest and is priced correctly, and the government does not get seigniorage payments. In the international context, McKinnon (1969, pp. 17-23) and Grubel (1969, pp. 269-72) argued that competition will drive international seigniorage payments to zero.

Here I examine the possibility that seigniorage payments between countries will occur in a long run steady state, even when paying interest on money is feasible and money does not enter as an argument in the utility function. I thus show the possibility of a steady state equilibrium with an "over priced" asset whose rate of return is dominated by another asset. These two ways of describing the main result are equivalent because seigniorage is obtained when selling an "over priced" asset.

I use a model in which markets are incomplete, trade occurs in a sequence of Walrasian markets and uncertainty about demand causes price dispersion.

Price dispersion allows for the distinction between the rate of return on the asset and its "liquidity". The rate of return depends on the asset and not on the individual who holds it. Liquidity is an individual specific attribute. It depends on the probability that you will use the asset to buy at the low price.

For the sake of concreteness I assume two countries: The home country (US) and the foreign country (Japan or the rest of the world). There are two assets: US government debt and Japanese government debt. These assets will be called dollars and yen for short.

In the model risk neutral sellers choose both the price, and the asset that they will accept as payment. They may choose a low price or a high price and they may quote their price in dollars or in yen. In the equilibrium we study high price sellers choose to accept both assets, but low price sellers choose to accept dollars only. Since dollars are generally accepted, in equilibrium its rate of return is lower than the rate of return on the yen. Thus, you pay a "liquidity premium" for holding dollars (or you get an "illiquidity premium" for holding yen).

When choosing the payment asset, sellers take into account the probability that they will find the good at the low price when they spend it. This is relevant because the liquidity advantage of the dollar is realized only when the individual finds the good at the low price. An individual who typically buys in the high demand state has a low chance of finding the good at the low price and will therefore require a relatively small "illiquidity premium" to hold yen. In equilibrium only the agents who typically buy in the high demand state will hold both assets.

I assume that the demand of the Japanese is erratic and plays the role of "aggregate demand shifter". The demand of the Americans is stable. Since the Americans are more likely to buy at the low aggregate demand state they are willing to pay a relatively high premium for holding the generally accepted asset. In equilibrium the dollar promises a lower rate of return but is more "liquid" than the yen. For the

Japanese the liquidity of the dollar exactly compensates for its lower rate of return and in equilibrium they hold both assets. The Americans are willing to pay a higher "liquidity premium" and accept dollars only.

As was mentioned above, I use price dispersion to model liquidity. In Prescott's (1975) "hotels" model there is price dispersion. Versions of the Prescott model have been studied by, among others, Bryant (1980), Rotemberg and Summers (1990) and Dana (1998). Here I use a flexible price version: The uncertain and sequential trade (UST) model in Eden (1990, 1994) and Lucas and Woodford (1993).

## 2. THE MODEL

I consider a single good overlapping generations model. There are two countries. The demand in the home country (US) is stable (predictable). The demand in the foreign country (Japan) is unstable. I start with the case of autarky assuming a single asset.

### Autarky:

A new generation is born each period. Individuals live for two-periods. They work in the first period of their life and if they want they consume in the second.

The utility function for the representative agent born at  $t$  is:  $E\{\theta_{t+1}\beta c_{t+1} - A_t v(L_t)\}$ , where  $c_{t+1}$  is the amount of second period consumption,  $\beta$  is a discount factor and  $L_t$  is the amount of first period labor. Expectations are taken over two independently distributed random variables:  $\theta_{t+1}$  is a "taste shock" and  $A_t$  is a "productivity shock". The taste shock is the driving force behind the results. It is assumed that

$\theta_{t+1}$  is an i.i.d random variable that can take the realizations 1 with probability  $\pi$  and 0 otherwise. The shock to technology does not play an important role in our model, but it does not add much to the complexity of the model and is relevant for potential applications.

A unit of labor produces  $A_t$  units of output so that  $A_t L_t$  is output. The cost of supplying labor ( $Av[L]$ ) depends on productivity because the home production alternative depends on it. This assumption leads to a constant labor supply. (An alternative is to allow for an income effect that cancels the substitution effect). To simplify, I assume that the gross rate of change in productivity  $\varepsilon = A_t/A_{t-1}$  is an i.i.d random variable with  $E(\varepsilon) = 1$ . Assuming an arbitrarily given mean  $E(\varepsilon)$  will not change the results. For simplicity I also assume:  $v(L) = (\frac{1}{2})(L)^2$ .

The realization of the productivity shock  $A_t$  is known before the choice of output but the taste shock  $\theta$  is known only after the choice of output. Output produced will be sold only when  $\theta = 1$ . As in Abel (1985), when the old generation experiences  $\theta = 0$  they transfer their balances to the young generation as an accidental bequest. An alternative formulation may assume that agents derive utility from bequest as in Barro (1974), but the weight they assign to the utility of future generations is random. The main results will not change if this more general specification is employed.

There is a single asset (government debt) called yen. The representative young agent born at time  $t$  takes the yen prices of the consumption good in the current period ( $P_t$ ) as given. He also forms expectations about next period prices. If the old generation experiences  $\theta_t = 1$ , he sells his output and gets  $P_t A_t L_t$  yen for it. He then deposits the revenue in a government owned bank that pays interest at the nominal

rate  $i$ .<sup>2</sup> His pre-transfer next period balances when  $\theta_t = 1$  are thus:  $P_t A_t L_t (1 + i)$ . When  $\theta_t = 0$ , he gets  $M_{t+1}$  yen as accidental bequest. In addition he gets a deterministic transfer payment ( $G_{t+1}$ ). The expected next period post-transfer balances are:

$$(1) \quad B_{t+1} = \pi P_t A_t L_t (1 + i) + (1 - \pi) M_{t+1} + G_{t+1}$$

The worker will use these balances in the next period if  $\theta_{t+1} = 1$ . He therefore chooses  $L$  by solving:

$$(2) \quad \max_L - A_t v(L_t) + \pi \beta E \left( \frac{B_{t+1}}{P_{t+1}} \right) \quad \text{s.t. (1)}.$$

The expectations in (2) are over  $P_{t+1}$ . The first order condition for this problem is:

$$(3) \quad A_t v'(L_t) = \pi^2 \beta A_t R_{t+1},$$

where  $R_{t+1} = E\{P_t(1+i)/P_{t+1}\}$  is the expected gross real interest rate. We may think of  $\pi^2 \beta A_t R_{t+1}$  as the (expected discounted) real wage. The term  $\pi^2$  plays a role because a unit produced yields utility to the producer only if it is sold (with probability  $\pi$ ) and only if he will want to consume (also with probability  $\pi$ ). The probability of this joint event is  $\pi^2$ .

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<sup>2</sup> We may think of a check or a debit card transaction in which the money is transferred directly from one interest paying account to another. Alternatively, we may assume that the government pays, in addition to the lump sum transfer, a proportional transfer of  $i$  yen per yen.

The real wage is therefore  $\beta A_t R$  with probability  $\pi^2$  and zero otherwise. The first order condition (3) says that the marginal cost must equal the expected real wage.

We require market clearing when  $\theta_t = 1$ . That is,

$$(4) \quad P_t A_t L_t = M_t + G_t,$$

where  $M_t$  is the pre-transfer (post-interest) asset supply at time  $t$ .

I focus on an equilibrium in which inflation is constant and the nominal price of a unit of labor ( $A_t$  units of output) is proportional to the post-transfer asset supply. I thus assume a normalized price  $p$  such that:

$$(5) \quad P_t A_t = p(M_t + G_t)$$

Substituting (5) in the first order condition (3) leads to:

$$(6) \quad L = \pi^2 \beta R,$$

where now  $R = (1 + i)/(1 + \mu)$  is the constant expected gross real interest rate and  $1 + \mu = (M_{t+1} + G_{t+1})/(M_t + G_t)$  is the deterministic gross rate of change in the asset supply. Note that since the cost of labor is proportional to productivity, labor supply does not depend on the realization of the productivity shock,  $A_t$ . Note also that the government can vary  $R$  (and  $L$ ) by varying  $\mu$  and  $i$ .

With the risk of repetition I now set the problem in magnitudes that are normalized by the post-transfer asset supply. This will become

useful later. A normalized yen (NY) is  $M_t + G_t$  regular yen. The nominal price of a unit of labor ( $A_t$  units of output) is  $p = P_t A_t / (M_t + G_t)$  NY. When the price of  $A_t$  units is half NY it means that you have to pay half of the post-transfer asset supply to get  $A_t$  units. Since the asset supply changes over time we must renormalize every period. A normalized yen (NY) in the current period that is carried to the next period is worth  $\omega = (M_t + G_t) / (M_{t+1} + G_{t+1}) = (1 + \mu)^{-1}$  in terms of next period's NYs.

A worker (young agent) who sells a unit of labor ( $A_t$  units of output) for  $p$  NYs will have in the next period  $p(1 + i)\omega = pR$  normalized yen. The expected real wage conditional on selling is  $pRX$ , where  $X$  is the expected purchasing power of a normalized yen. To define  $X$  note that next period  $p$  normalized yen will buy  $A_{t+1}$  units and 1 normalized yen will buy  $A_{t+1}/p$  units. Since  $E_t(A_{t+1}) = A_t$ , the expected purchasing power of a yen is:

$$(7) \quad X_t = \pi \frac{A_t}{P}$$

When  $\theta_t = 1$  the worker sells his output ( $AL$  units) and gets on average  $\omega(pL)(1 + i)X = (pL)RX$  units of consumption in period  $t+1$ . In addition he gets a transfer payment of  $g$  normalized yen that will buy on average  $gX$  units.<sup>3</sup> His expected consumption when  $\theta_t = 1$  is therefore:  $(pLR + g)X$  units. When  $\theta_t = 0$  the worker does not sell his output but receives a bequest. The value of the bequest plus the transfer payment

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<sup>3</sup> In the Appendix (Lemma 1) I show that  $g = (\mu - i) / (1 + \mu)$ .

is 1 (= the post-transfer asset supply). The worker's maximization problem is therefore:

$$(8) \quad \max_L \pi\beta(pLR + g)X + (1 - \pi)\beta X - Av(L)$$

The first order condition that an interior solution to (8) must satisfy is:

$$(9) \quad Av'(L) = \beta\pi pRX = \beta\pi^2 RA,$$

where the second equality uses (7). The market clears when demand is strictly positive ( $\theta = 1$ ):

$$(10) \quad pL = 1.$$

Note that the equilibrium conditions (9) and (10) are the same as (4) and (6) but their derivation does not require algebra.

The sum of (the buyer's and the seller's) period  $t$  expected utilities is:

$$(11) \quad \text{Welfare} = A_t \{\beta\pi L_t - v(L_t)\} = A_t W_t,$$

where  $W_t = \beta\pi L_t - v(L_t)$  is welfare when  $A_t = 1$ . Since  $E(A_{t+1}) = A_t$ , (11) is also the steady state expected utility of the representative young agent.

Substituting the equilibrium level of labor  $L = \beta\pi^2R$  in (11), we get:  $\Omega(\pi, R) = \beta\pi(\beta\pi^2R) - v(\beta\pi^2R)$ . When  $R \leq 1/\pi$ , this function is decreasing in  $\pi$  and increasing in  $R$ . Maximum welfare is attained at  $\bar{R} = 1/\pi$ .

A planner's problem: To gain some insight, I now consider a planner who solves:

$$(12) \quad \max_L AW = A\{\pi\beta L - v(L)\}$$

The first order condition for this problem is:

$$(13) \quad v'(L) = \pi\beta$$

Since in equilibrium  $v'(L) = \beta\pi^2R_t$  efficiency requires:

$$(14) \quad \beta\pi^2R_t = \pi\beta \text{ or } \bar{R} = 1/\pi$$

Note that when  $\pi < 1$ , efficiency requires a strictly positive interest rate ( $R > 1$ ). This result is similar to the well-known result by Friedman (1969) but here, as in other OG models, the optimal real interest rate does not depend on the discount factor. The argument for  $R > 1$  is however, analogous to Friedman's argument. When  $R = 1$  there is a difference between the social and the private value of a unit produced. From the social point of view, a unit produced will be consumed with probability  $\pi$ . Therefore its social value is  $\pi\beta$ . From the individual's point of view a unit produced yields utility only if he

sells it and only if he wants to consume. This joint event occurs with probability  $\pi^2$ . Therefore when  $R = 1$ , a unit produced is worth to the individual only  $\beta\pi^2$  units of consumption.  $R > 1$  is required to correct for the difference between the social and the individual's point of view.

Note also that  $\bar{R}\pi = 1$  where  $\bar{R}$  is the optimal choice of  $R$ . This says that the monetary authorities should compensate the seller for the risk of not wanting to consume.

Since discounting does not play an important role in the analysis I assume, in what follows,  $\beta = 1$ . To illustrate, Table 1 calculates the equilibrium magnitudes for different values of  $R$  and  $\pi$ .

Table 1: Autarky ( $\beta = 1$ )

$\pi$	$R$	$L$	Welfare/A = W
1	1	1	0.5
0.9	$1/\pi$	$\pi$	0.405
0.9	1	0.81	0.401

I assume that in the US demand is stable and  $\theta = 1$  with probability 1. With a slight abuse of notation I assume that in Japan  $\theta = 1$  with probability  $\pi < 1$ . I use stars to denote the foreign country variables. I now turn to the welfare implications of trade, starting from the case of a small open economy.

A small open economy: To build intuition I assume now that Japan is relatively small and Japanese can sell and buy at the dollar price  $P$  that remains constant over time. To simplify, I assume:  $A^* = 1$ .

When  $R^* = 1/\pi$ , Japanese sellers are indifferent between dollars and yen. They can always sell for dollars and get a gross real return of unity. If they post the price in yen they will sell with probability  $\pi$  but since  $R^* = 1/\pi$  the expected gross real return is also unity. We can thus have equilibrium in which Japanese accept yen only, Americans accept dollars only and there is no trade.

The Japanese government can do better than autarky by adopting full dollarization and allowing for export to the US whenever there is no demand in Japan. To see this claim note that dollarization does not change the expected real wage and the Japanese labor supply ( $L^*$ ). The Japanese will sell at the price of  $P$  dollars with certainty earning  $PL^*$  dollars. In addition they may get a bequest of  $PL^*$  dollars. Thus under dollarization the expected consumption of the Japanese conditional on wanting to consume is:  $\pi(PL^*/P) + (1 - \pi)(2PL^*/P) = (2 - \pi)L^*$ . This is greater than the conditional expected consumption under autarky ( $= L^*$ ).

I now relax the small open economy assumption and show that when  $R = 1$ , the adoption of the dollar by Japan will indeed increase welfare in Japan but will also reduce welfare in the US.

### 3. A FULLY INTEGRATED WORLD ECONOMY

In the autarkic case we had two markets. The market in the home country opened with certainty and the market in the foreign country opened with probability  $\pi$ . In the fully integrated world we will also

have two markets but unlike the autarkic case, agents from both countries will participate in both markets.

I assume costless transportation. Productivity is  $A_t$  in the home country and  $A_t^* = bA_t$  in the foreign country, where  $b > 0$  is a known constant. As before, the gross rate of change  $\varepsilon = A_{t+1}/A_t$  is i.i.d with  $E_t(\varepsilon) = 1$ .

I start with the case of a single asset: The dollar. (Thus, there is full dolarization in Japan). The supply of dollars grows at the rate of  $\mu$  because of interest and transfer payments. While all holders of dollars get interest payments, only home country buyers get transfer payments.

After receiving interest and transfer payments the representative buyer in the home country holds  $m$  normalized dollars and the representative buyer in the foreign country holds  $1 - m$  normalized dollar, where a normalized dollar (ND) is the post transfer supply of dollars. In assume a steady state in which  $m$  does not change over time.

Trade occurs sequentially (on the internet).<sup>4</sup> At the beginning of the period buyers who want to buy form a line. When  $\theta = 0$ , only US buyers get in line. When  $\theta = 1$  buyers from both countries get in line.

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<sup>4</sup> A real version of this model that allows for transportation costs is in Eden (forthcoming). In the real version I distinguish between the case in which goods must be displayed on location before the beginning of trade to the case in which orders are placed first and delivery occurs later. Here I focus on the second delivery to order case. We may think, for example, of the market for resorts. Buyers from all over the world may make reservations on the internet. Those who make early reservations may get relatively cheap vacations. Other examples may be trade in intermediate goods.

The place in the line is determined by a lottery that treats all active buyers symmetrically. I assume that any segment of the line represents the population of active buyers.

Active buyers arrive at the market place one by one according to their place in the line and buy at the cheapest available price offer.

The amount that will be spent is  $m$  ND if only the home country buyers want to consume and  $1$  ND if all buyers want to consume. We say that the first  $m$  NDs buy in the first market at the price of  $p_1$  ND per  $A_t$  units. If  $\theta = 1$  an additional amount of  $1 - m$  NDs will arrive, open the second market and buy at the price  $p_2$  (per  $A_t$  units). The use of normalized prices assumes that the regular dollar prices are proportional to the asset supply:  $P_{st}A_t = p_s(M_t + G_t)$ .

When aggregate demand is low and only one market opens the probability of buying at the first market price is unity and the expected purchasing power of a normalized dollar is:  $A_t/p_1$ . When demand is high and two markets open the probability of buying at the first market price is  $m$  (= the fraction of dollars that will buy in the first market). When two markets open, the expected purchasing power of a normalized dollar is:  $mA_t/p_1 + (1-m)A_t/p_2$ . I use  $z_{st}$  to denote the expected purchasing power of a normalized dollar if exactly  $s$  markets open:

$$(15) \quad z_{1t} = \frac{A_t}{p_1} \quad \text{and} \quad z_{2t} = A_t \left( \frac{m}{p_1} + \frac{1-m}{p_2} \right)$$

The unconditional expected purchasing power of a normalized dollar is:

$$(16) \quad Z_t = (1 - \pi)z_{1t} + \pi z_{2t} , \text{ for a home country buyer and}$$

$$Z_t^* = \pi z_{2t} , \text{ for a foreign country buyer.}$$

Note that a buyer in the home country will buy regardless of the realization of  $\theta$  and therefore  $Z$  is a weighted average of  $z_1$  and  $z_2$ . A foreign buyer will buy only if  $\theta = 1$ . In this case two markets will open and therefore  $Z^*$  is a weighted average between zero and  $z_2$ .

Sellers (workers) take prices as given. They know that they can sell (in the first market) at the price  $p_1$  with probability 1 and (in the second market) at the price  $p_2$  with probability  $\pi$ . I use  $k_s A_t$  to denote the supply of the home country seller to market  $s$ . The home country seller solves:

$$(17) \quad \max_{k_s} - Av(k_1+k_2) + (1-\pi)(p_1 k_1)RZ + \pi(p_1 k_1 + p_2 k_2)RZ + gZ.$$

The first term in (17) is the cost of producing  $k_1 + k_2$  units. The next two terms are the expected real wage. When only one market opens the seller sells the output of  $k_1$  units of labor and his revenues is  $p_1 k_1$  ND. When both markets open the seller's revenues are  $p_1 k_1 + p_2 k_2$  NDs. Revenues are invested at the gross real rate  $R$ . To convert next period's balances to expected consumption we multiply by  $Z$ .

The representative young agent in the foreign country supplies  $k_s^* A_t^* = k_s^* b A_t$  units to market  $s$ . If he sells it he gets  $p_s b k_s^*$  ND. He therefore solves:

$$(18) \quad \max_{k_s^*} - bAv(k_1^* + k_2^*) + (1-\pi)[b(p_1 k_1^*)R + (1-m)]Z^* + \pi b(p_1 k_1^* + p_2 k_2^*)RZ^*$$

Note that the expected purchasing power function is now  $Z^*$  instead of  $Z$ ; the foreign agent does not get a transfer payment; but in the low demand state he gets a bequest.

A steady state requires that the home country seller supplies to the first market only ( $k_1 = L, k_2 = 0$ ). Under this assumption, the post transfer balances held by the buyer in the home country do not change over time and are given by:

$$(19) \quad m = p_1LR + g.$$

To state the first order condition for the problem (17) [(18)] note that the expected real revenue per unit of labor (real wage) is  $p_1RZ$  ( $bp_1RZ^*$ ) if the unit is supplied to the first market and  $\pi p_2RZ$  ( $\pi bp_2RZ^*$ ) if it is supplied to the second market. At the optimum the marginal cost ( $Av'[L] = AL$ ) must equal the expected real wage:

$$(20) \quad AL = p_1RZ = \pi p_2RZ; \quad bAL^* = bp_1RZ^* = \pi bp_2RZ^*$$

A steady state equilibrium is a solution ( $L, L^*, k_1^*, p_1, p_2, m$ ) to (19) - (20) and the market clearing conditions:

$$(21) \quad p_1(L + bk_1^*) = m; \quad p_2b(k_2^* = L^* - k_1^*) = 1 - m.$$

Claim 1: (a) There exists unique steady state equilibrium for the single-asset world, (b) An increase in the relative productivity of the foreigners (the parameter  $b$ ) reduces the steady state level of  $m$ .

The proof of this and all other claims is in the Appendix. The comparative static is intuitive: Since labor supplies do not depend on the parameter  $b$  the foreign country's share of income and wealth increases with its relative productivity.

Table 2 illustrates the steady state solutions for two values of  $\mu$ , assuming  $\pi = 0.9$ ,  $i = 0$ ,  $b = 1$ . The last two columns are the steady state welfare in each country computed by:

$$W = c - (\frac{1}{2})L^2, \quad W^* = \pi(L + L^* - c_2) - (\frac{1}{2})(L^*)^2, \quad \text{where } c = (1 - \pi)c_1 + \pi c_2, \\ c_1 = m/p_1 \text{ and } c_2 = m[(m/p_1 + (1-m)/p_2].$$

Table 2: The fully integrated single asset world ( $\pi = 0.9$ ;  $i = 0$ ,  $b = 1$ )

$\mu$ (R)	m	L	$L^*$	W	$W^*$
0 (R=1)	0.501	0.955	0.855	0.456	0.447
0.05 (R=1/1.05)	0.526	0.912	0.817	0.498	0.404

Comparing Tables 2 and 1 reveals that when  $\mu = 0$  both employment and welfare in the home country are higher under autarky. Buyers in the home country suffer from the price dispersion introduced by the foreigners because sometimes they cannot buy at the cheaper price. Imposing a moderate inflation tax works in the direction of compensating the home country.

non-steady state equilibria: The assumption that US sellers specialize in market 1 is required for a steady state analysis in which  $m$  does not change over time. Since this assumption plays an important role, I now

turn to discuss non-steady state equilibria. (See the real version of this model in Eden [forthcoming] for a more comprehensive treatment).

I assume small transportation costs of  $\tau$  ND per unit. When buyers go on the internet they see the location of the seller and take transportation costs into account. A buyer will thus prefer to buy at  $p$  ND from a home country seller rather than at  $p^*$  ND from a foreign seller if:  $p \leq p^* + \tau$ .

I now consider an alternative in which Japanese specialize in market 1 and Americans supply to both markets. The American seller posts the prices:  $p_1, p_2$ . Since he is indifferent between the two markets we must have:  $p_1 = \pi p_2$ . The Japanese seller can guarantee the making of a sale only if he posts the price:  $p^* \leq p_1 - \tau$ . To see this point note that in the low demand state all buyers are from the US and they will prefer an offer at the price  $p_1$  from a US seller to an offer at the price  $p^* > p_1 - \tau$  from a Japanese seller. Note also that the Japanese seller can get the price  $p_2 - \tau$  in the high demand state from Japanese buyers who did not make a buy in market 1. Since  $p_1 = \pi p_2$ , implies  $p_1 - \tau < \pi(p_2 - \tau)$ , the Japanese seller strictly prefers market 2 and we cannot have equilibrium in which he specializes in market 1.

A similar argument can also be used to rule out equilibria in which both sellers supply to both markets. I could not rule out non-steady state equilibria in which the Japanese specialize in market 2. But the main results will hold also in this more complicated case. In what follows I abstract from transportation costs.

## 3.1 A TWO ASSETS WORLD

I now introduce an additional asset: the yen. The post-transfer, post-interest-payments supply of dollars (yen) at time  $t$  is  $M_t + G_t$  ( $M_t^* + G_t^*$ ). The deterministic rate of growth of the dollar (yen) supply is  $\mu$  ( $\mu^*$ ).

In the steady state, the yen and the dollar rates of return are constant and there exist normalized dollar prices,  $p_s$ , and normalized yen prices  $p_s^*$  such that:

$$(22) \quad P_{st}A_t = p_s(M_t + G_t); \quad P_{st}^*A_t = p_s^*(M_t^* + G_t^*).$$

The dollar price of yen ( $e_t$ ) is determined in a foreign exchange market that opens before the realization of the time  $t$  shocks. Since nothing happens between the selling of the goods and the opening of the foreign exchange market in the next period, we require:

$$(23) \quad e_{t+1}P_{st}^* = P_{st}$$

Equations (22) and (23) lead to:

$$(24) \quad \frac{e_{t+1}}{e_t} = \frac{1 + \mu}{1 + \mu^*}$$

Thus as in standard models, the rate of growth of the exchange rate depends on the money supplies growth rates. Note that (24) implies

that the dollar value of the yen supply is a constant fraction  $\alpha$  of the dollar supply:

$$(25) \quad \alpha = \frac{e_t(M_t^* + G_t^*)}{M_t + G_t} = \frac{e_{t-1}(M_{t-1}^* + G_{t-1}^*)}{M_{t-1} + G_{t-1}}$$

In the steady state Americans hold  $m$  normalized dollars and  $n$  normalized yen (a fraction  $m$  of the dollar supply and a fraction  $n$  of the yen supply) where  $0 \leq m, n \leq 1$ .

Agents form expectations about the probability that each asset will be accepted as payment for goods. They expect that if they did not make a buy in the first market they will be able to use both assets to buy in the second market. Their expectations with respect to acceptance in the first market are as follows. In the state of high demand the probability of buying with yen (dollars) is  $n$  ( $m$ ). In the state of low demand agents expect that they will be able to buy with yen (dollars) if some of the units supplied to the first market are offered for yen (dollars). In equilibrium this condition will be satisfied if  $n > 0$  ( $m > 0$ ).

Omitting the index  $t$ , the expected purchasing power of a normalized yen if exactly  $s$  markets open is:

$$(26) \quad x_1 = \frac{A}{p_1^*} \text{ if } n > 0 \text{ and } \frac{A}{p_2^*} \text{ otherwise; } x_2 = A \left( \frac{n}{p_1^*} + \frac{1-n}{p_2^*} \right)$$

Similarly, the expected purchasing power of a normalized dollar is:

$$(27) \quad z_1 = \frac{A}{p_1} \text{ if } m > 0 \text{ and } \frac{A}{p_2} \text{ otherwise; } z_2 = A \left( \frac{m}{p_1} + \frac{1-m}{p_2} \right)$$

The unconditional expected purchasing power of 1 NY (ND) is:

$$(28) \quad X = (1 - \pi)x_1 + \pi x_2 ; \quad X^* = \pi x_2$$

$$Z = (1 - \pi)z_1 + \pi z_2 ; \quad Z^* = \pi z_2$$

The young agent chooses the price tag on each unit. He may choose among the following four alternatives:  $p_1^*$ ,  $p_1$ ,  $p_2^*$  and  $p_2$ . When a price is expressed in yen (dollars) it means that the seller will accept only yen (dollars) for the unit.

In terms of our hypothetical markets, the young American chooses the amount supplied to market  $s$  ( $Ak_s$ ) and the fraction of the supply that will be offered for dollars ( $0 \leq \psi_s \leq 1$ ). The choice of the payment asset satisfies:

$$(29) \quad \psi_s = 0 \text{ if } p_s R Z < p_s^* R^* X ; \quad \psi_s = 1 \text{ if } p_s R Z > p_s^* R^* X \text{ and}$$

$$0 \leq \psi_s \leq 1 \text{ if } p_s R Z = p_s^* R^* X.$$

This says that the seller will post a yen price if the expected real wage when posting a yen price ( $p_s^* R^* X$ ) is greater than the expected real wage when posting a dollar price ( $p_s R Z$ ). Similarly, for the Japanese we have:

$$(30) \quad \psi_s^* = 0 \text{ if } p_s R Z^* < p_s^* R^* X^* ; \quad \psi_s^* = 1 \text{ if } p_s R Z^* > p_s^* R^* X^* \text{ and}$$

$$0 \leq \psi_s^* \leq 1 \text{ if } p_s R Z^* = p_s^* R^* X^*$$

The American seller chooses the supplies  $Ak_s$  according to the maximum expected real wage in market  $s$ :  $\max\{p_s RZ, p_s^* R^* X\}$ . This is described by the solution to the following problem.

$$(31) \max_{k_s} - Av(k_1+k_2) + k_1 \max\{p_1 RZ, p_1^* R^* X\} + \pi k_2 \max\{p_2 RZ, p_2^* R^* X\} + gZ$$

Similarly, for the Japanese we have:

$$(32) \max_{k_s^*} - bAv(k_1^* + k_2^*) + bk_1^* \max\{p_1 RZ^*, p_1^* R^* X^*\} + \pi b k_2^* \max\{p_2 RZ^*, p_2^* R^* X^*\} \\ + \pi g^* X^* + (1 - \pi)[(1 - m)Z^* + (1 - n)X^*]$$

Since Americans hold  $m$  ND and  $n$  NY, these are the minimum amounts that will be spent. Since the probability of making a sale in the first market is unity, the clearing of the first market requires (using  $v_s = 1 - \psi_s$ ):

$$(33) \quad p_1^*(v_1 k_1 + v_1^* b k_1^*) = n; \quad p_1(\psi_1 k_1 + \psi_1^* b k_1^*) = m$$

When demand is high then additional  $1 - m$  ND and  $1 - n$  NY will buy in the second market. The clearing of the second market requires:

$$(34) \quad p_2^*(v_2 k_2 + v_2^* b k_2^*) = 1 - n; \quad p_2(\psi_2 k_2 + \psi_2^* b k_2^*) = 1 - m$$

The market clearing conditions insure that if you did not make a buy in the first market you can make a buy in the second. Furthermore, if  $n > 0$  ( $m > 0$ ) you will be able to use yen (dollars) to buy in the

first market in the low demand state. The expectations (26)-(27) are thus consistent with market clearing (rational).

In addition to market clearing we require that in the steady state the portfolio will not change over time. This requires  $k_2 = 0$  ( $L = k_1$ ) and

$$(35) \quad R^* p_1^* (\psi_1 L) = n; \quad R p_1 (\psi_1 L) + g = m$$

The vector:

$$(p_1, p_2, p_1^*, p_2^*, R, R^*, m, n, X, Z, X^*, Z^*, k_1, k_2, k_1^*, k_2^*, \psi_1, \psi_2, \psi_1^*, \psi_2^*)$$

is a steady state equilibrium if it satisfies (26)-(30) and (34)-(35) and given  $(p_1, p_2, p_1^*, p_2^*, R, R^*, X, Z, X^*, Z^*), (k_1, k_2 = 0)$  solve (31) and  $(k_1^*, k_2^*)$  solve (32).

I assume (without proving) that a steady state in which both sellers accept both assets as payments for goods does exist and show the following Claim.

Claim 2: In a steady state equilibrium in which both sellers accept both assets ( $0 < \psi_1 < 1, 0 < \psi_1^* < 1, 0 < \psi_2^* < 1$ ) we must have:

$$(a) p_1 = \pi p_2, (b) p_1^* = \pi p_2^*, (c) m = n, \text{ and } (d) R = R^*.$$

The proof is in the Appendix. The result that a steady state with an interior solution to  $\psi = (\psi_1, \psi_1^*, \psi_2^*)$  requires  $m = n$  is surprising, given our assumption of risk neutrality. It says that there is no "home bias": If Americans hold a fraction  $m$  of the dollar supply they must also hold a fraction  $m$  of the yen supply. This equilibrium is similar to

the single asset equilibrium analyzed in the previous section except for the transfer payments that are now more symmetric.

I now turn to discuss a steady state equilibrium in which only dollars are accepted in the first market<sup>5</sup>:

$$(36) \quad \psi_1 = \psi_1^* = 1 \text{ and } 0 \leq \psi_2^* \leq 1.$$

Proposition 1: When  $\alpha \leq b$ , there exists a unique steady state equilibrium that satisfies (36). This steady state has the following properties:

- (a)  $L \geq L^*$  and  $R^* \geq R$  with the inequalities being strict when  $\pi < 1$ ;
- (b) An increase in  $\alpha$  leads to a decrease in  $1 - m$ , and an increase in labor supplies in both countries;
- (c) US sellers strictly prefer dollars to the equivalent yen amount.

The condition  $\alpha \leq b$  is required to guarantee existence. When  $b = 1$ , it says that the dollar value of the yen supply is not greater than the dollar supply. The intuition for (a) - (c) is as follows.

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<sup>5</sup> I tried other specification of  $\psi$ . Equilibrium with  $\psi_1 = 1$  and  $\psi_1^* = \psi_2^* = 0$  implies no trade. Expressing the market clearing conditions in dollar terms and assuming  $p_1 = \pi p_2$  leads to:  $p_1 L^* = \pi \alpha$  and  $p_1 L = 1$ . Therefore  $L^*/L = \pi \alpha$ . Since  $\pi \alpha$  is exogenous in our model this poses a problem for proving existence of equilibrium.

A steady state in which the Japanese accept yen only ( $0 < \psi_1 < 1$ ,  $\psi_1^* = \psi_2^* = 0$ ) requires:  $k_1^* = 0$  when  $R^* = 1$ . This Claim follows from conditions (33) and (35) and leads to problems in proving existence of equilibrium with  $R^* = 1$ .

(a) Foreign workers may not want to consume and therefore have less incentive to work. The yen rate of return must be higher because lower price sellers do not accept it. (b) When  $\alpha$  increases foreign agents substitute yen for dollars and  $1 - m$  goes down. As a result the dollar promises a higher chance of buying in the first market and a higher yen rate of return is required to compensate for the difference in liquidity. The higher rate of return on the yen leads to a higher expected real wage in Japan. The expected real wage in the US also goes up as a result of the increase in  $m$  and the increase in the probability that US buyers will buy at the cheaper price. The increase in the expected real wage leads to an increase in labor supply in both countries. (c) The "liquidity premium" on the dollar is sufficient to make Japanese sellers accept both assets. US sellers are willing to pay a higher "liquidity premium" on holding dollars because they buy in both states and the advantage of the dollar is larger in the low demand state (where the probability of buying at the cheaper price is unity for the dollar and zero for the yen).

Note that sellers make portfolio choices in the goods market. Since nothing happens between the end of trade in the goods market and the trade in foreign exchange, there are no transactions in the foreign exchange market.

Uncovered interest parity does not hold in our model. Proposition 1 says:  $R = (1 + i)/(1 + \mu) < (1 + i^*)/(1 + \mu^*) = R^*$ . Using (24), this implies:  $(1 + i)/(1 + i^*) < (1 + \mu)/(1 + \mu^*) = e_{t+1}/e_t$ . We can have for example,  $\mu = \mu^*$  and  $i^* > i$ . In this case, the exchange rate does not change over time but the nominal interest rate on the foreign asset is higher. We can also have both  $i^* > i$  and  $\mu > \mu^*$ . This is the "forward premium

puzzle" found in the empirical literature, where the low interest rate currency tends to depreciate. (See Burnside, Eichenbaum, Kleshchelski and Rebelo [2006] for a recent discussion).

Are there arbitrage opportunities? The standard argument is that when there is an "over priced" asset one can make money by holding a negative amount of it. But this assumes that the speculator is not interested in consumption. If he is interested in consumption he may find that he has to buy at the high price with his relatively illiquid asset.<sup>6</sup>

I now turn to a numerical example. Table 3 computes the equilibrium magnitudes for various  $\mu$  and  $\alpha$  assuming  $i = i^* = 0$ ,  $\pi = 0.9$  and  $b = 1$ . In the first four rows  $\mu = 0$  and  $\alpha$  takes four values:  $\alpha = 0, 0.1, 0.8, 1$ . When  $\alpha = 1$  we get autarky. Reducing  $\alpha$  increases welfare in the foreign country and decreases welfare in the home country. Note that  $\alpha > 0$  requires  $R^* > 1$  ( $\mu^* < 0$ ). Furthermore, an increase in  $\alpha$  leads to an increase in  $m$  and an increase in the probability that a dollar will buy in the first market. Therefore an

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<sup>6</sup> What if a buyer offers yen in the first market? Our general equilibrium formulation does not allow for communication between the buyer and the seller: The seller posts a price which is a take it or leave it offer. But a yen may fail to buy at the low price even if we allow for some communication. This is because the seller knows that only Japanese hold yen and therefore after observing a yen offer he may update upward the probability that this is the high demand state. He may then say that he is stocked out at the low price and offer the good at the high price. Thus our equilibrium may be robust to some negotiation between the buyer and the seller.

increase in  $\alpha$  requires a higher liquidity premium and hence a higher  $R^*$  (lower  $\mu^*$ ).

When  $\mu > 0$ , increasing  $\alpha$  (and holding  $\mu$  constant) has an ambiguous effect on welfare. It reduces both the inflation tax paid by foreigners and the probability that a foreign buyer will buy at the low price. The first inflation tax effect works to improve welfare in the foreign country and reduce welfare in the home country. The second, term of trade effect, works in the opposite direction. The inflation tax effect dominates when  $\mu$  is large.

Increasing  $\mu$  (and holding  $\alpha$  constant) has also two effects on welfare in the home country. It increases the inflation tax collected from foreigners (when  $\alpha < 1$ ) and it creates a distortion in the labor supply choice. When  $\alpha$  is low the inflation tax effect dominates and therefore an increase in  $\mu$  increases welfare in the home country and reduces welfare in the foreign country. When  $\alpha$  is large the distortion effect dominates and an increase in  $\mu$  reduces welfare in both countries.

Table 3: The two assets case ( $\pi = 0.9$ ,  $i = i^* = 0$ ,  $b = 1$ )

$\mu$	$\alpha$	m	$\mu^*$	L	L*	W	W*
0	0	0.501		0.955	0.855	0.456	0.447
0	0.1	0.551	-0.06	0.960	0.860	0.460	0.443
0	0.8	0.900	-0.09	0.991	0.891	0.491	0.414
	1	1	-0.1	1	0.9	0.5	0.405
0.05	0	0.526		0.912	0.817	0.498	0.404
0.05	0.1	0.574	-0.01	0.916	0.821	0.495	0.407
0.05	0.8	0.905	-0.05	0.944	0.849	0.495	0.407
0.05	1	1	-0.05	0.952	0.857	0.499	0.404

\* The first two columns are the choice of the two policy-makers:  $\mu$ ,  $\alpha$ . We then have the following endogenous variables: the fraction of the post transfer dollar supply held by the buyers in the home country (m), the equilibrium rate of change in the yen supply ( $\mu^*$ ), labor supply in the home country (L), labor supply in the foreign country L\* and welfare in the two countries (W, W\*).

Figures 1 and 2 describe welfare in both countries as a function of  $\pi$ . The measure plotted is welfare relative to the no-taste shock case (Since in the no shock case,  $W = \frac{1}{2}$ , I plot  $2W$ ,  $2W^*$ ). This is done for  $\mu = 0$  and two values for  $\alpha$ :  $\alpha = 0.1$  and  $\alpha = 0.8$ . Note that a decrease in  $\pi$  has an adverse effect on welfare in both countries. Welfare in the US is lower and welfare in Japan is higher for small  $\alpha$ . This is special to the case  $\mu = 0$  when no inflation tax is imposed.

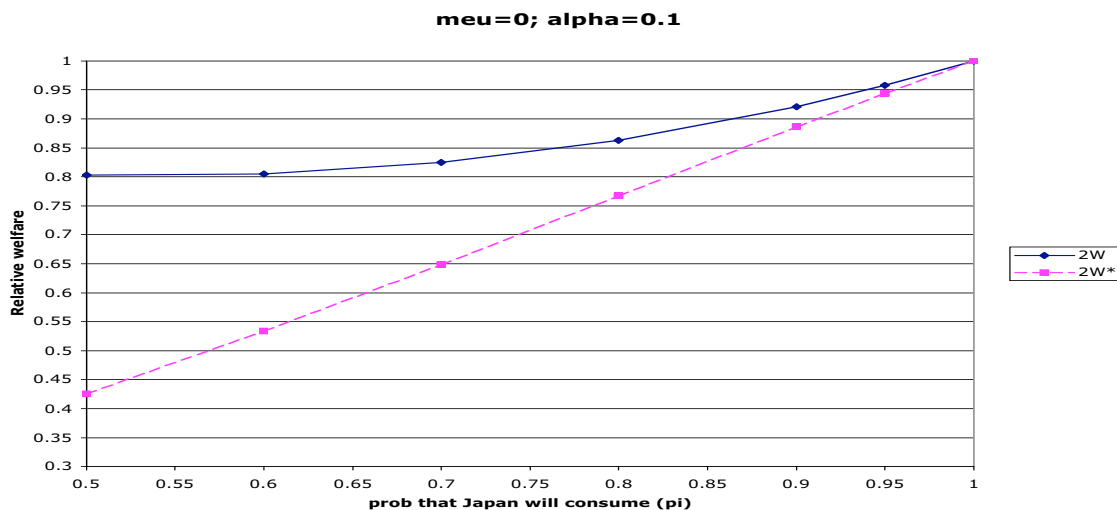


Figure 1: Welfare relative to the no-shock case ( $W/0.5, W^*/0.5$ ) when  $\mu=0$  and  $\alpha=0.1$

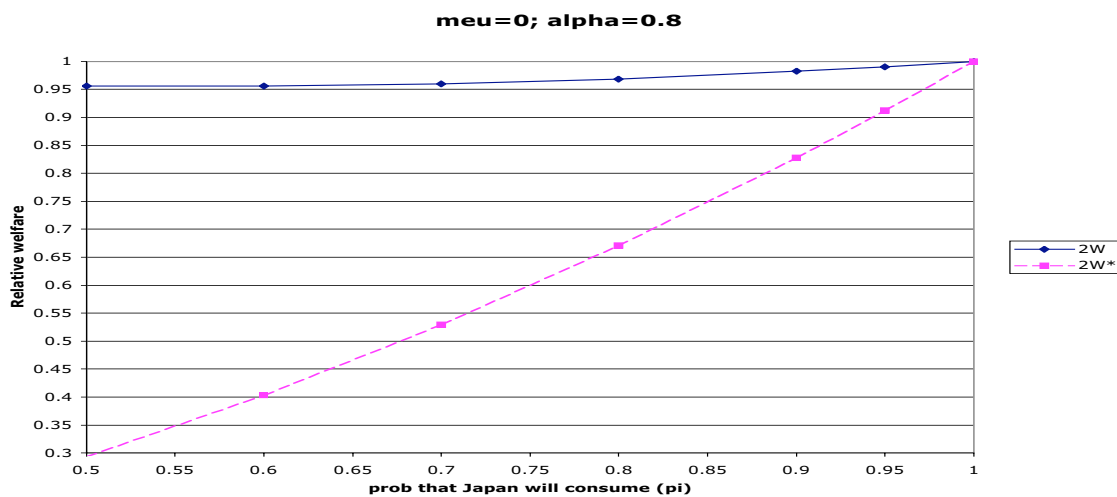


Figure 2: Welfare relative to the no-shock case ( $W/0.5, W^*/0.5$ ) when  $\mu=0$  and  $\alpha=0.8$

### A Sequential policy game

I now turn to a brief description of a sequential game between the policy makers. Since the rest of the world consists of many countries, I assume that the US moves first and chooses  $\mu$ , knowing the reaction function of the rest of the world.

Figure 3 illustrates the reaction  $\alpha(\mu; \pi)$  of the representative foreign government for the US choice of  $\mu$  under the assumptions  $i=i^*=0$

and  $b=1$ . This is done for two cases:  $\pi = 0.9$  and  $\pi = 0.95$ . The foreign country trade-off is between the terms of trade (the probability of buying at the low price) and the inflation tax. When  $\mu = 0$  there is no inflation tax and therefore the foreign country focus on the terms of trade which are best when  $\alpha = 0$ . When  $\mu$  is positive a higher  $\alpha$  means less inflation tax but also less favorable terms of trade. When  $\mu$  is sufficiently high the inflation tax dominates and the foreign country chooses  $\alpha = 1$ . Note that when  $\pi$  increases from 0.9 to 0.95 the term of trade effect becomes less important and the foreign government chooses higher  $\alpha$  for any given  $\mu$  to avoid the inflation tax. In the limit case when  $\pi = 1$ , the foreign government will choose  $\alpha = 1$  regardless of  $\mu$ .

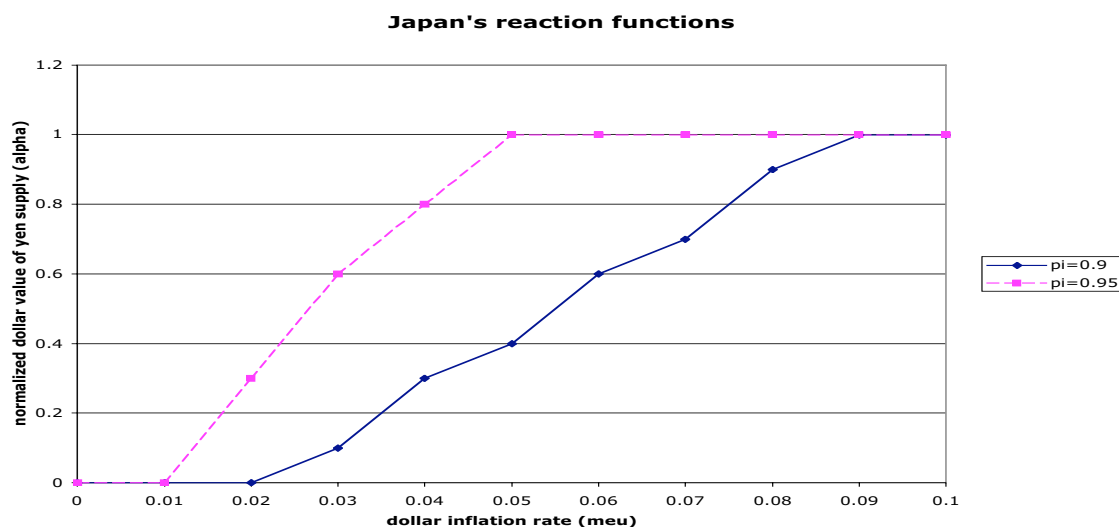


Figure 3:  $\alpha(\mu)$  for  $\pi = 0.9$  (the solid line) and  $\pi = 0.95$

The optimal choice of the home country is  $\mu = 0.08$  when  $\pi = 0.9$  and  $\mu = 0.05$  when  $\pi = 0.95$ . In the first case the optimal reaction is  $\alpha(0.08; 0.9) = 0.9$ . In the second the optimal reaction is

$\alpha(0.05; 0.95) = 1$ . It seems that the optimal  $\mu$  is "too high" relative to recent observations. A successful calibration of the model will require some modification. For example, we may add a "transaction motive" for holding money. This will increase the welfare cost of inflation and reduce the optimal  $\mu$ .

#### 4. POTENTIAL APPLICATIONS

Candidates for the home country in our model are: The US, Canada and some major European countries. Candidates for the foreign country in our model are: Japan and some countries that are not in the G-7.

To evaluate these choices we need to assess the relative stability of the two groups of countries, where stability is measured by the predictability of demand. Predictability of GDP may serve as a proxy. This is only a proxy because GDP in our model is determined by both technology and demand.

Recently Stock and Watson (2003, Table 2) estimated the predictability of GDP for the G-7 countries. Their estimates imply that since 1984 the one-step ahead forecast RMSE for Japan was higher than the RMSE for the US by 90%. The IMF Tables provide information about 181 countries.<sup>7</sup> During the period 1984–2004 the median rate of change of real GDP in countries that do not belong to the G-7 was 3.2 and the median standard deviation was 3.9. For the G-7 the median rate of growth

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<sup>7</sup> IMF World Economic Outlook Database for September 2006.

<http://www.imf.org/external/pubs/ft/weo/2006/02/data/index.aspx>

was 2.4 and the median standard deviation was 1.5. It thus seems that Japan and countries that are not in the G-7 are relatively unstable.<sup>8</sup>

Apparent seigniorage payments: Does the US get seigniorage payments from the rest of the world? In a recent article Gourinchas and Rey (GR, 2005) found strong evidence of sizeable excess returns of gross US assets over gross US liabilities. They found that during the period 1952 - 2004 the average annualized real rate of return on gross liabilities was 3.61% while the average annualized real rate of return on gross assets was 5.72%. The difference of 2.11% is considerable. This difference is especially large when looking at the post Bretton-Woods period: 1973 - 2004. The post Bretton-Woods average asset return is 6.82% while the corresponding total liability return is only 3.50%. The excess return in the post Bretton Woods era is thus 3.32%.

In an Appendix that is in Eden (2006), I provide some preliminary calculations of the seigniorage that the US may expect to receive from foreigners. The calculations assume risk neutrality, expected excess return equal to the post Bretton-Woods average (3.32%) and quantities at their 2004 levels. If we adopt the narrow definition of seigniorage (payments on cash) we get roughly 0.2% of US GDP. If we adopt a broad definition we get 2% of US GDP. The broad definition includes payments both to the US government and to US private agents. The US government

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<sup>8</sup> I also looked at a balanced sample of countries that have complete information. After eliminating the G-7 countries this yields a sample of 145 countries. The average growth rate and the average standard deviation for this sample are: 3.6 and 4.6. The medians are: 3.3 and 3.8.

may expect to collect about 0.7% of US GDP from securities and cash held by foreigners.

Rates of return and seigniorage: The US government gets seigniorage if it sells "over priced" bonds. Under risk neutrality bonds are "over priced" if they promise an expected rate of return that is less than the highest expected rate of return alternative. Since US government bonds promise an expected rate of return that is less than the highest alternative (equity or FDIs) and since Japan holds these bonds, Japan pays seigniorage to the US. This is true regardless of whether the best alternative is in Japan or elsewhere.

Indeed US private agents who hold US government bonds also pay seigniorage to the US government. This paper does not explain why US private agents are willing to hold US government bonds (the equity premium puzzle). It does attempt to explain why Japanese agents are willing to hold US government bonds when higher expected return alternatives are available.

Other implications: Our model is consistent with the observation that the US is cheap relative to the prediction of income-price regressions. See Balassa (1964), Samuelson (1964) and Rogoff (1996).

As in Rose and Wincoop (2001), the adoption of a common currency increases trade in our model. This is different from Bacchetta and van Wincoop (2000) who argue that exchange-rate stability is not necessarily associated with more trade.

Americans work hard in our model because they are relatively certain about the prospect of enjoying the fruits of their labor. This is not unlike the tax explanation in Prescott (2004).

Dollarization: Although the paper focus on a broad definition of money it has bearing on the issues of dollarization and currency unions that typically focus on narrow definitions of money. Fischer (1982) argues that countries choose to have national monies to avoid paying seigniorage to a foreign government. Here we showed that an unstable demand country may gain from full or partial dollarization even if it pays moderate seigniorage.

Transaction costs and the ability to commit play a major role in Alesina and Barro (2001, 2002) analysis of dollarization and currency unions. They argue that seigniorage should be part of the overall negotiations. This may be feasible in the case of currency unions they consider. But here we discuss the holding of dollar denominated assets by agents from all (193) countries. Cooperation in this case is more difficult and therefore a sequential game in which the US moves first seems appropriate.

## 5. CONCLUDING REMARKS

We used price dispersion to model liquidity. The idea that liquidity may play a role in explaining assets returns is of-course not new. Recently, McGrattan and Prescott (2003) argue that short term US government securities provide liquidity and are therefore overpriced.

Cochrane (2003) argued that some stocks are over-priced because they provide liquidity.

Our model illustrates the possibility that the apparent seigniorage paid to the US by the rest of the world may continue in a steady-state equilibrium if the US demand will continue to be relatively predictable. This is different from standard steady state analysis in which trade surplus is required to pay the interest on the debt. See for example, Blanchard Giavazzi and Sa (2005).

Our analysis has some common elements with Caballero et al. (2006). They attribute the increase in the importance of US assets to an unexpected reduction in the growth rate of European and Japanese output and (or) a collapse of the asset markets in the rest of the world.

Devereux and Engel (2003) find that the implications of risk for foreign trade are highly sensitive to the choice of currency at which prices are set. In their model prices are rigid and firms satisfy demand. Here the choice of currency is endogenous, there is no incentive to change prices during trade and sellers are not committed to satisfy demand (low price sellers are stocked out in the high demand state).

Our approach is also related to the random matching models pioneered by Kiyotaki and Wright (1993) and used among others by Matsuyama, Kiyotaki and Matsui (1993), Zhou (1997), Wright and Trejos (2001) and Liu and Shi (2005). In both the random matching models and the UST model uncertainty about trading opportunities plays a key role. In the random matching models agents are uncertain about whether they will meet someone that they can actually trade with. But whenever a meeting takes place it is bilateral. In the UST model sellers are also uncertain about the arrival of trading partners but whenever a meeting

occurs there are a large number of agents on both sides of the market. As a result there is a difference between the assumed price determination mechanisms. In the random matching models prices are either fixed or are determined by bargaining (as in Trejos and Wright [1995] and Shi [1995]). In the UST model prices clear markets that open.

Our overlapping generations model does not distinguish between bonds and money (interest bearing debt and non-interest bearing debt). The framework in Lagos and Wright (2005) may be a good way of doing it. In their framework, random matching occurs during the "day" and Walrasian auction occurs during the "night". We may replace the Walrasian night auction with sequential trade. That is, after interacting in a decentralized market with anonymous bilateral matching during the day, agents go on the internet and place orders as in our model. During the night it is easy to transfer funds from one account type to another and therefore we may assume that in fact everyone accepts bonds.

In our model we get a steady state equilibrium with two monies even when  $R \neq R^*$ . This is different from Karekan and Wallace (1981) and is not consistent with Gresham's law.

The paper may be viewed as a first step in using the UST model to address a major issue in monetary and financial economics: The rate of return dominance and closely related puzzles. The introduction of physical capital is the obvious second step that will allow us to address the equity premium puzzle directly.

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## APPENDIX

Proof of Claim 1:

The first order conditions (20) imply:

$$(A1) \quad p_1 = \pi p_2.$$

We substitute (A1) in (15)-(16) to get:

$$z_1 = A/p_1 ; z_2 = mA/p_1 + (1-m)\pi A/p_1 \text{ and}$$

$$(A2) \quad z = (A/p_1)[1 - \pi + \pi^2 + \pi(1 - \pi)m]; \quad z^* = (A/p_1)[\pi^2 + \pi(1 - \pi)m]$$

Substituting (A2) in (20) yields:

$$(A3) \quad L = R[1 - \pi + \pi^2 + \pi(1 - \pi)m] ; \quad L^* = R[\pi^2 + \pi(1 - \pi)m]$$

From (19) we get:  $p_1 L = (m - g)/R$ . Using (A3) leads to:

$$(A4) \quad p_1 = \frac{m - g}{R^2[1 - \pi + \pi^2 + \pi(1 - \pi)m]}$$

Substituting  $p_1 L = (m - g)/R$  in the market clearing condition

$p_1(L + b k_1^*) = m$  leads to:  $p_1 b k_1^* = m - (m - g)/R$ . We now substitute this in the market clearing condition  $p_1 b(L^* - k_1^*) = \pi(1 - m)$  to get:

$p_1 b L^* = \pi(1 - m) + m - (m - g)/R$ . Using (A3) leads to:

$$(A5) \quad p_1 = \frac{\pi(1-m) + m - \frac{m-g}{R}}{bR[\pi^2 + \pi(1-\pi)m]}$$

Equating (A4) to (A5) leads to:

$$(A6) \quad \frac{b[\pi^2 + \pi(1-\pi)m]}{R[1-\pi + \pi^2 + \pi(1-\pi)m]} = \frac{\pi(1-m) + m - \frac{m-g}{R}}{m-g}$$

Lemma 1:  $g = \frac{\mu - i}{1 + \mu}$

Proof: To show that we use the following steps:

$$M_{t+1} = (M_t + G_t)(1 + i)$$

$$(M_{t+1} + G_{t+1}) / (M_t + G_t) = 1 + \mu$$

$$[(M_t + G_t)(1 + i) + G_{t+1}] / (M_t + G_t) = 1 + \mu$$

$$(1 + i) + G_{t+1} / (M_t + G_t) = 1 + \mu$$

$$G_{t+1} / (M_t + G_t) = (1 + \mu) - (1 + i) = \mu - i$$

$$G_{t+1} / (M_t + G_t)(1 + \mu) = (\mu - i) / (1 + \mu). \quad \square$$

Lemma 2: When  $m = 1$ , the right hand side of (A6) is zero.

Proof: When  $m = 1$  we can write the right hand side of (A6) as:

$\frac{R-1+g}{R(1-g)}$ . Using the definition of  $R$  and Lemma 1 we get:

$$R - 1 = \frac{1+i}{1+\mu} - 1 = \frac{i-\mu}{1+\mu} = -g. \quad \text{Therefore } \frac{R-1+g}{R(1-g)} = 0. \quad \square$$

Lemma 3: There exists a unique solution to (A6).

Proof: The right hand side (RHS) of (A6) is decreasing when  $m - g \geq 0$ .

When  $m - g$  is small (and positive) it is large. Using Lemma 2 we get

that when  $m = 1$  the RHS of (A6) is zero. The LHS of (A6) is

increasing and when  $m = 1$  it is equal to  $\pi b/R$ . Therefore there exists a unique solution  $\bar{m}$  in Figure A1. An increase in  $b$  will shift the LHS curve up and reduce  $m$  (increase  $1 - m$ ).  $\square$

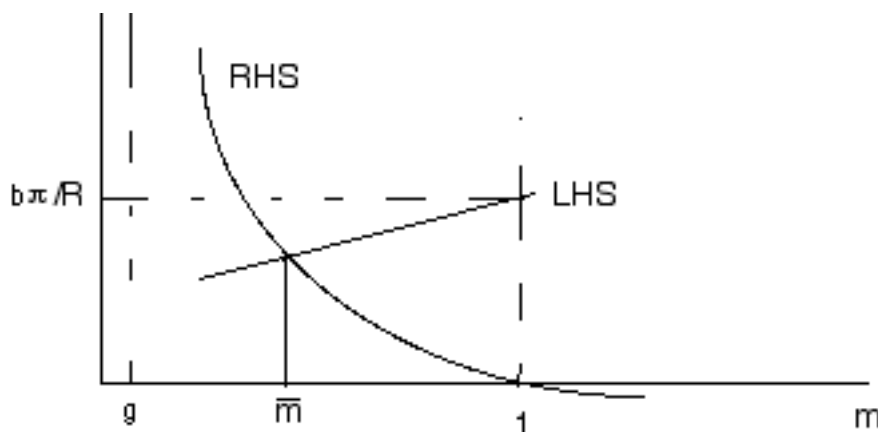


Figure A1

We now substitute the solution  $\bar{m}$  in (A3) to solve for the steady state magnitudes  $L$  and  $L^*$ . We proceed by solving for  $p_1$  from (A5) and  $p_2 = \pi p_1$ . To solve for  $k_1^*$  we use the market clearing condition:  
 $p_2 b(L^* - k_1^*) = 1 - m$ .  $\square$

Proof of Claim 2: To show (a) and (b) note that when  $0 < \psi_1^* < 1$ ,  $0 < \psi_2^* < 1$ , (30) implies:

$$(A7) \quad p_1 R Z^* = p_1^* R^* X^* \text{ and } p_2 R Z^* = p_2^* R^* X^*.$$

Therefore, when  $p_1 > \pi p_2$  we have  $p_1 R Z^* > \pi p_2 R Z^*$  and  $p_1^* R^* X^* > \pi p_2^* R^* X^*$ . The expected real price is higher in the first market and therefore the Japanese will specialize in the first market and choose  $k_2^* = 0$ . This is not consistent with the clearing of the first market.  $p_1 < \pi p_2$  implies

$p_1 R Z < \pi p_2 R Z$  which is not consistent with the choice of the American seller ( $k_2 = 0$ ).

To show (c) note that  $0 < \psi_1 < 1$  and (29) imply:

$$(A8) \quad p_1 R Z = p_1^* R^* X.$$

This and (A7) leads to:

$$(A9) \quad X/Z = X^*/Z^*$$

Substituting  $p_1 = \pi p_2$  and  $p_1^* = \pi p_2^*$  in (26) - (28) leads to:

$$(A10) \quad X/Z = \frac{n(1-\pi)+1}{m(1-\pi)+1} = \frac{n(1-\pi)+\pi}{m(1-\pi)+\pi} = X^*/Z^*$$

This implies  $n = m$ .

To show (d) I substitute  $p_1 = \pi p_2$  and  $p_1^* = \pi p_2^*$  in (26) - (28).

This leads to:

$$(A11) \quad p_1 R Z = AR[1 + m(1 - \pi)]; \quad p_1^* R^* X = AR^*[1 + n(1 - \pi)]$$

Since  $m = n$ , (A11) and (A8) imply  $R = R^*$ .  $\square$

Proof of Proposition 1: I start from the equilibrium conditions under (36). Under (36),  $n = 0$  and the expected purchasing power of a normalized dollar is given by (16). Since yen can buy goods in the

second market only, the unconditional expected purchasing power of a normalized yen is:

$$(A12) \quad X = A/p_2^* ; \quad X^* = \pi(A/p_2^*)$$

The first order conditions (20) describe the labor supply choices under the assumption that sellers accept dollars only. Here we add conditions that justify the assumed choice of assets. We require that US sellers cannot benefit by selling in yen:

$$(A13) \quad AL = Rp_1Z = \pi Rp_2Z \geq R^* p_1^* X = \pi R^* p_2^* X$$

And we require that Japanese sellers are indifferent between dollars and yen:

$$(A14) \quad AL^* = Rp_1Z^* = \pi Rp_2Z^* = \pi R^* p_2^* X^*$$

In the steady state Japanese hold a portfolio of both assets that is worth  $1 - m + \alpha$  normalized dollars. We may therefore write the market clearing conditions as follows.

$$(A15) \quad p_1(L + bk_1^*) = m ;$$

$$(A16) \quad p_2b(L^* - k_1^*) = 1 - m + \alpha$$

The steady state requirement (35) can now be written as:

$$(A17) \quad p_1 L = (m - g)/R.$$

I start by solving (A12) - (A17) for the steady state level of  $m$ . Substituting (A17) in (A15) leads to:  $p_1 b k_1^* = m - (m - g)/R$ . I now substitute this and (A1) in (A16) to get:  
 $p_1 b L^* = \pi(1 - m) + \pi\alpha + m - (m - g)/R$ . We also verify that equations (A3) and (A4) still hold. Using (A3) leads to:

$$(A18) \quad p_1 = \frac{\pi(1 - m) + \pi\alpha + m - \frac{m-g}{R}}{bR[\pi^2 + \pi(1 - \pi)m]}$$

Equating (A4) to (A18) leads to:

$$(A19) \quad \frac{b[\pi^2 + \pi(1 - \pi)m]}{R[1 - \pi + \pi^2 + \pi(1 - \pi)m]} = \frac{\pi(1 - m) + \pi\alpha + m - \frac{m-g}{R}}{m - g}$$

I now turn to show that there exists a unique solution to (A19). The left hand side of (A19) is the same as the LHS of (A6).

The RHS of (A19) is decreasing and when  $m = 1$  it is equal to (using Lemma 2)  $\pi\alpha/(1 - g)$ . It was shown in the proof of Lemma 1 that  $1 - g = R$ . Therefore when  $\alpha \leq b$ , the RHS is less than the LHS when  $m = 1$  and there exists a unique solution,  $\bar{m}$  in Figure A2.

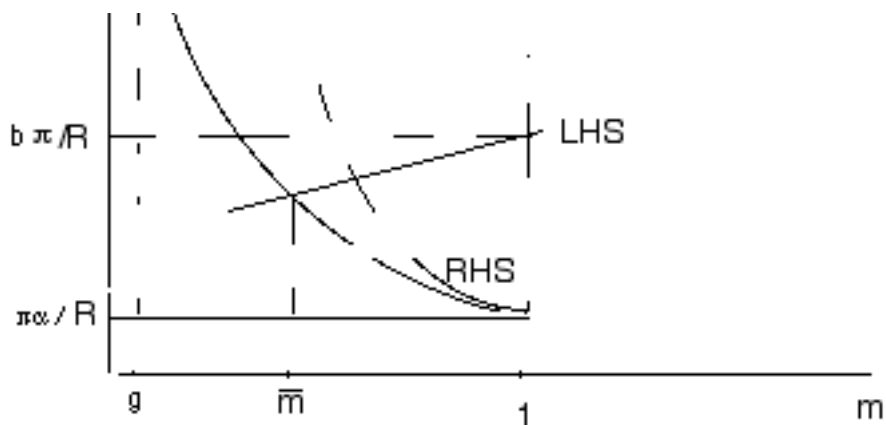


Figure A2

We now use the solution  $\bar{m}$  to solve for the steady state magnitudes. We have thus shown existence and uniqueness.

I now choose  $\omega^*$  so that the Japanese seller is indifferent between yen and dollars. From (A12) and (A14) we get:

$$(A20) \quad R p_2 z^* = R^* p_2^* X^* = \pi A R^*$$

We use (A1) and (A2) to get:  $R p_2 z^* = A R [\pi + (1 - \pi)m]$ . Substituting this in (A20) leads to:

$$(A21) \quad R^*(m) = R \left( 1 + \frac{(1 - \pi)m}{\pi} \right)$$

Condition (A21) implies that  $R^*(m)$  is an increasing function and  $R^* > R$ . Note that (A3) implies  $L \geq L^*$ . We have thus shown part (a).

To show (b) note that an increase in  $\alpha$  increases the RHS of (A19) for all  $m$  and therefore shifts the RHS curve in Figure B2 to the right. This leads to an increase in the steady state level of  $m$ . Note also that

(A3) implies that  $L$  and  $L^*$  are monotonic in  $m$ . Therefore as  $\alpha$  grows and  $m$  grows and labor supplies in both countries grow.

I now turn to show that the US seller strictly prefers dollars. A US seller that sells for dollars will have the expected real wage:

$$(A22) \quad R p_1 Z = AR[1 - \pi + \pi^2 + \pi(1 - \pi)m]$$

The expected real wage when selling in yen is:

$$(A23) \quad R^* p_1^* X = R^* \pi p_2^* (A/p_2^*) = \pi AR^* = \pi AR \left( 1 + \frac{(1-\pi)m}{\pi} \right)$$

where the last equality uses (A21). Subtracting (A23) from (A22) leads to:

$$(A24) \quad R p_1 Z - R^* p_1^* X = AR(1 - m)(1 + \pi^2 - 2\pi) \geq 0.$$

When  $\pi < 0$ , this difference is strictly positive and decreasing in  $\pi$ . We have thus shown (c).  $\square$