STICKY PRICES IN A CASH-IN-ADVANCE MODEL: DOES MONEY MATTER?

Benjamin Eden[1]

The University of Haifa and Vanderbilt University

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It is shown that monetary shocks may have large welfare cost even when the observed correlation between money and output is low. This is shown in a sticky price model in which sellers choose both prices and quantities optimally.

Mailing address: Department of Economics, The University of Haifa, Haifa 31905, Israel.

E-mail: b.eden@econ.haifa.ac.il

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1. INTRODUCTION

The importance of controlling the money supply has been debated for a long time. Friedman and Schwartz (1963) argued that money plays an important role in causing the business cycle and since money works with a long and variable lag a k% rule is optimal. This has been questioned first by Tobin (1970) who argued that money might respond to the business cycle rather than causing it and later by the real business cycle literature. Lately there has been a renewed interest in monetary policy partly due to empirical work that finds a significant effect of monetary policy on real variables. See for example, Christiano, Eichenbaum and Evans (1999) and Clarida, Gali and Gertler (1999) for useful surveys.

The literature thus implicitly assumes a connection between the importance of money in explaining the business cycle and the welfare cost of money supply shocks. Here I examine this connection using sticky-prices, cash-in-advance models.

I start with a model in which sellers choose prices optimally whenever these choices are made and then satisfy demand at the pre-announced prices. I then relax the demand-satisfying assumption.

This is different from the standard new Keynesian economics literature in two respects. I use the cash-in-advance constraint instead of the money in the utility function approach and allow sellers/producers to choose quantities optimally. These two changes combined make a difference. In a cash-in-advance model, money earned today is spent in the next period. Therefore when a seller is committed to a price of say P_t dollars per unit, an increase in the money supply means that next period price level, P_{t+1}, will be high and therefore the relevant real price P_t/P_{t+1} is low. This works in the direction of a negative relationship between money and labor supply. In money in the utility function model this effect is missing because money earned today is typically spent today at prices that do not respond to the current money shock.

Since here sellers choose both prices and quantities optimally, the paper may be viewed as an attempt at integrating the disequilibrium literature pioneered by Patinkin (1965), Clower (1965) and Barro and Grossman (1971) with the new Keynesian economics literature that began with Svensson (1986) and Blanchard and Kiyotaki (1987).

It is shown that the amount of monopoly power required for justifying the demand-satisfying assumption critically depend on the labor supply elasticity and that when this elasticity is not high satisfying demand may not be optimal even for low and moderate inflation rates. Once we relax the demand-satisfying assumption we may get little or no effect of money on output but nevertheless money supply shocks may impose a large welfare cost.

2. A DEMAND-SATISFYING MODEL

This model is based on the work of Blanchard and Kiyotaki (1987) and Woodford (2001). They assume that money is in the utility function. Here I use a cash-in-advance model.

There are a large number of N infinitely lived households, where a household is a worker/shopper pair. The shopper takes the available cash and spends all of it. The worker produces and sells his output for cash. At the end of the period both members of the household reunite and consume whatever the shopper has bought.

Each household produces a different good and consumes all goods. Normalizing N = 1, the single period utility function is of the Dixit-Stiglitz (1977) type:

$$[\Sigma_{j=0}^{1} (y_j)^{\gamma}]^{1/\gamma} - \upsilon(L), \tag{1}$$

where $0 < \gamma < 1$, $\upsilon(L) = (^1/_\delta)L^\delta$ and $\delta > 1$.

Household i starts the period with m_i normalized dollars and gets a transfer of x normalized dollars, where a normalized dollar is the beginning of the period money supply.[2] The amount of transfer x is an i.i.d random variable with a density function $\phi(x)$. It is assumed that $\beta - 1 \leq x \leq g$ so the money growth rate cannot be below the Friedman rule.[3]

The buyer takes the normalized prices $(p_0, ..., p_1)$ as given and spends the entire available amount of $m_i + x$ normalized dollars on all goods. Buyer i solves:

$$\max_{y_j} [\Sigma_{j=0}^{1} (y_j)^{\gamma}]^{1/\gamma} \text{ s.t. } \Sigma_{j=0}^{1} p_j y_j = m_i + x. \tag{2}$$

The first order conditions for the buyer's problem (2) are:

$$y_j = y_1(p_j/p_1)^{\theta}, \tag{3}$$

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[2] Thus, we divide all nominal magnitudes by the pre-transfer money supply.

[3] When $x \geq \beta - 1$, the assumption that the buyer spends everything can be derived as a result. Otherwise the buyer will not spend and equilibrium does not exist.

where $\theta = 1/(\gamma - 1) < 0$. We now substitute (3) in the budget constraint ($\Sigma_j\ p_j y_j = m_i + x$) to get:

(4) $$y_1 = (m_i + x)(p_1)^{\theta}/\Sigma_j\ (p_j)^{1+\theta}.$$

Using $z = 1/\Sigma_j\ (p_j)^{1+\theta}$ and symmetry, household's i demand for product j is $y_j = z(p_j)^{\theta}(m_i + x)$ and his utility from consumption is:

(5) $$F(m_i + x,\ p_{-i},\ p_i) = \{\Sigma_j[(m_i + x)z(p_j)^{\theta}]^{\gamma}\}^{1/\gamma},$$

where $p_{-i} = (p_0,...,p_{i-1},p_{i+1},...,p_1)$ is the prices posted by other sellers. Since there are many agents the effect of any single price on F is small and will be neglected. We therefore write $F(m + x,\ p_{-i})$ instead of $F(m + x,\ p_{-i},\ p_i)$.

Our normalization implies $\Sigma_j\ m_j = 1$. Nominal spending per household is therefore $1 + x$ and the aggregate demand for product i is:

(6) $$y_i = z(p_i)^{\theta}\Sigma_j\ (m_j + x) = z(p_i)^{\theta}(1 + x).$$

Using the aggregate demand (6), we compute next period's balances (in terms of next period's normalized dollars):

(7) $$m' = p_i y_i/(1 + x) = z(p_i)^{1+\theta}.$$

Note that next period's money does not depend on x. It depends only on the relative price: $(p_i)^{1+\theta}/\Sigma_j\ (p_j)^{1+\theta}$.

The individual seller takes p_{-i} and z as given and assumes that the normalized prices charged by others will not change over time. He chooses his price p_i by solving the following Bellman's equation:

$$(8) \qquad V(m;\, p_{-i}) = E_X\{F(m + x,\, p_{-i})\}$$

$$+ \max_{p_i} E_X\{-\,\upsilon[(1 + x)z(p_i)^{\theta}]\} + \beta V(z(p_i)^{1+\theta};\, p_{-i}),$$

where E_X denotes expectations with respect to x. The first order condition for this problem is:

$$(9) \qquad E_X\{\upsilon'[(1 + x)z(p_i)^{\theta}](1 + x)(p_i)^{-1}\} = \beta V'(z(p_i)^{\theta})\gamma.$$

To provide an intuitive explanation of (9) we consider the effect of a small increase in the price p_i for a given realization of x. By increasing the price p_i the seller reduces demand and production. The change in output (obtained by taking a derivative of [6]) is: $\theta z(p_i)^{\theta-1}(1 + x)$. The cost reduction benefit is therefore the absolute value of: $(\upsilon')\theta z(p_i)^{\theta-1}(1 + x)$. The increase in the price p_i also reduces next period's balances. The change in next period balances (obtained by taking a derivative of [7]) is: $(1 + \theta)z(p_i)^{\theta}$. The loss of utility associated with that is the absolute value of: $(\beta V')(1 + \theta)z(p_i)^{\theta}$. Since at the optimum, the cost reduction benefits must equal the loss of utility due to the loss of revenues we get (9).

In <u>equilibrium</u> $p_j = p$ for all j, $z = 1/\Sigma_j\,(p_j)^{1+\theta} = p^{-(1+\theta)}$ and $zp^{\theta} = p^{-1}$. Substituting this in (5) we get the utility from consumption:

(10) $[\Sigma_j \, (z(m + x)(p_j)^{\theta})^{\gamma}]^{1/\gamma} = [(z(m + x)p^{\theta})^{\gamma}]^{1/\gamma} = (m + x)/p.$

It follows that V' = 1/p. Substituting $zp^{\theta} = p^{-1}$ and V' = 1/p in (9), we get the equilibrium condition:

$$E_X\{\upsilon'[(1 + x)/p](1 + x)\} = \beta\gamma. \quad (11)$$

Since $\upsilon'' > 0$, the left hand side of (11), $a(p) = E_X\{\upsilon'[(1 + x)/p](1 + x)\}$, is decreasing in p and we get a unique solution as in Figure 1.[4]

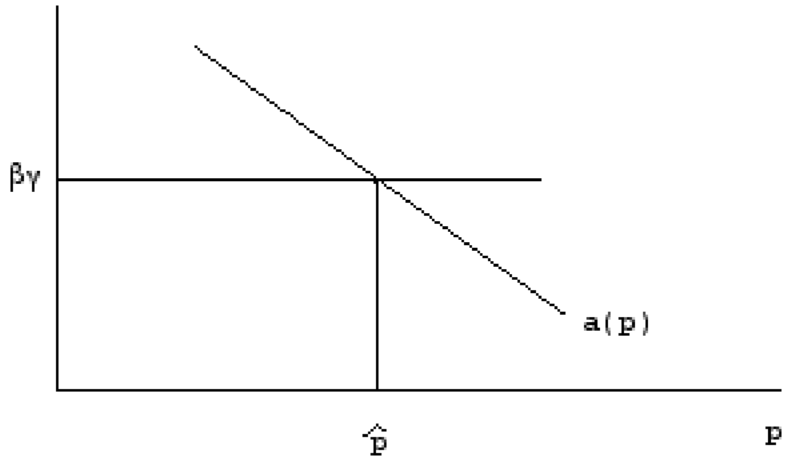

Figure 1

To interpret (11) we use $MC(x) = \upsilon'[(1 + x)/p]$ for the marginal cost and $MB(x) = \beta V'p/(1 + x) = \beta/(1 + x)$ for the marginal benefit and write (11) as:

$$E_X\{MC(x)/MB(x)\} = \gamma. \quad (12)$$

[4] Existence requires $\upsilon'(0) = 0$ and $\upsilon'(\infty) = \infty$. This is the case for our assumed functional form: $\upsilon(L) = (1/\delta)L^{\delta}$.

Thus γ is the average ratio of marginal cost to marginal benefits (one over the markup) and low γ means high monopoly power.[5]

We can use Figure 1 to do comparative static. An increase in γ (and a reduction of monopoly power) leads to a reduction in the equilibrium level of p and to an increase in average production. An increase in β also leads to an increase in output because of the delay in the payment for the labor effort assumed in the cash-in-advance model.

Substituting $\upsilon(L) = (1/\delta)L^{\delta}$ and $\upsilon'(L) = L^{\delta-1}$ in (11) leads to: $p = \{E_X(1 + x)^{\delta}/\beta\gamma\}^{1/\delta-1}$. The equilibrium labor supply is therefore:

(13) $$L(x) = (1 + x)/p = (\beta\gamma)^{1/\delta-1}(1 + x)/\{E_X(1 + x)^{\delta}\}^{1/\delta-1}.$$

The elasticity of labor supply with respect to 1 + x is unity.

Optimal monetary policy:

A social planner in this environment will solve:

(14) $$\max_{y_j} [\Sigma_j (y_j)^{\gamma}]^{1/\gamma} - \Sigma_j [\upsilon(y_j) = (1/\delta)(y_j)^{\delta}]$$

Imposing symmetry we can write (14) as max $y - (1/\delta)y^{\delta}$. The solution to this problem is: $y_j = 1$ for all j.

The optimal choice of a non-random policy x solves[6]:

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[5] The demand elasticity can be obtained by looking at the log of (4). It is: $|\theta = 1/(\gamma - 1)|$. Thus low γ implies low demand elasticity.

[6] We cannot benefit from allowing a random choice of x because the cost function is convex.

(15) $\max_x \; L(x) - (1/\delta)[L(x)]^\delta$ s.t. (13) and $x \geq \beta - 1$.

The optimal policy is the Friedman rule: $x = \beta - 1$. To see this claim note that at the Friedman rule (13) implies $L = \gamma^{1/\delta-1} < 1$. Therefore the marginal cost is less than the social benefit and increasing x is not desirable.

Are the implied quantities optimal?

After observing the realization of x and after the commitment to price was already made, the seller will want to satisfy demand if the marginal cost is less than the marginal benefit:

(16) $$\upsilon'[(1 + x)/p] \leq \beta/(1 + x).$$

Substituting $p = \{E_X(1 + x)^\delta/\beta\gamma\}^{1/\delta-1}$ and $\upsilon'(L) = L^{\delta-1}$ in (16) leads to the following claim.

Claim 1: Satisfying demand is optimal if:

(17) $$E_X(1 + x)^\delta/(1 + x)^\delta \geq \gamma \quad \text{for all } x \leq g.$$

Claim 1 says that satisfying demand is optimal when the money supply increase is largely anticipated. To advance this interpretation, we look at the expression:
$\gamma(1 + x)^\delta - E_X(1 + x)^\delta$. The first term in this expression depends on the realization x while the second is an expected average term. We may therefore think of this term as a somewhat unconventional

definition of money surprise and read Claim 1 as saying that satisfying demand is not optimal whenever a strictly positive surprise occurs.

Note that when γ is close to unity satisfying demand when $x = g$ is not optimal. In general, we can use the condition in Claim 1 to see the effect of changes in various parameters on the desire to satisfy demand. This is done in Claim 2.

Claim 2[7]: The following changes may cause a departure from demand-satisfying behavior: (a) an increase in γ; (b) an increase in δ; (c) replacing x by a random variable x' with a lower δth moment: $E_{x'}(1 + x')^{\delta} < E_X(1 + x)^{\delta}$ and $\beta-1 \le x' \le g$; (d) replacing x by a random variable x' with a higher upper bound: $E_{x'}(1 + x')^{\delta} = E_X(1 + x)^{\delta}$ and $\beta-1 \le x' \le g'$ where $g' > g$.

Proof: To show (a) note that an increase in γ leads to an increase in the maximum money surprise: $\gamma(1 + g)^{\delta} - E_X(1 + x)^{\delta}$ and a violation of (17) may occur.

To show (b) note that $E_X(1 + x)^{\delta}/(1 + g)^{\delta}$ is a decreasing function of δ. To see this, consider the effect of an increase in δ by 1 on the expression $E_X(1 + x)^{\delta}/(1 + g)^{\delta}$. The numerator goes up by a factor less than $(1 + g)$ while the denominator goes up by the factor $(1 + g)$.

To show (c) note that

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[7] Since $(1 + x)^{\delta}$ is an increasing function it is well known that (c) implies $\text{prob}(x \ge z) \ge \text{prob}(y \ge z)$ for all z (x dominates y in the first order sense).

$E_{x'}(1 + x')^{\delta}/(1 + g)^{\delta} < E_x(1 + x)^{\delta}/(1 + g)^{\delta}$ and therefore violation of (17) may occur if we replace x by x'. Similarly, we show (d) by noting that $E_{x'}(1 + x')^{\delta}/(1 + g')^{\delta} < E_x(1 + x)^{\delta}/(1 + g)^{\delta}$. □

The intuition for Claim 2 is as follows. (a) When monopoly power is low, marginal cost is larger than marginal benefits for large x; (b) When δ is high labor supply elasticity $(1/(\delta - 1))$ is low and the seller does not want to accommodate demand when the realization of x is large; (c) When a small probability event of high money supply realization occurs the surprise is large and the sellers do not want to accommodate. The same intuition applies also to (d).

A discrete example:

To get a feeling for the magnitudes we now turn to a numerical example in which x may take two possible realizations: $x = \beta - 1$ and $x = g$. I use $\beta = 0.96$ and calculate the critical value of γ for which $E_x(1 + x)^{\delta}/(1 + g)^{\delta} = \gamma$ and (17) holds with equality. (Thus if the true γ is greater than the critical value then satisfying demand when $x = g$ is not optimal). Table 1 makes these calculations for three alternative values of δ and alternative probability distributions for x.

Table 1*: Critical values of γ (demand satisfying is optimal if the true γ is less than the critical value)

g =	0.05	0.1	0.15	0.2
x = β–1 with probability = 0.5 and x = g otherwise.				
Expected inflation	0.005	0.030	0.055	0.08
standard deviation	0.045	0.07	0.095	0.12
Critical values of γ:				
For δ = 2:	0.92	0.88	0.85	0.82
For δ = 3:	0.88	0.83	0.79	0.76
For δ = 11:	0.69	0.61	0.57	0.54
x = β–1 with probability = 0.9 and x = g otherwise.				
Expected inflation =	-0.031	-0.026	-0.021	-0.016
standard deviation	0.027	0.042	0.057	0.072
Critical values of γ:				
For δ = 2:	0.85	0.79	0.73	0.68
For δ = 3:	0.79	0.70	0.62	0.56
For δ = 11:	0.44	0.30	0.22	0.18

* This Table calculates the levels of γ for which (17) holds with equality. The calculations assume $\beta = 0.96$. The rate of change in the money supply is $x = \beta-1$ or $x = g$ where g = 0.05, 0.1, 0.15 and 0.2. The probability that $x = \beta-1$ is 0.5 and 0.9. The calculations are made for $\delta = 2,3$ and 11 which correspond to labor supply elasticities of $1/(\delta-1) = 1$, 0.5 and 0.1.

The example shows that when labor elasticity and the probability that $x = \beta-1$ are low, satisfying demand is not optimal if the true γ is say 0.8.

I now use the numerical example to present the equilibrium magnitudes for the case in which $x = \beta-1$ occurs with probability 0.5. To focus on the case in which satisfying demand is not optimal I use $\gamma = 0.95$. As can be seen from Table 2, the expected markup as typically measured (1/marginal cost), EM is in the range

1.11 - 3.68.[8] The expected welfare cost of deviating from the Friedman rule ($x = \beta - 1$ with probability 1) is calculated as a percentage of the labor supply (output). It is in the range 0.4% - 5.9%.

Table 2*: The demand-satisfying model

g =	0.05	0.1	0.15	0.2
Labor supply elasticity = 1 (δ = 2)				
p =	1.11	1.17	1.23	1.29
L(β–1) =	0.76	0.82	0.78	0.74
L(g) =	0.95	0.94	0.93	0.93
EM =	1.11	1.14	1.23	1.21
Welfare cost =	0.005	0.009	0.015	0.022
Labor supply elasticity = 0.5 (δ = 3)				
p =	1.06	1.10	1.15	1.20
L(β–1) =	0.91	0.87	0.84	0.80
L(g) =	0.99	1.00	1.00	1.00
EM =	1.12	1.16	1.21	1.27
Welfare cost =	0.004	0.008	0.015	0.022
Labor supply elasticity = 0.1 (δ = 11)				
p =	1.03	1.07	1.11	1.16
L(β–1) =	0.94	0.90	0.86	0.83
L(g) =	1.02	1.03	1.03	1.03
EM =	1.36	1.81	2.53	3.68
Welfare cost =	0.010	0.023	0.040	0.059

* $x = \beta - 1$ or $x = g$ with equal probabilities; $\gamma = 0.95$ and $\beta = 0.96$. The variables are normalized price (p), labor in the low money supply realization (L[β-1]), labor in the high money supply realization (L[g]), the expected markup, (EM), the expected welfare

[8] The typical measure of markup assumes that the seller can use his revenues to buy goods in the same period. This is not the case in our cash-in-advance model. We use this measure here for the sake of possible comparison with estimated markups.

cost of departing from the optimal policy as a percentage of labor supply (Welfare cost). Here are the details of the calculations.

$EM = 0.5[L(\beta-1)]^{1-\delta} + 0.5[L(g)]^{1-\delta}$ is the expected markup.

The welfare under the optimal policy ($x = \beta-1$ with probability 1) is:

$W_{max} = \gamma^{1/(\delta-1)} - (1/\delta)\gamma^{\delta/(\delta-1)}$. The expected welfare cost as a fraction of labor supply is:

$WC = 0.5\{W_{max} - [L(\beta-1) - (1/\delta)[L(\beta-1)]^{\delta}]\}/L(\beta-1)$

$+ 0.5\{W_{max} - [L(g) - (1/\delta)[L(g)]^{\delta}]\}/L(g)$.

3. STICKY PRICES WITH OPTIMAL CHOICE OF QUANTITIES: THE PRODUCTION TO ORDER CASE

I now examine the robustness of the above model to the relaxation of the demand-satisfying assumption. I start by focusing on the choice of quantities and assume that the normalized price, p, is exogenously given and constant over time (this means that the regular dollar price $P_t = pM_t$ responds with a one period lag to changes in the money supply). Since we do not allow for a choice of price we do not need the differentiated commodities structure. I therefore assume at this stage a single consumption good and a risk neutral utility function: $c - \upsilon(L)$, where c is the quantity consumed.

Figure 2 describes the sequence of events. The money supply transfer is realized after the price ($P_t = pM_t$) is exogenously set. Then sellers receive orders and choose production to satisfy some or all of the orders.

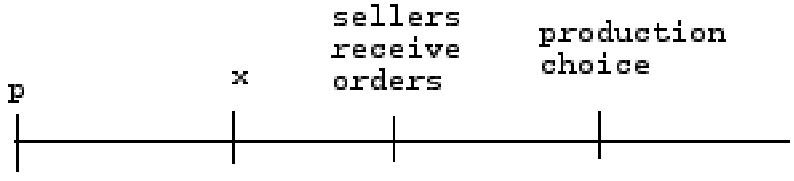

Figure 2

As before the typical buyer receives a transfer payment of x normalized dollars, where $\beta - 1 \leq x \leq g$ is an i.i.d random variable with a density function $\phi(x)$.

In equilibrium, the typical seller receives an order of 1 + x normalized dollars. The revenue of the representative seller is therefore min(pL, 1 + x) if he chooses to produce L units.

Buyers arrive sequentially in an order that is determined randomly by an i.i.d lottery. Buyers who arrive late may not be able to buy. The probability that the buyer will make a buy depends on the realization x and is denoted by $\Pi(x)$. The household takes p and the probability $\Pi(x)$ as given and solves the following Bellman's equation:

$$(17)\quad V(m;\, p) = \int_{\beta-1}^{g} \Pi(x)[(m + x)/p]\phi(x)dx + \int_{\beta-1}^{g} \{\max_L \; - \upsilon(L)$$

$$+ \Pi(x)\beta V[\min(pL,\, 1 + x)/(1 + x);\, p]$$

$$+ [1 - \Pi(x)]\beta V[(m + x + \min(pL,\, 1 + x))/(1 + x);\, p]\}\phi(x)dx.$$

The first row is the expected consumption for a household that starts with m normalized dollars. Then we have the expected value of the labor choices that are made after observing the realization of x.

The constant marginal utility of money, V', is:

$$(18)\; V' = \int_{\beta-1}^{g} \{\Pi(x)/p + [1 - \Pi(x)]\beta V'/(1 + x)\}\phi(x)dx = \pi/p(1 - \sigma\beta),$$

where $\pi = \int_{\beta-1}^{g} \Pi(x)\phi(x)dx$ and $\sigma = \int_{\beta-1}^{g} \{[1 - \Pi(x)]/(1 + x)\}\phi(x)dx$. To derive (18) note that an additional unit of money will yield 1/p utils if it buys in the current period. If it does not buy it will be carried to the next period yielding $\beta V'/(1 + x)$ utils.

Since the utility function is linear in consumption and V' is a constant we can write the labor supply choice problem (for a given x) as: $\max_L - \upsilon(L) + \beta V[\min(pL, 1 + x)/(1 + x)]$. The first order conditions for this problem are:

(19) $$pL \leq 1 + x \ ;$$

(20) $\upsilon'(L) \leq p\beta V'/(1 + x) = A/(1 + x)$ with equality when $pL < 1 + x$,

where $A = \beta\pi/(1 - \sigma\beta)$ and (18) is used to substitute for V'.

Condition (19) says that it is not optimal to produce more than the quantity demanded. Condition (20) says that the marginal cost must be less than the marginal benefit from selling the good and must equal to it when there is excess demand.

Equilibrium for a given normalized price p is a pair of functions $[L(x), \Pi(x)]$ and a scalar V' such that

(a) Given $\Pi(x)$, the first order conditions (18) - (20) are satisfied;

(b) $\Pi(x) = \min\{1, pL(x)/(1 + x)\}$.

The requirement (b) says that in case of excess demand, the probability of making a buy is equal to the ratio of nominal supply to nominal demand.

<u>To solve for equilibrium</u> we guess a cut-off point ζ such that $\Pi(x) = 1$ if $x \leq \zeta$ and $\Pi(x) = pL(x)/(1 + x)$ otherwise. To solve for ζ

we start by treating A as a constant and define S(x; A) by the solution to:

$\upsilon'(L) = A/(1 + x)$. Using $\upsilon'(L) = L^{\delta-1}$ leads to

$S(x; A) = [A/(1 + x)]^{1/(\delta-1)}$.

The notional supply S(x; A) is a decreasing function of x as in Figure 3. The intuition is that when x increases, the relevant real price $P_t/P_{t+1} = pM_t/pM_t(1 + x) = 1/(1 + x)$, decreases. The demand is the upward sloping line (1 + x)/p in Figure 3. Supply equals demand when $x = \zeta$.

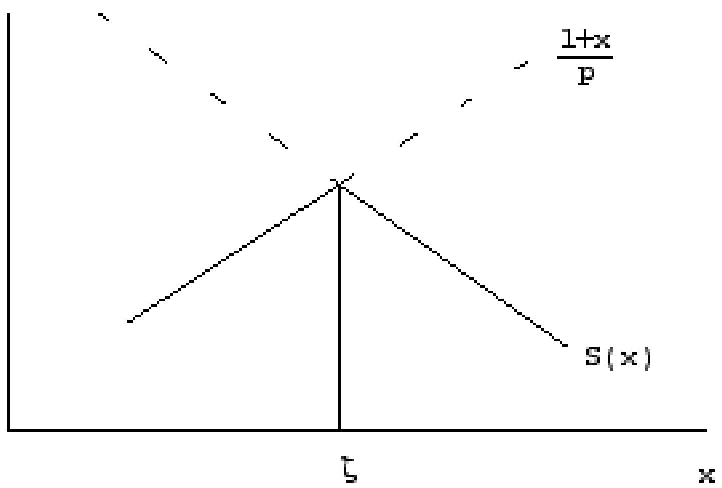

Figure 3

The actual amount traded is the minimum between supply and demand:

(21) $$L(x; A) = \min\{S(x; A), (1 + x)/p\}.$$

Note that $L(x; A) = (1 + x)/p$ when $x \le \zeta$ and $L(x; A) = S(x; A)$ when $x > \zeta$. It therefore has a "tent like shape" as the solid line in Figure 3.

The cutoff point ζ is given by the solution to:

$1 + \zeta = pL(\zeta)$. Using $S(x, A) = [A/(1 + x)]^{1/(\delta-1)}$ and (21) leads to: $\zeta(p) = p^{(\delta-1)/\delta}A^{1/\delta} - 1$. The probability of making a buy is given by:

$$(22) \qquad \pi(A, p) = \int_{\beta-1}^{g} \Pi(x)\phi(x)dx =$$

$$\text{Prob}[x \leq \zeta(p)] + \int_{\zeta(p)}^{g} [pA^{1/(\delta-1)}(1 + x)^{\delta/(1-\delta)}]\phi(x)dx.$$

where for $x \geq \zeta(p)$ we use (20) to get: $\Pi(x) = pL(x)/(1 + x) = pA^{1/(\delta-1)}(1 + x)^{\delta/(1-\delta)}$. This is also used to compute:

$$(23) \qquad \sigma(A, p) = \int_{\beta-1}^{g} \{[1 - \Pi(x)]/(1 + x)\}\phi(x)dx$$

$$= \int_{\zeta(p)}^{g} [1 - pA^{1/(\delta-1)}(1 + x)^{\delta/(1-\delta)}](1 + x)^{-1}\phi(x)dx.$$

We look for a solution (fixed point) to the following equation:

$$(24) \qquad A = \beta\pi(A, p)/[1 - \sigma(A, p)\beta] \geq \beta^{\delta}p^{1-\delta}.$$

The inequality is required to insure that $\zeta(p) = p^{(\delta-1)/\delta}A^{1/\delta} - 1 \geq \beta - 1$.

I now turn to a discrete example and add the choice of price.

<u>Endogenous price in a discrete example:</u>

We now use the discrete example in which x can take two possible realizations $x = \beta - 1$ and $x = g$ with equal probabilities of occurrence. We use the differentiated commodity structure of section

2 and formulate the price choice problem in the above discrete example under the assumption that satisfying demand is optimal when $x = \beta - 1$ but is not optimal when $x = g$. For a more general treatment see Appendix A.

Using equation (4) we derive the demand for product i when $x = \beta - 1$. This is: $\beta z(p_i)^\theta$ where z is an appropriate average of all other prices and $\theta = 1/(\gamma - 1) < 0$. The revenues when $x = \beta - 1$ and demand is satisfied are: $p_i[\beta z(p_i)^\theta]/\beta = z(p_i)^{1+\theta}$. The utility derived when $x = \beta - 1$ is thus: $- \upsilon[\beta z(p_i)^\theta] + \beta V[z(p_i)^{1+\theta}]$.

When $x = g$ there is excess demand and the individual seller can find a buyer who could not buy any other product. This buyer will spend his money only if:

$$1/p_i \geq b, \tag{25}$$

where $b = \beta V'/(1 + g)$ is the value of a dollar carried to the next period. When (25) is satisfied, household's i revenues are given by $p_iL/(1 + g)$ next period's normalized dollars if buyer i makes a buy and by $(m + x + p_iL)/(1 + g)$ otherwise.

The household takes z and b as given and solves the following Bellman's equation:

$$V(m) = (1/2)F(m + \beta - 1) + (1/2)\Pi(g)F(m + g) \tag{26}$$

$$+ \max_{p_i} \{(1/2)\{- \upsilon[\beta z(p_i)^\theta] + \beta V[z(p_i)^{1+\theta}]\}$$

$$+ (1/2)I(1/p_i \geq b)\{\max_L - \upsilon(L) + \Pi(g)\beta V[p_iL/(1 + g)] +$$

$$+ [1 - \Pi(g)]\beta V[(m + x + p_iL)/(1 + g)]\}\},$$

where $I(1/p_i \geq b) = 1$ if $1/p_i \geq b$ and zero otherwise and F( ) is the expected utility made by the buyer, defined by (5).

The first expression, $(1/2)F(m + \beta - 1) + (1/2)\Pi(g)F(m + g)$, is the expected current utility from consumption. When $x = \beta - 1$ demand is satisfied and the buyer spends the available cash with certainty. When $x = g$ the buyer spend his cash with probability $\Pi(g)$.

The first maximization problem is with respect to the price p_i. When $x = g$ demand is not satisfied and the choice of L is described by the second maximization problem.

The first order condition for the labor choice problem in (26) is: $\upsilon'(L) = L^{\delta-1} = \beta V' p_i/(1 + g)$ or $L = [\beta V' p_i/(1 + g)]^{1/(\delta-1)}$. Using this and risk neutrality we may write the price choice problem in (26) as:

$$(27) \qquad \max_{p_i} \; -\upsilon[\beta z(p_i)^{\theta}] + \beta V[z(p_i)^{1+\theta}]$$

$$-\upsilon\{[\beta V' p_i/(1 + g)]^{1/(\delta-1)}\} + \beta V\{p_i^{\delta/(\delta-1)}(1 + g)^{\delta/(1-\delta)}(\beta V')^{1/(\delta-1)}\}$$

It is useful to write the first order condition to the problem (27) as the sum of two terms. When the household increases its price it will typically experience a loss of utility when $x = \beta - 1$ and a gain in utility when $x = g$. The intuition is that when x is low the price is ex-post "too high" and the opposite is true when x is high. Following this intuition we take the derivative of the utility conditional on $x = \beta - 1$, $-\upsilon[\beta z(p_i)^{\theta}] + \beta V[z(p_i)^{1+\theta}]$, with respect to p_i:

$$(28) \qquad C = -\theta[\beta z(p_i)^{\theta}]^{\delta-1}\beta z(p_i)^{\theta-1} + (1 + \theta)(\beta V')z(p_i)^{\theta}.$$

We also take the derivative of the utility conditional on $x = g$, $-\upsilon\{[\beta V' p_i/(1 + g)]^{1/(\delta-1)}\} + \beta V\{p_i^{\delta/(\delta-1)}(1 + g)^{\delta/(1-\delta)}(\beta V')^{1/(\delta-1)}\}$, with respect to p_i:

(29) $$B = - (1/(\delta-1))[\beta V' p_i/(1 + g)]^{2+\delta/(1-\delta)}$$

$$+ (\delta/(\delta-1))p_i^{1/(\delta-1)}[\beta V'/(1 + g)]^{\delta/(\delta-1)}.$$

Typically $C < 0$ and $B > 0$ and at the optimum the first order condition $C + B = 0$ is satisfied. We now turn to define equilibrium for this example.

Equilibrium for the discrete example is a vector $[\Pi(g), b, z, p, L(g), V']$ such that:

(a) Given $\Pi(g)$, b and z, the price p and the labor supply in the high demand state $L(g)$ solve the household's problem (26) and V' is the resulting constant marginal utility of money;

(b) $z = p^{-1-\theta}$, $b = \beta V'/(1 + g)$ and $\Pi(g) = pL(g)/(1 + g) < 1$.

Table 3 calculates the optimal price using the first order condition $C + B = 0$.[9] It turns out that the constraint: $1/p_i \geq b$ is not binding. The optimal price is lower than the demand-satisfying price. At the demand-satisfying price, $C + B < 0$. Since C is the same in both models it follows that the benefit from increasing price that occurs when $x = g$ is larger in the demand-satisfying model. This

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[9] We substitute $z = p^{-1-\theta}$, $z(p_i)^{\theta} = p^{-1}$ and $z(p_i)^{\theta-1} = p^{-2}$ in (28) to get: $C = - \theta\beta^{\delta}p^{-1-\delta} + (1 + \theta)(\beta V')p^{-1}$.

benefit is the reduction in "unwanted production" that occurs when the seller must satisfy demand.

Table 3*: The production to order model

g =	0.05	0.1	0.15	0.2
Labor supply elasticity = 1 (δ = 2)				
p =	1.05	1.05	1.05	1.06
π =	0.96	0.91	0.87	0.84
A =	0.956	0.948	0.937	0.925
L(β-1) =	0.91	0.91	0.91	0.91
L(g) =	0.91	0.86	0.82	0.77
EM =	1.10	1.13	1.16	1.20
Elasticity =	-0.03	-0.39	-0.54	-0.61
Welfare cost=	0.0029	0.0061	0.01	0.018
Labor supply elasticity = 0.5 (δ = 3)				
p =	1.01	1.01	1.01	1.02
π =	0.96	0.93	0.90	0.87
A =	0.957	0.950	0.942	0.933
L(β-1) =	0.95	0.95	0.95	0.94
L(g) =	0.95	0.93	0.90	0.88
EM =	1.10	1.13	1.17	1.20
Elasticity =	0.05	-0.15	-0.23	-0.27
Welfare cost=	0.002	0.003	0.006	0.009
Labor supply elasticity = 0.1 (δ = 11)				
p =	0.97	0.97	0.97	0.97
π =	0.96	0.93	0.91	0.89
A =	0.956	0.951	0.945	0.938
L(β-1) =	0.99	0.99	0.99	0.99
L(g) =	0.99	0.99	0.98	0.98
EM =	1.10	1.13	1.17	1.20
Elasticity =	0.01	-0.03	-0.04	-0.05
Welfare cost=	0.0003	0.0007	0.001	0.002

* This Table assumes that $x = \beta-1$ with probability 0.5 and $x = g$ otherwise; $\gamma = 0.95$ and $\beta = 0.96$. It calculates the normalized price (p) the unconditional probability of making a buy (π) the equilibrium level of $A = p\beta V'$, labor supply for the two possible realizations (L[β-1], L[g]), the elasticity of labor supply with respect to x and the welfare cost of departing from the Friedman rule. The formulas used are:

$p = \{E_X(1 + x)^{\delta}/\beta\gamma\}^{1/\delta-1} = \{[0.5\beta^{\delta} + 0.5(1+g)^{\delta}]/\beta\gamma\}^{1/\delta-1};$

$\pi(A, p) = 0.5 + 0.5p(A)^{1/\delta-1}(1 + g)^{\delta/1-\delta};$

$\sigma(A, p) = 0.5[1 - p(A)^{1/\delta-1}(1 + g)^{\delta/1-\delta}]/(1 + g)$

$A = \beta\pi(A, p)/[1 - \sigma(A, p)\beta]$

$L(\beta-1) = \beta/p;\ L(g) = [A/(1 + g)]^{1/\delta-1};$

Elasticity = $[(L(g)/L(\beta-1)) - 1]/[((1 + g)/\beta) - 1]$.

Welfare cost is calculated by the formula in Table 2.

The elasticity of output with respect to x tends to be negative and is in the range -0.61 to 0.05. The welfare cost is in the range 0.03% - 1.8%.[10] This is lower than the welfare cost under the demand-satisfying model which was in the range 0.4% - 5.9%. The highest welfare cost in the production to order model occurs when labor supply elasticity is high while in the demand-satisfying model it occurs when the labor supply elasticity is low.

4. THE PRODUCTION TO MARKET CASE

I now consider the case in which producers do not receive orders and the amount produced is simply put on the market. In this case some of the output produced may not be sold and money surprises affect capacity utilization. It is shown that the change in the assumption about selling leads to a qualitatively different equilibrium labor supply function.

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[10] I also made the calculations in Table 3 using the demand satisfying prices in Table 2. The welfare cost when using the demand satisfying prices is in the range 0.5% - 5.7%. This is more than twice the welfare cost when using the optimal prices suggesting that the demand satisfying prices are poor approximations for the optimal prices.

As in the previous section I start by assuming that the normalized price, p, is exogenously given. After observing the realization of the money supply (x) the seller chooses how much to produce and put his output on the market for sale. Buyers arrive and buy part or all of the available supply. Figure 4 describes the sequence of events.

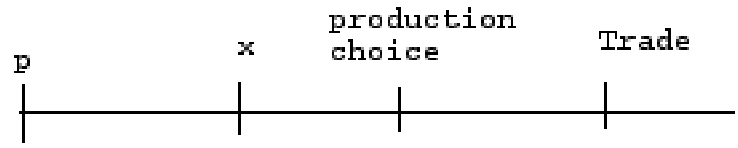

Figure 4

As in the previous section buyers who arrive late may not make a buy and $\Pi(x)$ denotes the probability of making a buy. It is assumed that all sellers sell the same fraction, $0 \leq \Omega(x) \leq 1$, of their output. The average revenue received by the seller for a unit produced is therefore: $W(x) = p\Omega(x)$. The amount of money that the household will have at the beginning of next period is: $LW(x)/(1 + x)$ if the buyer made a buy and $[LW(x) + m + x]/(1 + x)$ if the buyer did not make a buy.

Given the functions $\Pi(x)$, $W(x)$ the Bellman equation which describes the household's choice problem is:

$$(31) \qquad V(m; p) = \int_{\beta-1}^{g} [(m + x)\Pi(x)/p]\phi(x)dx$$

$$+ \int_{\beta-1}^{g} \{\max_L \; - \upsilon(L) + [\Pi(x)]\beta V[LW(x)/(1 + x); p]$$

$$+ [1 - \Pi(x)]\beta V[(m + x + LW(x))/(1 + x); p]\}\phi(x)dx.$$

Since the utility function is linear in consumption and V' is a constant, we can write the maximization problem in (31) as: $\max_L \; - \upsilon(L) + \beta V[LW(x)/(1 + x)]$. The first order condition for this problem is:

$$(32) \qquad \upsilon'(L) = [W(x)/(1 + x)]\beta V' = A\Omega(x)/(1 + x),$$

where the second equality uses $\beta V' = \beta\pi/p(1 - \sigma\beta) = A/p$ and $W(x) = p\Omega(x)$.

Equilibrium for a given normalized price p is a vector of functions $[L(x), \Pi(x), \Omega(x)]$ and a triplet $[\pi = \int_{\beta-1}^{g}\Pi(x)\phi(x)dx,\ \sigma = \int_{\beta-1}^{\infty}\{[1 - \Pi(x)]/(1 + x)\}\phi(x)dx,$ $A = \beta\pi/(1-\sigma\beta)]$ such that:

(a) $\upsilon'(L) = A\Omega(x)/(1 + x)$

(b) $\Pi(x) = \min\{1, pL(x)/(1 + x)\}$ = probability of making a buy;

(c) $\Omega(x) = \min\{1, (1 + x)/pL(x)\}$ = fraction of output sold.

<u>Solving for equilibrium:</u> As in the previous production to order case, the labor supply in the excess demand region $x \geq \zeta$ is given by the solution, L(x), to $\upsilon'(L) = L^{\delta-1} = A/(1 + x)$. The cutoff point ζ is given by the solution to: $1 + \zeta = pL(\zeta)$ which is: $\zeta(p) = p^{(\delta-1)/\delta}A^{1/\delta} - 1$. In the excess demand range $\Omega(x) = 1$. When $x \leq \zeta$ there is excess supply, $\Omega(x) = (1 + x)/pL(x) \leq 1$ and we can write (32) as:

$$(33) \qquad \upsilon'(L) = A/pL \text{ for } x \leq \zeta.$$

The solution to (33) does not depend on x and is given by:

$\hat{L} = (A/p)^{1/\delta}$. The intuition is as follows. An increase in x has two effects on the wage in terms of next period's normalized dollars: The increase in the fraction of output sold, $\Omega(x) = (1 + x)/p\hat{L}$, and the reduction in the value of a current normalized dollar. These two effects exactly offset each other and therefore the effective wage in terms of next period's normalized dollars, $W(x)/(1 + x) = 1/\hat{L}$ does not depend on x.

The equilibrium labor supply function L(x) is therefore flat for $x \leq \zeta$ and then declines as in Figure 5.

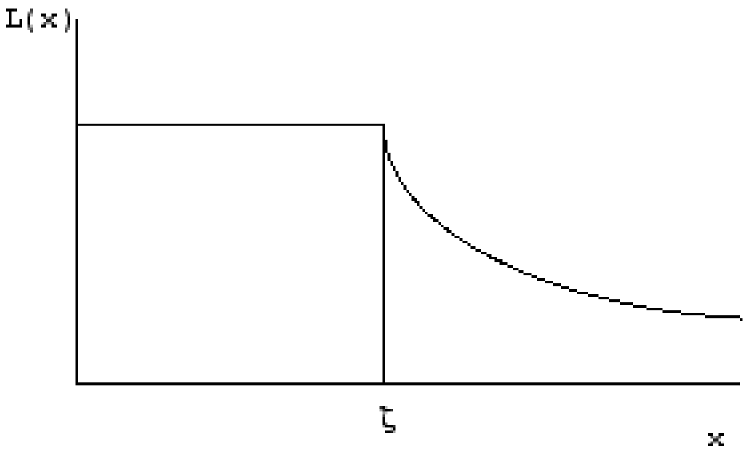

Figure 5

Endogenous price in a discrete example:

We use the above discrete example to endogenize the price. For a more general treatment see Appendices A and B.

We start by specifying the fraction of output sold by an individual seller as a function of his relative price. For this purpose we define average capacity utilization by:

$CU(x) = \min\{(1 + x)/pL(x), 1\}$, where p and L(x) are average (across sellers) price and labor supply.

In the low demand state there is excess supply and average capacity utilization is: $CU(\beta-1) = \beta/p\hat{L} < 1$. It is assumed that if seller i post the price p_i he will sell a fraction:

$$(34) \qquad \Omega(p/p_i, \beta-1) = \min\{[\exp(\alpha p/p_i)/\exp(\alpha)]CU(\beta-1), 1\},$$

of his output in the low demand state, where $\alpha > 0$ is a given parameter. The specification (34) assumes that when $p_i = p$, seller i's capacity utilization is equal to the average, CU. In this case the parameter α is the elasticity of Ω with respect to the relative price p/p_i. This elasticity should vary with CU. When CU is relatively high, a one percent increase in the relative price p/p_i should have a relatively small effect on the percentage of output sold. I therefore allow

$$(35) \qquad \alpha = \alpha(CU, p).$$

The household takes CU, p and $\alpha(CU, p)$ as given parameters.

The effective wage in the low demand state in terms of next period's normalized dollars is:

$$(36) \quad W(p/p_i, \beta-1)/\beta = p_i\Omega(p/p_i, \beta-1)/\beta = [\exp(\alpha p/p_i)/\exp(\alpha)]p_i/p\hat{L}.$$

In the high demand state there is excess demand and average capacity utilization is: $CU(g) = 1$. In this case it does not matter whether production is made to order or to market. In both cases a seller can find a buyer who could not find any other product and will buy from the seller if $1/p_i \geq b$, where $b = \beta V'/(1 + g)$ is the value of a dollar carried to the next period. It is therefore assumed that when constraint (25) is satisfied the fraction of output sold in the high demand state is: $\Omega(p/p_i, g) = 1$. The effective wage in the high demand state is therefore:

$W(p/p_i, g)/(1+g) = p_i\Omega(g, p/p_i)/(1+g) = p_i/(1+g)$.

Household i takes the average price posted by others, p, the value of a normalized dollar that is not spent in the high demand state, b, the effective wage function, $W(p/p_i, \beta-1)$ and the probability of making a buy in the high demand state, $\Pi(g)$ as given. It solves the following Bellman equation:

$$V(m) = (1/2)F(m + \beta - 1) + (1/2)\Pi(g)F(m + g) \tag{37}$$

$$+ \max_{p_i} \{(1/2)\{\max_L - \upsilon(L) + \beta V[W(p/p_i, \beta-1)L/\beta] \text{ s.t. } (36)\}$$

$$+ (1/2)I(1/p_i \geq b)\{\max_L - \upsilon(L) + \Pi(g)\beta V[p_i L/(1 + g)] +$$

$$+ [1 - \Pi(g)]\beta V[(m + x + p_i L)/(1 + g)]\}\},$$

where as before I(statement) = 1 if the statement is true and I(statement) = 0 otherwise. The function F( ) is the expected utility that the buyer makes defined by (5).

Equilibrium for the discrete example is a vector $[p, \Pi(g), \Omega(p/p_i, \beta-1), W(p/p_i, \beta-1), b, L(\beta-1), L(g), V']$ such that:

(a) Given $[p, \Pi(g), \Omega(p/p_i, \beta-1), W(p/p_i, \beta-1), b]$, the price p and the labor supplies $L(\beta-1)$ and $L(g)$ solve the household's problem (37) and V' is the resulting constant marginal utility of money;

(b) $\Omega(p/p_i, \beta-1)$ satisfies (34), $W(p/p_i, \beta-1)$ satisfies (36), $b = \beta V'/(1 + g)$ and $\Pi(g) = pL(g)/(1 + g) < 1$.

The choice of labor in the high demand state is the same as in the production to order case. We focus here on the choice of labor in the low demand state. The first order condition that $L(\beta-1)$ must satisfy is:

$$(38)\quad \upsilon'(L) = L^{\delta-1} = \beta V'[W(p/p_i,\ \beta-1)/\beta] = A[\exp(\alpha p/p_i)/\exp(\alpha)]p_i/p^2\hat{L}.$$

This leads to $L = \{A[\exp(\alpha p/p_i)/\exp(\alpha)]p_i/p^2\hat{L}\}^{1/(\delta-1)}$. The utility when demand is low can therefore be written as:

$$(39)\qquad U(p/p_i,\ \beta-1) = -\ \upsilon(L) + \beta V(LW(p/p_i,\ \beta-1)/\beta) =$$

$$= -\ (1/\delta)\{A[\exp(\alpha p/p_i)/\exp(\alpha)]p_i/p^2\hat{L}\}^{\delta/(\delta-1)}$$

$$+\ \beta V(\{A[\exp(\alpha p/p_i)/\exp(\alpha)]p_i/p^2\hat{L}\}^{1/(\delta-1)}[\exp(\alpha p/p_i)/\exp(\alpha)]p_i/p\hat{L}).$$

In equilibrium $p_i = p$ and, using (33), the derivative of (39) is:

$$(40)\quad C = (1/(\delta - 1))(1 - \alpha)p^{-1}(\hat{L})^{\delta-1}\{-\ \hat{L} + 1/\hat{L}\} + (1 - \alpha)(\hat{L})^{\delta/(\delta-1)},$$

where $C = \partial U/\partial p_i$. Note that when $\hat{L} \leq 1$, and $\alpha > 1$, $C < 0$.

The first order condition that governs the choice of price is given by:

$$(41)\qquad C + B = 0,$$

where B is the derivative of welfare in the high demand state with respect to price. B is the same as in the production to order case and is given by (29).

To solve the model we must assume a specific functional form $\alpha(CU)$. Rather than doing that I treat α as an endogenous variable and ask what is the value of α required to get the production to order prices in Table 3. Using (29) for B, (40) for C and (33) we get:

$$(42) \qquad \alpha = 1 + B/\{(1/(\delta - 1))p^{-1}(\hat{L})^{\delta-1}\{- \hat{L} + 1/\hat{L}\} + (\hat{L})^{\delta/(\delta-1)}\}.$$

Since $CU = \beta/p\hat{L}$ we can write (42) as a function of CU and p.

Table 4 uses the production to order prices in Table 3 and treat α as an endogenous variable, using (29) and (42) to compute it.

Note that the resulting α is a decreasing function of CU. Note also that prices do not react to changes in g (regime changes) but nevertheless capacity utilization is an increasing function of g. The reason is that an increase in g lowers the probability of making a buy (π) and therefore it lowers the marginal utility of money V'. As a result the benefits from working go down and less excess capacity is produced.

The expected welfare cost is much larger than in the production to order case (Table 3). It is now in the range of 0.5 - 3.1 percent of labor supply.

Table 4*: The production to market model

g =	0.05	0.1	0.15	0.2
Labor supply elasticity = 1 (δ = 2)				
p =	1.05	1.05	1.05	1.06
CU=	0.957	0.962	0.967	0.971
α =	1.61	1.42	1.27	1.24
EM =	1.07	1.11	1.14	1.18
Elasticity =	-0.49	-0.64	-0.69	-0.70
Welfare cost=	0.023	0.024	0.027	0.031
Labor supply elasticity = 0.5 (δ = 3)				
p =	1.01	1.01	1.01	1.02
CU =	0.968	0.969	0.971	0.972
α =	1.82	1.74	1.65	1.58
EM =	1.07	1.10	1.13	1.17
Elasticity =	-0.30	-0.35	-0.37	-0.37
Welfare cost=	0.017	0.017	0.019	0.021
Labor supply elasticity = 0.1 (δ = 11)				
p =	0.97	0.97	0.97	0.97
CU =	0.9909	0.9911	0.9913	0.9915
α =	1.93	1.88	1.83	1.79
EM =	1.06	1.09	1.12	1.16
Elasticity =	-0.09	-0.09	-0.09	-0.08
Welfare cost=	0.005	0.005	0.005	0.006

* This example assumes x = β with probability 0.5 and x = g otherwise. β = 0.96. It uses the computation of A and p from Table 3. Labor supply when x = g, L(g) is the same as in Table 3 but labor supply when x = β-1 is larger and is given by: $L(\beta-1) = (A/p)^{1/\delta}$. The elasticity is the percentage change in labor devided by the percentage change in x: $[(L(g)/L(\beta-1))-1]/[(1+g)/\beta)-1]$. Capacity utilization is the percentage of output sold when x = β: CU = β/pL(β-1). Welfare cost is calculated as a percentage of labor supply according to the formula in Table 2.

It was shown that if we relax the demand-satisfying assumption we may get a negative rather than a positive relationship between money and output. This should reduce our confidence in the sticky price model as a possible explanation for the observed positive money/output relationship.

Alternatively we may consider a model in which it is optimal to satisfy demand for some goods but not for other goods. Some goods are produced to order and some are produced to market. In such a combined model, we may get a positive relationship between money and employment but the welfare cost of deviating from the Friedman rule does not depend on the elasticity of labor with respect to x. We may get a low elasticity and a relatively high welfare cost if production to market is relatively important. For a formulation of the combined model see Appendix A.

The welfare cost of departing from the Friedman rule:

In his presidential address (January 2003) and in his 1987 book Lucas has argued that monetary policy cannot change trend output and that the possible gains from eliminating deviations from trend output is tiny. Lucas focus on the gain in expected utility from consumption. Here we have risk neutrality so this gain is literally zero. But there are substantial costs of departing from the Friedman rule.

Perfectly anticipated inflation is costly and affect labor supply, because households spend the money they earn with a lag and as a result inflation affect the real wage.

Random fluctuations in x (holding the mean of x constant) cause fluctuations in labor supply.[11] The fluctuations in x may therefore cause additional harm because smooth production is cheaper than non-smooth production with the same mean. (It is cheaper to equate the marginal cost over time). This is especially true for the demand-satisfying model because in this model the elasticity of labor with respect to x is large and fluctuations in x cause large fluctuations in L.

In the third, production to market model, the welfare cost of random fluctuations in x is large. This is because of the waste that occurs in the excess supply state. To understand why this cost is large relative to the cost of non-smooth production it is useful to think about the case in which the markup is close to unity. In this case small fluctuations in L are costless: The representative agent is willing to add (or subtract) a unit of labor in exchange for a unit of consumption. This is very different from the production to market case in which the seller supplies additional labor and gets nothing for it.

The cost of not making a sale in the production to market model is similar to the cost of involuntary unemployment in the old Keynesian literature. The old literature assumed that the leisure gained by unemployed workers is valueless. Here leisure is valued but output which is not sold is in a sense a waste of time.

Table 5 illustrates the welfare calculations for the regime (policy) in which $x = \beta - 1 = -0.04$ with probability 0.5 and $x = 0.1$ otherwise. The expected inflation in this regime is 3%.

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[11] They may also change the mean labor supply but this effect is very small in the first two models.

Table 5*: The welfare cost of 3% average inflation rate (x = β-1 with probability 0.5 and x = 0.1 otherwise)

δ =	2	3	11
Labor supply elasticity	1	0.5	0.1
Second best L =	0.95	0.97	0.99
Expected inflation L =	0.89	0.94	0.99
Welfare cost of expected inflation =	0.6%	0.3%	0.1%
<u>Demad satisfying model</u>			
Elasticity =	1	1	1
Welfare cost =	0.9%	0.8%	2.3%
<u>Production to order</u>			
Elasticity =	- 0.39	- 0.15	- 0.03
Welfare cost=	0.61%	0.3%	0.07%
<u>Production to market with production to order prices</u>			
Elasticity =	- 0.64	-0.35	-0.09
Welfare cost=	2.4%	1.7%	0.5%
<u>Production to market with demand-satisfying prices</u>			
Elasticity =	- 0.27	- 0.15	-0.03
Welfare cost=	5.2%	4.6%	4.6%

* This Table uses the elasticity and welfare cost calculations from previous Tables for the case g = 0.1. It also add some calculations about expected inflation. The second best L is the quantity that will be produced when the Friedman rule is implemented. Expected inflation L is the labor supply when the expected inflation of 3% occurs with probability 1.

The welfare cost of expected inflation is the cost of increasing the money supply at the rate of 3% with probability 1 relative to the Friedman rule alternative. It is in the range of 0.1% to 0.6% and its importance declines with δ. In the extreme case when labor is inelastically supplied this cost is zero. The welfare cost in the demand-satisfying model are in the range of 0.8% - 2.3%. The welfare cost in the production to order case are similar to the welfare cost of expected inflation suggesting that random fluctuations in x do not cause significant additional harm in this model. The welfare cost in the production to market case is in the range of 0.5% to 2.4% if we choose α that will lead to the production to order prices and in the range of 4.6% to 5.2% if the demand-satisfying prices (from Table 2) are used.

The expected cost of inflation in the combined model of the Appendix is some weighted average of the numbers in Table 5 where the average is across models. It seems like a formidable task to estimate this cost. The point I want to make here is that this may potentially be a large number. If for example, the average rate of unemployment is 2% above the efficient (natural) rate and capital utilization is on average 2% less than its efficient level, then we have a loss of 2% of output. If half of this loss is due to policy "mistakes" than there is a potential gain of 1% of output which is a big number.

5. CONCLUSIONS

Satisfying demand is not optimal when the monopoly power and the labor supply elasticity are not large and there is a large element of surprise in the money supply increase.

When sellers are allowed to choose quantities optimally, more money leads to a lower real wage and less output in the region of excess demand. In the region of excess supply, more money leads to more output of goods produced to order but may have no effect on the output of goods produced to market.

The welfare cost of random variations in x may be large in the production to market case because of the waste that occurs in low demand states. But there is no positive relationship between money and the output of goods produced to market.

In the combined model discussed in the Appendix the representative household produces three goods: One to order, one to market and one to satisfy demand. (We choose the parameters in a way that it is always optimal to satisfy the demand for the third good). When production to market is relatively important, we may get a weak positive relationship between money and output in this model and a large welfare cost. This may occur when production to market is relatively important.

It may be useful to extend this model to an environment in which a feedback rule is optimal and examine the effect of relaxing the demand-satisfying assumption on the optimal feedback rule. I plan to address this issue in another paper.

APPENDIX A: A COMBINED MODEL

In the real world some sellers satisfy demand, some goods are produced to order and some are produced to market. I now combine the three models in the text to get a unified general framework.

I use the Dixit-Stiglitz differentiated goods framework in section 2 and assume a large number of N infinitely lived households each producing three types of differentiated goods and consuming all 3N goods. The utility function of the representative household is:

$$[\Sigma_{j=1}^{3N} (y_j)^{\gamma}]^{1/\gamma} - \Sigma_{\tau=1}^{3} \upsilon_{\tau}(L_{\tau}) \tag{A1}$$

where y_j is the quantity consumed of good j, $0 < \gamma < 1$ is the demand elasticity parameter, L_τ is the amount of labor used to produce type τ good and $\upsilon_\tau(\ )$ is the utility cost of doing it. The transfer payment can take values in the range: $\beta-1 \le x \le g$, where g is not large.

I use a dual indices system. When adopting the sellers' point of view it is useful to talk about type τ good produced by seller j and write the utility function (A1) as:

$$[\Sigma_{\tau=1}^{3}\Sigma_{j=1}^{N} (y_{j\tau})^{\gamma}]^{1/\gamma} - \Sigma_{\tau=1}^{3} \upsilon_{\tau}(L_{\tau}). \tag{A1'}$$

I start from the buyer's point of view using the single index system of (A1).

Goods of type 1 are produced to order and goods of type 2 are produced to market. The demand of type 3 goods is always satisfied. To justify this last assumption we choose the cost functions $\upsilon_\tau(L_\tau)$

appropriately. For example we may choose $\upsilon_3(L_3) = 0$ and in this case it is always optimal to satisfy the demand for good 3.[12]

Buyers and orders arrive sequentially. Buyers who arrive late may not find all goods. Buyers index goods by availability. They expect that good j is rationed when $x > \zeta_j$, where $\zeta_j \le \zeta_{j+1}$.

Buyers find all goods when $x \le \zeta_1$. When $\zeta_1 < x \le \zeta_2$ buyers who arrive early find all goods but buyers who arrive late find only 3N - 1 goods (they do not find good 1). When $\zeta_2 < x \le \zeta_3$ buyers who arrive early find all 3N goods. Those who arrive second find 3N - 1 goods and those who arrive last find only 3N - 2 goods.

From the buyer's point of view there are 2N + 1 possible characterizations of the availability of goods. When the availability index is 1 all goods are available. When the availability index is 2 only goods indexed $j \ge 2$ are available. And in general, When the availability index is $1 \le s \le 2N + 1$ only goods indexed $j \ge s$ are available. (When the availability index is 2N + 1 only type 3 goods are available).

A buyer who chooses to spend d normalized dollars in market condition s solves:

$$\text{(A2)} \qquad F(d, s) = \max_{y_j} \; [\Sigma_{j=s}^{3N} (y_j)^{\gamma}]^{1/\gamma} \quad \text{s.t. } \Sigma_{j=s}^{3N} p_j y_j = d.$$

[12] In the context of the numerical example of Table 1 we may assume that the labor supply elasticity for type 3 good is 1 ($\delta = 2$) and the labor supply elasticities of type 1 and 2 goods is 0.1 ($\delta = 11$). If $\gamma = 0.8$, then it is optimal to satisfy the demand of type 3 goods but it is not always optimal to satisfy the demand of types $\tau < 3$ goods. Alternatively, we may allow for differences in monopoly power across the three types of goods but this seems more complicated.

Here F(d, s) is the expected current utility when the buyer finds, upon arrival, market condition s. Let $z_s = 1/[\Sigma_{j=s}^{3N} (p_j)^{1+\theta}]$ where as before $\theta = 1/(\gamma - 1) < 0$. The buyer's demand for good j in market condition s can be derived in a way that is similar to (6) in the text and is given by:

(A3) $$(p_j)^{\theta} d(x, s) z_s,$$

where d(x, s) is the amount that the buyer will spend in market condition s.

Let $\pi(x, s)$ denote the fraction of the buyers who upon arrival find market condition s. These functions will be specified in detail later. We now use them to aggregate the demands (A3) for good j. This leads to:

(A4) $$(p_j)^{\theta} z(x),$$

where $z(x) = N\Sigma_{s=1}^{2N+1} \pi(x, s) d(x, s) z_s$. Since N is large, we may neglect the individual seller's effect on z(x).

We now turn to the seller's problem using a pair of indices (i, τ) for the good from the seller's point of view.[13] Seller i takes z(x) as given and choose type 1 labor $L_{i1}(x)$ under the constraint that he does not produce more than the quantity demanded:

[13] Since the same good is indexed in two ways we can match indices and use $j = j(i,\tau)$ to denote the index of type τ good produced by seller i from the buyers' point of view.

(A5) $$L_{i1}(x) \leq (p_{i1})^{\theta} z(x).$$

The fraction of type 2 output that will be sold on the market depends on the price and the state and is denoted by $\Omega(p_{i2}, x)$. This function will be specified shortly.

The household takes the probability that its buyer will find market condition s, $\pi(x, s)$, the demand factor, $z(x)$, and the fraction of type 2 output that will be sold, $\Omega(p_{i2}, x)$, as given. This assumption should be regarded as an approximation that is good when N is large. Given these functions the household solves the following Bellman's equation:

(A6) $$V(m) = \max_{p_{i\tau}} \int_{\beta-1}^{g} \{\max_{L_{i1}\, L_{i2}} \Sigma_s \, \pi(x, s)$$

$$\{\max_d F(d, s) - \Sigma_{\tau=1}^{3} \upsilon(L_{i\tau})$$

$$+ \beta V[[m + x - d + p_{i1}L_{i1} + p_{i2}\Omega(p_{i2}, x)L_{i2} + (p_{i3})^{\theta+1} z(x)]/(1 + x)]$$

$$\text{s.t. } d \leq m + x, \text{ (A2) and (A5)}\}\}\phi(x)dx$$

This says that the household chooses the prices p_τ before it observes x. Then the worker observes x and chooses labor supplies L_τ. Finally, the buyer chooses the amount of spending, d, after observing x and his market condition. The term $F(d, s) - \Sigma_{\tau=1}^{3} \upsilon_\tau(L_\tau)$ is the current utility. The amount saved by the buyer is $m + x - d$ current normalized dollars. The type 1 good is produced subject to the constraint (A5). The revenues from selling the type 1 goods are p_1L_1. Since only a fraction $\Omega(p_2, x)$ of the type 2 good is sold, the

revenues from selling it are $p_2\Omega(p_2, x)L_2$. The quantity demanded from the type 3 good is $(p_3)^\theta z(x)$ and since the demand for type 3 good is always satisfied the revenues from selling the type 3 good are $(p_3)^{\theta+1}z(x)$. The household's next period balances (in terms of next period's normalized dollars) are thus:
$m' = [m + x - d + p_{i1}L_{i1} + p_{i2}\Omega(p_{i2}, x)L_{i2} + (p_{i3})^{\theta+1}z(x)]/(1 + x)$. The expression $\beta V(m')$ is the future utility term.

We now adopt the availability indices from the buyer's point of view and turn to the specification of the functions $\pi(x, s)$. The demand for good 1 when all goods are available is:
$Nd(x, 1)z_1(p_1)^\theta$. When $x \le \zeta_1$ all buyers find all goods and $\pi(x, 1) = 1$. ($\pi[x, s] = 0$ for $s > 1$). When $x > \zeta_1$ only a fraction $p_1L_1(x)/Nd(x, 1)z_1(p_1)^\theta$ of the buyers find all goods. Thus,

(A7)
$$\pi(x, 1) = 1 \text{ if } x \le \zeta_1 ;$$
$$\pi(x, 1) = p_1L_1(x)/Nd(x, 1)z_1(p_1)^\theta \text{ if } x > \zeta_1.$$

Similarly, let
$D_2(x) = N\{\pi(x, 1)d(x, 1)z_1(p_2)^\theta + [1 - \pi(x, 1)]d(x, 2)z_2(p_2)^\theta\}$ denotes the demand for good 2 when goods indexed $j \ge 2$ are not rationed. The fraction of buyers in market condition 2 is:

(A8)
$$\pi(x, 2) = 1 - \pi(x, 1) \text{ if } x \le \zeta_2;$$
$$\pi(x, 2) = p_2L_2(x)/D_2(x) - \pi(x, 1) \text{ if } x > \zeta_2.$$

This says that when $x \le \zeta_2$ a fraction $\pi(x, 1)$ of the buyers finds all goods and the rest finds N - 1 goods, because at this range of x at most good 1 is rationed. When $x > \zeta_2$ only a fraction $p_2L_2(x)/D_2(x)$ of the demand for good 2 is satisfied and therefore a fraction

$p_2L_2(x)/D_2(x)$ of the buyers finds either all N goods or N-1 goods. We subtract the fraction that finds all N goods ($\pi[x, 1]$) to arrive at the fraction that finds N - 1 goods.

In general, the demand for good $s \leq N$ when goods indexed $j \geq s$ are not rationed is: $D_s(x) = N\Sigma_{j=1}^{s-1} \pi(x, j)d(x, j)z_j(p_s)^\theta + N[1 - \Sigma_{j=1}^{s-1} \pi(x, j)]d(x, s)z_s(p_s)^\theta$. And the fraction of buyers in market condition $s \leq 2N$ is:

$$\text{(A9)} \qquad \pi(x, s) = 1 - \Sigma_{j=1}^{s-1} \pi(x, j) \quad \text{if } x \leq \zeta_s;$$
$$\pi(x, s) = p_sL_s(x)/D_s(x) - \Sigma_{j=1}^{s-1} \pi(x, j) \quad \text{if } x > \zeta_s.$$

The fraction of buyers who are in market condition $s = 2N + 1$ is:

$$\text{(A10)} \qquad \pi(x, 2N+1) = 1 - \Sigma_{j=1}^{2N} \pi(x, j).$$

I now turn to the specification of the fraction of the type 2 good that will be sold, $\Omega(p_{i2}, x)$. Here I use the pair of indices from the seller's point of view to name a good. Let I(statement) = 1 when the statement is true and zero otherwise. The average capacity utilization (the fraction of output sold) of goods which are supplied in excess of demand (all goods with $\zeta_{i2} \geq x$) is the ratio of the nominal demand for these goods to the nominal supply:

$$\text{(A11)} \quad CU(x) = \Sigma_{i=1}^{N} I(x \leq \zeta_{i2})(p_{i2})^{\theta+1}z(x) \;/\; \Sigma_{i=1}^{N} I(x \leq \zeta_{i2})p_{i2}L_{i2}(x).$$

The average price of these units is:

$$\text{(A12)} \quad p(x) = \Sigma_{i=1}^{N} I(x \leq \zeta_{i2})p_{i2}L_{i2}(x) \;/\; \Sigma_{i=1}^{N} I(x \leq \zeta_{i2})L_{i2}(x).$$

We assume that the quantity sold is determined by a function $\Omega[p_{i2}, p(x), CU(x), x]$ which is decreasing in p_{i2}, increasing in p(x) and in CU(x). Furthermore we require that a seller that set his price equal to the average price will sell the average fraction:

$\Omega(p, p, CU, x) = CU$.

I use $\Omega(p_{i2}, x) = \Omega[p_{i2}, p(x), CU(x), x]$ for short and define equilibrium as follows.

Equilibrium is a vector of functions

$\{d(x, s), \pi(x, s), L_{i\tau}(x), z(x), p(x), CU(x), \Omega[p_{i2}, p(x), CU(x), x]\}$ and a vector of scalars $(p_{i1}, p_{i2}, p_{i3}, \zeta_{i1}, \zeta_{i2})$ such that

(a) $p_{i\tau}L_{i\tau} \geq (p_{i\tau})^{\theta+1}z(x)$ for $x \leq \zeta_{i\tau}$ and $p_{i\tau}L_{i\tau} < (p_{i\tau})^{\theta+1}z(x)$ otherwise $(\tau = 1,2)$;

(b) Given $\pi(x, s)$, $z(x)$ and $\Omega(p_2, x) = \Omega[p_{i2}, p(x), CU(x), x]$, the magnitudes $[p_{i\tau}, \zeta_{i\tau}, L_{i\tau}(x), d(x, s)]$ solve (A6) for m = 1;

(c) $\pi(x, s)$ is calculated by (A7) - (A10),

$z(x) = N\Sigma_s \pi(x, s)d(x, s)z_s$, CU(x) is calculated by (A11) and p(x) by (A12).

A symmetric equilibrium is an equilibrium with the added property: $L_{i\tau}(x) = L_\tau(x)$, $p_{i\tau} = p_\tau$ and $\zeta_{i\tau} = \zeta_\tau$ for all i.

We have assumed that the individual seller ignores his effect on average per household magnitudes and on the market condition that his buyer will face. We now formulate the sellers' problem as a symmetric game. This formulation does not require the above approximation and is useful because we can apply a general existence proof of Nash equilibrium.

A symmetric game:

There are N sellers. Seller i has a non-empty set A_i of actions with a typical element: $a_i = [p_{i1}, p_{i2}, p_{i3}; L_{i1}(x), L_{i2}(x), L_{i3}(x)]$.

There is a set of action profiles or outcome A with a typical element $a = (a_1, \ldots, a_N)$.

We assume $d(x, s) = 1 + x$ and compute for each element in A the functions: $z(x)$, $\pi(x, s)$, and $\Omega(p_{i2}, x)$. We then solve the problem (A6) assuming $d = m + x$.

It is assumed that $0 \le p_{i\tau} \le 1/V'$, where V' is obtained from the solution to (A6). (Since we assume risk neutrality, V' is a constant). Under this assumption, $1/p_{i\tau} \ge \beta V'/(1 + x)$, the cash-in-advance constraint is always binding and $d(x, s) = 1 + x$. The labor supplies are restricted by:
$0 \le L_{i1}(x) \le (p_{i1})^{\theta} z(x)$; $0 \le L_{i2}(x) \le 1$; $0 \le L_{i3}(x) \le 1$. Thus the sets A_i are compact and convex.

Using the solution V( ) for the problem in (A6) we can define the following preference relations on A:

$$u_i(a) = G(a_{-i}, a_i) = \int_{\beta-1}^{g} \{- \Sigma_{\tau=1}^{3} \upsilon[L_{i\tau}(x)] \tag{A16}$$

$$+ \beta V[[p_{i1}L_{i1}(x) + p_{i2}\Omega(p_{i2}, x)L_{i2}(x) + (p_{i3})^{\theta+1} z(x)]/(1 + x)]\}\phi(x)dx$$

The preference relations $G(a_{-i}, a_i)$ is quasi concave in a_i and continuous. Thus our symmetric game satisfies the conditions of Proposition 20.3 in Osborne and Rubinstein's (1994, page 20) and therefore there exists a Nash equilibrium for this game. Since our game is symmetric we can apply exercise 20.4 in (Osborne and Rubinstein) to show existence of a symmetric Nash equilibrium in which all sellers choose the same strategy: $[p_1, p_2, p_3; L_1(x), L_2(x), L_3(x)]$.

APPENDIX B: MONOPOLISTIC COMPETITION WITH RANDOM UTILITY MAXIMIZING CONSUMERS

In the text and in Appendix A the fraction of goods sold was determined by an ad-hock function of the relative price and x. Here I use McFadden's random utility maximization framework to construct a model that determines the probability that a unit produced to market will be sold.[14] For the purpose of this Appendix and to simplify notation I assume that there is a single good that is produced to market (type 2) and the buyer spends his entire holding of money (1 + x). I also assume that all prices satisfy condition (25) in the text: $1/p_i \geq \beta V'/(1 + x)$ for all i.

Buyers come to the market observe all remaining price offers and choose a single offer. They may choose a relatively expensive offer by "mistake". These "mistakes" may reflect unobserved attributes of the goods such as location. As a result of these mistakes a unit may be sold even if its price is higher than the price of other units.

When s units are available, the probability of choosing unit i is: $Prob(i) = w_i/\Sigma_{j=1}^{s} w_j$, where,

(B1) $$w_i = \exp[\alpha(P/p_i)],$$

[14] For a survey of this approach see McFadden (2000).

$\alpha > 0$ is a parameter, p_i is the price of unit i and P is the average price of all offers (units).

Units are sold sequentially. The first unit is sold in the first round or market. Then a second unit is sold in the second market and so on. We now compute the probability of selling unit i at the price p_i when there are s-1 other units offered at the price p.

The average price when there are s - 1 units offered at the price p and one unit offered at the price p_i is:

(B2) $$P(p_i, p, s) = [(s - 1)p + p_i]/s.$$

To simplify notation, I write $P(s) = P(p_i, p, s)$ to denote this average. The probability of selling unit i in the first market is:

(B3) $$\omega_1(p_i, p, s) = w_i/\Sigma_{j=1}^{s} w_j = \exp[\alpha P(s)/p_i]/D(s),$$

where $D(s) = \{(s-1)\exp[\alpha P(s)/p] + \exp[\alpha P(s)/p_i]\}$. If the unit was not sold in the first market it may be sold in the second market. Since s - 1 units are offered in the second market, the probability that the unit will be sold in the second market is:

(B4) $$\omega_2 = (1 - \omega_1)w_i/\Sigma_{j=1}^{s-1} w_j = (1 - \omega_1)\exp[\alpha P(s-1)/p_i]/D(s-1).$$

Using $\omega_0 = 0$, we can write the probability that the unit will be sold in the jth market as:

(B5) $$\omega_j = \Pi_{k=0}^{j-1}(1-\omega_k)\exp[\alpha P(s-j+1)/p_i]/D(s-j+1).$$

In equilibrium, the number of units on the market is $s(x) = L(x)$. The number of units demanded is: $(1 + x)/p$. The number of rounds is: $S = \mathrm{Min}\{(1 + x)/p, s(x)\}$. The probability of selling a unit in excess supply situations, when $S = (1 + x)/p < s(x)$, is:

(B6) $$\omega(p_i, p, s) = \Sigma_{j=1}^{S} \omega_j.$$

In excess demand situations when $S \geq s(x)$, the probability of selling the unit is 1 because if the unit was not sold in the first s-1 rounds it will be sold in round s when it is the only unit on the market. The probability of making a sale is therefore:

(B7) $$\Omega[p_i, p, s(x), x] = \omega[p_i, p, s(x)]$$

when $s(x) > (1 + x)/p$ and 1 otherwise.

To use (B7) as a possible story for the allocation rule Ω assumed in Appendix A we need to make two assumptions: (a) the individual seller ignores any effect it may have on the average supply per household s(x) and (b) we may treat the probability of selling a unit as the fraction of output sold. The first assumption may be justified if the supply of each individual seller is small relative to the aggregate. The second assumption may be justified by risk neutrality.

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