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*Hierarchical Human Capital and Economic Growth: Theory and Evidence**

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Abstract

The sequential process by which basic human capital is transformed into more advanced varieties generates a hierarchical structure that is distinct from physical capital. For example, production of a unit of PhD human capital typically requires the prior sequential production of primary, secondary, and tertiary level human capital. In contrast, advanced physical capital may be produced directly from primordial material and need not, in general, pass through a sequence of more primitive productive incarnations. In this paper we imbed an N-level human capital hierarchy in a growth model and demonstrate that hierarchical structure generates an optimal investment programs with phases of stock depletion and expansion in the stocks of the various levels of human capital. We then take the implications of the model to data from a diverse sample of countries and find patterns of stock expansion and contraction predicted by the theoretical model. We also illustrate how accounting for hierarchical human capital formation might contribute to the empirical growth literature.

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1. Introduction

In the process of human capital formation individuals generally pass through a sequence of pre-determined levels: from primary level, to secondary, and occasionally beyond. This hierarchical system is ubiquitous in modern education systems and is therefore a key element in the process of economic development. Nevertheless, most theoretical formulations ignore this hierarchical structure and simplify the dynamics of human capital by using an aggregate production technology similar to those used to represent the production of other capital goods (See for example Bénabou (1996, 2003) or Lucas (1988)).[1]

In this paper, we present a model of hierarchical human capital accumulation and show how it can be used in both theoretical and empirical growth applications. The empirical implications are potentially important as we demonstrate that disregard of the hierarchical nature of human capital accumulation in growth equations estimates will likely lead to biased coefficients. This result speaks directly to the perplexing negative human capital coefficients that appear regularly in the empirical growth literature.[2]

As defined here, a human capital hierarchy is an ordered sequence of qualitatively distinct human capital types that are "built" in sequential, cumulative fashion. For example, a unit of primary human capital is created by the transformation of primordial human material. To obtain a unit of secondary level human capital the primary unit (which is itself potentially productive) is subject to a further transformation. More generally, in an N-level hierarchy the creation of level i human capital requires level $i-1$ human capital as an input.

Of course, it is possible for an individual with a tertiary education to work productively in a capacity that utilizes only their secondary (or primary) level human capital. However, assuming that at a moment in time a person can only be employed in one capacity (e.g., cannot work *simultaneously* as a street sweeper and an engineer) they enter the production function in one capacity or the other. Moreover, since the transformation of human capital from level i to level $i+1$ is costly, the optimal program will never entail utilizing human capital in less than its highest capacity. As we will focus on initial conditions where advanced human capital is relatively scarce, we need not be concerned that the optimal program will require deconstruction of a physicist to a laborer. We will elaborate further on these issues when the formal model is introduced.

Two fundamental properties of the human capital that occupies our hierarchy are qualitative distinctiveness and intermediate productivity. Qualitative distinctiveness implies that "advanced" human capital cannot be acquired by simply collecting enough "basic" human capital. In a formal sense, this means that the different levels of human capital enter the production function as distinct inputs. As an illustration of this qualitative distinctiveness, consider the case where a unit of basic human capital corresponds to a primary school graduate and advanced human capital to a PhD. physicist. It is clear that ten (or for that matter a hundred) primary school graduates do not constitute a physicist. However, a physicist *can be created* from a primary school graduate given the requisite additional investment. The primary school graduate is the raw material, which when combined with the advanced investment technology (i.e., a University education), yields a unit of advanced human capital.

The second fundamental property of human capital in our model is intermediate productivity. As the human putty traverses the hierarchy it may cease transformation at any stage and enter the production function. A "half-built" PhD (a secondary school graduate) is productive in a way that a half-built airplane is not.[3] An implication of this structure is a stock dependence between adjacent levels in the hierarchy. Specifically, investment in a particular hierarchy level generates a depletion effect on the next lower hierarchy level. As the optimal program entails distinct regimes of focused human capital investment (on different levels of human capital) this effect migrates through the hierarchy generating a pattern of stock depletion and expansion.

The evolution of the human capital hierarchy depends critically on initial conditions. In this paper we focus both empirically and analytically on initial conditions consistent with those found in many developing countries, and in the western industrialized countries at earlier stages of development – relative scarcity of high level human capital. While this orientation unifies our empirical and analytical focus we wish to emphasize that the implications of hierarchical structure for growth remain relevant at all stages of development.

The reminder of the paper is organized as follows. Section 2 develops the general N-level hierarchical human capital growth model and characterizes the steady-state. Section 3 utilizes the International Data Base on Educational Attainment to explore the empirical implications of the hierarchical model. Section 4 provides interpretation and concludes.

2. Investment in Hierarchical Human Capital: The General Model

Consider a continuous time setting with an N-level human capital hierarchy.[4] Denote the stock of human capital of hierarchy level i as H_i, where $i = 1, . . ., N$. Interpret H_1 as the most basic and H_N as pinnacle human capital. Each of these stocks can be interpreted as the population with the corresponding level of education as the highest attained. Let $H(t)$ be the $Nx1$ vector of human capital stocks at time t. We suppress the t argument when the clarity constraint permits. These human capital stocks are used to produce flow output through time, Y, as described by the production function:

$$(1) \qquad Y = f(H), \qquad f_i > 0, \;\; f_{ii} \leq 0, f_{ij} \geq 0; \;\; i,j = 1, . . ,N.$$

Let x_i denote investment in human capital level i with $x_i \geq 0$, for all i. These investments can be interpreted as education expenditures. The following equations of motion reveal the structure of the hierarchy:[5]

$$(2a) \qquad \dot{H}_i = x_i - x_{i+1} \qquad for \;\; i = 1, . . ., N\text{-}1$$

$$(2b) \qquad \dot{H}_N = x_N.$$

Note the depletion effect of the hierarchical system. As reflected in the equations of motion, x_i depletes H_{i-1} for $i= 2, . . ., N$. So that focus can be directed to the effect the hierarchical structure, the relative price (opportunity cost) of investment in H_i is normalized to 1 for all i.[6]

Output can be consumed or invested. Consumption, denoted by c yields utility flow $U(c)$ where $U'(c) > 0$ and $U''(c) < 0$. Again, to lay bare the implications of hierarchical structure there is no borrowing, lending, or depreciation. Hence,

$$(3) \qquad c = f - \sum_{i=1}^{N} x_i \;\; \geq 0.$$

What distinguishes this problem from the standard investment problem is the relationship between human capital stocks. As noted, any increase in the stock of H_i for $i > 1$ involves an equal decrease in the stock of H_{i-1}. For example, suppose H_i denotes the population with baccalaureate degree as the highest education level attained and H_{i-1} those with high school. Then an increase in H_i is matched by an equal decrease in H_{i-1}, all else equal. Of course, college graduates retain their high school degrees, but at moment in time they work in one capacity or another – as noted, an optimal program will not waste resources transforming human capital unless it will be employed at its highest level. Increments to H_1 can be thought of as coming from an underlying stock of an unproductive resource, e.g., untrained children, sufficiently large that it imposes no binding constraint.

Our model is solved from a planner's perspective. Since the model contains no externalities the decentralized solution will be identical to our centralized solution. Letting r denote the discount rate the planners' goal is to maximize the present discounted value of utility:

$$\text{(4)} \qquad \max_{c \geq 0,\ x_i \geq 0} \int_0^{\infty} U(c) e^{-rt} dt \qquad i = 1, \ldots, N;$$

Subject to: *(1) – (3), initial conditions* $H_i(0) > 0$, *and* $x_i \geq 0$.

The present-value Hamiltonian for this problem is:

$$\text{(5)} \qquad \max_{x_i} \mathcal{H} = U(c) + \sum_{i=1}^{N-1} \lambda_i [x_i - x_{i+1}] + \lambda_N x_N + \sum_{i=1}^{N} \theta_i x_i,$$

where the λ_i's are the costate variables, and θ_i's are non-negativity multipliers. Noting that $dc/dx_i = -1$ for all i, the first order conditions are:

$$\text{(6)} \qquad -U' + \lambda_1 + \theta_1 = 0$$

$$\text{(7)} \qquad -U' - \lambda_i + \lambda_{i+1} + \theta_{i+1} = 0, \qquad i = 1, \ldots, N-1$$

$$\text{(8)} \qquad \dot{\lambda}_i = r\lambda_i - U' f_i \qquad i = 1, \ldots, N$$

As a point of reference we first solve for an interior solution ($\theta_i = 0$). Recursion of equation 7 and further manipulation of *(6)-(8)* then yields the following pattern:

$$(9) \qquad \lambda_i = i\,U'.$$

Differentiating with respect to time yields:

$$(10) \qquad \dot{\lambda}_i = iU''\dot{c} \qquad i = 1,\ldots, N.$$

Using *(8) – (10)* provides the following relationship:

$$(11) \qquad \frac{U''}{U'}\dot{c} = \left(\frac{1}{i}\right)\left[ir - f_i\right] \qquad i = 1,\ldots,N,$$

which in turn yields:

$$(12) \qquad \frac{f_i}{i} = \frac{f_j}{j} \qquad \forall\, i, j = 1,\ldots, N.$$

Equation *(12)* is the interior steady-state marginal productivity relationship in the hierarchical environment.[7] Instead of equating the value of inputs' marginal products, as in non-hierarchical settings, the optimal program requires equating the *ratios* of marginal products and hierarchy positions. The hierarchy positions in the denominators of *(12)* reflect the cost of the multiple transformations required to traverse the human capital hierarchy. Thus to satisfy *(12)*, *level N* human capital must have a marginal product *N* times greater than *level 1* human capital in the steady state.

To obtain further insight into *(12)* consider any adjacent ratio pair: $f_i / i = f_{i+1} /(i+1)$ for $i = 1, 2, \ldots, N-1$. Cross multiplying and subtracting 1 from both sides yields: $\frac{f_{i+1} - f_i}{f_i} = \frac{1}{i}$. So the percentage change in marginal product from traversing each level of the hierarchy is one over the hierarchy level in the steady state. An additional manipulation establishes the following relationship in the steady state: $\frac{f_i}{i} = f_{i+1} - f_i$. Substituting this expression into *(12)* yields:

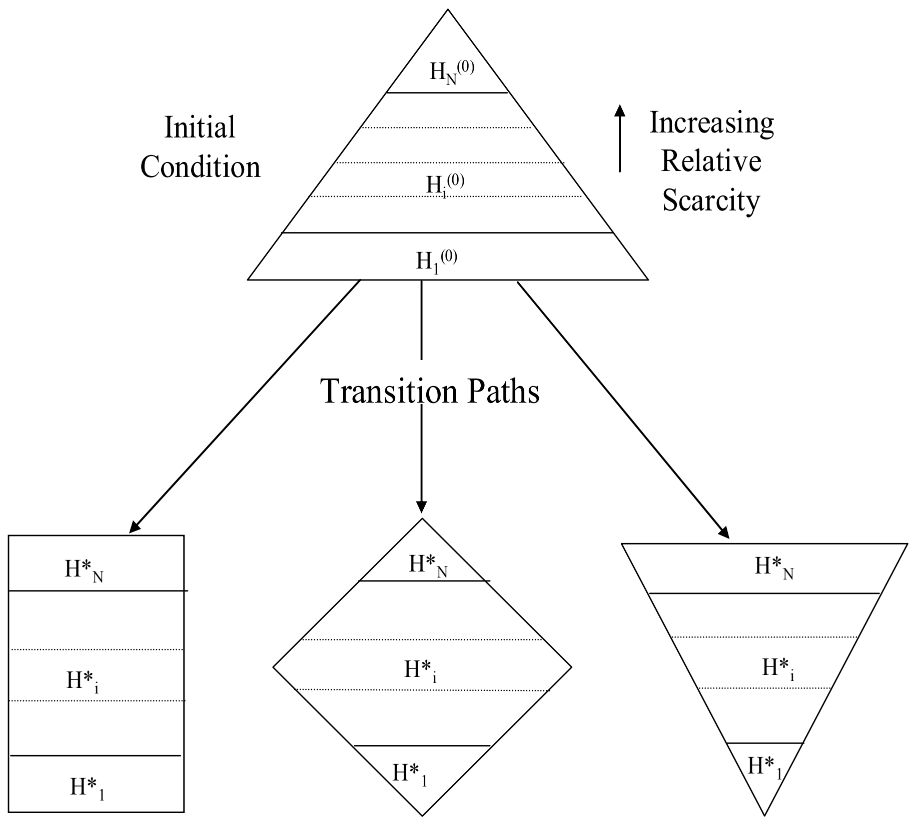
(13) $$f_2 - f_1 = f_3 - f_2 = \ldots\ldots = f_N - f_{N-1}.$$

Equation *(13)* provides a further illustration of the depletion effect in human capital hierarchy. When a unit of human capital is transformed to from level i to level $i+1$ the net change in output is $f_{i+1} - f_i$. The stocks of human capital that satisfy *(12)* or *(13)* depend, of course, on the properties of the production function. We denote a vector of steady state human capital stocks that satisfy *(12)* (and hence *13*) by H^*, with H_i^* the $i^{\underline{th}}$ element of H^*.

The steady state condition *(12)* has a visual representation that will be useful for understanding the evolution of income distribution on the transition path. In particular, one can represent alternative steady state human capital distributions with "pyramids" of various shapes. For example, a traditional pyramid with a wide base and narrow pinnacle represents a large steady-state stock of basic human capital (H_1) and relatively smaller stocks at progressively higher levels in the human capital hierarchy: $H_1^* > H_2^* > \ \ldots > H_N^*$. This would be the shape of the steady state human capital pyramid if *f(H)* were Cobb-Douglas with equal coefficients across hierarchy levels. The Cobb-Douglas case is presented in the appendix. Alternatively, if productivity increases sufficiently as one ascends the hierarchy, the steady state pyramid may be inverted: $H_1^* < H_2^* < \ \ldots < H_N^*$. Other production functions could yield a steady state hierarchy with large stocks of mid-level human capital (H_i) and smaller stocks of "extreme" human capital types: $H_1^* < \ldots \ < H_{i-1}^* < \ H_i^* > \ H_{i+1}^* > \ \ldots H_N^*$.

Just as the steady-state human capital hierarchy can be represented by a "pyramid," so too can the initial conditions. The initial condition is simply a distribution of human capital (which can be interpreted as population) across the hierarchy. Let Let $\underline{H}(0)$ be the vector of initial stocks, and $H_i(0)$ the initial stock of level i human capital. We assume that at $\underline{H}(0)$, human capital stocks are not at their steady state relationships and *(12)* becomes a set of inequalities. The evolution of the human capital levels along the transition path is dictated by the relationship between initial and steady state pyramids. Figure 1 below illustrates an initial condition with increasing relative scarcity at higher hierarchy levels and several alternative steady state configurations. We will interpret these shapes subsequently in the context of alternative development paradigms.

Figure 1



Some Potential Steady State Configurations

2.1 Transitional Dynamics

Before moving to the data we consider the transition dynamics and begin by characterizing classes of initial conditions where *(12)* is not satisfied. For an N level hierarchy there are $N!$ strict inequality orderings of *(12)*:

(14)

$$\text{(i).}\quad \frac{f_1(\underline{H}(0))}{1} < \frac{f_2(\underline{H}(0))}{2} < \ldots\ldots < \frac{f_{N-1}(\underline{H}(0))}{N-1} < \frac{f_N(\underline{H}(0))}{N}$$

$$\text{(ii).}\quad \frac{f_2(\underline{H}(0))}{2} < \frac{f_1(\underline{H}(0))}{1} < \frac{f_3(\underline{H}(0))}{3} < \ldots\ldots < \frac{f_N(\underline{H}(0))}{N}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\text{(N!).}\quad \frac{f_N(\underline{H}(0))}{N} < \frac{f_{N-1}(\underline{H}(0))}{N-1} < \ldots\ldots < \frac{f_2(\underline{H}(0))}{2} < \frac{f_1(\underline{H}(0))}{1}.$$

Each of these *N!* orderings can be interpreted as a class of initial conditions. Associated with these inequality orderings are inequality orderings of net gains of human capital transformation in equation *(13)*. We focus on the following:

(15) $$f_2 - f_1 < f_3 - f_2 < \ldots\ldots < f_N - f_{N-1}.$$

Initial condition *15* reflects increasing relative scarcity of high-level human capital vis-à-vis low level and is associated with the wide-base/narrow-pinnacle pyramid illustrated in Figure 1. We will show in the next section that this configuration is consistent with current conditions in less developed countries and in high-income countries at earlier stages of development.

For any initial condition that generates an inequality ordering (such as *(15)*), the optimal transition investment program follows a similar pattern. Specifically, the optimal transition program in this environment is "bang-bang" with *N-1* phases of transition investment.[8] A simple arbitrage argument provides the intuition for the bang-bang property. That is, if over some interval a feasible program does not invest exclusively in the highest net return human capital, there exists an alternative feasible program with investment reallocated to the high return human capital that arrives at an identical future state with higher utility. Therefore if (*15)* holds, the optimal program begins with investment in level *N* human capital exclusively. During this initial phase of transition (which we denote *Phase 1*), f_N falls and f_{N-1} rises (due to the depletion effect) until the level *N* net return ($f_n - f_{N-1}$) equals the level *N-1* net return ($f_{N-1} - f_{N-2}$). At this point, *Phase*

2 of transition begins with investment in *N* and *N-1* to maintain $(f_N - f_{N-1}) = (f_{N-1} - f_{N-2})$ or equivalently $f_N / N = f_{N-1}/(N-1)$. Note that during this phase the next net return in the inequality chain, $(f_{N-2} - f_{N-3})$, rises due to the depletion effect on *N-2*. More generally, given initial condition *(15)* the following qualitative expression describes controls in Phase *k* of transition:

$$(16) \qquad x_i > 0 \ \ for \ \ i \geq N+1-k; \qquad x_i = 0 \ \ for \ \ i < N+1-k, \qquad (k \leq N-1)$$

Returning to Phase 1 this control pattern implies the following stock evolution:

$$(17) \qquad \dot{H}_N = -\dot{H}_{N-1}; \qquad \dot{H}_i = 0 \quad for \ \ i = 1, 2, \ldots, N-2.$$

Note that the relationship between the stocks of H_N and H_{N-1} in *(17)* is a pure hierarchical effect -- there is no traditional depreciation in this model so that focus can be directed to the implications of hierarchical structure. Moving again from the specific to the general, stock evolution during Phase k of transition can be described as follows:

$$(18) \qquad \dot{H}_i = 0 \quad for \ \ i < N-1-k; \ \ \dot{H}_{i-1} = -\dot{H}_i < 0 \quad for \ \ i = N-k;$$

$$\dot{H}_i > 0 \ | \ \frac{f_i}{i} = \frac{f_{i+1}}{i+1} \qquad for \ \ i > N-k.$$

We can therefore partition the set of human stocks at each moment into three subsets: those with positive, zero, and negative growth rates. We denote the subsets with positive and zero growth rates respectively as $\dot{H}^+$, and $\dot{H}^o$. Recall that at each moment on the transition path only the hierarchy level adjacent to (below) the lowest indexed element of $\dot{H}^+$ has a negative growth rate. This singularity arises because, with the "bang-bang" solution, only one hierarchy level at a time is subject to an *uncompensated depletion effect*. Hierarchy level *i* is experiencing uncompensated depletion when $\dot{H}_i = -\dot{H}_{i+1} < 0$ and $x_i = 0$. That is, when the stock reduction that accompanies investment in the next highest hierarchy level is not offset by any stock augmenting investment. Therefore if we assign the index number *d* to the hierarchy level experiencing uncompensated depletion then $\dot{H}_d = -\dot{H}_{d+1}$, where $\dot{H}_{d+1} \in \dot{H}^+$, and at each moment there is a single hierarchy level (*d*) experiencing uncompensated depletion.

3. Evidence and implications of Hierarchical Human Capital formation

We now turn our attention to the empirical implications of our model. First, we analyze whether the observed changes in the distribution of human capital across a sample of countries are consistent with the patterns predicted by the theoretical model. Second, we illustrate how accounting for hierarchical human capital formation might contribute to the empirical growth literature that often fails to find a significant relationship between conventional measures of human capital and rates of economic growth. As noted in the introduction, numerous authors have cited the consistently negative coefficients on human capital in growth regressions as perplexing.[9] Of course, these negative coefficients must be interpreted as relative to the excluded education categories. In developing countries, this would typically be primarily the uneducated, which explains the incredulity at the persistent negative coefficients. We will illustrate that the *depletion effect* associated with hierarchical structure generates a bias that may partial explain these perplexing results.

3.1 Data Description

As described in the previous sections, a natural interpretation of the model is to think of the stocks of different human capital as corresponding to those segments of the population with different levels of schooling. We therefore utilize data on schooling attainment from the International Data Base on Educational Attainment as presented by Barro and Lee (1993, 2000). The Barro-Lee (BL) data set classifies the total population according to the highest level of schooling attained and completed. They provide this classification for the total population of ages 15 and older (both men and women), and also for the total population aged 25 and older. Here we have used the sample of the total population aged 15 and older only, as we believe this sample is more representative of the labor force. All data is presented as shares of the total population.

The Barro-Lee data set provides information about four main educational categories across a number of countries for five year intervals between 1960 and 2000. The categories are:

- Fraction of the population with no schooling
- Fraction of the population with primary schooling (attained and completed)

- Fraction of the population with secondary schooling (attained and completed)
- Fraction of the population with higher education (attained and completed). Where higher education could be university, college, or technical level

We note that the educational groups could be used directly (as opposed to shares) and the principal results presented below are not significantly altered. We also note that the "human capital" categories in this data set are not perfectly congruent with the theoretical model. In our theoretical model an individual attains a higher level of human capital only when that level of education is completed; whereas in the Barro-Lee data set individuals who have attained (but not completed) a higher level of education are included in that level (and excluded from the previous level's stock). We therefore modify the Barro-Lee categories so that the share measures of human capital levels include only those individuals with completed education. When this is done we end up with measures of human capital stocks disaggregated into four levels: Ha, Hb, Hc, and Hd; where the level of human capital increases alphabetically. The stocks are hierarchical in the sense that an increase in Hd, for example, generates a decrease in Hc, Hb, or Ha.

Starting from the Barro-Lee categories, the measures of human capital stocks are constructed as follows:

Ha : Fraction of the population with no education or schooling + Fraction of the population with incomplete primary schooling

Hb : Fraction of the population with complete primary education + Fraction of the population with incomplete secondary education

Hc : Fraction of the population with complete secondary education + Fraction of the population with incomplete higher education

Hd : Fraction of the population with completed higher education.

Additional data was obtained from the World Bank World Development Indicators (2004). These variables include per-capita GDP levels (GDP), annual growth rates (Growth), investment as share of GDP (Investment), and rates of population growth (POP). Since we are interested in studying the evolution of the human capital stocks in economies with different initial conditions, all countries were grouped into the following four regions:[10]

Latin America: Argentina, Bahamas, Barbados, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominica, Dominican Republic, Ecuador, E1 Salvador, Guatemala, Guyana, Haiti, Honduras, Jamaica, Mexico, , Nicaragua, Panama, Paraguay, Peru, Saint Lucia,Saint Vincent and the Grenadines, Suriname, Trinidad and Tobago, Uruguay, and Venezuela

Africa: Algeria, Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Congo, Cote d'Ivoire, Egypt, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya, Lesotho, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Morocco, Mozambique, Niger, Nigeria, Rwanda, Senegal, Seychelles, Sierra Leone, Somalia, South Africa, Sudan, Swaziland, Togo, Tunisia, Uganda, Zambia, and Zimbabwe

Asia: Afghanistan, Australia, Bahrain, Bangladesh, China, Fiji, Hong Kong, India, Indonesia, Iran, Iraq, Israel, Japan, Jordan, Kuwait, Malaysia, Myanmar, Nepal, New Zealand, Oman, Pakistan, Papua New Guinea, Philippines, Saudi Arabia, Singapore, Sri Lanka, Syria Taiwan, Thailand, and the United Arab Emirates

Europe: Austria, Belgium, Cyprus, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Spain, Sweden, Switzerland, and United Kingdom

Other: Canada, USA

3.2 Mapping the evolution of Human Capital

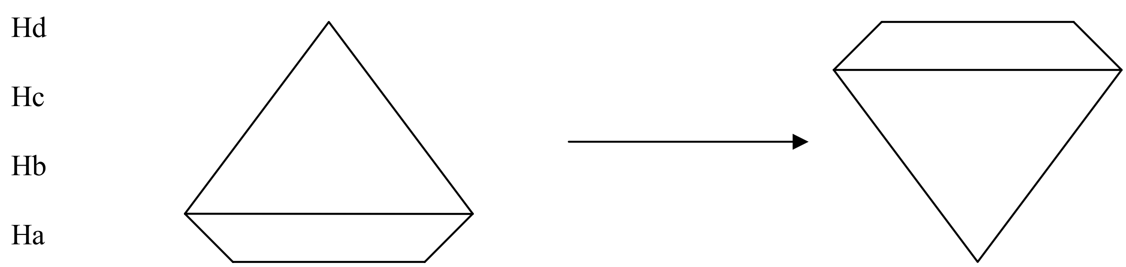
In this section we compare the observed changes in the distribution of human capital across a sample of countries with the patterns predicted by the theoretical model. One way to summarize the evolution of the different human capital stocks for a single country (or group of countries), is to arrange them into a matrix that shows the human capital stocks across columns and the years across rows. Table 1 below, for example, illustrates for the US:

Table 1 –Evolution the Human Capital Hierarchy – US

US

year	Ha	Hb	Hc	Hd
1960	21.60	47.40	24.00	7.00
1965	9.30	56.60	26.40	7.70
1970	5.30	55.70	30.10	8.80
1975	11.80	43.50	34.10	10.70
1980	3.00	19.90	64.50	12.90
1985	3.80	30.90	49.90	15.40
1990	5.60	29.40	43.10	21.80
1995	4.70	29.10	43.60	22.50
2000	5.10	26.80	43.60	24.50

Since each row in the matrix corresponds to a distribution of human capital at a given time, one could alternatively follow the evolution of human capital levels graphically. In the case of the USA, the graphic representation of the evolution takes the form of a pyramid that inverts itself between 1960 and 2000:



The US is an extreme case both in the levels of human capital achieved and in the speed of the transition. No other country has achieved such a high level of higher education capital, and thus, the US is our benchmark for comparison. The corresponding matrices for the other four regions of the world are presented in the tables below.

Table 2 –Evolution the Human Capital Hierarchy – Europe, Latin America, Africa

Europe

	Mean(Ha)	mean(Hb)	mean(Hc)	mean(Hd)
1960	33.63	54.63	9.86	1.88
1965	32.58	54.70	10.83	1.89
1970	27.81	55.76	14.16	2.28
1975	28.41	52.22	16.22	3.16
1980	24.75	50.02	21.39	3.86
1985	24.08	49.85	21.41	4.67
1990	20.35	47.11	27.20	5.46
1995	19.45	44.37	29.52	6.67
2000	18.23	42.89	30.93	7.94

Asia

	mean(Ha)	mean(Hb)	mean(Hc)	mean(Hd)
1960	70.23	21.60	6.96	1.19
1965	68.85	22.86	7.05	1.20
1970	64.60	25.80	8.00	1.56
1975	61.35	26.26	10.36	2.02
1980	55.93	28.64	12.78	2.65
1985	52.71	30.49	13.72	3.09
1990	48.91	31.54	15.67	3.80
1995	46.01	32.44	17.10	4.44
2000	43.61	33.14	18.19	5.06

Latin America

	mean(Ha)	mean(Hb)	mean(Hc)	mean(Hd)
1960	70.21	24.63	4.40	0.74
1965	71.11	23.58	4.46	0.85
1970	66.05	26.40	6.52	1.04
1975	62.74	28.35	7.53	1.35
1980	57.22	31.64	9.27	1.87
1985	56.76	30.55	10.08	2.62
1990	53.75	30.86	11.70	3.64
1995	50.69	31.73	13.21	4.30
2000	48.90	31.84	14.23	5.03

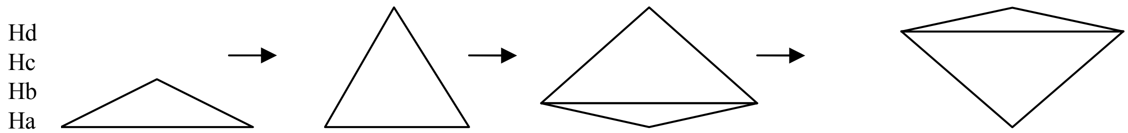
Table 2 (Continued) –Evolution The Human Capital Hierarchy – Europe, Latin America, Africa

Africa

	mean(Ha)	mean(Hb)	mean(Hc)	mean(Hd)
1960	89.30	9.37	1.04	0.23
1965	88.88	9.81	1.03	0.20
1970	86.88	11.35	1.45	0.25
1975	85.24	12.49	1.94	0.30
1980	82.60	14.71	2.32	0.40
1985	78.70	17.46	3.36	0.44
1990	76.02	19.22	3.92	0.69
1995	73.35	20.80	4.92	0.92
2000	71.69	21.87	5.36	1.07

The theoretical sections emphasized scenarios in which initial conditions exhibited relative scarcity of the higher levels of human capital. From the data we can say that this scenario is actually present in all developing regions from 1960 to 2000 and, to a lesser extent, in Europe and the USA in earlier years. The tables also reveal a significant disparity in average education levels across time. After the USA, Europe is the region exhibiting the most human capital accumulation, followed by Asia, Latin America and Africa. Taken together the dynamics of human capital accumulation revealed by the tables are consistent with our hierarchical model.

The model predicts that, given a similar technology and initial conditions, the transition dynamics of human capital across regions should follow a common pattern (see equations 15-18). Such a common pattern is reflected in the tables above. Specifically, note that the European human capital distribution in 2000 is similar to that of the USA for 1960. In turn, the human capital distribution of Asia and Latin America in 2000 resembles that of Europe in 1960. Finally, note that the human capital distribution in Africa for 2000 is almost identical to those of Asia and Latin America in 1960. Taken together the different regions of the world seem to be converging towards the same state and, in the process, delineate a common path of human capital accumulation. Graphically, we capture this pattern below by following the evolution of human capital across regions from less developed to more developed (Africa, Asia/LA, Europe and USA):



This common transition pattern can be traced to equations 12 and 15 in our model. Equation 12 describes the existing steady state and 15 the transition to that steady state. One way to think of the pattern exhibited in the data in relation to our theoretical model is that the transitional paths followed by the different regions of world are determined by similar initial conditions and similar steady states that emerge naturally as technologies of production disseminate throughout the world and human capital is accumulated. Further evidence emerges by comparing Asia and Latin America. Although Asia started with a small advantage over Latin America in more advanced human capital, the initial conditions on 1960 are remarkably close (refer again to Table 2). According to the theoretical model, if there is a common steady state (eq. 12) then similar initial conditions (eq. 15) would lead to similar transition patterns. As can be seen from the tables, this is exactly the case for these two regions.

3.2 The effects of hierarchical human capital on growth accounting

We now illustrate how accounting for hierarchical human capital formation might contribute to the empirical growth literature. Within this literature, the typical econometric specification follows those of Mankiw, Romer and Weil (1992) and Barro (1991). They estimate the relationship between the rate of economic growth and a vector of independent control variables that include the initial level of income **(GDP60)**, the rate of population growth **(POP)**, the share of investment to GDP **(INV)**, and some measure of human capital **(H).** Such a specification can be summarized as follows:

(19) $$\text{Growth} = \alpha + \varphi \text{GDP60} + \gamma\, \text{POP} + \beta\, \text{INV} + \theta\, \text{H} + \varepsilon\,.$$

Though the estimated coefficient for the human capital variable is expected to be positive and significant; aggregate cross-country studies reporting an insignificant (or even negative) effect of human capital on the rate of economic growth are not rare (see, for example, Benhabib and Spiegel (1994), Islam (1995), Prichett (1996) and Kalaitzidakis et al. (2001), Bils and Klenow (2000), among others). In most of these studies, however, the variables used to measure human capital are the average years of schooling of the population (or labor force), or the percentage of the population that has completed some specific level of education (e.g., secondary level). Only occasionally do empirical growth studies include more than two educational categories as independent variables.

The hierarchical nature of human capital accumulation suggests that the standard econometric specification noted above will lead to biased estimations of human capital's contribution to growth. If the empirical estimation includes only one measure of human capital (say, secondary) then the estimated coefficient for this variable will be biased as it would also be capturing a negative effect of the simultaneous drop in the stock of primary education capital. The fundamental problem is that all levels of human capital are likely to contribute to production and, at the same time, they are perfectly collinear when the hierarchical structure of human capital is acknowledged. Thus, one category must be excluded from the regressions. In general, the estimated coefficients for the different human capital stocks *do not* measure the contribution to growth of that specific human capital level, but rather the contribution of an included human capital level *net* of stock depletion effect on an omitted category lower in the hierarchy. To demonstrate this intuition more formally consider the following growth equation:

$$\text{(20)} \quad \text{Growth} = \alpha + \varphi \text{GDP60} + \gamma \text{ POP} + \beta \text{ INV} + \theta_1 \text{ Ha} + \theta_2 \text{ Hb} + \theta_3 \text{ Hc} + \theta_4 \text{ Hd} + \varepsilon.$$

The hierarchical formation of human capital requires:

$$\text{(21)} \quad \text{Ha} + \text{Hb} + \text{Hc} + \text{Hd} = 1 ,$$

Then, a correct econometric specification would be:

$$\text{(22)} \quad \text{Growth} = \alpha + \varphi \text{GDP60} + \gamma \text{ POP} + \beta \text{ INV} + (\theta_2 - \theta_1)\text{Hb} + (\theta_3 - \theta_1) \text{ Hc} + (\theta_4 - \theta_1) \text{ Hd} + \varepsilon,$$

and the estimated coefficients should be interpreted accordingly.

The estimation of this type of econometric specification, where only one of the human capital categories is excluded, is presented in Tables 3 and 4. Table 3 presents the estimations for which the Human capital stocks in Table 2 (Ha-Hd) are used. Table 4 presents the estimations in which the Barro-Lee "completed" education categories are used. In both tables, five year averages were used to estimate fixed effects regressions. Columns 3-6 of both tables present the results of conducting the regressions for alternative sub-samples of poor and rich countries separately; where the median per-capita GDP was used to separate the samples.

Table 3. Dependent variable: per capita GDP growth

	All countries		Poor countries		Rich countries	
	(1)	(2)	(3)	(4)	(5)	(6)
Initial GDP	$-4*10^{-6}$ (-4.62)	$-4*10^{-6}$ (-4.83)	$-5*10^{-5}$ (-4.55)	$-6*10^{-5}$ (-4.70)	$-3*10^{-6}$ (-3.56)	$-3*10^{-6}$ (-3.77)
Investment	0.0016 (4.35)	0.0016 (4.39)	0.0024 (4.63)	0.0024 (4.56)	0.0011 (2.28)	0.0011 (2.32)
POP	0.006 (3.02)	0.006 (3.14)	0.009 (3.98)	0.009 (4.01)	-0.011 (-2.52)	-0.010 (-2.31)
Ha	**-0.005** **(-4.55)**		**-0.012** **(-2.97)**		**-0.004** **(-3.67)**	
Hb	**-0.006** **(-4.40)**	**-0.0001** **(-0.46)**	**-0.012** **(-2.75)**	**-0.0003** **(-0.48)**	**-0.004** **(-3.40)**	**0.0002** **(0.58)**
Hc	**-0.004** **(-3.52)**	**0.0009** **(2.64)**	**-0.011** **(-2.46)**	**0.0006** **(0.52)**	**-0.003** **(-2.84)**	**0.0009** **(2.36)**
Hd		**0.006** **(4.79)**		**0.012** **(3.01)**		**0.004** **(3.90)**

T-statistics are in parentheses.

Table 4. Dependent variable: per capita GDP growth – Barro-Lee Ed. Categories

	All countries		Poor countries		Rich countries	
	(1)	(2)	(3)	(4)	(5)	(6)
Initial GDP	$-2*10^{-6}$ (-2.52)	$-4*10^{-6}$ (-4.53)	$-4*10^{-5}$ (-3.65)	$-6*10^{-5}$ (-4.87)	$-1*10^{-6}$ (-1.83)	$-3*10^{-6}$ (-3.50)
Investment	0.0016 (4.10)	0.0016 (4.24)	0.0026 (4.67)	0.0024 (4.52)	0.0010 (1.94)	0.0010 (2.11)
POP	0.006 (2.71)	0.006 (3.12)	0.009 (3.56)	0.009 (3.97)	-0.013 (-2.68)	-0.011 (-2.47)
Noed	**-0.0007** **(-2.61)**		**-0.0003** **(-0.77)**		**-0.0008** **(-1.71)**	
Ped	**-0.0009** **(-2.08)**	**-0.0002** **(-0.48)**	**0.0011** **(1.02)**	**0.0008** **(0.78)**	**-0.0005** **(-1.22)**	**0.0002** **(0.49)**
Sed	**0.0002** **(0.51)**	**0.0007** **(2.06)**	**0.0012** **(0.85)**	**0.0002** **(0.24)**	**0.0016** **(0.45)**	**0.0007** **(1.97)**
Hed		**0.007** **(5.07)**		**0.014** **(3.51)**		**0.0055** **(4.06)**

T-statistics are in parentheses.

As expected, Table 3 results indicate that the estimated coefficient for Hd is greater than Hc and, in turn, the estimated coefficient for Hc is greater than that of Hb. More importantly, however, when the Hd category is excluded in Column (1), the estimated coefficients for all other measures of human capital become negative and significant. The natural explanation for this result in light of our model is that the estimated coefficients are biased downwards by the omitted human capital level since they measure only the net contribution between the specific human capital levels and the omitted one. Indeed, when the hierarchical structure of human capital is acknowledged the negative coefficients in Columns (1) are anticipated since more advanced human capital should have a greater contribution to growth. Table 4 illustrates this potential bias exists in a similar manner when the Barro-Lee human capital categories are employed.

When the sample is divided into poor and rich countries, the previous results remain unaltered, but additional insight is gained with respect to the transition process. In our theoretical model the transition from a state of relative scarce high level human capital to a state of relative abundance is motivated by the *net* difference in the contribution to output of higher and lower levels of human capital. As can be seen in tables 3 and 4, the net impact of higher level human capital on the rate of growth is indeed greater for the poor countries sub-sample than for the rich countries sub-sample.

It is important to note that the data used in this empirical section is recorded in five year intervals and therefore does not capture all sequential periods of time. Thus the empirical estimates cannot capture the complete human capital transition path (in contrast to the theoretical model). In the theoretical model, for example, an increase in Hd causes *an immediate* decrease in Hc. As the transition path progresses, this initial drop is followed by additional investment in Hc and a depletion of lower human capital categories. As the focus of human capital investment migrates through the hierarchy the final increases in the stocks of the more advanced human capital types (Hd, Hc, Hb) will be fueled by a depletion of the most basic human capital (Ha). Table 5 below presents the correlation matrix of the four human capital categories that illustrates this long term relationship. Note that as suggested by the model Ha is negatively correlated with all higher levels of human capital.

Table 5. Correlation Matrix of Human Capital Categories

	Ha	Hb	Hc	Hd
Ha	1.000			
Hb	-0.869	1.000		
Hc	-0.852	0.492	1.000	
Hd	-0.742	0.446	0.760	1.000

4. Summary and Conclusion

Although human capital formation has received much attention in the literature, the subtle differences that distinguish human capital formation from physical capital formation have been widely ignored. In this paper we present a growth model in which the hierarchical structure of human capital formation is explicitly represented. Though the model simplifies many features of the environment, the abstraction facilitates illumination of the intra-hierarchy depletion effect. Hierarchical structure has important implications at both the theoretical and empirical levels. At a theoretical level we show that hierarchical structure influences the optimal investment program both during transition and in the steady state. Our characterization of the transition and steady-state demonstrates that disregard of the hierarchical structure will lead to sub-optimal human capital investment programs.

At the empirical level we demonstrate that disregard of the hierarchical nature of human capital accumulation will likely lead to biased coefficients in growth equations estimates. Indeed, our paradigm provides perhaps the simplest explanation for the perplexing negative human capital coefficients that appear regularly in the empirical growth literature.

Modeling the formation of human capital as a hierarchical process allows analysis of the relationship between the modern educational system and macroeconomic variables of interest in a way that the typical growth model cannot. Natural extensions of this paper include the analysis of optimal educational policies and optimal budget allocations of public expenses across educational categories.

Appendix – A Cobb-Douglas Example

Assume the following standard functional forms for utility and production respectively:

$$(A1) \qquad U(c) = \frac{c^{(1-v)} - 1}{1-v}, \qquad Y = \prod_{i=1}^{N} H_i^{\alpha_i} .$$

If the generalized Cobb-Douglas production function of *(A1)* exhibits constant returns to scale, and there is no depreciation or exogenous population growth, equations *(11), (12)*and *(A1)* can be used to derive the following expression for the (endogenous) steady-state growth rate of consumption for an *N*-level hierarchy:

$$(A2). \qquad \gamma_c = \frac{1}{v}\left[\prod_{i=1}^{N}\left(\frac{\alpha_i}{i}\right)^{\alpha_i} - r\right],$$

where α_i is the Cobb-Douglas productivity weight of human captial level *i*. Again note the relationship between the steady-state consumption growth rate and hierarchy size *(i's)*. One interesting implication of *(A2)* is that the steady-state consumption growth rate is declining in hierarchy size. The functional forms above together with equation *(12)* yield explicit following steady-state human capital ratios:

$$(A3) \qquad \frac{H_i}{H_j} = \frac{j\alpha_i}{i\alpha_j}, \qquad \forall\ i, j = 1,\ldots, N.$$

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Endnotes

1 Driskill and Horowitz (2002) develop a model with a simple 2-level human capital hierarchy. However, the 2-level specification precludes realistic transition dynamics and most importantly does not permit illumination of the empirical biases in prior empirical literature.

2 For example, Islam (1995) states: "Whenever researchers have attempted to incorporate the temporal dimension of human capital variables into growth regression, outcomes of either statistical insignificance or negative sign have surfaced." We provide additional cites and motivation in the empirical section (see also endnote 9 below).

3 Of course, intermediate physical products are occasionally productive – but these are exceptions rather than rules. An example of a productive intermediate product is logs, which could be used as telephone poles, or transformed into 2x4s. Intermediate human product (e.g., a secondary school graduate), however, is generally productive.

4 In adopting a continuous time formulation we abstract from "time to build" or training time.

5 The implicit human capital production function may be written as $h(x_i, h_{i-1}) = min[x_i, h_{i-1}]$ with x_i the physical input and h_{i-1} the type human capital. Efficiency requires $x_i = h_{i-1}$, yielding *2a*.

6 This assumption can easily be relaxed. However, the uniform price of investment lays bare the implications of the hierarchy without the obfuscating effect of differential investment costs.

7 A sufficient condition for $x_i > 0$ in the steady state is simply $f_i > i$, for all i. That is, marginal benefit exceeds marginal "cost."

8 The program is bang-bang because of the lack of adjust costs (see Kamien and Schwartz 1981). First order conditions (6)-(8) therefore have $\theta_i > 0$ for some *i's* during transition. There are some technical subtleties that may arise on the transition path that are tangential to our focus on transition path income inequality, and we hence ignore. For example, for a sufficiently productive technology investment in multiple hierarchy levels may occur simultaneously. In addition, the relationship between *(14)* and *(15)* is more subtle than that between *(12)* and *(13)* since *(15)* was derived using adjacent pairs, and the ordering of inequalities in (15) need not be index number adjacent.

9 See also Pritchett 2001 and 1995 who states: ". . . the estimated impact of growth of human capital on conventional growth accounting measures of total factor productivity demand is large, strongly significant, and *negative.*"

10 Not all variables are available for all countries. Because of that, the number of countries used varies depending on the number of variables involved on the empirical exercise.