RESERVE REQUIREMENTS AND OUTPUT FLUCTUATIONS

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ABSTRACT

When trade is uncertain and sequential, a fractional reserve banking system may give rise to endogenous monetary shocks. These endogenous monetary shocks lead to fluctuations in capacity utilization and waste. When fluctuations in the currency/deposit ratio are the important source of the monetary shocks, a high reserve requirement on checkable accounts can minimize this waste and the central bank should not pay full interest on reserves. When fluctuations in the fraction of credit-card transactions are important then the reserve requirement may be low but full interest should be paid on reserves.

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1. INTRODUCTION

There has been an ongoing trend to reduce reserve requirements on transaction accounts.[1] In some industrial countries they have even been completely eliminated. This striking policy trend has been the result of several factors. The development of financial instruments has vastly increased the number of non-bank institutions that provide very similar services to those of banks, without being subject to reserve requirements. Moreover, central banks found it ever harder to enforce the reserve requirements on banks, which devised ways to circumvent these requirements by transferring funds from reservable accounts to non-reservable ones.[2] As a result, the required reserves in the US have declined from about $60 billions in 1994 to roughly $38 billion in mid 2000.[3]

Central banks have been concerned with the declining role of reserve requirements, in particular in conjunction with their ability to stabilize short-term interest rates. Countries that have reacted to the aforementioned pressures by eliminating the reserve

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[1] In the US. reserve requirements were reduced from about 20% in the late '40s to their current level of less than 10%, on average. See Feinman (1993) for a history of reserve requirements.

[2] The dramatic growth of the “sweep accounts” has started in mid-1995. See Sellon and Weiner (1996).

[3] An additional development was the dramatic decline in the deposits banks hold with the Fed, from about $25 billion in 1994 to a current level of $6 billion. The bulk of the required reserves is now held in the form of vault cash (87% as compared to about 55% in 1994), which presumably would have been held also absent the reserve requirements.

Source of the data: Federal Reserve Statistical Release, http://www.federalreserve.gov/releases/H3/hist/

requirements have developed appropriate procedures to conduct monetary policy.[4] In the US the reaction to the same pressures seems to be different. Rather than eliminating the reserve requirements, legislation has been introduced to pay interest on reserves in order to eliminate the incentive to circumvent the reserve requirement.[5]

Surprisingly enough there is very little discussion of this policy trend. Text-books try to provide some explanation. For example, Barro (1993, page 478) argues that high reserve requirements are associated with a large spread between borrowing and lending rates and less intermediation between borrowers and lenders. As a result resource allocation becomes less efficient.

On the other hand, Friedman (1959) argues that reserve requirements do not affect intermediation. He envisaged that under 100% reserve requirement there will be two institutions. One that stores deposits and provides checking services for a fee and one that does the intermediation between lenders and borrowers.[6] Friedman

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[4] Even where reserve requirements have been eliminated, banks are still required to hold “settlement accounts”. This feature allows central banks to affect short-term interest rates (see Sellon and Weiner (1997)).

[5] See the testimony of Governor Meyer before the committee on banking and financial services on May 3, 2000. See the internet site: http://www.federalreserve.gov/boarddocs/testimony/2000/20000503.htm

[6] In describing "how a 100% reserves would work", Friedmans says (page 69-70) "The effect of this proposal would be to require our present commercial banks to divide themselves into two separate institutions. One would be a pure depository institution, a literal warehouse for money. It would accept deposits payable on demand or transferable by check. ...The other institution that would be formed would be an investment trust or brokarage firm. It would acquire capital by selling shares or debentures and would use the capital to make loans or acquire investments."

recommends 100% reserve requirement to improve the control of the money supply and reduce fluctuations in real output.

Here we examine Friedman's hypothesis about the relationship between reserve requirements and output fluctuations. We focus on the role of banks as creators of money rather than their role in expanding credit. This modeling choice reflects our belief that in the modern world, the main impact of reserve requirements is on money creation rather than credit expansion.

This emphasis sets our model apart from most previous treatments of reserve requirements. Typically, in the model economies these requirements are imposed on activities related to intertemporal decisions. See for example, Sargent and Wallace (1982), Chari *et al* (1995) and Haslag and Young (1998). In such environments the requirement that banks hold non-interest paying reserves against loans (or time-deposits) creates a gap between the lending and borrowing interest rates and affects saving decisions. Alternatively, banks have been viewed as providing risk-sharing services (Diamond and Dybvig (1983)). Imposing high reserve requirements in this case may destroy this beneficial service. However, as Jacklin (1987) has argued, other institutions may replace the Diamond and Dybvig banks as providers of insurance.

To model the monetary role of banks, we impose a money-in-advance constraint on (some of) the consumers in our model economy. In general, some consumers may be able to use credit in order to buy goods, but others must use money. Money users potentially consist of two types of people – those who may use checks and those who must use cash.[7] The various groups of consumers face different effective prices on consumption. Credit users face the lowest price since they incur no interest cost at all. In a fractional reserve system check users are paid some interest on their deposits and face a higher effective consumption price than credit users. Cash users, who receive no interest face the highest effective price. The different prices face by the different groups of consumers affect their respective demand. Consequently, if the size of the various groups fluctuates, the demand for consumption will also fluctuate.

These fluctuation in demand may have real effects because trade is uncertain and sequential as in Eden (1994), Lucas and Woodford (1994), Bental and Eden (1996), Williamson (1996) and Woodford (1996). In particular, sellers choose their output before demand materializes. Moreover, the realization of the demand is revealed to them in a sequential fashion. They know the money base, and can compute the minimal amount of purchasing power that will arrive with certainty. However, more purchasing power may arrive if the number of buyers who are cash constrained is low (a low currency/deposit ratio), or if the number of credit users is high. Accordingly, at the beginning of the period, sellers make a contingent plan which specifies the amount that will be sold to each batch of purchasing power that may arrive during the trading period. Since the probability that all potential batches will arrive is low, sequential trade typically results in waste: Output is sold in full only when

[7] Notice that while different buyers use different media of exchange, the purchased goods are the same. This is different from the cash-good credit-good environment in Lucas and Stokey (1987). In their environment goods are defined by the medium of exchange which is used for their purchase.

the highest possible realization of demand occurs and all batches arrive. Therefore, measures that reduces the uncertainty about the severity of liquidity constraints improve capacity utilization and reduce waste.

To simplify the discussion, we consider the case in which there are only two instruments for buying goods: cash and checks. Demand fluctuations are minimized when the two instruments implies the same interest cost for buying consumption. Since it is not possible to pay interest on cash, this equality can be achieved by imposing a 100% reserve requirement on checking accounts and paying no interest on reserves. The model is then used to discuss the case in which the two instruments are checks and credit-cards. The central bank can achieve an equality between the interest cost of these two instruments by paying interest on reserves. If all three instruments are used at the same time, demand uncertainty can be completely removed only under the Friedman zero interest rule or under regulations which prohibit the use of "near moneys". In the absence of such measures, cash constrained consumers face the highest effective price of consumption, while credit users face the lowest price. The policy maker thus faces a tradeoff between two types of distortions: One arising as a result of the difference in the effective consumption price between cash and checks users and one arising as a result of the difference between checks and credit users.

In our model policy makers can affect the effective consumption price faced by check users. If the amount of cash purchases is relatively small, it seems likely that output fluctuations will be reduced if some reserve requirement is maintained, and interest is paid on these reserves. If fluctuations in cash purchases are important, then the central bank should impose a 100% reserve requirement and pay no interest on reserves.

2. LIQUIDITY CONSTRAINTS AND DEMAND UNCERTAINTY

We introduce first a simple version of our set-up which demonstrates that uncertainty about the degree at which agents are liquidity constrained can lead to waste. We demonstrate this point in an uncertain and sequential trade (UST) model in which interest rates are exogenously set to zero and agents spending decision is trivial.

There are N households consisting of two infinitely lived individuals: a worker and a buyer. To simplify, we assume that the single period utility function of the representative household is given by $c - v(L)$ where c denotes consumption and L denotes labor input of the worker (which is also equal to output). The cost function $v(\ )$ has the standard properties ($v' > 0$ and $v'' > 0$ everywhere). The household's discount factor is given by $0 < \beta < 1$.

The amount of cash available to household h at the beginning of period t is M_t^h dollars. In addition, all households get the same perfectly anticipated lump sum cash transfer of G_t dollars. The average post transfer amount of high powered money is: $M_t = G_t + (1/N)\Sigma_{h=1}^{N} M_t^h$. The rate of change in high powered money is constant over time and is given by: $M_{t+1}/M_t = 1 + \mu$.

At the beginning of the period the worker goes to work. The buyer takes the cash (initial holdings plus transfer) and goes to the bank. At the bank, the buyer participates in a lottery that determines whether he gets credit. Buyers who get credit approval can take a loan of $b = 1/rr - 1$ dollars for each dollar deposited in a checking account, where $0 < rr \leq 1$ is the reserve requirement. The loan must be paid at the end of the period. It is assumed that rr is sufficiently large, so that at the end of the period all borrowers can pay the loan out of the cash revenues of the worker. The nominal interest on checkable deposits and on the loan is zero.

Since the interest rate is zero, it is quite obvious that buyers who get credit open a checking account and use checks to buy goods, while buyers who do not get credit use cash to buy goods.

The fraction of buyers who get credit and pay in checks is an i.i.d random variable ϕ which can take S possible realizations: $\phi_1 < \phi_2 < ... < \phi_S$. When $\phi = \phi_s$ the probability that a buyer will get credit is ϕ_s. We denote the probability that $\phi = \phi_s$ by Π_s and the probability that $\phi \geq \phi_s$ by q_s.

It is assumed that all buyers spend as much as they can at the goods market. The buyer spends $(M_t^h + G_t)/rr$ if he gets credit and $M_t^h + G_t$ if he does not get credit.

Using lower case letters to denote nominal magnitudes divided by $M_t + G_t$ (normalized nominal magnitudes) we can write the average amount that will be spent per household as:

$$\phi/rr + (1 - \phi) = 1 + b\phi, \qquad (1)$$

where $b = 1/rr - 1 > 0$, is the amount of loan per dollar deposited in the checking account. Note that the nominal spending is equal to M1 and is monotonic in ϕ.

It is assumed that buyers arrive at the goods market sequentially and therefore money arrives sequentially. From the sellers' (workers') point of view, purchasing power arrives sequentially in batches. The first batch is the minimum amount of spending: $1 + b\phi_1$ normalized dollars. After the first batch was spent, there can be two possible events: Either an additional batch of $b(\phi_2 - \phi_1)$ normalized dollars arrives or trade ends for the current period. The size of the second batch is computed as the difference between the second lowest realization of nominal spending $(1 + b\phi_2)$ and the lowest realization $(1 + b\phi_1)$.

In general, after trade in market s has been completed there may be two possibilities. Either an additional batch of $b(\phi_{s+1} - \phi_s)$ normalized dollars arrives and opens market s+1 or trade ends for the period.

The worker produces a total of L units and makes a contingent plan on how to sell these units to the various batches of purchasing power that may arrive. This contingent plan is described as an allocation to the S potential markets: k_s units to market s.

The normalized price (the price divided by the post transfer money supply) in market s is p_s. We use $\omega = (1 + \mu)^{-1}$ to convert current normalized dollars into next period's normalized dollars.[8]

Given that s markets open this period, the expected purchasing power of a normalized dollar (= $M_t + G_t$ regular dollars) held after the visit to the bank and before entering the goods market is:

(2) $$z_s = \Sigma_{j=1}^{s}(\upsilon_j^s/p_j),$$

where υ_j^s is the probability that the dollar will buy in market j when s markets open: $\upsilon_1^s = (1 + b\phi_1)/(1 + b\phi_s)$ and

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[8] When the money supply increases at the rate of μ the number of regular dollars in a current normalized dollar will be less than the number of regular dollars in next period's normalized dollars: It will equal a fraction $(1 + \mu)^{-1}$ of the number of regular dollars in the next period's normalized dollar.

$\upsilon_j^S = b(\phi_j + \phi_{j-1})/(1 + b\phi_S)$ for $j > 1$.

The Bellman equation which describes the household's decision problem is:

$$(3) \qquad V(m) = \max_{k_s} \{(m + g)\Sigma_{s=1}^{S} \Pi_s[\phi_s(1 + b) + (1 - \phi_s)]z_s - v(\Sigma_{s=1}^{S} k_s) + \beta\Sigma_{s=1}^{S} \Pi_s\{\phi_s V[\omega\Sigma_{j=1}^{s} p_j k_j - b\omega(m + g)] + (1 - \phi_s)V[\omega\Sigma_{j=1}^{s} p_j k_j]\}\}.$$

The first term in the right hand side of (3) is the expected consumption from holding m normalized dollars and receiving a transfer of g normalized dollars. The buyer will spend $(m + g)(1 + b)$ normalized dollars if his credit is approved and $(m + g)$ normalized dollars otherwise. Since the probability of getting a credit approval is ϕ_s, his expected spending is: $(m + g)\Sigma_{s=1}^{S} \Pi_s[\phi_s(1 + b) + (1-\phi_s)]$ normalized dollars. The expected purchasing power of a normalized dollar held prior to learning about the outcome of the credit approval lottery is therefore: $\Sigma_{s=1}^{S} \Pi_s[\phi_s(1 + b) + (1 - \phi_s)]z_s$.

The second term is the cost of supplying $L = \Sigma_{s=1}^{S} k_s$ units.

The third term is the expected future utility. The revenue of the worker is $\Sigma_{j=1}^{S} p_j k_j$ normalized dollars and a household which did not get credit will end the period with this amount. A household which got credit will end the period (after paying the debt) with $\Sigma_{j=1}^{S} p_j k_j - b(m + g)$ current period normalized dollars.

To state the first order conditions for (3) let us define the expected utils from a normalized dollar held at the beginning of the period (before taking the credit lottery) by the solution to:

$$(4) \qquad z = \Sigma_{s=1}^{S} \Pi_s[\phi_s(1 + b) + (1-\phi_s)]z_s - \beta b\omega z\Sigma_{s=1}^{S} \Pi_s\phi_s.$$

The first term is the expected current consumption that a normalized dollar will buy. The second term is the expected discounted cost of the debt: When credit is approved, with probability $\Sigma_{s=1}^{S} \Pi_s \phi_s$, the loan is b normalized dollars, which will become $b\omega$ next period normalized dollars, and will buy on average $b\omega z$ consumption units.

The first order conditions for (3) can now be stated in the familiar way:

(5) $$q_s \beta p_s \omega z = \beta p_1 \omega z = v'(\Sigma_{s=1}^{S} k_s).$$

This says that the expected consumption from supplying a unit to market s is the same as the expected consumption from supplying a unit to market 1 and is equal to the marginal cost.

Equilibrium requires (5) and the market clearing conditions:

(6) $$p_1 k_1 = 1 + b\phi_1 \text{ and } p_s k_s = b(\phi_s - \phi_{s-1}) \quad \text{for all } s > 1.$$

We can see, therefore, that in equilibrium output is allocated to all S markets that may potentially open. Accordingly, output allocated to a market that is not open will not be sold and will be lost.

<u>The case of 100% reserve requirements:</u>

When rr = 1, b = 0 and only the first market is active. In this case capacity is fully utilized, there is a single price, p, and the purchasing power of a normalized dollar is z = 1/p. The first order condition (5) is now:

(7) $$v'(k) = \beta\omega.$$

Equilibrium requires in addition to (7) the market clearing condition: pk = 1.

First best: Efficiency in this model requires full capacity utilization and v'(k) = 1. This can be achieved by choosing the two policy parameters ω and rr.

Full capacity utilization can be achieved by setting rr = 1. The condition v'(k) = 1 can be achieved by applying the Friedman rule and choosing: $\omega = \beta^{-1}$.

Second best: Suppose that for a reason exogenous to this model ω is set at a level lower than β^{-1}.[9] In this case we can still achieve the condition for full capacity utilization by setting rr = 1.

We now turn to a model which endogenizes interest rates and highlights the role of interest rates in spending choices.

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[9] When taste shocks are important, the Friedman rule may not be optimal. Suppose that agents who experience a negative taste shock do not want to consume in the current period. Under the Friedman rule, these agents have no incentive to lend money and therefore taste shocks may affect nominal spending and may lead to waste. It may thus be desirable to have a small positive nominal interest rate which will provide an incentive for the non-consumers to lend their money. This will lead to an equilibrium in which the amount of money spent is non random and capacity is fully utilized. For related arguments, see Eden (1986) and Williamson (1996).

3. THE MODEL

In the previous section the household made labor supply and marketing choices but did not make any interesting financial choices. Here we focus on financial choices and abstract from the labor supply choice. We focus on the second best problem of maximizing capacity utilization when the Friedman rule is not implemented.

As before, it is assumed that the buyer visits a bank before he goes to the goods market. At the bank he chooses the amount of spendable funds that he will carry to the goods market. This choice may be different across buyer types. Some buyers can buy only with cash and may face a relatively high interest cost. Other buyers can use checks or credit cards and may face a lower interest cost. As a result, the amount of spendable funds that will be taken to the goods market may be different across types, and uncertainty about the number of buyers of each type may lead to uncertainty about nominal spending in the goods market.

We consider an economy with two assets that can be used to buy goods: cash and checks. (We then use the model to discuss the case in which some buyers can buy with credit cards). Buyers who stay in their own neighborhood can pay with either cash or checks. Buyers who travel to other neighborhoods must use cash. The fraction of buyers who stay in their own neighborhood is random. Therefore, in a fractional reserve system, there is uncertainty about the currency/deposit ratio and about M1. As in the previous section, this uncertainty causes waste.

3.1 The story

We consider a discrete time economy with infinitely lived households. Each household consists of two members: a worker and a buyer. The households evenly populate two identical islands. Households turn out to be one of three types. Some households will consume at the current period and some will not. This will allow for the distinction between time and demand deposits. We assume that a constant fraction, α, of the households are non-consumers.

Out of the households who will consume in the current period some will shop in their home island and some will travel to the other island. As before some buyers will use credit and some will not but here buyers choose the amount of credit. Buyers who travel will choose not to take credit while buyers who stay in their home island will choose to get credit.

We use here $\tilde{\phi}$ to denote the random fraction of the households which do consume and shop in their home island.[10] This fraction is an identically and independently distributed random variable, which takes S possible realizations:
$0 < \phi_1 < \phi_2 < ...< \phi_S$. The probability that $\tilde{\phi} = \phi_s$, is denoted by Π_s and the probability that $\tilde{\phi} \geq \phi_s$ is denoted by q_s. The identity of the households who belong to each type is determined every period by an i.i.d. lottery.

All agents first trade in a securities market. Then they go to a bank and learn their type. Finally, they go to the goods market and learn the price (market) at which they can buy.

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[10] Here $\tilde{\phi}$ plays the same role as ϕ in the previous section. Other symbols have counterparts in the previous section.

Travelers can use only cash to buy goods. Non-travelers can use checks and cash to buy goods. In equilibrium, with a fractional reserve banking system, only non-travelers will use checks and therefore the amount of inside money depends on the number of non-travelers. Accordingly, total purchasing power (M1) depends on the realization of $\tilde{\phi}$.

We start from describing the arrival of purchasing power from the sellers' point of view.

3.2 Firms

From the point of view of the representative firm, demand arrives sequentially in batches. The number of batches that will arrive is denoted by the random variable $\tilde{s}$, where $\tilde{s}$ takes values from 1 to S. The amount of dollars in each batch is determined endogenously. The number of batches that will arrive depends on the realization of $\tilde{\phi}$. In particular, the probability that $\tilde{s} = s$ is: $\Pi_s = \text{prob}(\tilde{\phi} = \phi_s)$. The only information that is revealed by the arrival of batch j is that $\tilde{s} \geq j$.

The representative firm hires labor, l, and produces according to a linear production function k = l, where k denotes total capacity. Units of capacity can be costlessly converted to units of output at the rate of one to one.

The firm knows that it can sell to batch s at the price P(s), if batch s arrives. It makes a contingent plan: k(s) units of output will be sold to batch s if it arrives. Unsold units are wasted.

We say that the arrival of the first batch opens the first market. The arrival of each additional batch opens an additional market. Using this language, the firm allocates total supply among the S potential markets. Thus,

(8) $$l = \Sigma_s k(s).$$

Units allocated to market j bring P(j) dollars if market j opens and zero if it does not. The nominal revenue if exactly s markets open is:

(9) $$y(s) = \Sigma_{j\leq s} P(j)k(j).$$

At the beginning of the period there are complete markets for contingent claims, to be described below. The price at the beginning of the period of a dollar that will be delivered at the end of the period if exactly s markets open is n_s. The nominal wage is given by W and is paid at the beginning of the period.

The firm chooses k(s) to maximize the present value of profits. It solves:

(10) $$\max Y = \Sigma_s n_s[y(s)] - Wl$$

s.t.

$$y(s) = \Sigma_{j\leq s} P(j)k(j);$$

$$l = \Sigma_s k(s);$$

$$k(s) \geq 0.$$

3.3 Banks

The representative bank faces three interest rates: i_D for checkable deposits, i_T for time deposits and i_L for loans.

Let DD(s), TD(s) and LL(s) denote the amounts of checkable (demand) deposits, time deposits and loans that the bank has in state s. The bank's profits in state s are given by:

(11) $$z(s) = i_L LL(s) - i_D DD(s) - i_T TD(s).$$

Let,

(12) $$Z = \Sigma_S\, n_S z(s);\ D = \Sigma_S n_S DD(s);\ T = \Sigma_S n_S TD(s);\ L = \Sigma_S n_S LL(s),$$

denote the expected value of the corresponding quantities when using "risk neutral probabilities". (We later show that in equilibrium $n_S = \Pi_S$ and these are standard mathematical expectations).

While banks cannot observe the state s at the time they operate, they can still control the expected values D, T and L. This assumption is motivated by the following Bertrand type argument. We assume that if the bank sets the market interest rates it will get the market average quantities. By deviating slightly from the market rates and setting appropriate quantity limits, a bank can attract any amount it wants.

There is a regulator who can infer from the operating procedures of the bank the expected values (12). It is assumed that the regulator imposes an "average" reserve requirement on demand deposits[11]:

[11] In practice, central banks control reserve requirement by computing periodic averages. Our formulation allows for excess reserves. In our formulation excess reserves are zero on average, but a different regulation which impose heavy penalties on ex-post

(13) $$(D + T - L)/D \geq rr,$$

where $0 < rr \leq 1$ is the average reserve requirement. There is no reserve requirement on time deposits. The bank chooses D, T and L to solve:

(14) $\max Z = i_L L - i_D D - i_T T$; s.t. (13) and non-negativity constraints.

In equilibrium, $i_L \geq i_D$ and therefore (13) will hold with equality. Substituting (13) into (14) allows us to write the profit of the bank as:

(15) $$[(1 - rr)i_L - i_D]D + (i_L - i_T)T.$$

In equilibrium D and T must be finite and positive and therefore:

(16) $$(1 - rr)i_L = i_D \text{ and } i_L = i_T.$$

3.4 Households

The objective function of the household is given by

(17) $$\Sigma_t \beta^t \theta_t u(c_t),$$

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reserves which are below the required level may lead to positive excess reserves on average.

where c_t is consumption at time t, θ_t is an i.i.d. random variable that may take the value of 1 (if the household wants to consume) and 0 (otherwise) and $0 < \beta < 1$ is a discount factor. The single period utility function u( ) is differentiable and strictly concave with $u'(0) = \infty$. The amount of consumption depends on the realizations of three shocks: the aggregate shock, ($\tilde{s}_t$), the (idiosyncratic) market at which the buyer participates ($\tilde{j}_t \leq \tilde{s}_t$) and the (idiosyncratic) type of the household ($\tilde{\tau}_t$). The type of the household is determined both by its desire to consume and the traveling status of the buyer. The buyer is of type 0 if he wants to consume ($\theta = 1$) and he is non-traveler, he is of type 1 if he wants to consume and he travels and of type 2 if he does not want to consume ($\theta = 0$).

The household starts the period with A_t dollars. It owns a firm and a bank which are valued at Y_t and Z_t dollars, respectively. It sells (inellastically) a unit of labor for W_t dollars. It first goes to the securities market and buys or sells (from and to the "market") contingent dollars that will be delivered at the end of the period. The contingencies are on the realizations of the aggregate shock, ($\tilde{s}_t$), the (idiosyncratic) market at which the buyer participates ($\tilde{j}_t$) and the (idiosyncratic) type of the buyer ($\tilde{\tau}_t$).

The price of a dollar that will be delivered if the realization of ($\tilde{s}_t, \tilde{j}_t, \tilde{\tau}_t$) is (s,j,τ) is denoted by $n_t(s,j,\tau)$ and the number of dollars that will be delivered in this case is $\zeta_t(s,j,\tau)$. Note that $\zeta_t(s,j,\tau)$ is defined only for $s \geq j$. For notational convenience we set $\zeta_t(s,j,\tau) = 0$ for $s < j$. The total cost of these contingent claims is thus,

$\Sigma_s\Sigma_j\Sigma_\tau\, n_t(s,j,\tau)\zeta_t(s,j,\tau)$. The amount of money that the household carries after the end of transactions at the securities market is:

(18) $$BD_t = A_t + Y_t + Z_t + W_t - \Sigma_s\Sigma_j\Sigma_\tau\, n_t(s,j,\tau)\zeta_t(s,j,\tau).$$

After the end of trade in the securities market, one member of the household goes to work (the worker) and the other member goes to the bank (the buyer). At the bank, the buyer learns his type, τ, and chooses the amount of spendable dollars, $SD_t(\tau)$.

After the end of bank transactions buyers go to their shopping island (non-travelers stay in their island of origin and travelers go to the other island). In each island, buyers form a line. The place of an individual buyer in the line is exogenously determined by an i.i.d. lottery. Buyers arrive at the goods market sequentially according to their place in line. Buyers cannot resell goods.

Upon arrival at the market-place, buyers find out the lowest price, $P_t(j)$, at which goods are still available. They thus learn that they participate in market j. A buyer of type τ who participates in market j, chooses to spend $E_t(j,\tau)$ dollars which buy:

(19) $$c_t(j,\tau) = E_t(j,\tau)/P_t(j),$$

units of consumption. The money in advance constraint is:

(20) $$E_t(j,\tau) \leq SD_t(\tau).$$

The asset transition equation for the household is:

(21) $$A_{t+1}(s,j,\tau) = (1+i_{BDt})BD_t - i_{SDt}(\tau)SD_t(\tau_t) - E_t(j,\tau) + \zeta_t(s,j,\tau),$$

where i_{BDt} and i_{SDt} are shadow interest rates: i_{BDt} is the interest applicable to BD_t and $i_{SDt}(\tau_t)$ is the interest cost of a spendable dollar, which is type dependent.

The household chooses $\zeta_t(s,j,\tau)$, $SD_t(\tau)$ and $E_t(j,\tau)$ to maximize the expected value of (17) with respect to (18) - (21). A dynamic programming formulation of the household's maximization problem is in Appendix A. We now turn to specify the shadow interest rates as a function of the bank's rates: i_L, i_T and i_D.

The shadow interest rates:

The shadow interest rate on BD can be computed by holding SD constant and adding a dollar to BD. If the buyer borrows from the bank (SD > BD), a dollar added to BD will reduce the amount of loans by one dollar and will cut the interest cost by i_L. If he does not borrow, a dollar added to BD will simply be deposited at the interest $i_T = i_L$. Thus, the shadow interest rate applicable to BD is: $i_{BD} = i_L$.

We now turn to specify the interest cost of a spendable dollar. For type 2 consumers, SD = 0 and the specification of the interest cost is superfluous. For types 1 and 0 the interest cost depends on the exact specification of the money in advance constraint for travelers and non-travelers.

The (generic) buyer chooses the amount of loans, ll, and allocates the total of BD + ll between cash, cu, demand deposits, dd, and time deposits, td. Thus,

$$cu + dd + td = BD + ll. \tag{22}$$

A buyer who travels must satisfy the cash-in-advance constraint:

(23) $$E \le cu \ ,$$

where E is the nominal expenditures on goods. A buyer who does not travel, must satisfy the less stringent constraint:

(24) $$E \le cu + dd.$$

The money in advance constraint (20) takes the form in (23) for a traveler and (24) for a non-traveler. Accordingly, the amount of spendable dollars, SD, is the right hand side of (23) for a traveler and of (24) for a non-traveler.

A traveler who wants to add a dollar to the spendable amount and has no time deposits ($SD \ge BD$) will have to borrow the additional dollar and add it to cash. The interest cost is i_L. If he has time deposits ($SD < BD$), he will withdraw the dollar from time deposits and loose i_L. Thus, $i_{SD}(1) = i_L$. A non-traveler who wants to add a dollar to his spendable amount, will take a loan and deposit the dollar in a checkable account if $SD \ge BD$. The net interest cost for doing that is $i_L - i_D$. If $SD < BD$ he will transfer the dollar from time to demand deposits and the interest cost is also $i_L - i_D$. Thus,

(25) $$i_{SD}(0) = i_L - i_D; \ i_{SD}(1) = i_L.$$

This difference in the shadow price of a spendable dollar turns out to be crucial for generating endogenous fluctuations in M1.

3.5 Equilibrium

We assume that when $\tilde{\phi} = \phi_s$ the first s markets open. The amount of money that will be spent in each market is determined endogenously in the following way.

Assuming that the fraction of travelers and non-travelers is the same in all markets, total amount of expenditures per seller if $\tilde{\phi} = \phi_s$ is given by:

(26) $$TE(s) = (1 - \alpha)[\phi_s \Sigma_{j \leq s} E(j,0) + (1 - \phi_s)\Sigma_{j \leq s} E(j,1)].$$

Without loss of generality, we assume $E(j,0) \geq E(j,1)$. (Otherwise we redefine indices). This implies: $TE(s) \leq TE(s+1)$.

We say that the minimum amount of dollars that will arrive, TE(1), are spent in market 1. If more than TE(1) dollars arrive, then market 2 opens. If more than TE(2) dollars arrive, then market 3 opens and so on. The nominal demand per seller in market s is:

(27) $$\Delta(s) = TE(s) - TE(s-1),$$

where we set: TE(0) = 0.

Note that the amount of dollars that will be spent in each market is endogenous and depends on the choices of $E(s,\tau)$. In the special case in which $E(j,0) = E(j,1)$ for all j, TE(s) = TE(1) for all s and

$\Delta(s) = 0$ for all s > 1. This case of full capacity utilization occurs when the average reserve ratio, rr, is unity, because in this case (16) implies that $i_D = 0$ and (25) implies $i_{SD}(0) = i_{SD}(1) = i_L$. However, if $E(j,0) > E(j,1)$ for all j, then $\Delta(s) > 0$ for all s.

The probability that a dollar will be spent at market j when exactly s markets open is:

(28) $$\upsilon_j^s = \Delta(j)/TE(s).$$

Market clearing requires:

(29) $$\Delta(s) = P(s)k(s), \quad \text{for all } s.$$

We assume that the representative household starts with a nominal wealth A = H, where H is outside money and define equilibrium as follows.

A <u>stationary symmetric equilibrium</u> for the reserve requirement rr ($0 \le rr \le 1$) and A = H, is a vector [W, n_s, P(s), k(s), y(s), i_L, i_D, i_T, $i_{SD}(\tau)$, LL(s), TD(s), DD(s), z(s), L, T, D, Y, Z, n(s,j,τ), ζ(s,j,τ), BD, SD(τ), E(j,τ), Δ(s), υ_j^s, A'(s,j,τ); s,j = 1,...,S; and τ = 0,1,2] such that:

(a) Δ(s) and υ_j^s satisfy (30) - (32), y(s), z(s) are defined by (13) and (15); Y is defined by (10); L, T, D, Z are defined by (16), $i_{SD}(\tau)$ satisfy (29) and $A'(s,j,\tau) = (1+i_L)BD - i_{SD}(\tau)SD(\tau) - E(j,\tau) + \zeta(s,j,\tau)$.

(b) Maximizing behavior

Given (W, n(s,j,τ), i_L, i_D, P(s), Y, Z) the quantities ζ(s,j,τ), BD, SD(τ), E(j,τ) solve the household's maximization problem (maximizing the expected value of (21) subject to (18)-(25): see Appendix A for a complete dynamic programming formulation);

Given (W, n_s, P(s)), the quantities l and k(s) solve the firm's problem (14);

Given (i_L, i_D, i_T) the expected quantities (L, D, T) solve the bank's problem (18).

(c) Market clearing

Securities:

$$\Sigma_{j\leq s}\, \upsilon_j^s\, [(1-\alpha)\phi_s\zeta(s,j,0) + (1-\alpha)(1-\phi_s)\zeta(s,j,1) + \alpha\zeta(s,j,2)]$$

= z(s) + y(s), for all s;

The left hand side is the total amount of dollars claimed when s markets open and the right hand side is the supply of dollars in this case.

Money:

BD = H;

$$\Sigma_{j\leq s}\, \upsilon_j^s\, [(1-\alpha)\phi_s A'(s,j,0) + (1-\alpha)(1-\phi_s)A'(s,j,1) + \alpha A'(s,j,2)] = H,$$

for all s;

This says that H will always be willingly held. The first requirement insures that H is willingly held after the end of transactions in the securities market. The second requirement insures that outside money is willingly held by the household at the end of the period. (The first order conditions for the banks and the travelers insure that money is willingly held during the period).

Banks:

$LL(s) = (1 - \alpha)[\phi_s \max\{0, SD(0) - H\} + (1 - \phi_s)\max\{0, SD(1) - H\}];$

$DD(s) = \phi_s(1 - \alpha)SD(0);$

$TD(s) = \alpha H + (1 - \alpha)\{\phi_s \max(0, H - SD(0)) + (1 - \phi_s)\max(0, H - SD(1))\}$

On the left hand side are banks' supplies. On the right hand side are aggregate demands. Since non-consumers do not take loans we aggregate the demand for loans of types 0 and 1 only (first condition). Since $i_T \geq i_D$, travelers use time deposits rather than demand deposits, in case they choose SD < BD. Therefore, only non-travelers use checkable deposits (second condition). The last condition aggregates demand for time deposits over all types.

Goods: $\Delta(s) = P(s)k(s)$, for all s;

Markets which are opened are cleared.

Labor: $l = 1$.

Stationarity of wealth distribution:

$A'(s,j,\tau) = H$ for all s, j, τ.

In Appendix B we show the following main results.

Proposition 1: There exists a unique stationary symmetric equilibrium.

Proposition 2: The allocation obtained when rr = 1 is Pareto efficient.[12]

The intuition for the second result is that setting rr = 1 eliminates the endogenous fluctuations in M1 and leads to full capacity utilization. In detail, when rr = 1, $i_D = 0$ and $i_{SD}(0) = i_{SD}(1) = i_L$. Therefore, E(j,0) = E(j,1) for all j, TE(s) = TE(1) for all s and

$\Delta(s) = 0$ for all s > 1. Market clearing implies that all the capacity is supplied to the first market and since this market always open, capacity is fully utilized. When rr < 1, $i_D > 0$ and

$i_{SD}(0) = i_L - i_D \neq i_{SD}(1) = i_L$. In this case, E(j,0) > E(j,1) for all j, and $\Delta(s) > 0$ for all s. Strictly positive capacity will be supplied to all markets and capacity in markets which do not open is wasted.

4. CONCLUSIONS

We have shown that endogenous fluctuations in M1 lead to fluctuations in output, as argued by Friedman. In our UST model fluctuations in the currency/deposit ratio create endogenous monetary shocks. These fluctuations are non-neutral here for the same reason that fluctuations in the money supply are non-neutral in other UST models: actual trade occurs before all the information about the current money supply and demand is revealed. To insure full capacity

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[12] This result uses the assumption that labor supply is inelastic. Otherwise, the Friedman zero nominal interest rate rule is required to achieve efficiency.

utilization, sellers must know the current demand. In our model, this is achieved by imposing a 100% reserve requirement without paying interest on reserves.

However, when the nominal interest rate is positive, there are incentives to circumvent the money-in-advance constraint. The use of credit cards is a good example.[13] In terms of the model we may view credit cards as a way of using time deposits to buy goods. In general, there will be three types of buyers: cash users, check users and credit-card users. And the difference in the interest cost of consumption cannot be entirely eliminated. If we adopt the 100% reserve requirement and pay no interest on reserves, we eliminate the cost difference between cash users and check users but maximize the cost difference between these two types and credit users. Paying interest on reserves will eliminate the difference in cost between credit-card users and check users but cash users will pay more.[14]

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[13] In our model, prohibiting the use of credit cards combined with the 100% reserve requirement, will ensure full capacity utilization. But it is not clear whether such regulations can be enforced.

[14] Zero reserve requirements, if possible, may achieve a similar outcome.

APPENDIX A

Dynamic programming formulation

Here we specify the dynamic programming problem faced by the household.

There are three sessions of trade. At the first session there is trade in securities and labor. The household brings from the previous period a nominal wealth A, and after completion of transactions (choosing ζ, receiving profits from the firm and the bank, and selling labor) its wealth at the end of the period is: A_1 dollars. The buyer then goes to the bank and chooses SD, which changes his wealth to A_2. Finally, the buyer goes to the goods market and chooses E, changing the wealth to A_3 which is carried over to the next period as A'. Thus, A_{i-1} denotes the (random, end of period) wealth at the beginning of session i. We use v_i to denote the maximum expected utility in session i, which depends on A_{i-1}.

Using the logic of dynamic programming we start from the last session.

At the goods market:

We use $A_2(\tilde{s},j,\tau)$ to denote the end of period wealth of the household at the beginning of trade in the goods market j. This value depends on the yet unknown realization of $\tilde{s}$ and the indices j and τ (which are known at this stage) because contracts signed at previous stages are contingent on these variables. Note that $A_2(\tilde{s},j,\tau)$ is defined only for realizations $s \geq j$. For notational convenience we set $A_2(s,j,\tau) = 0$ for $s < j$. The same convention is adopted below for similar cases.

We use the vector:

$A_2(\bullet,j,\tau) = \{A_2(1,j,\tau), A_2(2,j,\tau), \ldots, A_2(S,j,\tau)\}$, to denote all possible realizations of $A_2(\tilde{s},j,\tau)$. The buyer faces the price $P(j)$ and chooses to spend $E(j,\tau)$ dollars subject to the constraint: $E(j,\tau) \leq SD(\tau)$. The end of period nominal wealth after spending is given by $A'(\tilde{s},j,\tau) = A_2(\tilde{s},j,\tau) - E(j,\tau)$. This amount yields next period the expected utility $EV(A'(\tilde{s},j,\tau))$. We require that bankruptcies do not occur so that $A'(\tilde{s},j,\tau)$ is positive.

The buyer who found out his type in the previous stage, has used Bayes law to update the probability of state s in a way which will be described below. As a result, buyer of type τ assigns the probability $\pi_s(\tau)$ to the event: $\tilde{\phi} = \phi_s$. When the buyer finds that he participates in market j and that $\tilde{s} \geq j$, he updates the probability again: $Prob(\tilde{s} = s|\tilde{s} \geq j, \tau) = (\pi_s(\tau)/q_j)$. Taking $SD(\tau)$ and $A_2(\bullet,j,\tau)$ as given the buyer chooses $E(j,\tau)$ to solve:

(A1) $$v_3(A_2(\bullet,j,\tau),SD(\tau),j) =$$

$$\max \{\theta u(E(j,\tau)/P(j)) + \beta\Sigma_{s\geq j} (\pi_s(\tau)/q_j)V(A'(s,j,\tau))\}$$

s.t

$$0 \leq E(j,\tau) \leq SD(\tau) ;$$

$$A'(s,j,\tau) = A_2(s,j,\tau) - E(j,\tau) \geq 0.$$

At the bank:

When the buyer learns his type, he uses Bayes rule to update the probability that $\tilde{\phi} = \phi_s$. This probability conditional on τ, is:

(A2) $$\pi_s(0) = \{[(\Pi_s\phi_s)/\psi],$$

$$\pi_s(1) = [(\Pi_s(1-\phi_s))/(1-\psi)] \text{ and } \pi_s(2) = \Pi_s,$$

where $\psi = \Sigma_s \Pi_s \phi_s$ is the probability of being a non-traveler given $\theta = 1$.[15]

Before transacting at the bank, the end of period wealth is $A_1(\tilde{s},\tilde{j},\tau)$. After the completion of transactions at the bank, the end of period wealth is:

(A3) $$A_2(\tilde{s},\tilde{j},\tau) = A_1(\tilde{s},\tilde{j},\tau) - i_{SD}(\tau)SD(\tau).$$

At the goods market, the expected utility of the household which participates in market j is: $v_3(A_2(\bullet,j,\tau),SD(\tau),j)$. However, at the banking stage, $\tilde{j}$ is still a random variable. To compute expectations, we use υ_j^s to denote the probability that the buyer will participate in market j given that $s \geq j$ markets open. Using this notation the probability that a buyer of type τ assigns to the event that he will participate in market j is given by $f_j(\tau) = [\Sigma_{s\geq j}\ \upsilon_j^s \pi_s(\tau)]$. (Note that the index j is not relevant for type 2 but we include it for notational convenience). Therefore the maximum expected utility at the beginning of the third session is:
$Ev_3(A_2(\bullet,\tilde{j},\tau),SD(\tau),\tilde{j}) = \Sigma_j\ f_j(\tau)v_3(A_2(\bullet,j,\tau),SD(\tau),j)$.

At the bank, the buyer chooses $SD \geq 0$ to solve:

(A4) $$v_2(A_1(\bullet,\bullet,\tau),\tau) = \max_{SD} \Sigma_j\ f_j(\tau)v_3(A_2(\bullet,j,\tau),SD(\tau),j)$$
$$\text{s.t. (A3)},$$

where $A_1(\bullet,\bullet,\tau)$ is the matrix of all possible realizations of

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[15] For example,
$prob(\tilde{\phi} = \phi_s |\ \tau = 0) = prob(\{\tilde{\phi} = \phi_s\}\cap\{\tau = 0\})/prob(\tau = 0) = (\Pi_s\phi_s)/\psi$.

$A_1(\tilde{s},\tilde{j},\tau)$.

<u>At the securities market:</u>

The household starts with A dollars and after receiving the profits from the firm and the bank and selling labor it has $A + Y + Z + W$ dollars. It then chooses BD and ζ out of the budget constraint (22) in the text to maximize the expected value of $v_2(A_1(\bullet,\bullet,\tilde{\tau}), \tilde{\tau})$. The shadow interest rate for BD is i_L and therefore the asset transition equation is:

$$\text{(A5)} \qquad A_1(\tilde{s},\tilde{j},\tilde{\tau}) = (1 + i_L)BD + \zeta(\tilde{s},\tilde{j},\tilde{\tau}).$$

Before learning its type, the household chooses $\zeta(\tilde{s},\tilde{j},\tilde{\tau})$ to maximize $Ev_2(A_1(\bullet,\bullet,\tilde{\tau}), \tilde{\tau})]$. The household thus solves:

$$\text{(A6)} \quad V(A) = \max\ (1 - \alpha)[\psi v_2(A_1(\bullet,\bullet,0), 0) + (1-\psi)v_2(A_1(\bullet,\bullet,1), 1)] + \alpha v_2(A_1(\bullet,\bullet,2), 2)$$

s.t. (18) in the text and (A5),

where as before ψ denotes the probability that a buyer will not travel given that he wants to consume.

<u>Existence and Characterization of equilibrium</u>

We start by valuing an additional dollar at the beginning of the period under the assumption that an equilibrium exists. This is:

(A7) $V'(A) =$

$$\Sigma_s \Pi_s \Sigma_{j \leq s} \upsilon_j^s \{(1 - \alpha)\phi_s\{\beta i_D V'(A'(s,j,0)) + u'(E(j,0)/P(j))/P(j)\}$$

$$+ (1 - \alpha)(1-\phi_s)u'(E(j,1)/P(j))/P(j)$$

$$+ \alpha\beta(1 + i_L)V'(A'(s,j,2))\}$$

This follows from the envelope argument applied to (A6). The intuition is as follows.

A type 0 buyer cannot do better than deposit the dollar in a checkable account and spend it. This follows from the fact that we have an interior solution. In detail, spending the dollar on consumption will be the strictly preferred option in case the money-in-advance constraint is binding. Since the buyer always buys a strictly positive amount of consumption, when the money-in-advance constraint is not binding the buyer is indifferent between spending the dollar on consumption and carrying it over to the next period. So in either case we may assume that the dollar is spent on consumption. Since the dollar is deposited, the end of period wealth increases by i_D dollars due to the interest on demand deposits and this is valued by: $\beta i_D V'(A'(s,j,0))$. Since the event $\{\theta = 1$, s markets open, and the buyer is a non-traveler who participates in market $j \leq s\}$ occurs with probability $\Pi_s \upsilon_j^s (1 - \alpha)\phi_s$, the second line in (A7) is the total value of the additional dollar to a non-traveler.

A type 1 buyer will take the dollar as cash and spend it. Since the event $\{\theta = 1$, s markets open and the buyer is a traveler who participates in market $j \leq s\}$ occurs with probability $\Pi_s \upsilon_j^s (1 - \alpha)(1 - \phi_s)$, the value from doing it is the third line of (A7).

A type 2 buyer will deposit the dollar in a time deposit at an interest rate i_L. His end of period wealth increases by $(1 + i_L)$

dollars and his expected utility by $\beta(1 + i_L)V'(A'(s,j,2)$. Since the event {$\theta$ = 0, s markets open, and the buyer (fictitiously) participates in market $j \leq s$} occurs with probability $\alpha\Pi_s\upsilon_j^s$, the last expression under the summation on the right hand side of (A7) is the value of an additional dollar to a type 2 buyer.

Since u is concave, it can be shown (following Stokey and Lucas [1989]) that V(A) is concave. Concavity and the market clearing condition:

$$\Sigma_j\upsilon_j^s[(1-\alpha)\phi_s A'(s,j,0) + (1-\alpha)(1-\phi_s)A'(s,j,1) + \alpha A'(s,j,2)] = H$$

for all s,

leads to stationarity: $A'(s,j,\tau) = H$ for all s, j, τ. This follows from the fact that the sum of the weights in the above market clearing condition is unity: $\Sigma_j\upsilon_j^s[(1-\alpha)\phi_s + (1-\alpha)(1-\phi_s) + \alpha] = 1$, for all s.

Stationarity and (A7) imply:

(A8) $$V'(H) = \Sigma_s\Pi_s\Sigma_{j\leq s}\upsilon_j^s\{\phi_s u'(E(j,0)/P(j))[1/P(j)] + (1-\phi_s)u'(E(j,1)/P(j))[1/P(j)]\}/\Gamma,$$

where $\Gamma = [1 - \alpha\beta(1+i_L) - (1 - \alpha)i_D\beta\psi]/(1 - \alpha)$.

Next we use the first order conditions that govern the choice of $SD(\tau)$ to show that in equilibrium:

(A9) $$\Sigma_s\pi_s(\tau)\Sigma_{j\leq s}\ \upsilon_j^s\ u'(E(j,\tau)/P(j))[1/P(j)] = \beta[1 + i_{SD}(\tau)]V'(H).$$

Condition (A9) uses the following reasoning. At the optimum the buyer is indifferent between taking an additional spendable dollar and actually spending it to not doing so (this is true even when he is not constrained by the money in advance constraint, see the reasoning for (A7)). If he spends an additional dollar he will get the additional expected utility from consumption calculated by the left hand side of (A9). The cost of doing so, which is on the right hand side of (A9), arises because he will have $1 + i_{SD}(\tau)$ dollars less at the end of the period. Consistency of conditions (A9) and (A8) requires $(1 + i_L) = 1/\beta$. We will argue soon that this must hold in equilibrium.

The first order conditions that govern the choice of E in the goods market (invoking stationarity) imply:

$$\text{(A10)} \qquad u'(E(j,\tau)/P(j))/P(j) \geq \beta V'(H); \quad \text{with equality if } E(j,\tau) < SD(\tau).$$

Stationarity and the concavity of V( ) imply nominal prices which are actuarially fair:

$$\text{(A11)} \quad n(s,j,0) = \beta(1 - \alpha)\phi_s\Pi_s\upsilon_j^s; \qquad n(s,j,1) = \beta(1 - \alpha)(1-\phi_s)\Pi_s\upsilon_j^s;$$
$$n(s,j,2) = \beta\alpha\Pi_s\upsilon_j^s; \qquad n_s = \Sigma_j\Sigma_\tau\, n(s,j,\tau) = \beta\Pi_s; \qquad \Sigma_s\, n_s = \beta.$$

The absence of arbitrage opportunities implies,

$$\text{(A12)} \qquad (1 + i_L) = 1/\beta.$$

To see why $(1 + i_L) = 1/\beta$, note that the price of a dollar in the next period is $\Sigma_s\, n_s = \beta$ and the implied gross interest rate in the securities market is $1/\beta$. Suppose now that $(1 + i_L) > 1/\beta$. Then a household can choose large BD by selling claims on dollars in the securities market and deposit (BD - SD) at the bank as time deposits, making an unbounded amount of money with certainty. If $(1 + i_L) <$

$1/\beta$, it will choose large negative BD by buying claims on dollars and take loans from the bank to get the desired level of SD.

From (13) in the text and (A12) it follows immediately that:

$$i_L - i_D = rr(\beta^{-1} - 1). \tag{A13}$$

Thus, the interest spread is increasing in the reserve requirement, rr.

The first order condition for an interior solution to the firm's problem (14), implies:

$$q_s P(s) = P(1) = W/\beta. \tag{A14}$$

In Appendix B we use the above result to show that:

Proposition 1: There exists a unique stationary symmetric equilibrium.

We now turn to discuss the optimal choice of reserve requirements. We first show,

Claim 1: If $SD(\tau) \geq SD(\tau')$, then $E(j,\tau) \geq E(j,\tau')$ and vice versa.

This follows from (A10).

Claim 2: When $rr < 1$, $SD(1) < SD(0)$.

To show this Claim, suppose $SD(1) \geq SD(0)$. Then $E(j,1) \geq E(j,0)$ by Claim 1. It follows that travelers spend more both on consumption and on interest than non-travelers. Since prices are actuarially fair (see, [A11]) the strategy of consuming more when traveling is worse than a strategy of consuming an amount that does not depend on the traveling status. To see this point, note that if the mean is the same, concavity of u( ) works in favor of the alternative. Moreover, average consumption is higher under the alternative because interest costs, i_{SD}, are lower. Thus, by contradiction, SD(1) < SD(0).

When rr = 1, (16) implies $i_D = 0$ and (25) implies that both types face the same shadow interest rate for SD: $i_{SD}(0) = i_{SD}(1) = i_L$. Therefore,

Claim 3: When rr = 1, SD(1) = SD(0) and E(s,0) = E(s,1) for all s.

This leads to:

Proposition 2: The allocation obtained when rr = 1 is Pareto efficient.

When rr = 1, Claim 3 and the definitions of Δ in (31) imply that only the first market is active. Therefore, in equilibrium k(1) = 1, k(s) = 0 for all s > 1. Consumption per household is unity and does not depend on the traveling status.

APPENDIX B

We compute a stationary and symmetric equilibrium in the following way. We first arbitrarily choose a vector SD = [SD(0), SD(1)] and compute prices, consumption and the marginal utility of a dollar as functions of SD. We then use these functions to solve for a vector SD that satisfies the first order conditions at the banking session. It turns out that if SD is a solution then λSD is also a solution for all $\lambda > 0$. We use the reserve requirement and BD = H, to scale the SD vector and to show the existence of a unique stationary and symmetric equilibrium.

Proof of Proposition 1

We first define equilibrium in the goods market for a given vector SD = [SD(0), SD(1)].

The vector [(P(1),...,P(S), k(1),...,k(S), E(0,1),...,E(0,S), E(1,1),...,E(1,S), υ_j^s, V'] is an equilibrium in the goods market if:

(B1) $$q_s P(s) = P(1)$$

(B2) $$\Sigma_s \, k(s) = 1$$

(B3) $\Delta(s)/P(s) = k(s)$, where $\Delta(s)$ is from (31) in the text;

(B4) $$E(j,\tau) \leq SD(\tau)$$

(B5) $u'(E(j,\tau)/P(j))/P(j) \geq \beta V'$ with equality if $E(j,\tau) < SD(\tau)$.

(B6) $$V' = \Sigma_s \Pi_s \Sigma_{j\leq s} \, \upsilon_j^s \{(1-\phi_s)u'(E(j,1)/P(j))/P(j) + \phi_s u'(E(j,0)/P(j))/P(j)\}$$

where (after substituting [A12]) $\Gamma = 1 - i_D\beta\psi$, and υ_j^s is given by (28) in the text.

Claim B1: For any SD > 0, there exists a unique equilibrium in the goods market: [(P(1;SD),...,P(S;SD), k(1;SD),...,k(S;SD), E(0,1;SD),...,E(0,S;SD), E(1,1;SD),...,E(1,S;SD), υ_j^S(SD), V'(SD)].

Proof: Let p denote the expected revenue per unit in the goods market. We choose p > 0 arbitrarily and set:

$$P(s;p) = p/q_s. \tag{B7}$$

Lemma B1: Given SD and p, there exists a solution, E(j, τ; SD, p) to:

$$u'(E(j,\tau)/P(j;p))/P(j;p) \geq \beta\Sigma_s\Pi_s\Sigma_{i\leq s}\ \upsilon_i^S\{(1-\phi_s)u'(E(i,1)/P(i;p))/P(i;p) + \phi_s u'(E(i,0)/P(i;p))/P(i;p)\}/\Gamma \tag{B8}$$

with equality if E(j,τ) < SD(τ).

Note that to get (B8) we substitute (B6) into (B5) and therefore (B8) insures that both conditions are satisfied. To show existence of a solution to (B8), we choose κ as our guess for V' and define:

$$\kappa' = \Sigma_s\Pi_s\Sigma_{j\leq s}\ \upsilon_j^S\{(1-\phi_s)\min[u'(SD(1)/P(j;p))/P(j;p),\ \beta\kappa] + \phi_s\min[u'(SD(0)/P(j;p))/P(j;p),\ \beta\kappa]\}/(\beta\Gamma). \tag{B9}$$

If κ is small then $\kappa' = \kappa/\Gamma > \kappa$ since $\Gamma < 1$. If κ is sufficiently large, then $\kappa' = \Sigma_s\Pi_s\Sigma_{j\leq s}\ \upsilon_j^S\{(1-\phi_s)u'(SD(1)/P(j;p))/P(j;p) + \phi_s u'(SD(0)/P(j;p))/P(j;p)\}/(\beta\Gamma) < \kappa$.

By continuity, there exists a fixed point κ(SD, p) of (B9). Since the mapping is monotone, κ(SD, p) is unique.

We now set $E(j, \tau; SD, p) = SD(\tau)$ for all j and τ such that: $u'(SD(\tau)/P(j;p))/P(j;p) \geq \beta\kappa(SD, p)$. Otherwise, $E(j, \tau; SD, p)$ is given by the solution to: $u'(E(j,\tau)/P(j;p))/P(j;p) = \beta\kappa(SD, p)$. Thus we have shown Lemma B1.

Let $TE(s;SD,p)$ and $\Delta(s;SD,p)$ be defined by (30) and (31) when using $E(j, \tau) = E(j, \tau; SD, p)$. Then,

Lemma B2: $\Sigma_S q_S\Delta(s;SD,p)/p$ is decreasing in p.

To show this claim note that:

(a) $E(s, \tau; \lambda SD, \lambda p) = \lambda E(s, \tau; SD, p)$;

(b) $E(s, \tau; SD, p)$ is increasing in SD.

From the definition of Δ in the text (27) and (a) and (b) it follows that:

(a') $\Delta(s;\lambda SD,\lambda p) = \lambda\Delta(s;SD,p)$;

(b') $\Sigma_S q_S\Delta(s;SD,p)$ is increasing in SD.

From (a') and (b') we get for $\lambda > 1$:

$\Sigma_S q_S\Delta(s;SD,\lambda p)/\lambda p < \Sigma_S q_S\Delta(s;\lambda SD,\lambda p)/\lambda p = \Sigma_S q_S\Delta(s;SD,p)/p$.

This completes the proof of Lemma B2.

To continue with the construction of equilibrium in the goods market, we note that the real demand in market s at the prices $P(s;p)$, is: $k^d(s;p) = \Delta(s;SD,p)/P(s;p) = q_S\Delta(s;SD,p)/p$. The total real demand is:

$K^d(p) = \Sigma_S k^d(s;p) = \Sigma_S q_S\Delta(s;SD,p)/p$. Since total supply is unity, market clearing requires:

(B10) $$K^d(p) = 1.$$

By Lemma B2, $K^d(p)$ is continuously decreasing. When p is arbitrarily large, K^d is arbitrarily small and vice versa. This leads to a unique solution of the expected revenue per unit: p(SD). We can now compute equilibrium magnitudes. This completes the proof of Claim B1.

To compute a stationary symmetric equilibrium we use the following goods market equilibrium magnitudes:

$P(s;SD) = P(s;p(SD))$;

$E(j, \tau;SD) = E(j, \tau;SD,p(SD))$;

$V'(SD) = \kappa(SD, p(SD))$;

$\upsilon_j^s(SD) = \Delta(j;SD,p(SD))/TE(s;SD,p(SD))$

We now look for a vector SD that will satisfy the first order condition at the banking session, given P(s;SD), E(j, τ;SD) and V'(SD). We denote the expected marginal utility of a dollar to a type τ buyer, given that s markets open and the dollar is actually spent, by:

(B11) $$X(s,\tau,SD) = \Sigma_{j\leq s}\ \upsilon_j^s(SD)\ u'(E(j,\tau;SD)/P(j;SD))/P(j;SD).$$

The first order conditions at the banking session (A6) are:

(B12a) $$\Sigma_s[(\Pi_s\phi_s)/\psi]X(s,0,SD) = \beta(1 + i_{SD}(0))V'(SD)$$

(B12b) $$\Sigma_s[(\Pi_s(1-\phi_s))/(1-\psi)]X(s,1,SD) = \beta(1 + i_{SD}(1))V'(SD)$$

<u>Lemma B3:</u> There exists a vector SD that solves (B12).

Note that if SD solves (B12) then λSD is also a solution for any $\lambda > 0$. Note also that (B6) is a linear combination of (B12a) and (B12b). To see this multiply (B12a) by ψ and (B12b) by $(1-\psi)$ and add the two while using $(1 + i_L) = 1/\beta$, to get (B6), which holds by construction. We can therefore look at a single equation, say (B12a), normalize SD(1) = 1 and solve for SD(0). Let us rewrite (B12a) as:

$$\text{(B13)} \quad \Sigma_S[(\Pi_S\phi_S)/\psi]X(s,0,[SD(0), 1]) = \beta(1 + i_{SD}(0))V'([SD(0), 1]).$$

Note that when SD(0) is large, the consumption of type 1 goes to zero and X(s,0,[SD(0), 1]) is large because we assume: $u'(0) = \infty$. Since V' is a linear combination of X(s,0,[SD(0), 1]) and X(s,1,[SD(0), 1]), it follows that V' is large. In particular it is larger than the LHS of (B13). The opposite holds when SD(0) is small. Thus there exists a solution: $\hat{SD} = [\hat{SD}(0), 1]$. This completes the proof of Lemma A3.

We now scale $\hat{SD}$ to satisfy the reserve requirements. For this purpose, we characterize all combinations of SD = [SD(0), SD(1)] that satisfy the reserve requirement. In equilibrium when BD = H, the non-consumers (a fraction α of the population) deposit BD in time deposits. In addition, consumers deposit any amount beyond SD in time deposits. Thus,

$$\text{(B14)} \quad T = \alpha H + (1 - \alpha)\{\psi \max(0, H - SD(0)) + (1 - \psi)\max(0, H - SD(1))\}.$$

Consumers who choose $SD(\tau) > H$, take loans and therefore:

$$\text{(B15)} \quad L = (1 - \alpha)\psi \max(0, SD(0) - H)$$

$$+ (1 - \alpha)(1 - \psi)\max(0, SD(1) - H).$$

Only non-travelers use demand deposits and therefore:

(B16) $$D = (1 - \alpha)\psi SD(0).$$

In equilibrium there will be no excess reserves (on average) and therefore (using [17] in the text):

(B17) $$(1 - rr)D + T = L.$$

Substituting (B14)-(B16) into (B17) yields:

(B18) $$\Psi_0 SD(0) + \Psi_1 SD(1) = 1.$$

where, $\Psi_0 = (1 - \alpha)rr\psi/H$; $\Psi_1 = (1 - \alpha)(1 - \psi)/H$. Thus we can scale the solution $\hat{SD}$ by $1/(\Psi_0\hat{SD}(0) + \Psi_1)$ to get a stationary symmetric equilibrium. With this we have shown, existence and uniqueness of a stationary symmetric equilibrium.

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