Technical Appendix to
"Search, Bargaining, and Agency in the Market for Legal Services"

Andrew F. Daughety and Jennifer F. Reinganum

In this Technical Appendix, we provide: (1) the detailed analysis of equilibrium beliefs and behavior in the continuation equilibrium between L2 and C, for the case of unconstrained F and for the case wherein F is constrained to be zero; (2) the equilibrium refinement arguments using D1 to select the equilibrium that involves the smallest likelihood of a second search in the case wherein F is unconstrained; (3) the equilibrium refinement arguments using D1 to justify the use of skeptical beliefs when F = 0; (4) the equilibrium refinement arguments using D1 to eliminate a separating equilibrium at the lower root of equation (5) when F = 0; and (5) some results for the regime in which no transfers are allowed, for the case of a continuum of types, $A \in [\underline{A}, \overline{A}]$.

1. Equilibrium Beliefs and Behavior in the Continuation Game Between C and L2

Analysis of the Continuation Game Between C and L2 when F is Unconstrained

Recall that $B_2(\alpha_2, F_2 \mid B(\alpha_1, F_1))$ denotes C's posterior belief, after having arrived at L2 with beliefs $B_1(\alpha_1, F_1)$ and having received the demand (α_2, F_2) from L2, where:

$$B_2(\alpha_2, F_2 \mid \underline{A}) = \underline{A} \text{ for } (\alpha_2, F_2) \in u(\underline{A}); \text{ for all other } (\alpha_2, F_2),\ B_2(\alpha_2, F_2 \mid \underline{A}) \in \{\underline{A}, \overline{A}\}.$$

$$B_2(\alpha_2, F_2 \mid \overline{A}) = \overline{A} \text{ for } (\alpha_2, F_2) \in \text{epi}(u(\underline{A})) \cap \text{hypo}(u(\overline{A})); \text{ for all other } (\alpha_2, F_2),\ B_2(\alpha_2, F_2 \mid \overline{A}) \in \{\underline{A}, \overline{A}\}.$$

We can now characterize the client's optimal behavior in response to L2's demand (α_2, F_2). Suppose that $B_1(\alpha_1, F_1) = \underline{A}$. If L2's demand (α_2, F_2) is on $u(\underline{A})$, then C's beliefs are confirmed and she is willing to accept any such demand. This follows since she is indifferent between every point on the curve $u(\underline{A})$ and the point $(1, \Pi^L(1, \underline{A}))$, which is what she expects to obtain from the auction. If L2's demand (α_2, F_2) is above $u(\underline{A})$ but below $u(\overline{A})$, then C will accept it if she continues to believe $\underline{A}$ and she will conduct the auction if she revises her belief upward to $\overline{A}$[1]. If L2's demand (α_2, F_2) is below $u(\underline{A})$, then C will initiate the auction regardless of her beliefs. If L2's demand (α_2, F_2) is on the curve $u(\overline{A})$, then she is indifferent between accepting it and conducting the auction if she revises her belief upward to $\overline{A}$, and she will accept this demand if she continues to believe $\underline{A}$. Finally, if L2's demand is above $u(\overline{A})$, then C will accept regardless of her beliefs.

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[1] If C arrives at L2 with $B_1(\alpha_1, F_1) = \underline{A}$, but revises her beliefs upward to $\overline{A}$ for any (α_2, F_2) above $u(\underline{A})$ but on or below $u(\overline{A})$, then there is another equilibrium in which C always conducts the auction. This results in the same payoffs for L1 and C, and the same equilibrium for the overall game.

Alternatively, suppose that $B_1(\alpha_1, F_1) = \overline{A}$. If L2's demand (α_2, F_2) is on $u(\overline{A})$, then C's beliefs are confirmed and she is willing to accept (or reject) any such demand. This follows since she is indifferent between every point on the curve $u(\overline{A})$ and the point $(1, \Pi^L(1, \overline{A}))$, which is what she expects to obtain from the auction (and the most she could ever hope to obtain). If L2's demand (α_2, F_2) is on or above $u(\underline{A})$ but below $u(\overline{A})$, then C continues to believe $\overline{A}$ and thus she will conduct the auction. Again, if L2's demand (α_2, F_2) is below $u(\underline{A})$, C will initiate the auction regardless of her beliefs and if L2's demand (α_2, F_2) is above $u(\overline{A})$, C will accept the demand regardless of her beliefs.

We now characterize L2's optimal demand (α_2, F_2), given C's beliefs upon approaching L2, $B_1(\alpha_1, F_1)$, and given L2's own observation of the expected case value. In principle, there are four possibilities (though only the first one will occur in the overall equilibrium). First, suppose that L2 observes $\underline{A}$ and C believes $B_1(\alpha_1, F_1) = \underline{A}$. Then L2 can obtain a payoff of zero by demanding $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$, which the client accepts, or by offering (α_2, F_2) below $u(\underline{A})$, which the client rejects in favor of an auction. An offer of (α_2, F_2) on $u(\underline{A})$, but different from $(1, \Pi^L(1, \underline{A}))$, is acceptable to C but yields a negative payoff for L2; this is also true for offers (α_2, F_2) on or above $u(\overline{A})$. An offer of (α_2, F_2) above $u(\underline{A})$ but below $u(\overline{A})$ yields a payoff of zero to L2 if C believes $\overline{A}$ and therefore rejects it, or a negative payoff if C believes $\underline{A}$ and therefore accepts it. Thus, it is clear that $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$ is an optimal demand by L2 when the expected value of the case is $\underline{A}$, C believes $B_1(\alpha_1, F_1) = \underline{A}$ and C accepts this demand; C obtains a payoff of $\Pi^L(1, \underline{A})$.

Second, suppose that L2 observes $\overline{A}$ but C believes $B_1(\alpha_1, F_1) = \underline{A}$. Then L2 can obtain a payoff of $\Pi^L(1, \overline{A}) - \Pi^L(1, \underline{A})$ by demanding $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$, which the client accepts (because it confirms her beliefs). Although C would also be willing to accept any other offer on $u(\underline{A})$, these would yield lower profits for L2. Similarly, any offer above $u(\underline{A})$ and below $u(\overline{A})$ would either provoke an auction (from which L2 expects to obtain a payoff of zero) if C were to revise her beliefs upward, or be accepted (yielding lower profits to L2) if C were to maintain her beliefs. Offers above $u(\overline{A})$ would be accepted but yield lower profits to L2, while offers below $u(\underline{A})$ would result in an auction, yielding profits to L2 of zero. Thus, it is clear that $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$ is the optimal demand by L2 when the expected case value is $\overline{A}$ but C believes $B_1(\alpha_1, F_1) = \underline{A}$, and C accepts this demand; C obtains a payoff of $\Pi^L(1, \underline{A})$. **To summarize:** when the client approaches L2 believing $B_1(\alpha_1, F_1) = \underline{A}$, then an optimal demand for L2 is to confirm C's beliefs by offering $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$, prompting C to accept L2's demand.

Third, suppose that L2 observes $\underline{A}$ but C believes $B_1(\alpha_1, F_1) = \overline{A}$. Then L2 can do no better than to provoke an auction (from which L2 expects a payoff of zero). This is because any offer on $u(\overline{A})$ would yield a negative payoff to L2, and C's beliefs are skeptical in that she does not revise her beliefs downward when L2 makes a demand below $u(\overline{A})$. Notice that even if C did revise her beliefs down to $\underline{A}$ for, say, some (α_2, F_2) below $u(\overline{A})$ but on or above $u(\underline{A})$, and were to accept such a demand (with positive probability), L2 would still strictly prefer the auction with the single exception of the demand $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$, at which L2 is indifferent between acceptance and the auction.

Fourth, suppose that L2 observes $\overline{A}$ and C believes $B_1(\alpha_1, F_1) = \overline{A}$. Then L2 can obtain a payoff of zero by demanding $(\alpha_2, F_2) = (1, \Pi^L(1, \overline{A}))$, which confirms C's beliefs and which she accepts; or by offering (α_2, F_2) below $u(\overline{A})$, which the client rejects in favor of an auction (since she continues to believe that the expected value of the case is $\overline{A}$). An offer of (α_2, F_2) on $u(\overline{A})$, but different from $(1, \Pi^L(1, \overline{A}))$, is acceptable to C but yields a negative payoff for L2; this is also true for offers (α_2, F_2) above $u(\overline{A})$. Thus, it is clear that $(\alpha_2, F_2) = (1, \Pi^L(1, \overline{A}))$ is an optimal demand by L2 when the expected value of the case is $\overline{A}$, C believes $B_1(\alpha_1, F_1) = \overline{A}$, and C accepts this demand; C obtains a payoff of $\Pi^L(1, \overline{A})$.

Notice that this last situation is where C's skeptical beliefs come into play. To see why these are the "right" out-of-equilibrium beliefs, consider what would happen if C were to revise her beliefs downward following a demand (α_2, F_2) below $u(\overline{A})$ but on or above $u(\underline{A})$, and thus accept such a demand (with positive probability). The result would be that L2 could make a positive profit by deviating to such a demand and falsely persuading C that her case has a low expected value when it actually has a high expected value. Moreover, <u>only</u> an L2 of type $\overline{A}$ could profit from such a deviation, since we showed immediately above that if L2 observed $\underline{A}$ but C believed $B_1(\alpha_1, F_1) = \overline{A}$, then L2 could never make a positive profit from such a deviation, even if she were able to convince C that the expected case value was $\underline{A}$. Thus, C should rationally attribute such a deviation to an L2 of type $\overline{A}$.

Analysis of the Continuation Game Between C and L2 when F = 0

Recall that $B_2(\alpha_2 \mid B(\alpha_1))$ denotes C's posterior belief, after having arrived at L2 with beliefs $B_1(\alpha_1)$ and having received the demand α_2 from L2, where:

$B_2(\alpha_2 \mid \underline{A}) = \underline{A}$ for $\alpha_2 = \alpha^C(\underline{A})$; for all other α_2, $B_2(\alpha_2 \mid \underline{A}) \in \{\underline{A}, \overline{A}\}$.

$B_2(\alpha_2 \mid \overline{A}) = \overline{A}$ for all α_2.

We can now characterize the client's optimal behavior in response to L2's demand α_2. First, suppose that $B_1(\alpha_1) = \underline{A}$. If L2's demand is $\alpha_2 = \alpha^C(\underline{A})$, then C's beliefs are confirmed and she is being offered her most-preferred contingent fee, so she is willing to accept this demand. If L2's demand α_2 is anything except $\alpha^C(\underline{A})$ and C continues to believe that $B_2(\alpha_2 \mid \underline{A}) = \underline{A}$, then C rejects the demand α_2 in favor of an auction, from which she expects a payoff of $\Pi^C(\alpha^C(\underline{A}), \underline{A})$. If L2's demand α_2 is anything except $\alpha^C(\underline{A})$ and C revises her belief upward to $B_2(\alpha_2 \mid \underline{A}) = \overline{A}$, then C is willing to accept the demand $\alpha_2 = \alpha^C(\overline{A})$, but she rejects all other demands α_2 in favor of an auction, from which she expects a payoff of $\Pi^C(\alpha^C(\overline{A}), \overline{A})$. Now suppose that $B_1(\alpha_1) = \overline{A}$. Since C does not revise her beliefs, the only demand that C is willing to accept is $\alpha^C(\overline{A})$; all others will be rejected in favor of an auction, from which she expects a payoff of $\Pi^C(\alpha^C(\overline{A}), \overline{A})$.

We now characterize L2's optimal demand α_2, given C's beliefs upon approaching L2, $B_1(\alpha_1)$, and given L2's own observation of the expected case value. In principle, there are four possibilities (though only the first one will occur in the overall equilibrium). First, suppose that L2 observes $\underline{A}$ and C believes $B_1(\alpha_1) = \underline{A}$. Then L2 can obtain a payoff of $\Pi^L(\alpha^C(\underline{A}), \underline{A})$ by demanding $\alpha^C(\underline{A})$, which confirms the client's beliefs, and which she accepts. This is strictly better than his other options and their resulting payoffs: 1) he can obtain a payoff of $\Pi^L(\alpha^C(\overline{A}), \underline{A})$ by demanding $\alpha^C(\overline{A})$ if C is thereby persuaded to revise her beliefs upward to $\overline{A}$ and to accept this demand, but $\alpha^C(\overline{A}) < \alpha^C(\underline{A})$ and the lawyer prefers the higher contingent fee $\alpha^C(\underline{A})$; 2) he can obtain a payoff of $.5\Pi^L(\alpha^C(\underline{A}), \underline{A})$ by provoking an auction using a demand that C associates with $\underline{A}$; or 3) he can obtain a payoff of $.5\Pi^L(\alpha^C(\overline{A}), \underline{A})$ by provoking an auction using a demand that C associates with $\overline{A}$.

Second, suppose that L2 observes $\overline{A}$ but C believes $B_1(\alpha_1) = \underline{A}$. Then L2 can obtain a payoff of $\Pi^L(\alpha^C(\underline{A}), \overline{A})$ by demanding $\alpha_2 = \alpha^C(\underline{A})$, which confirms C's beliefs and which she accepts. Again, this is strictly better than L2's other options: 1) he can obtain a payoff of $\Pi^L(\alpha^C(\overline{A}), \overline{A})$ by demanding $\alpha^C(\overline{A})$ if C is thereby persuaded to revise her beliefs upward to $\overline{A}$ and to accept this demand, but $\alpha^C(\overline{A}) < \alpha^C(\underline{A})$ and the lawyer prefers the higher contingent fee $\alpha^C(\underline{A})$; 2) he can obtain a payoff of $.5\Pi^L(\alpha^C(\underline{A}), \overline{A})$ by provoking an auction using a demand that C associates with $\underline{A}$; or 3) he can obtain a payoff of $.5\Pi^L(\alpha^C(\overline{A}), \overline{A})$ by provoking an auction using a demand that C associates with $\overline{A}$. **To summarize:** when the client approaches L2 believing $B_1(\alpha_1) = \underline{A}$, then L2's unique optimal strategy is to confirm C's beliefs by offering $\alpha_2 = \alpha^C(\underline{A})$, prompting C to accept L2's demand.

Third, suppose that L2 observes $\underline{A}$ but C believes $B_1(\alpha_1) = \overline{A}$. Since there is nothing L2 can do to change C's beliefs, L2's best demand is $\alpha_2 = \alpha^C(\overline{A})$, which confirms C's beliefs and which is accepted, yielding a payoff of $\Pi^L(\alpha^C(\overline{A}), \underline{A})$ for L2 and a payoff of $\Pi^C(\alpha^C(\overline{A}), \underline{A})$ for C. Any other demand would be rejected but C would continue to believe $\overline{A}$, so L2 would expect a payoff of

$.5\Pi^L(\alpha^C(\overline{A}), \underline{A})$ from the auction. Fourth, suppose that L2 observes $\overline{A}$ and C believes $B_1(\alpha_1) = \overline{A}$. Again, there is nothing L2 can do to change C's beliefs, so L2's best demand is $\alpha_2 = \alpha^C(\overline{A})$, which confirms C's beliefs and which is accepted, yielding a payoff of $\Pi^L(\alpha^C(\overline{A}), \overline{A})$ for L2 and a payoff of $\Pi^C(\alpha^C(\overline{A}), \overline{A})$ for C. Any other demand would be rejected but C would continue to believe $\overline{A}$, so L2 would expect a payoff of $.5\Pi^L(\alpha^C(\overline{A}), \overline{A})$ from the auction.

To see why C's skeptical beliefs are the "right" out-of-equilibrium beliefs when $B_1(\alpha_1) = \overline{A}$, consider what would happen if C were to revise her beliefs downward to $\underline{A}$ following an (out-of-equilibrium) demand of $\alpha_2 = \alpha^C(\underline{A})$ and accept this demand.[2] The result would be that either type of L2 could make a positive profit by deviating to $\alpha_2 = \alpha^C(\underline{A})$, but type $\overline{A}$ would benefit more than type $\underline{A}$, since $\Pi^L(\alpha^C(\underline{A}), \overline{A}) - \Pi^L(\alpha^C(\overline{A}), \overline{A}) - [\, \Pi^L(\alpha^C(\underline{A}), \underline{A}) - \Pi^L(\alpha^C(\overline{A}), \underline{A})]$ has the same sign as $\Pi^L_{12} > 0$. Thus, using a D1-type argument (essentially, type $\overline{A}$ would be willing to defect for a higher rejection probability than $\underline{A}$) implies that C should rationally attribute a deviation to the out-of-equilibrium demand $\alpha_2 = \alpha^C(\underline{A})$ to an L2 of type $\overline{A}$.

Finally, we note that there is another equilibrium in the continuation game between L2 and C wherein C rejects L2's demand in favor of an auction whenever she is indifferent between these two choices. This would not change C's equilibrium payoffs but it would change those of the lawyers, since L1 could conceivably end up obtaining the case even if C searches twice and this would affect his initial demand. We do not consider this equilibrium further in the text, but an analysis is provided for a continuum of types later in this Technical Appendix.

2. Equilibrium Refinement using D1 to Select the Least-cost Separating Equilibrium when F is Unconstrained

We argued in the text that the lawyer with an $\overline{A}$-type case must use the equilibrium strategy $(1, \varphi(1, \overline{A}))$, and this demand will be accepted for sure. We also argued that any separating

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[2] One could also imagine L2 using a demand α_2 other than $\alpha^C(\underline{A})$ simply to persuade C to revise her beliefs downward to $\underline{A}$, even though this demand provokes an auction in which L2 expects to obtain a payoff of $.5\Pi^L(\alpha^C(\underline{A}), A)$, where A is the expected value of the case observed by L2. This would not provide a profitable deviation for either $\underline{A}$ or $\overline{A}$ as long as $\Pi^L(\alpha^C(\overline{A}), A) \geq .5\Pi^L(\alpha^C(\underline{A}), A)$ for $A = \underline{A}, \overline{A}$. This inequality surely holds when $\alpha^C(\underline{A})$ and $\alpha^C(\overline{A})$ are not too far apart (recall that they are equal for our power-function example $p(x) = \lambda x^\theta$, and they will be sufficiently close provided that $\underline{A}$ and $\overline{A}$ are sufficiently close).

equilibrium contingent fee for the lawyer with an $\underline{A}$-type case – say, $\hat{\alpha}$, which must be accompanied by the transfer $\varphi(\hat{\alpha}, \underline{A})$ – must belong to the interval $[\alpha^k, 1]$, where α^k is defined implicitly by $s = \Pi^L(\alpha^k, \bar{A}) - \varphi(\alpha^k, \underline{A})$. Recall that the minimal acceptance probability (to deter mimicry by $\bar{A}$) at α^k is 1; for all $\alpha \in (\alpha^k, 1]$, the minimal acceptance probability (to deter mimicry by $\bar{A}$) is strictly less than 1 (see Figure 2). In the text we denoted the separating equilibrium α-value for the $\underline{A}$-type as $\underline{\alpha}$*; in what follows we suppress the * so as to simplify the notational overhead.

In the text, we identify a particular separating equilibrium given by $\{(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A})), (1, \varphi(1, \bar{A}))\}$, with $1 - r(1, \varphi(1, \bar{A})) = 1$ and $1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A})) = s/[\Pi^L(\underline{\alpha}, \bar{A}) - \varphi(\underline{\alpha}, \underline{A})]$, where $\underline{\alpha}$ maximizes $s[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A})]/[\Pi^L(\alpha, \bar{A}) - \varphi(\alpha, \underline{A})]$. Although (as indicated above) the lawyer with an $\bar{A}$-type case must use the second component of the equilibrium strategy described above (and will be accepted for sure), there are many possible alternative equilibrium strategies of the form $(\alpha, \varphi(\alpha, \underline{A}))$ for the lawyer with an $\underline{A}$-type case, each one supported by an associated acceptance probability and out-of-equilibrium beliefs. Under the additional assumption that there is a unique global maximizer $\underline{\alpha}$, we claim that the selected equilibrium is the unique separating equilibrium outcome surviving refinement using D1. We state this additional assumption below and maintain it thereafter.

<u>Assumption T1</u>. Assume that $\underline{\alpha} = \text{argmax} \{ s[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A})]/[\Pi^L(\alpha, \bar{A}) - \varphi(\alpha, \underline{A})] \mid \alpha \in [\alpha^k, 1]\}$ is unique.[3]

First, we will show that $IC(\bar{A})$ must hold with equality in any separating equilibrium surviving D1. This implies that any D1 separating equilibrium must involve an acceptance probability on the downward-sloping portion of the curve labeled "minimal rejection curve" in Figure 2, where the $\bar{A}$-type is just-deterred from mimicry. Second, we will argue that no separating equilibrium involving a strategy $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))$ for which $\hat{\alpha} \neq \underline{\alpha}$ will survive D1. Finally, we will prove that the sole remaining candidate, $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))$, does survive D1.

<u>Claim 1</u>. $IC(\bar{A})$ must hold with equality in any separating equilibrium surviving D1.

<u>Proof</u>. Suppose there exists a separating equilibrium with strategies $\{(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A})), (1, \varphi(1, \bar{A}))\}$ with associated acceptance probabilities of $1 - r(1, \varphi(1, \bar{A})) = 1$ and $1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))$. Moreover, suppose that $IC(\bar{A})$ is slack. Then $1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A})) < 1$ (since it must be strictly less than the

[3] We conjecture that if there were multiple optima, then the following arguments would still apply, and only the separating equilibria involving these values of the contingent fee would survive refinement using D1.

minimal acceptance probability to deter mimicry by $\overline{A}$). Then the $\overline{A}$- type's equilibrium payoff is:

$$s > [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})],$$

while the $\underline{A}$-type's equilibrium payoff is:

$$[1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})].$$

Now consider a defection to $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}) + \varepsilon)$, where ε is a small positive number. This defection must be met with a sufficiently high rejection probability in order to support the hypothesized separating equilibrium (which means the out-of-equilibrium beliefs must place a sufficiently high probability on type $\overline{A}$; in the text, we assumed the beliefs assigned a probability of 1 to type $\overline{A}$, leading to rejection). We will argue that the equilibrium refinement D1 requires that – starting from this putative equilibrium wherein IC($\overline{A}$) is slack – beliefs at this out-of-equilibrium demand be $\underline{A}$, leading to acceptance by the client. This, in turn, would induce the lawyer of type $\underline{A}$ to pursue the (profitable) defection and upset the original separating equilibrium at $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))$.

To see this, notice that the $\underline{A}$-type would be willing to defect to $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}) + \varepsilon)$ for any acceptance probability 1 - r such that:

$$(1 - r)[\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A}) - \varepsilon] \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})];$$

that is, for any probability:

$$1 - r \geq 1 - \tilde{r}(\underline{A}) \equiv [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A}) - \varepsilon].$$

However, for sufficiently small ε, the $\overline{A}$-type would <u>not</u> be willing to defect to $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}) + \varepsilon)$, since:

$$\lim_{\varepsilon \to 0} [1 - \tilde{r}(\underline{A})][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A}) - \varepsilon] = [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})] < s.$$

Thus, for sufficiently small ε, an observed demand of $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}) + \varepsilon)$ must (under D1) be inferred to have come from an $\underline{A}$-type. Since $\Pi^C(\hat{\alpha}, \underline{A}) + \varphi(\hat{\alpha}, \underline{A}) + \varepsilon > \Pi^L(1, \underline{A}) - s$, the client strictly prefers to accept the out-of-equilibrium demand under the belief $\underline{A}$. Finally, since $1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A})) < 1$, it follows that:

$$\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A}) - \varepsilon > [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]$$

for sufficiently small ε and thus $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}) + \varepsilon)$ provides a profitable deviation for type $\underline{A}$. QED

<u>Claim 2</u>. Any separating equilibrium involving $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))$, with $\hat{\alpha} \neq \underline{\alpha}$, does not survive D1.

Proof. Suppose, to the contrary, that there exists a separating equilibrium with strategies $\{(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A})), (1, \varphi(1, \overline{A}))\}$ and with associated acceptance probabilities of $1 - r(1, \varphi(1, \overline{A})) = 1$ and $1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))$, where $\hat{\alpha} \neq \underline{\alpha}$. This equilibrium is supported by out-of-equilibrium beliefs that assign a sufficiently high probability to the $\overline{A}$-type. We will argue that the $\underline{A}$-type can deviate to the demand $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}) + \varepsilon)$ for sufficiently small positive ε, be identified under D1 and accepted, and thereby profit from the deviation, upsetting the hypothesized separating equilibrium.

To see this, first note that since IC($\overline{A}$) is tight, the $\overline{A}$-type's equilibrium payoff is:

$$s = [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})],$$

while the $\underline{A}$-type's equilibrium payoff is:

$$[1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})].$$

Then the $\overline{A}$-type will be willing to defect to $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}) + \varepsilon)$ if the client's response, denoted in terms of the acceptance probability $1 - r$, is such that:

$$(1 - r)[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})];$$

that is, if:

$$(1 - r) \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] \equiv 1 - \tilde{r}(\overline{A}).$$

Similarly, the $\underline{A}$- type will be willing to defect to $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}) + \varepsilon)$ if the client's response, denoted in terms of the acceptance probability $1 - r$, is such that:

$$(1 - r)[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})];$$

that is, if:

$$(1 - r) \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] \equiv 1 - \tilde{r}(\underline{A}).$$

If $1 - \tilde{r}(\overline{A}) > 1 - \tilde{r}(\underline{A})$, then (according to D1), the out-of-equilibrium demand $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}) + \varepsilon)$ must be inferred to have been made by the $\underline{A}$ -type, since this type is willing to defect for the largest range of acceptance probabilities. The inequality $1 - \tilde{r}(\overline{A}) > 1 - \tilde{r}(\underline{A})$ holds if and only if:

$$[\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] > [\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon];$$

that is, if and only if:

$$[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon]/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] > [\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})].$$

Notice that the inequality is true for $\varepsilon = 0$ since $\underline{\alpha}$ (uniquely) maximizes the expression $[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A})]/[\Pi^L(\alpha\ \overline{A}) - \varphi(\alpha, \underline{A})]$. Thus, there is a sufficiently small (but still positive) ε for which the inequality still holds. Thus, such an out-of-equilibrium demand will be believed by the client to have come from the $\underline{A}$-type. Since $\Pi^C(\underline{\alpha}, \underline{A}) + \varphi(\underline{\alpha}, \underline{A}) + \varepsilon > \Pi^L(1, \underline{A}) - s$, the client will accept this out-of-equilibrium demand with probability 1. Finally, we claim that this will induce the $\underline{A}$-type to make this defection, thus upsetting the hypothesized separating equilibrium. To verify this final claim, recall that since we need only consider α-values in $[\alpha^k, 1]$, it follows that $s/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A})] \leq 1$. Note that:

$$\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) \geq s[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})]/ [\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})]$$

$$> s[\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})] = [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})].$$

The first inequality follows since $s/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})] \leq 1$ and the second (strict) inequality follows since $\underline{\alpha}$ uniquely maximizes the ratio of the terms in brackets. By defecting, the $\underline{A}$-type will obtain $\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon$. Since $\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) > [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]$, there is a sufficiently small ε for which $\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon > [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]$. QED

<u>Claim 3</u>. The separating equilibrium outcome given by $\{(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A})), (1, \varphi(1, \overline{A}))\}$, with $1 - r(1, \varphi(1, \overline{A})) = 1$ and $1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A})) = s/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})]$, where $\underline{\alpha}$ maximizes

$$s[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A})]/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A})],$$

survives refinement using D1.

<u>Proof</u>. In the specified equilibrium, the $\overline{A}$-type's equilibrium payoff is:

$$s = [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})],$$

while the $\underline{A}$-type's equilibrium payoff is:

$$[1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})].$$

Consider any out-of-equilibrium demand $(\alpha, \varphi(\alpha, \underline{A}) + \varepsilon)$, where $\alpha \neq \underline{\alpha}$ and $\varepsilon \geq 0$ or where $\alpha = \underline{\alpha}$ and $\varepsilon > 0$. Any out-of-equilibrium demand along $U(\underline{A})$ or between the loci $U(\overline{A})$ and $U(\underline{A})$ can be represented this way (see Figure 1). Demands on or above $U(\overline{A})$ are accepted regardless of beliefs and demands below $U(\underline{A})$ are rejected regardless of beliefs, and neither type is tempted to deviate to these out-of-equilibrium demands. So it is only the out-of-equilibrium demands between the loci (and along $U(\underline{A})$) that must be considered. Some of these demands are also immune to defection (that is, there is no response by the client that would tempt either type to defect); what we need to show is that if the $\underline{A}$-type is willing to defect to a particular out-of-equilibrium demand for some responses by the client, then the $\overline{A}$-type is willing to defect to that demand for a strictly greater range of client responses. D1 then requires that the beliefs assign the $\overline{A}$-type to the defection, which will lead to certain rejection by the client, which will, in turn, deter both types from defecting from the separating equilibrium involving $\underline{\alpha}$.

The $\overline{A}$-type will be willing to defect to $(\alpha, \varphi(\alpha, \underline{A}) + \varepsilon)$ if the client's response, denoted in terms of the acceptance probability $1 - r$, is such that:

$$(1 - r)[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] \geq [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})];$$

that is, if:

$$(1 - r) \geq [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] \equiv 1 - \tilde{r}(\overline{A}).$$

Similarly, the $\underline{A}$- type will be willing to defect to $(\alpha, \varphi(\alpha, \underline{A}) + \varepsilon)$ if the client's response, denoted in terms of the acceptance probability $1 - r$, is such that:

$$(1 - r)[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] \geq [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})];$$

that is, if:

$$(1 - r) \geq [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] \equiv 1 - \tilde{r}(\underline{A}).$$

If $1 - \tilde{r}(\underline{A}) > 1 - \tilde{r}(\overline{A})$, then (according to D1), the out-of-equilibrium demand $(\alpha, \varphi(\alpha, \underline{A}) + \varepsilon)$ must be inferred to have been made by the $\overline{A}$-type, since this type is willing to defect for the largest range of acceptance probabilities.

The inequality $1 - \tilde{r}(\underline{A}) > 1 - \tilde{r}(\overline{A})$ holds if and only if:

$$[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] > [\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon];$$

that is, if and only if:

$$[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})] > [\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon]/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon].$$

First, consider $\alpha \neq \underline{\alpha}$ and $\varepsilon = 0$; this is a deviation along the locus $U(\underline{A})$. Then the inequality holds since $\underline{\alpha}$ uniquely maximizes the ratio of the terms in brackets. Since the right-hand-side decreases as ε increases, the inequality also holds when $\alpha = \underline{\alpha}$ and $\varepsilon > 0$ and when $\alpha \neq \underline{\alpha}$ and $\varepsilon > 0$. QED

Finally, we note that in the text we specified the beliefs to be $B(\alpha, F) = \underline{A}$ for all (α, F) along the locus $U(\underline{A})$, even those involving $\alpha \neq \underline{\alpha}$. This was helpful for the purpose of exposition, it seems intuitively reasonable, and it suffices to support the separating equilibrium outcome involving $\underline{\alpha}$ (as specified above in Claim 3). However, as we have seen in the foregoing analysis, the out-of-equilibrium beliefs implied by D1 are somewhat harsher, requiring $B(\alpha, F) = \bar{A}$ (leading to rejection) for out-of-equilibrium values of α (i.e., for $\alpha \neq \underline{\alpha}$) along the locus $U(\underline{A})$. These harsher beliefs support the same separating equilibrium outcome.

3. Verification that Skeptical Beliefs Survive Refinement using D1 (uniquely) when Assumption 6 Holds (with a strict inequality) for the case of F = 0.

By "skeptical beliefs,"we mean that $B_1(\alpha) = \bar{A}$ for $\alpha \in (\alpha^L(\bar{A}), \alpha^L(\underline{A}))$. Recall that type $\bar{A}$'s equilibrium payoff is $\Pi^L(\alpha^L(\bar{A}), \bar{A})$ and type $\underline{A}$'s equilibrium payoff is $(1 - r^*)\Pi^L(\alpha^L(\underline{A}), \underline{A})$, where $1 - r^* = \Pi^L(\alpha^L(\bar{A}), \bar{A})/\Pi^L(\alpha^L(\underline{A}), \bar{A})$. Now consider an out-of-equilibrium demand $\alpha' \in (\alpha^L(\bar{A}), \alpha^L(\underline{A}))$. C will reject such a demand if she believes it comes from type $\bar{A}$ and she will accept it if she believes it comes from $\underline{A}$ (as long as $\alpha' \geq \alpha^C(\underline{A})$). Let ρ denote the probability that C believes the demand α' comes from type $\bar{A}$; then C will accept the demand α' with probability $1 - \rho$. Type $\bar{A}$ would be willing to defect from his equilibrium demand to the demand α' if $(1 - \rho)\Pi^L(\alpha', \bar{A}) \geq \Pi^L(\alpha^L(\bar{A}), \bar{A})$. Type $\underline{A}$ would be willing to defect to the demand α' if $(1 - \rho)\Pi^L(\alpha', \underline{A}) \geq (1 - r^*)\Pi^L(\alpha^L(\underline{A}), \underline{A})$. The minimum acceptance threshold for type $\bar{A}$ is $(1 - \rho(\bar{A})) \equiv [\Pi^L(\alpha^L(\bar{A}), \bar{A})/\Pi^L(\alpha', \bar{A})]$, while the minimum acceptance threshold for type $\underline{A}$ is $(1 - \rho(\underline{A})) \equiv (1 - r^*)[\Pi^L(\alpha^L(\underline{A}), \underline{A})/\Pi^L(\alpha', \underline{A})] = [\Pi^L(\alpha^L(\bar{A}), \bar{A})/\Pi^L(\alpha^L(\underline{A}), \bar{A})][\Pi^L(\alpha^L(\underline{A}), \underline{A})/\Pi^L(\alpha', \underline{A})]$, upon substituting for $1 - r^*$. According to D1, if the minimum acceptance threshold is strictly lower for $\bar{A}$ than for $\underline{A}$, then the out-of-equilibrium beliefs must associate the demand α' with type $\bar{A}$ (if the two thresholds are equal, it is allowable to associate the demand α' with type $\bar{A}$, but not required). After some algebraic manipulation, it can be shown that: $(1 - \rho(\bar{A})) < (=)\ (1 - \rho(\underline{A}))$ as $\Pi^L(\alpha', \underline{A})/\Pi^L(\alpha^L(\underline{A}), \underline{A}) > (=)\ \Pi^L(\alpha', \bar{A})/\Pi^L(\alpha^L(\underline{A}), \bar{A})$. Since $\alpha' < \alpha^L(\underline{A})$ and $\bar{A} > \underline{A}$, it follows from Assumption 6 that the inequality holds. Thus, the skeptical beliefs survive refinement using D1; when the inequality in Assumption 6 is strict, then such out-of-equilibrium demands α' must be assigned $B_1(\alpha') = \bar{A}$.

4. Equilibrium Refinement using D1 to Eliminate a Separating Equilibrium at the Lower Root of Equation (5).

Recall that equation (5) has two roots (where C is indifferent between accepting and visiting L2); the larger root is what we refer to as $\alpha^L(A)$ and this is L1's preferred solution under precontracting full information. Could there be another separating equilibrium (under asymmetric information) in which the lower root is used by one or both types? Consider type $\overline{A}$; in any separating equilibrium the lawyer of type $\overline{A}$ (the weak type) plays his full-information strategy, $\alpha^L(\overline{A})$, which is accepted for sure (so that he makes his full-information payoff). Now consider the lawyer of type $\underline{A}$, and let the lower root to equation (5) be denoted as $\alpha^\#$; can there be a separating equilibrium wherein type $\underline{A}$ demands $\alpha^\#$ and type $\overline{A}$ demands $\alpha^L(\overline{A})$? First, if $\alpha^\# < \alpha^L(\overline{A})$, then type $\underline{A}$ would prefer to defect to $\alpha^L(\overline{A})$, which is higher and is accepted for sure (this configuration occurs in the power-function example, so there cannot be a second separating equilibrium at the lower root for the power-function example). Second, if $\alpha^\# = \alpha^L(\overline{A})$, then this pair of demands does not separate types. Finally, if $\alpha^\# > \alpha^L(\overline{A})$, then C must reject $\alpha^\#$ with positive probability to deter type $\overline{A}$ from defecting to $\alpha^\#$. The corresponding IC constraints are (where r is the probability that the demand $\alpha^\#$ is rejected):

$$IC(\overline{A}): \quad \Pi^L(\alpha^L(\overline{A}), \overline{A}) \geq (1 - r)\Pi^L(\alpha^\#, \overline{A});$$

$$IC(\underline{A}): \quad (1 - r)\Pi^L(\alpha^\#, \underline{A}) \geq \Pi^L(\alpha^L(\overline{A}), \underline{A}).$$

Taken together, these imply that $1 - r \in [\Pi^L(\alpha^L(\overline{A}), \underline{A})/\Pi^L(\alpha^\#, \underline{A}), \Pi^L(\alpha^L(\overline{A}), \overline{A})/\Pi^L(\alpha^\#, \overline{A})]$. Assuming that C uses the lowest rejection probability consistent with deterring mimicry by $\overline{A}$, then the equilibrium rejection probability, denoted $r^\#$, is given by the upper endpoint of the interval: $1 - r^\# = \Pi^L(\alpha^L(\overline{A}), \overline{A})/(\Pi^L(\alpha^\#, \overline{A})$. Assume for the moment that $\alpha^\#$ does provide a second separating equilibrium (rather strange out-of-equilibrium beliefs are needed to support it, but that will be discussed later). We can compare the equilibrium payoffs to the L1 of type $\underline{A}$ at the two separating equilibria (C and the L1 of type $\underline{A}$ are indifferent). In the equilibrium wherein L1 plays $\alpha^L(\underline{A})$, his expected payoff is $(1 - r^*)\Pi^L(\alpha^L(\underline{A}), \underline{A})$; using r* as defined in the text, this becomes: $\pi(\alpha^L(\underline{A}) \equiv [\Pi^L(\alpha^L(\overline{A}), \overline{A})/\Pi^L(\alpha^L(\underline{A}), \overline{A})]\Pi^L(\alpha^L(\underline{A}), \underline{A})$. In the equilibrium wherein L1 plays $\alpha^\#$, his payoff is $\pi(\alpha^\#) \equiv (1 - r^{\#\prime})\Pi^L(\alpha^\#, \underline{A}) = [\Pi^L(\alpha^L(\overline{A}), \overline{A})/\Pi^L(\alpha^\#, \overline{A})]\Pi^L(\alpha^\#, \underline{A})$. Notice that (after simplification) $\pi(\alpha^L(\underline{A}))$ (>, =, <) $\pi(\alpha^\#)$ as $\Pi^L(\alpha^\#, \overline{A})/\Pi^L(\alpha^L(\underline{A}), \overline{A})$ (>, =, <) $\Pi^L(\alpha^\#, \underline{A})/\Pi^L(\alpha^L(\underline{A}), \underline{A})$. By Assumption 6, it follows that $\pi(\alpha^L(\underline{A})) \geq \pi(\alpha^\#)$ (with a strict inequality if the ratio in Assumption 6 is strictly increasing). Thus, we argue that C should expect the L1 of type $\underline{A}$ to demand $\alpha^L(\underline{A})$, and she should accord this the belief that $B(\alpha^L(\underline{A})) = \underline{A}$, and reject this demand with probability 1 - r* (this still deters mimicry by type $\overline{A}$). Alternatively, the L1 of type $\underline{A}$ can make a speech to C to this effect: "I am demanding $\alpha^L(\underline{A})$, and you should accord this the belief that $B(\alpha^L(\underline{A})) = \underline{A}$, and reject this demand with probability $1 - r^*$ (this still deters mimicry by type $\overline{A}$)."

An alternative way to select the equilibrium involving $\alpha^L(\underline{A})$ is to notice that, in order to support a separating equilibrium at $\alpha^\#$, C needs to reject demands in the interval $(\alpha^\#, \alpha^L(\underline{A}))$ with a sufficiently high probability. Since C would want to accept (reject) such a demand for sure if she believed it came from an L1 of type $\underline{A}$ (of type $\overline{A}$), she would have to assign a sufficiently high probability to such a demand coming from an L1 of type $\overline{A}$ in order to be willing to reject it with the requisite probability. We argue that, if the ratio in Assumption 6 is strictly increasing in A, then the beliefs required to support the separating equilibrium at $\alpha^\#$ do not survive refinement using D1. (Note that for the power function example, the ratio in Assumption 6 is constant in A; however, the "second" equilibrium at the lower root is directly eliminated in this case; see the discussion above). Therefore, in what follows, we assume that the ratio in Assumption 6 is strictly increasing in A.

Consider an out-of-equilibrium demand $\alpha' \in (\alpha^\#, \alpha^L(\underline{A}))$, and let ρ denote the probability that C believes the demand α' comes from type $\overline{A}$; then she will accept such a demand with probability $1 - \rho$. Type $\overline{A}$ would be willing to defect from his equilibrium demand $\alpha^L(\overline{A})$ to the demand α' if $(1 - \rho)\Pi^L(\alpha', \overline{A}) \geq \Pi^L(\alpha^L(\overline{A}), \overline{A})$. Type $\underline{A}$ would be willing to defect from his equilibrium demand $\alpha^\#$ to the demand α' if $(1 - \rho)\Pi^L(\alpha', \underline{A}) \geq (1 - r^\#)\Pi^L(\alpha^\#, \underline{A})$. The minimum acceptance threshold for type $\overline{A}$ is $(1 - \rho(\overline{A})) \equiv [\Pi^L(\alpha^L(\overline{A}), \overline{A})/\Pi^L(\alpha', \overline{A})]$, while the minimum acceptance threshold for type $\underline{A}$ is $(1 - \rho(\underline{A})) \equiv (1 - r^\#)[\Pi^L(\alpha^\#, \underline{A})/\Pi^L(\alpha', \underline{A})] = [\Pi^L(\alpha^L(\overline{A}), \overline{A})/(\Pi^L(\alpha^\#, \overline{A})][\Pi^L(\alpha^\#, \underline{A})/\Pi^L(\alpha', \underline{A})]$.

According to D1, if the minimum acceptance threshold is strictly higher for $\overline{A}$, then the out-of-equilibrium demand α' should be associated with type $\underline{A}$ (and thus accepted by C for sure, which would in turn induce defection from their equilibrium demands by both types, since $\alpha' > \alpha^\# > \alpha^L(\overline{A})$ and lawyers always prefer a higher contingent fee). After some algebraic manipulation, it can be shown that: $(1 - \rho(\overline{A})) > (1 - \rho(\underline{A}))$ as $\Pi^L(\alpha^\#, \overline{A})/\Pi^L(\alpha', \overline{A}) > \Pi^L(\alpha^\#, \underline{A})/\Pi^L(\alpha', \underline{A})$. Assumption 6 (with strictly increasing ratio) implies that this inequality holds.

5. Some Results on the Continuum-type Case for F = 0

When C accepts L2's demand (pre-empting the auction)

In what follows, we simply denote by α an arbitrary demand by L1. C departs from L1 with beliefs $B_1(\alpha)$. If L2 demands $\alpha^C(B_1(\alpha))$, then C is willing to forego the auction and accept L2's demand. Thus, the client expects that, if she searches again, she will obtain a payoff of $\Pi^C(\alpha^C(B_1(\alpha)), B_1(\alpha)) - s$. Her expected payoff if she rejects the offer α with probability r (and searches again) is:

$$(1 - r)\Pi^C(\alpha, B_1(\alpha)) + r[\Pi^C(\alpha^C(B_1(\alpha)), B_1(\alpha)) - s].$$

The client will accept the demand by L1 of α if $\Pi^C(\alpha, B_1(\alpha)) > \Pi^C(\alpha^C(B_1(\alpha)), B_1(\alpha))$ - s and reject the demand α if $\Pi^C(\alpha, B_1(\alpha)) < \Pi^C(\alpha^C(B_1(\alpha)), B_1(\alpha))$ - s. She will be indifferent between accepting and rejecting the demand α if $\Pi^C(\alpha, B_1(\alpha)) = \Pi^C(\alpha^C(B_1(\alpha)), B_1(\alpha))$ + s; in this event she will be willing to randomize between the strategies of accepting the demand of α and rejecting it in favor of seeking a second option. Randomizing is the means by which she can induce the types of lawyers to reveal themselves (that is, for the first lawyer's demand to reveal the expected value of the case).

From L1's point of view, the client will be using a rejection strategy that depends on the contingent fee he quotes: $r(\alpha)$. Anticipating that L2 will demand $\alpha^C(B_1(\alpha))$ and the client will accept this demand, L1 will only obtain C's case if she does not reject his demand in favor of a second search. Thus, L1's payoff from this interaction with C is simply $(1 - r(\alpha))\Pi^L(\alpha, A)$; although this lawyer will also obtain some cases for which he is L2, his contingent fee in those cases will be $\alpha^C(B_1(\alpha'))$, where α' is the demand made by some other lawyer, whose client did not accept it.

We are interested in a separating equilibrium, which consists of a rejection function $r(\alpha)$ that maximizes C's expected payoff, given her beliefs $B_1(\alpha)$, and a demand function that maximizes L1's expected payoff, given the rejection function employed by C. As will be shown below, the demand function will again be $\alpha^L(A)$. Finally, C's beliefs must be correct in equilibrium; that is, $B_1(\alpha^L(A)) = A$ for all $A \in [\underline{A}, \bar{A}]$. Note that the rejection function and beliefs must be defined for all $\alpha \in [0, 1]$, not just for equilibrium values of α.

In a separating equilibrium the following observations must hold. First, there will be a smallest and a largest contingent fee, denoted by $\underline{\alpha}$ and $\bar{\alpha}$, respectively. Second, the function $r(\alpha)$ must be increasing on $(\underline{\alpha}, \bar{\alpha})$; that is, the client must reject higher contingent fee offers with a higher probability since to do otherwise would invite mimicry and pooling. Third, since $r(\alpha)$ must be increasing on $(\underline{\alpha}, \bar{\alpha})$, it must be interior (i.e., $r(\alpha) \in (0, 1)$) on $(\underline{\alpha}, \bar{\alpha})$.

Fourth, this last point implies that the client must be made indifferent about searching again. Indifference implies that α must satisfy $\Pi^C(\alpha, B_1(\alpha)) = \Pi^C(\alpha^C(B_1(\alpha)), B_1(\alpha))$ - s. That is, there is a function $\alpha^0(A)$, such that $\Pi^C(\alpha^0(A), B_1(\alpha^0(A))) = \Pi^C(\alpha^C(B_1(\alpha^0(A))), B_1(\alpha^0(A))))$ - s. Consistency of beliefs requires that $B_1(\alpha^0(A)) = A$, so that the indifference requirement is that:

$$\Pi^C(\alpha^0(A), A) = \Pi^C(\alpha^C(A), A) - s, \tag{TA.1}$$

but this means that $\alpha^0(A)$ is identically equal to $\alpha^L(A)$. In other words, the full-information contingent-fee demand function, $\alpha^L(A)$, is also the separating equilibrium contingent-fee demand function. Since this function is downward-sloping, it provides the implied values of $\underline{\alpha}$ and $\bar{\alpha}$: $\underline{\alpha} = \alpha^L(\bar{A})$ and $\bar{\alpha} = \alpha^L(\underline{A})$. Fifth, the function $r(\alpha)$ must be continuous on $[\underline{\alpha}, \bar{\alpha})$, and continuous from the left at $\bar{\alpha}$. To see this, suppose to the contrary that there is a jump at some α in this interval; note that any jump must be upward since $r(\alpha)$ is increasing. But then type $A = (\alpha^L)^{-1}(\alpha)$ would prefer to cut his demand infinitesimally to gain a discrete reduction in the probability of rejection, which contradicts the fact that $\alpha^L(A)$ is the equilibrium contingent-fee offer function. Note that upward jumps to the right of $\bar{\alpha}$ are not ruled out and will, indeed, be part of the equilibrium. Finally, regardless of out-of-equilibrium beliefs, $r(\alpha) = 0$ for $\alpha^C(\bar{A}) \leq \alpha < \underline{\alpha}$ and $r(\alpha) = 1$ for $\alpha > \bar{\alpha}$. Since $r(\alpha)$ is continuous at $\underline{\alpha}$, it follows that $r(\underline{\alpha}) = 0$; this provides a boundary condition for the equilibrium rejection function.

Differentiating L1's payoff with respect to α yields the following first-order condition:

$$-r'(\alpha)\Pi^L(\alpha, A) + (1 - r(\alpha))\Pi_1^L(\alpha, A) = 0.$$

Substituting $A = B_1(\alpha)$ yields the differential equation $-r'(\alpha)\Pi^L(\alpha, B_1(\alpha)) + (1 - r(\alpha))\Pi_1^L(\alpha, B_1(\alpha)) = 0$. The solution through the boundary condition $r(\underline{\alpha}) = 0$ is:

$$r(\alpha) = 1 - \exp\left\{- \int [\Pi_1^L(t, B(t))/\Pi^L(t, B(t))]dt\right\}, \text{ where the integral is over } [\underline{\alpha}, \alpha]. \quad \text{(TA.2)}$$

This rejection function, along with the demand function $\alpha^L(A)$, provides a candidate for a separating equilibrium; it remains to verify that each of these components is a best response to the other.

Verification that ($\alpha^L(A)$, $r(\alpha)$) as defined in equations (TA.1) and (TA.2) provide the unique separating equilibrium outcome (when C accepts L2's offer)

We have to demonstrate that L1's and C's strategies are best replies to each other (when both anticipate that C will accept an offer of $\alpha^L(B_1(\alpha))$ from L2 rather than initiating the auction); moreover, we have to prove that $\alpha^L(A)$ is L1's unique optimum against $r(\alpha)$. If C expects L1 to play the strategy $\alpha^L(A)$, then C's beliefs will be $B_1(\alpha) = (\alpha^L)^{-1}(\alpha)$ and C will be indifferent between accepting and searching again. Therefore she is willing to randomize according to $r(\alpha)$ and thus $r(\alpha)$ is a best reply to $\alpha^L(A)$. If C observes an out-of-equilibrium $\alpha > \bar{\alpha}$, then she will reject this demand for sure (regardless of her beliefs). If C observes an out-of-equilibrium $\alpha < \underline{\alpha}$, she will accept it if $\alpha \geq \alpha^C(B_1(\alpha))$ and she will reject it otherwise.

Now suppose that C is expected to play the strategy $r(\alpha)$, where the function $B_1(\alpha) = (\alpha^L)^{-1}(\alpha)$; we will show that it is a unique best reply for L1 to play according to $\alpha^L(A)$. For future use, it is worth noting that equation (TA.2) implies that $r(\alpha) \in [0, 1)$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ and that $r'(\alpha) = (1 - r(\alpha))[\Pi_1^L(\alpha, B_1(\alpha))/\Pi^L(\alpha, B_1(\alpha))]$.

Recall that L1's payoff is $(1 - r(\alpha))\Pi^L(\alpha, A)$. First, we argue that any α outside the interval $[\underline{\alpha}, \bar{\alpha}]$ is strictly dominated by one inside the interval. Any $\alpha > \bar{\alpha}$ is rejected for sure and is thus strictly dominated by the demand $\alpha = \bar{\alpha}$ which is accepted with positive probability. Any demand $\alpha < \underline{\alpha}$ – whether it is accepted or rejected – is strictly dominated by the demand $\alpha = \underline{\alpha}$, which is accepted for sure. Next, notice that the first-order condition is given by:

$$-r'(\alpha)\Pi^L(\alpha, A) + (1 - r(\alpha))\Pi_1^L(\alpha, A)$$
$$= (1 - r(\alpha))\left\{\Pi_1^L(\alpha, A) - \Pi^L(\alpha, A)[\Pi_1^L(\alpha, B_1(\alpha))/\Pi^L(\alpha, B_1(\alpha))]\right\} = 0.$$

Since $1 - r(\alpha) > 0$, any stationary point equates the bracketed term to zero. Assumption 6 (which implies that the term in brackets is strictly decreasing in B_1 when $\Pi^L(\alpha, A)$ is strictly submodular) implies that there is a unique stationary point, at which $B_1(\alpha) = A$ or, equivalently, $\alpha = \alpha^L(A)$. The second-order condition for a maximum holds at the stationary point if and only if:

$$d\left\{\Pi^L_1(\alpha, A) - \Pi^L(\alpha, A)[\Pi^L_1(\alpha, B_1(\alpha))/\Pi^L(\alpha, B_1(\alpha))]\right\}/d\alpha < 0 \text{ at } B_1(\alpha) = A.$$

Differentiating and collecting terms implies that the inequality above holds if and only if:

$$- B_1'(\alpha)\partial[\Pi^L_1(\alpha, B_1(\alpha))/\Pi^L(\alpha, B_1(\alpha))]/\partial B_1 < 0 \text{ at } B_1(\alpha) = A.$$

Since $B_1'(\alpha) < 0$, Assumption 6 (assuming strict sub-modularity) implies that the inequality holds. Thus, the unique stationary point is a maximum. It is interior to the interval $[\underline{\alpha}, \overline{\alpha}]$ if A is interior to the interval $[\underline{A}, \overline{A}]$. Finally, this local interior maximum must be the global maximum because, if it were not (if there were a higher local maximum at either $\underline{\alpha}$ or $\overline{\alpha}$), then there would have to be an interior minimum between the two local maxima and we already know there is a unique stationary point. Thus, the function $\alpha = \alpha^L(A)$ provides the unique best reply for L1 to the strategy $r(\alpha)$. QED

When C Initiates the Auction after Visiting Two Lawyers

In the text we argued that there were two possible continuation equilibria after C visits two lawyers. We focused there on the one wherein L2 demands $\alpha^C(B_1(\alpha))$ and C accepts this demand. Alternatively, it is always a best response for C to initiate the auction following any demand by L2. If L1 anticipates this alternative continuation equilibrium, then when C rejects his demand in favor of a second opinion, L1 expects that he will be involved in the subsequent auction (wherein both lawyers will bid $\alpha^C(B_1(\alpha))$, and each will obtain the case with probability 1/2). L1's payoff in this game can be written as:

$$(1 - r(\alpha))\Pi^L(\alpha, A) + r(\alpha)\Pi^L(\alpha^C(B_1(\alpha)), A)/2.$$

That is, with probability $(1 - r(\alpha))$, C accepts the contingent fee α, in which case L1earns $\Pi^L(\alpha, A)$. But with probability $r(\alpha)$, C rejects the demand α and seeks a second option, initiating an auction for the right to represent the client. Since C's belief upon observing the offer α is that A is equal to $B_1(\alpha)$, the contingent fee that C will most prefer – and, therefore, the contingent fee that both lawyers will "bid" – is $\alpha^C(B_1(\alpha))$. Thus, each lawyer expects to obtain the case with probability 1/2, and to make $\Pi^L(\alpha^C(B_1(\alpha)), A)$ from the case should he obtain it.

Maximizing L1's payoff with respect to α yields the following first-order condition:

$$- r'(\alpha)[\Pi^L(\alpha, A) - \Pi^L(\alpha^C(B_1(\alpha)), A)/2] + (1 - r(\alpha))[\Pi^L_1(\alpha, A)]$$
$$+ (r(\alpha)/2)[\Pi^L_1(\alpha^C(B_1(\alpha)), A)\alpha^{C\prime}(B_1(\alpha))B_1'(\alpha)] = 0.$$

Using the substitution $A = B_1(\alpha)$ to eliminate A from the equation above yields:

$$- r'(\alpha)[\Pi^L(\alpha, B_1(\alpha)) - \Pi^L(\alpha^C(B_1(\alpha)), B_1(\alpha))/2] + (1 - r(\alpha))[\Pi^L_1(\alpha, B_1(\alpha))]$$

$$+ (r(\alpha)/2)[\Pi^L_1(\alpha^C(B_1(\alpha)), B_1(\alpha))\alpha^{C\prime}(B_1(\alpha))B_1{}'(\alpha)] = 0. \quad (TA.3)$$

Recall that $\alpha^L(A)$ is a decreasing function (and thus so is $B_1(\alpha)$), while $\alpha^C(A)$ is decreasing (or constant, as in the pwer-function example); moreover, recall that $\alpha^L(A) > \alpha^C(A)$ for all $A \in [\underline{A}, \overline{A}]$. Consequently, it follows that $\alpha > \alpha^C(B_1(\alpha))$ for all $\alpha \in [\underline{\alpha},\overline{\alpha}]$. This further implies that all of the bracketed terms in equation (TA.3) are positive for all $\alpha \in [\underline{\alpha},\overline{\alpha}]$.

The solution of the ordinary differential equation (TA.3) through the boundary condition $r(\underline{\alpha}) = 0$ is given by:

$$r(\alpha) = [\textstyle\int_{\underline{\alpha}}^{\alpha} g(t)z(t)dt]/g(\alpha),$$

where

$$g(\alpha) = \exp\{\textstyle\int_{\underline{\alpha}}^{\alpha} y(t)dt\},$$

$$y(t) = [\Pi^L_1(t, B_1(t)) - \Pi^L_1(\alpha^C(B_1(t)), B_1(t))\alpha^{C\prime}(B_1(t))B_1{}'(t)/2]/ [\Pi^L(t, B_1(t)) - \Pi^L(\alpha^C(B_1(t)), B_1(t))/2], \text{ and}$$

$$z(t) = \Pi^L_1(t, B_1(t))/[\Pi^L(t, B_1(t)) - \Pi^L(\alpha^C(B_1(t)), B_1(t))/2].$$

Since $g(\alpha)$ and $z(\alpha)$ are positive for all $\alpha \in [\underline{\alpha},\overline{\alpha}]$, it follows that $r(\alpha) > 0$ for all $\alpha \in (\underline{\alpha},\overline{\alpha}]$. From equation (TA.3), it is clear that as long as $r(\alpha) < 1$ then $r'(\alpha) > 0$. Thus, $r(\alpha)$ starts at 0 and increases from there, continuing to increase as long as it is interior. If it remains interior at $\overline{\alpha}$, then it provides a candidate for a separating equilibrium rejection function throughout the range of α (and thus throughout the range of A), and finishes with a jump upward to 1 for all $\alpha > \overline{\alpha}$. On the other hand, if $r(\alpha)$ reaches 1 for some $\alpha < \overline{\alpha}$, then there cannot be a fully-separating equilibrium throughout the range of A; this latter scenario can be avoided if $\overline{A}$ is not too large.

In order to prove that the computed rejection function and the demand function $\alpha^L(A)$ form a separating equilibrium, it needs to be verified that each component is a best reply to the other. An argument can be constructed along the lines of the verification argument above for the case wherein C eschewed the auction, although a different (from Assumption 6) restriction on the behavior of $\Pi^L(\alpha, A)$ is needed.

However, more headway can be made with the probability function $p(x) = \lambda x^\theta$, for $0 < \theta < 1$ and $\lambda > 0$. This is because $\alpha^C(A) = \theta$ for all A. Thus, L1's payoff can be written as: $(1 - r(\alpha))\Pi^L(\alpha, A) + r(\alpha)\Pi^L(\theta, A)/2$. The resulting first-order condition for L1 is simpler than in the general case.

$$- r'(\alpha)[\Pi^L(\alpha, A) - \Pi^L(\theta, A)/2] + (1 - r(\alpha))\Pi^L_1(\alpha, A) = 0.$$

In equilibrium (along the equilibrium demand function), $A = B_1(\alpha)$; using this substitution to eliminate A from the equation above yields:

$$- r'(\alpha)u(\alpha) + (1 - r(\alpha))v(\alpha) = 0,$$

where $u(\alpha) \equiv [\Pi^L(\alpha, B_1(\alpha)) - \Pi^L(\theta, B_1(\alpha))/2]$ and $v(\alpha) \equiv \Pi^L_1(\alpha, B_1(\alpha))$. The solution of this ordinary differential equation through the boundary condition $r(\underline{\alpha}) = 0$ is given by:

$$r(\alpha) = 1 - \exp\left\{- \int [v(t)/u(t)]dt\right\},$$ where the integral is over $[\underline{\alpha}, \alpha]$.

Note that any $\alpha \in [\underline{\alpha}, \alpha]$ is greater than θ since $\alpha^L(A) > \alpha^C(A) = \theta$ for all A. Since the ratio $v(\alpha)/u(\alpha) = [\alpha^{\theta/(1 - \theta)}/(1- \theta)]/[\alpha^{1/(1 - \theta)} - (½)\theta^{1/(1 - \theta)}]$, integration and simplification yields:

$$r(\alpha) = 1 - [\underline{\alpha}^{1/(1 - \theta)} - (½)\theta^{1/(1 - \theta)}]/[\alpha^{1/(1 - \theta)} - (½)\theta^{1/(1 - \theta)}].$$

We now verify that $\alpha^L(A)$ does maximize the payoff $(1 - r(\alpha))\Pi^L(\alpha, A) + r(\alpha)\Pi^L(\theta, A)/2$, using the rejection function $r(\alpha)$ derived above. For the power-function example, it turns out that this payoff factors into a function of α alone and a function of A alone. The derivative with respect to α equals zero for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$; that is, every A-type is indifferent about α in this range and thus the lawyer's equilibrium payoff has the value $\Pi^L(\underline{\alpha}, A)$ regardless of which $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ he demands. Now consider values of α outside this range. Values of $\alpha > \overline{\alpha}$ are rejected for sure, yielding a payoff of $\Pi^L(\theta, A)/2$, which is lower than $\Pi^L(\underline{\alpha}, A)$ (since $\underline{\alpha} > \theta$ and Π^L is increasing in its first argument). Values of $\alpha \in [\theta, \underline{\alpha}]$ are accepted for sure, yielding a payoff of $\Pi^L(\alpha, A)$, which is lower than $\Pi^L(\underline{\alpha}, A)$. Finally, values of $\alpha \in [0, \theta)$ are rejected for sure, yielding a payoff of $\Pi^L(\theta, A)/2$, which is lower than $\Pi^L(\underline{\alpha}, A)$. Thus, faced with the rejection function $r(\alpha)$ derived above, it is an optimal strategy for the A-type lawyer to demand $\alpha^L(A)$.