

Technical Appendix
 “A Dynamic Model of Lawsuit Joinder and Settlement”
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Derivation of f_{NQ}

To find an implicit condition so that $\bar{\delta}^N < \delta_Q$, first observe that if $\delta_Q \leq \underline{\delta}$, then it is immediate that $\bar{\delta}^N > \delta_Q$ since $\bar{\delta}^N > \underline{\delta}$. A sufficient condition for this to occur is if $\delta_Q = c_1/L_1 \leq (c_2 + f)/L_2 = \underline{\delta}$; that is, if $f \geq f^{\max} \equiv (c_1L_2 - c_2L_1)/L_1$. In this case, while all types $\delta_i \in [\bar{\delta}^N, \delta_1)$ regret having paid the fixed cost f , none would drop, as the expected value to continuing alone is non-negative. Clearly this is a overly strong requirement on f such that no cases, once filed, are dropped. To find a necessary condition, consider $f \in (0, f^{\max})$, so that $\delta_Q > \underline{\delta}$, and evaluate

$$Z^N(\delta, \delta; f) \equiv q_2[H(\delta) - H(\underline{\delta})][L_2\delta - c_2 - f] + [1 - q_2 + q_2H(\underline{\delta})][\max\{L_1\delta - c_1, 0\} - f]$$

at $\delta = \delta_Q$, where we have included the fee f as a parameter in Z^N (and recall that $\underline{\delta} = (c_2 + f)/L_2$).

Then $Z^N(\delta_Q, \delta_Q; f) = q_2[H(\delta_Q) - H(\underline{\delta})][L_2\delta_Q - c_2] - [1 - q_2 + q_2H(\underline{\delta})]f$. Since Z^N is strictly increasing in both arguments involving δ_Q , and since $Z^N(\bar{\delta}^N, \bar{\delta}^N; f) = 0$, it is clear that $\bar{\delta}^N (>, =, <) \delta_Q$ as $Z^N(\delta_Q, \delta_Q; f) (<, =, >) 0$. As $f \rightarrow 0$, $\bar{\delta}^N \rightarrow \underline{\delta} < \delta_Q$, where the inequality follows from the fact that $\delta_Q = c_1/L_1 > c_2/L_2$. Thus, for f sufficiently low, we have $Z^N(\delta_Q, \delta_Q; f) > 0$ and therefore $\bar{\delta}^N < \delta_Q$. Moreover, $\partial Z^N(\delta, \delta; f)/\partial f = -q_2h(\underline{\delta})[L_2\delta - c_2]/L_2 - [1 - q_2 + q_2H(\underline{\delta})] < 0$ for all $\delta \geq \underline{\delta}$. Therefore, there exists a unique value of $f \in (0, f^{\max})$, denoted f_{NQ} , such that $\bar{\delta}^N (>, =, <) \delta_Q$ as $f (>, =, <) f_{NQ}$.

Comparative Statics

Recall that $\underline{\delta} \equiv (c_2 + f)$ and $\bar{\delta}^N$ is defined by the following equation:

$$Z^N(\bar{\delta}^N, \bar{\delta}^N) = q_2[H(\bar{\delta}^N) - H(\underline{\delta})][L_2\bar{\delta}^N - c_2 - f] + [1 - q_2 + q_2H(\underline{\delta})][\max\{L_1\bar{\delta}^N - c_1, 0\} - f] = 0.$$

This means that $L_2\bar{\delta}^N - c_2 - f > 0$ and $\max\{L_1\bar{\delta}^N - c_1, 0\} - f < 0$. The comparative statics of $\bar{\delta}^N$ with respect to the parameters f , c_2 , L_2 , and q_2 are obvious. Recall that the function $Z^N(\bar{\delta}^N, \bar{\delta}^N)$ is strictly increasing in $\bar{\delta}^N$ and depends on the parameters f , c_2 , L_2 , and q_2 both directly and (possibly) indirectly through $\underline{\delta}$. Thus, for any parameter m ,

$$d\bar{\delta}^N/dm = -(\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial m)/[Z_1^N(\bar{\delta}^N, \bar{\delta}^N) + Z_1^N(\bar{\delta}^N, \bar{\delta}^N)],$$

so that $\bar{\delta}^N$ is an increasing function of any parameter m which decreases $Z^N(\bar{\delta}^N, \bar{\delta}^N)$, taking into account any indirect effects through $\underline{\delta}$. It is shown below that $\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial f < 0$ and $\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial c_2 < 0$, so the period 1 filing threshold increases (fewer cases are filed in period 1) with an increase in f or c_2 . On the other hand, $\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial L_2 > 0$ and $\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial q_2 > 0$, so the period 1 filing threshold decreases (more cases are filed in period 1) with an increase in L_2 .

$$\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial f = -(q_2h(\underline{\delta})/L_2)[L_2\bar{\delta}^N - c_2 - \max\{L_1\bar{\delta}^N - c_1, 0\}] - [1 - q_2 + q_2H(\bar{\delta}^N)] < 0.$$

$$\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial c_2 = -(q_2h(\underline{\delta})/L_2)[L_2\bar{\delta}^N - c_2 - \max\{L_1\bar{\delta}^N - c_1, 0\}] - q_2[H(\bar{\delta}^N) - H(\underline{\delta})] < 0.$$

$$\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial L_2 = [q_2 h(\underline{\delta})(c_2 + f)/(L_2)^2][L_2 \bar{\delta}^N - c_2 - \max\{L_1 \bar{\delta}^N - c_1, 0\}] + q_2 [H(\bar{\delta}^N) - H(\underline{\delta})] \bar{\delta}^N > 0.$$

$$\partial Z^N(\bar{\delta}^N, \bar{\delta}^N)/\partial q_2 = [H(\bar{\delta}^N) - H(\underline{\delta})][L_2 \bar{\delta}^N - c_2 - f] - [1 - H(\underline{\delta})][\max\{L_1 \bar{\delta}^N - c_1, 0\} - f] > 0,$$

where the inequality follows from the facts that $L_2 \bar{\delta}^N - c_2 - f > 0$ and $\max\{L_1 \bar{\delta}^N - c_1, 0\} - f < 0$.

Comparing the aggregate expected filing cost under preemptive versus deferred settlement

Claim. Suppose that H is the uniform distribution on $[0, \Delta]$ and that $f < f_{NQ}$ (and thus, $\bar{\delta} < \delta_Q$). Then the expected filing cost for a single harmed plaintiff (which is proportional to the expected number of cases filed) is higher under preemptive settlement than under deferred settlement.

Proof. Let EFC^* denote the expected filing cost under deferred settlement and let EFC^{S*} denote the expected filing cost under preemptive settlement. Then

$$EFC^* = f[1 - H(\bar{\delta}^N)](1 + q_2[H(\bar{\delta}^N) - H(\underline{\delta})]) \text{ and } EFC^{S*} = f[1 - H(\bar{\delta}^S)].$$

Using the uniform distribution:

$$EFC^{S*} > EFC^* \text{ if and only if } (\Delta - \bar{\delta}^S)/\Delta > [(\Delta - \bar{\delta}^N)/\Delta][1 + q_2(\bar{\delta}^N - \underline{\delta})/\Delta] \quad (TA1)$$

(TA1) holds if and only if $\Delta(\Delta - \bar{\delta}^S) > (\Delta - \bar{\delta}^N)[\Delta + \bar{\delta}^N - \underline{\delta} - (1 - q_2)(\bar{\delta}^N - \underline{\delta})]$, which holds if and only if

$$q_2 \bar{\delta}^N (\bar{\delta}^N - \underline{\delta}) > \Delta [\bar{\delta}^S - \underline{\delta} - (1 - q_2)(\bar{\delta}^N - \underline{\delta})]. \quad (TA2)$$

Using equations (5) and (8), the fact that $\bar{\delta}^N < \delta_Q$, and the uniform distribution yields the following relationship among the thresholds: $\bar{\delta}^S - \underline{\delta} = (\bar{\delta}^N - \underline{\delta})^2 / (\Delta - \underline{\delta})$. Substituting this into (TA2) implies that (TA2) holds if and only if $q_2 \bar{\delta}^N (\bar{\delta}^N - \underline{\delta}) > \Delta [((\bar{\delta}^N - \underline{\delta})^2 / (\Delta - \underline{\delta})) - (1 - q_2)(\bar{\delta}^N - \underline{\delta})]$, which holds if and only if $q_2 \bar{\delta}^N (\Delta - \underline{\delta}) > \Delta (\bar{\delta}^N - \underline{\delta}) - \Delta (1 - q_2)(\Delta - \underline{\delta})$, which holds if and only if $q_2 \underline{\delta} + (1 - q_2)\Delta > 0$, which is true. QED

Preferences Over Preemptive versus Deferred Settlement in the Partially-Unaware Case

It was claimed in the text that the plaintiff still prefers deferred to preemptive settlement, while the defendant prefers to have the option to make a preemptive settlement when plaintiff awareness is sufficiently low. To demonstrate these claims, first consider the preferences of an aware harmed victim. Using notation analogous to that in the text, an aware harmed victim's expected equilibrium payoff under deferred and preemptive settlement, respectively, is given by:

$$V_p^N(\delta_i) = \begin{cases} 0 & \delta_i \in [0, \underline{\delta}) \\ W_p^N(\delta_i, \bar{\delta}_p^N) & \delta_i \in [\underline{\delta}, \bar{\delta}_p^N) \\ F_p^N(\delta_i, \bar{\delta}_p^N) & \delta_i \in [\bar{\delta}_p^N, \infty); \end{cases}$$

and

$$V_{\rho}^S(\delta_i) = \begin{cases} 0 & \delta_i \in [0, \bar{\delta}^S) \\ F_{\rho}^S(\delta_i, \bar{\delta}^S) & \delta_i \in [\bar{\delta}^S, \infty). \end{cases}$$

In order to determine whether the plaintiff prefers deferred to preemptive settlement, we examine $V_{\rho}^N(\delta_i) - V_{\rho}^S(\delta_i)$. Since $\bar{\delta}^S < \bar{\delta}_{\rho}^N$ and $F_{\rho}^S(\delta_i, \bar{\delta}^S) = F_{\rho}^N(\delta_i, \bar{\delta}_{\rho}^N)$,¹ then the only places where $V_{\rho}^N(\delta_i)$ differs from $V_{\rho}^S(\delta_i)$ is on the two intervals $[\underline{\delta}, \bar{\delta}^S)$ and $[\bar{\delta}^S, \bar{\delta}_{\rho}^N)$, since the two payoff functions are the same on the other intervals. Furthermore, $V_{\rho}^N(\delta_i) - V_{\rho}^S(\delta_i) = W_{\rho}^N(\delta_i, \bar{\delta}_{\rho}^N) - 0 > 0$ on $[\underline{\delta}, \bar{\delta}^S)$, while $V_{\rho}^N(\delta_i) - V_{\rho}^S(\delta_i) = W_{\rho}^N(\delta_i, \bar{\delta}^N) - F_{\rho}^N(\delta_i, \bar{\delta}_{\rho}^N) > 0$ on $[\bar{\delta}^S, \bar{\delta}_{\rho}^N)$ since waiting is better than filing in the first period for these types. Thus, every type of aware harmed victim prefers deferred to preemptive settlement. It is clear that an unaware harmed victim prefers deferred to preemptive settlement, since deferred settlement involves a possibility that another victim may file suit and alert the unaware victim; by contrast, under preemptive settlement any (other) victim that files (early) ends up settling confidentially instead of alerting the unaware victim.

Now consider the *ex ante* preferences of the defendant. The defendant's expected payment is the *ex ante* expected number of harmed victims times the *ex ante* expected payment received by a harmed victim (this is tedious, but straightforward, to verify). We will now describe how to construct a harmed victim's expected receipts, taking into account that this victim may be aware or unaware. Under deferred settlement, a victim of type δ_i obtains the following payoffs:

$$\begin{aligned} \text{(a) } \delta_i \in [0, \underline{\delta}): & \quad 0 \\ \text{(b) } \delta_i \in [\underline{\delta}, \bar{\delta}_{\rho}^N): & \quad \rho q_2 [1 - H(\bar{\delta}_{\rho}^N)] [L_2 \delta_i - c_2] \\ \text{(c) } \delta_i \in [\bar{\delta}_{\rho}^N, \infty): & \quad \rho q_2 [1 - H(\underline{\delta})] [L_2 \delta_i - c_2] + \rho [1 - q_2 + q_2 H(\underline{\delta})] \max \{L_1 \delta_i - c_1, 0\} \\ & \quad + (1 - \rho) \rho q_2 [1 - H(\bar{\delta}_{\rho}^N)] [L_2 \delta_i - c_2]. \end{aligned}$$

These payoffs are explained as follows. A victim with $\delta_i \in [0, \underline{\delta})$ will never file suit, regardless of his level of awareness. A victim with $\delta_i \in [\underline{\delta}, \bar{\delta}_{\rho}^N)$ will wait in the first period, regardless of his level of awareness; consequently, he will file in period 2 if there is another victim, that victim is aware, and that victim has harm in excess of $\bar{\delta}_{\rho}^N$; in this case, the other victim will file in period 1 and victim i will join in period 2. Finally, a victim with $\delta_i \in [\bar{\delta}_{\rho}^N, \infty)$ will file in period 1 if he is aware (this explains the first two expressions in part (c) above); if he is unaware, he will wait in period 1 but he will file in period 2 if there is another victim, that victim is aware, and that victim has harm in excess of $\bar{\delta}_{\rho}^N$; this explains the third expression in part (c) above.

Under preemptive settlement, a victim of type δ_i obtains the following payoffs (after substituting for the equilibrium settlement offer):

¹ Recall that F_{ρ}^N and the reduced form of F_{ρ}^S are both independent of the other potential victim's strategy.

$$(d) \delta_i \in [0, \bar{\delta}^S): \quad 0$$

$$(e) \delta_i \in [\bar{\delta}^S, \infty): \quad \rho q_2 [1 - H(\bar{\delta})][L_2 \delta_i - c_2] + \rho [1 - q_2 + q_2 H(\bar{\delta})] \max\{L_1 \delta_i - c_1, 0\}$$

These payoffs are explained as follows. A victim with $\delta_i \in [0, \bar{\delta}^S)$ will not file suit in period 1, regardless of his level of awareness. Moreover, if there is another aware victim who files in period 1, this plaintiff will settle with the defendant and will thus be unavailable to be joined in period 2, and a victim with $[0, \bar{\delta}^S)$ will not proceed alone. A victim with $\delta_i \in [\bar{\delta}^S, \infty)$ will file in period 1 if he is aware; since this would permit him to alert any other victim, who would join in period 2 if her harm exceeds $\bar{\delta}$, victim i receives (via the settlement but gross of filing costs) the amount $q_2 [1 - H(\bar{\delta})][L_2 \delta_i - c_2] + [1 - q_2 + q_2 H(\bar{\delta})] \max\{L_1 \delta_i - c_1, 0\}$ with probability ρ .

A harmed victim's expected receipts under deferred and preemptive settlement, respectively, are found by integrating the payoffs described in (a)-(c) and (d)-(e), respectively, with respect to the distribution H . The defendant's *ex ante* expected payments are proportional to these expectations. The difference between the defendant's *ex ante* expected payments under deferred versus preemptive settlement are therefore proportional to $\rho\gamma(\rho)$, where

$$\begin{aligned} \gamma(\rho) \equiv & \int q_2 [1 - H(\bar{\delta}_\rho^N)] [L_2 \delta_i - c_2] h(\delta_i) d\delta_i && \text{(where the domain of integration is } [\bar{\delta}, \bar{\delta}^S]) \\ & + \int (1 - \rho) q_2 [1 - H(\bar{\delta}_\rho^N)] [L_2 \delta_i - c_2] h(\delta_i) d\delta_i && \text{(where the domain of integration is } [\bar{\delta}_\rho^N, \infty)) \\ & - \int \{q_2 [H(\bar{\delta}_\rho^N) - H(\bar{\delta})] [L_2 \delta_i - c_2] + [1 - q_2 + q_2 H(\bar{\delta})] \max\{L_1 \delta_i - c_1, 0\}\} h(\delta_i) d\delta_i, \end{aligned}$$

where the domain of integration for the final integral is $[\bar{\delta}^S, \bar{\delta}_\rho^N]$. Recall that $\partial \bar{\delta}_\rho^N / \partial \rho > 0$ and that $\bar{\delta}_\rho^N \rightarrow \bar{\delta}^S$ as $\rho \rightarrow 0$. Totally differentiating $\gamma(\rho)$ with respect to ρ implies that $\gamma(\rho)$ increases as ρ decreases. Moreover, the first two integrals converge to positive numbers as $\rho \rightarrow 0$, while the third integral converges to zero. Thus, there is a value ρ_0 that is close enough to zero (but still positive) at which $\gamma(\rho_0) = 0$; for any $\rho \in (0, \rho_0)$, it follows that $\rho\gamma(\rho) > 0$. Thus, for sufficiently small levels of plaintiff awareness ρ , the defendant expects to pay more under deferred than under preemptive settlement. Thus, D prefers to have the option to make a preemptive settlement when plaintiff awareness is sufficiently low.