Intergenerational Spillovers, Decentralization, and Durable Local Public Goods

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Introduction

The question

- The central question: how self-interested agents can be induced to invest optimally in future generations.
- More specifically, we investigate what sorts of institutional arrangements cause nonaltruistic agents to internalize intergenerational spillovers.
- We call goods that result in transfers from the old to the young *forward intergenerational goods* (FIGs).
- Our FIG: a durable local public good (DLPG).
  - Examples include such things as roads, public buildings, parks and R&D.
  - Essential feature: when the current generation produces them, the next generation necessarily receives a benefit in the form of an inherited public capital stock.
  - Unless this spillover is somehow internalized, DPGs will be systematically underprovided.

- Tiebout (1956):
  - local provision might lead to interjurisdictional competition that would in turn lead to an efficient outcome.
  - His focus:
    - Static coalition formation
    - Optimal sorting of agents by taste
    - Overcoming free-riding though tax/public good bundles offered by competing jurisdictions.
- In contrast, we ask : can this argument can be extended to show that interjurisdictional competition also prevents free riding (and the consequent underprovision of public goods) *between different generations*?

- Krugman recounting what Fama said:

  "people are very careful when they buy houses. It's typically the biggest investment they're going to make, so they look around very carefully and they compare prices. The bidding process is very detailed."

- Krugman himself

  "In the case of housing, buyers do carefully compare prices — with the prices of other houses. That is, they make sure that two-quart bottles of ketchup are the same price as one-quart bottles. As we've seen, however, they don't do a very good job of checking whether the overall level of housing prices makes sense. Yes, it was a bubble — and as Larry said way back when, the ketchup test just isn't enough."

- World consists of J equal-sized jurisdictions each of which has L identical "lots."
- Generation of size $L \cdot J$ born every period t, $t = 0, 1, ..., T$
- Each generation except the last lives two (2) periods.
    - Generation (cohort) t is the generation born at time t
    - Generation T lives only one period
- Each generation endowed when young with w units/agent of a homogeneous good.
    - w can be either consumed or transformed one-for-one into a local durable public good.
- When young, each member of generation t first decides in which jurisdiction j to live.
- After picking a jurisdiction, members of generation t in jurisdiction j collectively decide on a level of (flow) investment in a durable local public good g_t^j.

- Evolution of stock of DLPG:

$$G^j_{t+1} = \delta(G^j_t + g^j_t),\ \delta \in (0,1);\ G_0 \ given. \quad (1)$$

- Preferences: For each member of each generation t:

$$U^j_t = c_{t,t} + \rho c_{t,t+1} + V(G^j_t),\ \rho \in [0,1], V\prime > 0, V\prime\prime < 0. \quad (2)$$

- Notation:
  - U^j_t : utility of someone born at time t who resides in jurisdiction j, $j = 1, 2, ..., J$.
  - $c^j_{t,t}$: consumption of member of generation t at time t in jurisdiction j.
  - $c^j_{t,t+1}$: consumption of member of generation t at time $t+1$ in jurisdiction j.
  - G^j_t : the stock of a durable local public good in jurisdiction j at time t
  - $V(G^j_t)$: the utility for an individual in jurisdiction j at time t derived from G^j_t.

- Timing:
  - The young benefit *only* from the inherited stock of the DLPG.
  - Results not sensitive, but this assumption makes clear the intergenerational problem.
- Per capita resource constraint for each jurisdiction at time t:

$$w = c_{t,t} + c_{t-1,t} + \frac{g_t}{L}. \tag{3}$$

- Budget constraints for representative generation-t member of each jurisdiction:

$$w = c_{t,t} + p_t - \frac{g_t}{L}; \tag{4}$$

$$c_{t,t+1} = p_{t+1}. \tag{5}$$

- Note: consumption when old depends on selling your lot.

- We show that the solution to a planners problem is identical to the solution to a decentralized problem.
- For the decentralized problem, we make two key assumptions:
  - No Mobility Condition (NMC): in political equilibrium, given land prices, the utility for each young member of a generation must be the same across all jurisdictions. That is, we assume that free mobility of agents insures that in equilibrium there must be no movement across jurisdictions.
  - Small Jurisdictional Assumption (SJA): the price of a lot in any one jurisdiction is assumed to be unaffected by public good investments that take place in at least one other jurisdiction.

Planner's problem

Per capita

$$
\begin{aligned}
\max W &\equiv U_0 + ... + \rho^T U_T \\
s.t &: \quad w = c_{0,0} + \frac{g_0}{L}; \\
w &= c_{t-1,t} + c_{t,t} + \frac{g_t}{L}; t = 1, ..., T-1; \\
w &= c_{T-1,T} + c_{T,T}; \\
G_{t+1} &= \delta(G_t + g_t), \ \ t = 0, 1, ... T-1; \\
& G_0 \text{ given}
\end{aligned} \tag{6}
$$

Planner's solution

- The solution to the planner's problem for each jurisdiction is a sequence $\{G_t\}_{t=0}^{t=T}$ for which

$$\frac{1}{L} = \frac{\rho\delta}{1-\rho\delta} V\prime(G_t), \;\; t = 1, 2, ..., n; \tag{7}$$

  and for which

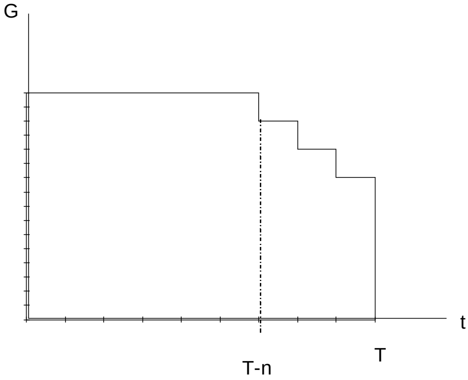
$$G_t = \delta G_{t-1}, \;\; t = n+1, n+2, ..., T;$$

  where the integer $n(\delta, \rho) \in (0, T)$.
- This says that for some n initial periods, the stock of the DLPG is kept constant by constant additions g_t to the capital stock.constant. After that, $g_t = 0$ and the stock depreciates.
- One interesting point: $T - n$ is independent of T. Thus, for any $t > 0$, there exists a T sufficiently large that

$$\frac{1}{L} = \frac{\rho\delta}{1-\rho\delta} V'(G_t).$$

Planner's solution

Picture



- The utility of a generation-T member of jurisdiction j is:

$$U_T^j = c_{T,T} + V(G_T^j) = w - p_T^j + V(G_T^j).$$

- The budget constraint for each such member is

$$c_{T,T} = w - p_T^j.$$

- Thus,

$$U_T^j = w - p_T^j + V(G_T^j)$$

Decentralized solution

Generation T

- NMC:

$$w - p_T^j + V(G_T^j) = w - p_T^{\bar{j}} + V(G_T^{\bar{j}}),$$
$$\bar{j} = 1, 2, ..., j-1, j+1, ..., J.$$

- Implies relative price equals relative LDPG values

$$p_T^j = p_T^{\bar{j}} + V(G_T^j) - V(G_T^{\bar{j}}).$$

- Note implication of SJA: changes in $G_T^{\bar{j}}$ don't affect p_T^j

$$\frac{\partial p_T^j}{\partial G_T^{\bar{j}}} = 0.$$

- Hence, for future use:

$$\frac{dp_T^j}{dG_T^j} = V\prime(G_T).$$

Decentralized solution

Generation T-1

- Utility for generation $T-1$ is
$$U = c^j_{T-1,T-1} + \rho c^j_{T-1,T} + V(G^j_{T-1}).$$
- Budget constraints for this generation are:
$$w = c^j_{T-1,T-1} + \frac{g^j_{T-1}}{L} + p^j_{T-1};$$
$$c^j_{T-1,T} = p^j_T$$
- Subbing into the utility function:
$$U = w - \frac{g^j_{T-1}}{L} - p^j_{T-1} + \rho p^j_T + V(G^j_{T-1}).$$
- FOC:
$$\frac{1}{L} - \theta_{T-1} = \rho \frac{dp^j_T}{dg^j_{T-1}};\ \theta_{T-1} g^j_{T-1} = 0;\ \theta_{T-1} \geq 0,\ g^j_{T-1} \geq 0$$

- The key: understanding $\frac{dp_T^j}{dg_{T-1}^j}$.
- From generation $T's$ NMC along with SJA, we have

$$\frac{dp_T^j}{dg_{T-1}^j} = V\prime(G_T^j)\frac{dG_T^j}{dg_{T-1}^j}.$$

- Because $G_T^j = \delta(G_{T-1}^j + g_{T-1}^j)$:

$$\frac{dG_T^j}{dg_{T-1}^j} = \delta.$$

- Thus

$$\frac{dp_T^j}{dg_{T-1}^j} = \delta V\prime(G_T^j)$$

- Hence

$$\frac{1}{L} - \theta_{T-1} = \rho\delta V\prime(G_T^j). \tag{8}$$

Decentralized solution

Generation T-1

- Rewrite FOC (8):

$$\theta_{T-1} = \frac{1}{L} - \rho\delta V\prime(\delta G^j_{T-1} + \delta\widehat{g}^j_{T-1}).$$

- If $\theta_{T-1} > 0$, $\widehat{g}^j_{T-1} = 0$, and

$$\theta_{T-1} = \frac{1}{L} - \rho\delta V\prime(\delta G^j_{T-1}).$$

- From above equation, with $\widehat{g}^j_{T-1} = 0$, there exists a small enough value of G^j_{T-1} such that $\theta_{T-1} = 0$. For G^j_{T-1} less than this amount, it must be that $\widehat{g}^j_{T-1} > 0$. Otherwise, θ_{T-1} would be negative, which is a contradiction of the K-T condition that $\theta_{T-1} \geq 0$.
- For expository ease, we focus on $g^j_{T-1} > 0$ $(\theta_{T-1} = 0)$.

Decentralized solution

Generation T-1

- Hence, in this case the FOC is

$$\frac{1}{L} = \rho\delta V\prime(G^j_T) = \rho\delta V\prime\delta(G^j_{T-1} + g^j_{T-1}). \tag{9}$$

- Thus (9) determines an optimal choice of investment, denoted $\widehat{g}^j_{T-1}(G^j_{T-1})$, and equilibrium stock $\widehat{G}^j_T\ (G^j_{T-1})$.
- Property of $\widehat{g}^j_{T-1}(G^j_{T-1})$ (from differentiation of (9)):

$$\frac{d\widehat{g}^j_{T-1}}{dG^j_{T-1}} = -1. \tag{10}$$

- Property of equilibrium level $\widehat{G}^j_T$:

$$\frac{d\widehat{G}^j_T}{dG^j_{T-1}} = \delta + \delta\overbrace{\frac{d\widehat{g}^j_{T-1}}{dG^j_{T-1}}}^{-1} = 0. \tag{11}$$

Decentralized solution

Generation T-2: the money shot

- FOC (Again, we focus on $\theta_{T-2} = 0$.)

$$\frac{1}{L} - \theta_{T-2} = \frac{1}{L} = \rho \frac{dp^j_{T-1}}{dg^j_{T-2}}.$$

- NMC of Generation $T-1$:

$$p^j_{T-1} = \rho p^j_T + V(G^j_{T-1}) - \frac{\widehat{g}^j_{T-1}}{L} + p^{\bar{j}}_{T-1} + \frac{\widehat{g}^{\bar{j}}_{T-1}}{L} - V(G^{\bar{j}}_{T-1}) - \rho p^{\bar{j}}_T.$$

- Interpretation: relative price a function of present discounted value of relative utility flows from DLPG's minus relative investment costs.

SJA implies

$$\frac{dp^j_{T-1}}{dg^j_{T-2}} = \frac{dp^j_T}{d\widehat{G}^j_T}\frac{d\widehat{G}^j_T}{dG^j_{T-1}}\frac{dG^j_{T-1}}{dg^j_{T-2}} + V\prime(G^j_{T-1})\frac{dG^j_{T-1}}{dg^j_{T-2}} - \frac{1}{L}\frac{d\widehat{g}^j_{T-1}}{dG^j_{T-1}}\frac{dG^j_{T-1}}{dg^j_{T-2}}. \tag{12}$$

Decentralized solution

Generation T-2: the money shot

- From (10) and (11) into (12), we have

$$\frac{dp^j_{T-1}}{dg^j_{T-2}} = \delta V\prime(G^j_{T-1}) + \frac{\delta}{L}. \tag{13}$$

- Thus, the FOC is

$$\frac{1}{L} = \left(\frac{\rho\delta}{1-\rho\delta}\right) V\prime(G^j_{T-1}) = \left(\frac{\rho\delta}{1-\rho\delta}\right) V\prime(\delta G^j_{T-2} + \delta g^j_{T-2}).$$

- This determines an optimal level of DLPG investment, $\widehat{g}^j_{T-2}(G^j_{T-2})$ and associated equilibrium stock $\widehat{G}^j_{T-1}(G^j_{T-2})$.
- Properties:

$$\frac{d\widehat{g}^j_{T-2}(G^j_{T-2})}{dG^j_{T-2}} = -1; \quad \frac{d\widehat{G}^j_{T-1}}{dG^j_{T-2}} = 0. \tag{14}$$

Decentralized solution

Miller time! Generation T-3 and beyond

- As with generation $T-2$, the first order condition for generation $T-k,\ k=3,4,\ldots$ is:

$$\frac{1}{L}=\rho\frac{dp^{j}_{T-(k-1)}}{dg^{j}_{T-k}}. \tag{15}$$

- The NMA for generation $T-(k-1)$ is:

$$\begin{aligned} p^{j}_{T-(k-1)} &= \rho p^{j}_{T-(k-2)}+V(G^{j}_{T-(k-1)})-\frac{\widehat{g}^{j}_{T-(k-1)}(G^{j}_{T-(k-1)})}{L} \\ &\quad +p^{\bar{j}}_{T-(k-2)}+\frac{\widehat{g}^{\bar{j}}_{T-(k-1)}}{L}-V(G^{\bar{j}}_{T-(k-1)})-\rho p^{\bar{j}}_{T-(k-2)}. \end{aligned}$$

- Can rewrite (solve forward) to show relative price a function of present discounted value of relative utility flows from DLPG's minus present discounted relative investment costs.

Decentralized solution

Miller time! Generation T-3 and beyond!

- Thus, by SJA,

$$\frac{dp^j_{T-(k-1)}}{dg^j_{T-k}} = \frac{dp^j_{T-(k-2)}}{dG^j_{T-(k-2)}} \overbrace{\frac{dG^j_{T-(k-2)}}{dG^j_{T-(k-1)}}}^{=0} \frac{dG^j_{T-(k-1)}}{dg^j_{T-k}}$$

$$+V'(G^j_{T-(k-1)}) \overbrace{\frac{dG^j_{T-(k-1)}}{dg^j_{T-k}}}^{=\delta} - \frac{1}{L} \overbrace{\frac{d\widehat{g}^j_{T-(k-1)}}{dG^j_{T-(k-1)}} \frac{dG^j_{T-(k-1)}}{dg^j_{T-k}}}^{=-\delta}.$$

Decentralized solution

Miller time! To generation T-3 and beyond.

- Hence,

$$\frac{1}{L} = \frac{\rho\delta}{(1+\rho\delta)} V\prime(G^j_{T-(k-1)}), k = 3, 4, ... T - 1. \quad (16)$$

- In general, we can show that the decentralized solution and the planner's solution are identical no matter; the number of periods before T for which $g^j_t = 0$ is identical with the planner's problem.
- For all time periods in which $\theta_{T-k} = 0$ and $g_{T-k} > 0$, this implies that there is an equilibrium price function

$$p^j_{T-k} = \frac{V(G^j_{T-k})}{1+\rho\delta} + C \quad (17)$$

that satisfies all first-order conditions. There are thus a family of solutions, each associated with an arbitrary constant (C).
- This solution is unique: at every stage we have a standard concave NLP.

- While relative prices are unique and efficient, the price level is indeterminant.
- Effects of a lower price level: except for first and last generation, each generation pays less when buying and receives less when selling.
- Clearly a stark implication dependent on our modelling of real estate market as an "assignment problem."
- Nontheless captures some element of recent experience.
- Importance of decentralization