Abstract—Pneumatic actuators are governed by nonlinear dynamics. Thus, robust precision motion control of pneumatic systems requires model-based control techniques such as sliding-mode and/or adaptive control. These controllers typically require full-state knowledge of the system, i.e., pressure, position, velocity, and acceleration. For measuring pressure states, pneumatic servo systems require two expensive pressure sensors per axis, and hence, it makes the system economically noncompetitive with most electromagnetic types of actuation. This paper presents the development of a Lyapunov-based pressure observer for pneumatically actuated systems. The pressure observer is energy-based and has the useful feature of not requiring a model for the load of the system, i.e., it is load-independent. This pressure observer is proven to be globally stable with the added feature of having a response bandwidth equal to that of the modeled pressure dynamics. A robust observer-based controller is developed to obtain a low-cost precision pneumatic servo system. Experimental results are presented that demonstrate and validate the effectiveness of the proposed observer.

Index Terms—Pneumatic actuators, pressure observer, sliding-mode control design.

I. INTRODUCTION

IN RECENT years, interest in pneumatic systems has grown given its high power density and appropriateness for untethered or mobile robotic systems [1] or given the ability to construct pneumatic components out of MRI-compatible materials [2], [3]. A schematic of a pneumatic servo system is depicted in Fig. 1. A typical setup consists of a pneumatic double-acting cylinder, a four-way proportional valve, a position sensor, and pressure sensors. In this system, the output position is controlled by a force that arises from the pressure differential across the piston in the cylinder. The dynamics of such a system that relates the control input at the valve to the position output is at least third order and is highly nonlinear including the nonlinear compressible gas dynamics on both sides of the actuator and the nonlinear mass flow across the valve as a function of upstream and downstream pressures. The mass flow rate dynamics, in particular, present a hard nonlinearity due to the saturation of the mass flow rate through the valve at sonic flow conditions.

The mass flow rate through the valve in “unchoked” flow conditions depends both on the upstream and downstream pressures and increases with the pressure difference. Once the velocity of air at the venturi of the valve orifice reaches the speed of sound, i.e., sonic, the mass flow rate is “choked” and is only a linear function of the upstream pressure. The transition of flow rate between the choked and unchoked condition is inevitable for any reasonable operating regime of desired motion control. The only potential way of avoiding this transition is to reduce the supply pressure to a very low level (~200 kPa). However, this low supply pressure results in unacceptably low actuation forces and a low output impedance. As such, pressure sensors are commonly employed in nonlinear model-based controllers of pneumatic servo systems in part to detect and compensate for the shift in dynamic behavior that occurs in the transition between choked and unchoked flow through the valve. Pressure sensors are also required due to the nonlinear compressible gas dynamics depending on the chamber volume, rate of change of volume, and mass flow rate. The pressure sensors ultimately serve as feedback to control the pressures on both sides of the actuator in the face of the nonlinear mass flow rate dynamics, the nonlinear compressible gas dynamics, and the motion dynamics of the load.

Pressures in the cylinder are commonly used as states in precision pneumatic servo actuation systems for model-based controllers. A typical precision pneumatic system employs two pressure sensors and a linear potentiometer to measure the states of the system. The requirement for pressure sensing in a pneumatic servo system is particularly burdensome because high-bandwidth, high-pressure sensors required for the control of pneumatic servo systems are expensive and large (relative to the actuator), with a typical cost between $250 and $500. Since

Fig. 1. Schematic of a pneumatic servo actuation system.
pneumatic systems require the use of two pressure sensors per axis, these sensors add $500–$1000 per axis of a pneumatic servo system. Coupled with valve and cylinder costs, pneumatic systems are not cost-competitive with power comparable electromagnetic types of actuation. If the requirement of pressure sensors can be eliminated by constructing observers to estimate these states, it will result in sizeable cost savings. Since the other states (i.e., motion output, velocity, and acceleration) are more readily measurable, the possibility exists to reconstruct the cylinder pressures by using the available knowledge of the inputs and the other states of the system. It should be noted that the requirement of pressure measurement can be avoided by the use of nonmodel-based controllers, such as the position–velocity–acceleration (PVA) controller structure [4]. Although such controllers have met with a certain amount of success, nonmodel-based controllers cannot address the often significant nonlinearities associated with pneumatic systems. It has been pointed out by Pandian et al. [5] that for precise and robust control performance, the use of pressure states is essential.

Despite a number of prior publications on control methodologies that require full-state measurement [6]–[12], few works explicitly consider the hardware costs associated with pneumatic systems. Pandian et al. [9] presented a sliding-mode controller for position control that showed good results at lower frequencies. Richer and Hurmuzlu [12] in their work presented the design of a sliding-mode force controller and showed a good response up to 20 Hz sinusoidal frequency. All these developed controllers concentrated on the position and/or force tracking accuracy and ignored the hardware costs associated with the control system. Many authors focused on the development of more energy efficient controllers to reduce the operating cost of the system. Sanville [13] suggested the use of a secondary reservoir in an open-loop system to collect exhaust air. This air was, in turn, utilized as an auxiliary low-pressure supply. Al-Dakkan et al. [14] presented a control methodology that provides significant energy savings. They used two three-way spool valves, instead of a conventional four-way proportional spool valve, and introduced a dynamic constraint equation that minimizes cylinder pressures resulting in lower energy consumption. Shen and Goldfrab [15] present an alternative energy saving method. In efforts to reduce hardware costs, Ye et al. [16], Kunt and Singh [17], Lai et al. [18], Royston and Singh [19], Paul et al. [20], and Shih and Hwang [21] demonstrated the viability of servo control of pneumatic actuators via solenoid ON/OFF valves in place of proportional valves.

Though all these efforts were made to reduce hardware and/or operating costs, the needed components of a precision pneumatic system still remain expensive relative to electromagnetic actuation systems. In the continuing efforts to achieve higher cost savings, Pandian et al. [2] in their work presented two methods for observing pressure in an effort to eliminate costly pressure sensors. In the first method, a continuous gain observer design, the pressure is measured in one chamber and the pressure in another chamber is observed—thereby eliminating one of the two pressure sensors. In this case, a choked flow condition is assumed by the authors. In addition, mass flow rate is assumed to be known while deriving the error equation. Both of these assumptions are restrictive since at a low pressure difference across the control valve, the flow rate is not choked. Also, the mass flow rate is a function of pressure whose value is to be estimated. In the second method, a sliding-mode pressure observer design, the same assumptions of the first method were used. In this method, the difference between the estimated and actual pressure in one chamber is treated as a disturbance and the pressure in another chamber is observed using a sliding-mode observer design. However, the convergence of the error to zero is not clear, as the disturbance, which is the nonhomogeneous part or driving term of the desired error dynamic differential equation, can lead to large steady-state error. In another development, Bigras and Khayati [22] presented a design of a pressure observer for a pneumatic cylinder system for which the connection ports provide a considerable restriction to the air supply. The observer was based on the measurement of actual pressure outside the cylinder, and hence, pressure sensors cannot be completely eliminated from the system. Wu et al. [23], based on a rank condition test, concluded that pressure states are not locally weakly observable within eight separate dynamic regimes from the measurement of motion output alone because of the existence of singular points such as when the system is at rest. However, Wu et al. [23] did not assess global observability and comment that the system could possibly posses a globally stable observer.

In this paper, a globally stable Lyapunov-based pressure observer design is presented. Preliminary versions of the results in this paper were also presented in [24] and [25]. The observer utilizes position and velocity states and the valve input. In this paper, it is both proven and shown experimentally that the error between the observed and actual states converges by including knowledge of valve spool position as well as the motion states of the system. As with any observer, experiments show some amount of inaccuracy in the observed values of the pressures. Therefore, a robust controller based on sliding-mode control theory is developed in this paper to compensate for the observer errors along with the uncertainties present in the system model, like friction, to obtain a low-cost pneumatic servo system devoid of pressure sensors. One feature of the observer making it convenient to link to a controller is that it has a bandwidth of pressure observation that is equivalent to the actual pressure dynamics. This feature allows the observer to be utilized within the context of a robust controller without the observer affecting the stability of the closed-loop controlled system.

This paper is organized as follows. In Section II, a model of the pneumatic system is presented. In Section III, the design and analytical properties of an energy-based Lyapunov observer is derived. Section IV presents the design of a sliding-mode controller for the servo control of the pneumatic system shown in Fig. 3. In Sections V and VI, the experimental setup, implementation, and results are discussed.

II. MODEL OF PNEUMATIC SERVO ACTUATOR

A model of a typical pneumatic servo actuator is reasonably standard and is derived in many texts and papers [26]–[29]. A
The complete model of the system was presented by Richer and Harmuzlu [30]. The salient features of the standard dynamic model are summarized in this paper for the development of the energy-based observer and the observer-based sliding-mode controller. The dynamic equation for the piston–rod–load assembly shown in Fig. 1 can be expressed as

\[ M \ddot{x} + B \dot{x} + F_c = P_a A_a - P_b A_b - P_{\text{atm}} A_r \]

where \( M \) (in kilograms) is the mass of the load; \( B \) (in Newtons per meter) is the viscous friction coefficient, \( F_c \) (in Newtons) is the Coulomb friction, \( P_a \) and \( P_b \) (in pascals) are the absolute pressure in each chamber of the cylinder, \( P_{\text{atm}} \) (in pascals) is the absolute environmental pressure, \( A_r \) (in square meters) is the cross-sectional area of the rod, and \( A_a \) and \( A_b \) (in square meters) are the effective piston areas in chambers “A” and “B,” respectively.

The dynamics of the chamber pressures \( P_a \) and \( P_b \) are as follows:

\[ \dot{P}_{a,b} = \frac{kRT}{V_{a,b}} \dot{m}_{a,b} - \frac{kV_{a,b}}{V_{a,b}} P_{a,b} \]

where \( k \) is the thermal coefficient characterizing a polytropic process with \( k \) ranging from unity for an isothermal process to \( \gamma \) for an adiabatic process; \( \gamma \) is the ratio of the specific heat at constant pressure \( C_p \) to the specific heat at constant volume \( C_v \); \( R \) (Joules per kilogram Kelvin) is the gas constant; \( V \) (in cubic meters) is the volume of the chamber; and subscripts “a” and “b” represents properties of chambers “A” and “B,” respectively. The sign convention used in this paper comes from the control volume power balance used to derive (2) where \( \dot{m} \) is positive while charging the cylinder and negative during discharge to the atmosphere.

The pressure dynamics of (2) are governed in part by the mass flow rate term, which, in turn, is algebraically influenced by the orifice area normalized mass flow rate \( \dot{m} \) given by

\[ \dot{m} = A_v \Psi(p_u, p_d) \]

with the orifice area normalized mass flow rate \( \Psi \) given by

\[ \Psi(p_u, p_d) = \begin{cases} \frac{C_f p_u}{\sqrt{T}} \sqrt{\frac{2}{\gamma + 1}} \left( \frac{\gamma}{\gamma - 1} \right)^{(\gamma + 1)/(\gamma - 1)}, & \text{if } \frac{p_d}{p_u} \leq \left( \frac{2}{\gamma + 1} \right)^{(\gamma - 1)/\gamma} \text{ (choked)} \bigg), \\ \frac{C_f p_u}{\sqrt{T}} \sqrt{\frac{2^\gamma}{R (\gamma + 1)}} \left( 1 - \frac{p_d}{p_u} \right)^{(\gamma - 1)/(\gamma)} \\ \text{otherwise (unchoked)} \bigg), & \text{otherwise} \end{cases} \]

where \( C_f \) (unitless) is the discharge coefficient of the valve—typically well characterized by the valve manufacturer; \( A_v \) (in square meters) is the flow orifice area of the valve; \( T \) (in Kelvin) is the stagnation temperature of the gas flowing; and \( p_u \) and \( p_d \) (in pascals) is the upstream and downstream pressure, respectively. It should be noted that during the charging process, \( p_u \) is the supply pressure \( P_s \) and \( p_d \) is the chamber pressure \( P \) of the cylinder. While in the case of the discharging, \( p_u \) is the chamber pressure and \( p_d \) is atmospheric pressure. This behavior can be captured as follows:

\[ \Psi(p_u, p_d) = \begin{cases} \Psi(P_s, P), & \text{for } A_e \geq 0 \\ \Psi(P, P_{\text{atm}}), & \text{for } A_e < 0 \end{cases} \]

where the signed flow orifice area \( A_v \) serves to switch between charging and discharging and also serves to switch the sign of the mass flow rate that is always calculated as positive in (3b).

The complete system dynamics of the pneumatic servo actuator are therefore characterized by the state vector \( x^T = [x \ \dot{P}_a \ P_b] \) and the input \( u = [A_{v_a} \ A_{v_b}] \) and described by the combination of (1)–(4), where a positive valve area indicates a connection to the supply pressure (charge) and a negative valve area indicates a connection to the atmosphere (discharge). The volume and rate of change of volume are algebraically related to the displacement and velocity of the piston, and therefore, do not give rise to independent states.

III. ENERGY-BASED LYAPUNOV PRESSURE OBSERVER

The pressure is estimated based on the following observer equations:

\[ \dot{\hat{P}}_a = \frac{RT}{V_a} \dot{\hat{m}}_a - \frac{\dot{V}_a}{V_a} \hat{P}_a \]

\[ \dot{\hat{P}}_b = \frac{RT}{V_b} \dot{\hat{m}}_b - \frac{\dot{V}_b}{V_b} \hat{P}_b \]

where \( \dot{\hat{P}} \) in the aforementioned equations represents the estimated pressure and \( \dot{\hat{m}} \) represents the estimated mass flow rate according to (3a) based on the estimated pressure and the known valve orifice area \( A_{v(a,b)} \). Given that the pressure observer equations are functions of the measured values of volumes and time rates of change of these volumes (via the position and velocity of the actuator and a known cylinder bore), the observer requires no model of the load dynamics. The load dynamics provide a dynamic relationship between the applied force, resulting from the chamber pressures \( P_a \) and \( P_b \) in the actuator, and the motion of the load. No knowledge of this relationship is needed; knowledge of the inertial, damping, friction, and compliance properties of the load are not needed. Explicit knowledge of disturbance forces on the load is also not required. All required knowledge of the load is implicitly contained in the measured terms \( V_{a,b}, \dot{V}_{a,b} \), and \( V_{a,b} \). Therefore, it can be stated that this pressure observer is load-independent.

Although (5) appears to be simply an open-loop estimation based on an isothermal assumption of the pressure dynamics of (2), the equations are actually closed-loop observers due to the relationship between \( \dot{\hat{P}} \) and \( \dot{\hat{m}} \). The fact that they also represent the modeled pressure dynamics allows this observer to be utilized with the confidence that they have the same fast dynamic response as the actual pressure dynamics—a concern that ensures that an observer be much faster than the closed-loop dynamics. In order to show the convergence between the actual
pressures and the estimated pressures obtained from the earlier equations, the following positive-definite candidate Lyapunov function is chosen:

\[ V = \frac{1}{2}(P_a V_a - \hat{P}_a V_a)^2 + \frac{1}{2}(P_b V_b - \hat{P}_b V_b)^2 \]  

(6)

where \( P_{(a,b)} \) and \( \hat{P}_{(a,b)} \) represent actual and estimated pressures in chambers “A” and “B.” It should be noted that the Lyapunov function chosen is based on the energy stored in the two sides of a pneumatic actuator, and represents a scaled difference between the actual and observed stored energies. The potential for a sealed volume of gas, with an initial volume \( V \) and pressure \( P \), to do work in the absence of any heat loss or gain (adiabatic) is

\[ E_{\text{adiabatic}} = \frac{PV}{1 - \gamma} \left( \left( \frac{P}{P_{\text{atm}}} \right)^{1 - \gamma}/\gamma - 1 \right). \]  

(7)

Likewise, the potential for a sealed volume of gas to do work isothermally is

\[ E_{\text{isothermal}} = {PV}_o \ln \left( \frac{P}{P_{\text{atm}}} \right). \]  

(8)

By utilizing a positive-definite expression that involves the same state variables as the energy expression, we are assured that the time rate of change of this expression will involve the same (physical) power terms as the actual system and will also include the control input terms of the system (the mass flow rates). Therefore, the Lyapunov function chosen in (6) involves the product \( PV \). Differentiating (6) results in

\[ \dot{V} = (P_a V_a - \hat{P}_a V_a)(\dot{P}_a V_a + P_a \dot{V}_a - \hat{P}_a V_a - \hat{P}_a \dot{V}_a) \]
\[ + (P_b V_b - \hat{P}_b V_b)(\dot{P}_b V_b + P_b \dot{V}_b - \hat{P}_b V_b - \hat{P}_b \dot{V}_b). \]  

(9)

If the process of charging and discharging of air in the cylinder is considered as isothermal (i.e., \( k = 1 \) in (2), the following substitutions can be made in (9)

\[ \dot{P}_{(a,b)} V_{(a,b)} + P_{(a,b)} \dot{V}_{(a,b)} = RT \hat{m}_{(a,b)} \]

and

\[ \dot{\hat{P}}_{(a,b)} V_{(a,b)} + \hat{P}_{(a,b)} \dot{V}_{(a,b)} = RT \hat{m}_{(a,b)}. \]  

(10)

The thermodynamic process of charging and discharging a pneumatic actuator is an active area of research. There are a number of publications that extensively discuss the process of gas expansion and compression in a pneumatic cylinder. It has been shown by some researchers that the charging process is dominantly isothermal [31], and some have concluded the discharging process also to be well approximated as isothermal [31], [32]. Therefore, an isothermal process is a reasonable assumption to make. Substitution of (10) in (9) yields

\[ \dot{V} = RT V_a (P_a - \hat{P}_a)(\hat{m}_a - m_a) + RT V_b (P_b - \hat{P}_b)(\hat{m}_b - \hat{m}_b). \]  

(11)

In the aforementioned equation, \( \dot{V} \) can be shown to be negative semidefinite. The term \((\dot{P}_{(a,b)} - \hat{P}_{(a,b)})(\hat{m}_{(a,b)} - m_{(a,b)})\) is always either negative semidefinite or negative definite for the charging and discharging process. The proof of this term being negative semidefinite is presented in Lemma 1. The proof of (11) being negative semidefinite, and consequently, the observer being stable, follows in Theorem 1.

**Lemma 1:** Given particular values of \( A_c, C_f, T, R, \) and \( \gamma \) in the mass flow rate relationship given by (3b) for either the actual \( \hat{m}(p_u, p_d) \) or estimated \( \hat{m}(\hat{p}_u, \hat{p}_d) \) functional values, the product \((P_{(a,b)} - \hat{P}_{(a,b)})(\hat{m}_{(a,b)} - m_{(a,b)})\) is negative semidefinite for the charging process and negative definite for the discharging process.

**Proof of Lemma 1:** The proof is presented for \((P_a - \hat{P}_a)(\hat{m}_a - \hat{m}_a)\). The same proof is applicable to \((P_b - \hat{P}_b)(\hat{m}_b - \hat{m}_b)\).

**A. Charging**

For the case of charging, the upstream pressure is identical and equal to the supply pressure for both the actual and estimated upstream pressures: \( P_u = \hat{p}_u = P_s \). The actual downstream pressure is given by \( P_u = P_a \), and the estimated downstream pressure is given by \( \hat{p}_d = P_a \). Accordingly, as given by (3b): \( \hat{m}_a = A_c \Psi(P_u, \hat{p}_d) = A_c \Psi(P_s, P_a) \) and \( \hat{m}_a = A_c \Psi(P_u, \hat{p}_u) = A_c \Psi(P_s, P_a) \), where, by definition, \( A_c > 0 \) for charging. For a fixed upstream pressure and varying downstream pressure, the function \( \hat{m}(p_u, p_d) \) is nonincreasing. Therefore, we have the following.

1. \( P_a > \hat{P}_a \) implies \( \hat{m}_a \leq m_a \) yielding \((P_a - \hat{P}_a)(\hat{m}_a - m_a) \leq 0\).
2. \( P_a < \hat{P}_a \) implies \( \hat{m}_a \geq m_a \) yielding \((P_a - \hat{P}_a)(\hat{m}_a - m_a) \leq 0\).
3. \( P_a = \hat{P}_a \) implies \((P_a - \hat{P}_a)(\hat{m}_a - m_a) = 0\).

An illustration of this is shown in Fig. 2(a). With respect to the observer error, the term \((P_a - \hat{P}_a)(\hat{m}_a - \hat{m}_a)\) is negative semidefinite for the charging process.

**B. Discharging**

In the case of discharging, the upstream pressure is now the variable of interest to be estimated. The following assumptions are made for the discharging case: \( \hat{p}_d = \hat{p}_d = P_{\text{atm}}, P_u = P_a \), and \( \hat{p}_u = \hat{P}_a \). As given by (3b) and the sign convention \( A_c < 0 \) for discharging: \( \hat{m}_a = A_c \Psi(p_u, \hat{p}_d) = - |A_c| \Psi(P_a, P_{\text{atm}}) \) and \( \hat{m}_a = A_c \Psi(p_u, \hat{p}_u) = - |A_c| \Psi(P_a, P_{\text{atm}}) \). For a fixed downstream pressure and varying upstream pressure during the discharging process, the function \( \hat{m}(p_u, p_d) \) is zero or negative and monotonically decreasing. Therefore, we have the following.

1. \( P_a > \hat{P}_a \) implies \( \hat{m}_a < \hat{m}_a \) yielding \((P_a - \hat{P}_a)(\hat{m}_a - m_a) < 0\).
2. \( P_a < \hat{P}_a \) implies \( \hat{m}_a > \hat{m}_a \) yielding \((P_a - \hat{P}_a)(\hat{m}_a - m_a) < 0\).
3. \( P_a = \hat{P}_a \) implies \((P_a - \hat{P}_a)(\hat{m}_a - \hat{m}_a) = 0\).

An illustration of this is shown in Fig. 2(b). With respect to the observer error, the term \((P_a - \hat{P}_a)(\hat{m}_a - \hat{m}_a)\) is negative definite for the discharging process. Note that \( \hat{m}_a = \hat{m}_a \) only if \( P_a = \hat{P}_a \), whereas this is not the case in the charging process.
Taken together or separately, the two pressure\(\dot{\hat{P}}_{a}\) and \(\dot{\hat{P}}_{b}\) until at which \(\dot{\hat{P}}_{a} - P_{a} > 0\) and \(\dot{\hat{P}}_{b} - P_{b} > 0\) always being \(< 0\).

\(\dot{\hat{P}}_{a} = 0\) proving the pair of observers globally Lyapunov stable for such cases, \(\dot{\hat{P}}_{a} = 3\) \(\lambda\) and \(\dot{\hat{P}}_{b} = 0\) is shown graphically. The other cases are similar.

\(\dot{\hat{P}}_{a} = \text{negative definite} \) by virtue of the function being nonincreasing. The case where \(\dot{\hat{P}}_{a} > P_{a}\) implies \(\dot{\hat{m}}_{a} < \dot{m}_{a}\) is shown graphically. The other cases are similar.

Proof of Theorem 1: Equation (6) is a once differentiable positive definite, radially unbounded, scalar Lyapunov function that is zero for any nonzero volume only if both estimated pressures are equal to the actual pressures. Following from Lemma 1, \(\dot{V}\) given by (11) always consists of a negative semidefinite term corresponding to the side undergoing the charging process, and a negative definite term corresponding to the side undergoing the discharging process. These two terms taken together result in a negative semidefinite \(\dot{V}\) proving the pair of observers globally Lyapunov stable. Given that Lemma 1 shows each side to be at least negative semidefinite and independent of the other side, each pressure observer is proven Lyapunov stable by separately considering the following two Lyapunov functions:

\[
V = \frac{1}{2} (P_{a}V_{a} - \hat{P}_{a}V_{a})^{2}
\]

\[
V = \frac{1}{2} (P_{b}V_{b} - \hat{P}_{b}V_{b})^{2}
\]

and employing the results of Lemma 1 to prove each \(\dot{V}\) negative semidefinite.

It is worth noting that if parametric errors exist in the characterization of mass flow, the observer can still be shown to be stable, but the observed pressure will have a steady-state error. This extension is left to the reader and can be done by visualizing two curves in Fig. 2(a) and (b) where one curve is the actual mass flow rate associated with \(\dot{m}_{a}\) and \(P_{a}\), and the second is the estimated mass flow rate associated with \(\dot{\hat{m}}_{a}\) and \(\hat{P}_{a}\). It can be seen that when \((P_{a} - \hat{P}_{a})(\dot{\hat{m}}_{a} - \dot{m}_{a}) > 0\) for such cases, \(\hat{P}_{a}\) will be driven away from \(P_{a}\) until \((\dot{\hat{m}}_{a} - \dot{m}_{a}) = 0\) at which point, a new stable equilibrium \(\dot{V} = 0\) will be established with a difference between \(P_{a}\) and \(\hat{P}_{a}\).

IV. SLIDING-MODE CONTROLLER

The proposed motion controller in this paper is based on sliding-mode control theory [33], [34]. Sliding-mode controllers are commonly used for their robustness properties in controlling nonlinear systems and generally well suited for pneumatic servo actuators due to the highly nonlinear behavior and uncertainties present in the model [35].

For the system shown in Fig. 3, the desired output is the position of the end-effector. The control input to the system is the area of the valve. In order to derive the control law, define a time-varying sliding surface with \(s = 0\) representing the manifold defining the desired stable motion tracking error dynamics

\[
s = \left(\begin{array}{c}
d \frac{d}{dt} + \lambda \\
\end{array}\right)^{-1} e
\]

where \(\lambda\) is a strictly positive number, \(n\) is the number of times the output must be differentiated to obtain the input term, and \(e\) is the error between the actual and desired position.

For \(n = 3\), the aforementioned equation can be rewritten as

\[
s = (\check{x} - \ddot{x}) + 2k\check{e} + \lambda^{2}e.
\]
where $K$ is a strictly positive gain and captures bounded uncertainties of the model and the pressure observer, and $\phi$ is the boundary layer thickness and selected to avoid excessive chattering across the sliding surface while maintaining the desired performance of the system.

The saturation function in (18) is defined by

$$\text{sat} \left( \frac{s}{\phi} \right) = \begin{cases} \text{sgn} \left( \frac{s}{\phi} \right), & \text{if } \left| \frac{s}{\phi} \right| \geq 1 \\ \frac{s}{\phi}, & \text{otherwise.} \end{cases} \quad (19)$$

V. EXPERIMENTAL SETUP

The sliding-mode controller, along with the developed observer, was implemented for the servo control of a commercial 2-DOF pneumatic robot shown in Fig. 3. For the experiments, only 1 DOF is used, which is a double acting pneumatic cylinder (Festo SLT-20-150-A-CC-B). Two four-way proportional spool valves (Festo MPYE-5-M5-010-B) constrained to operate together are used for controlling the charging and discharging process of both chambers of the cylinder. A linear potentiometer (Midori LP-150F) with a travel length of 150 mm is used to measure the position of the load. The velocity signal is obtained by an analog differentiator with a first-order roll-off at 50 Hz. Similarly, the acceleration signal is obtained by an analog differentiation of the velocity signal with a first-order roll-off at 50 Hz. Two pressure transducers (Festo SDE-16-10V/20 mA) are also used in the setup for the measurement of actual pressures. The control and the observer algorithms are implemented using Real Time Workshop by Mathworks on a 2.4-GHz, 512-MB RAM, Pentium-IV-processor-based PC. The communication between the computer and the experimental setup is established through the digital input and analog output channels of an A/D card (National Instruments PCI-6031E).

The pressure supply used for this experiment is 652 kPa (80 psig). Some of the parameters (e.g., area of piston, area of rod, stroke length, pay-load mass) for this experiment are known accurately. The discharge coefficient ($C_f$), which primarily represents frictional flow losses, is a function of the valve area among other parameters such as the size and shape of the valve opening, surface finish, and similar parameters. For this experiment, the average discharge coefficient was calculated based on the volumetric flowchart provided by the valve manufacturer. Other parameters, like the viscous friction coefficient, are difficult to measure. Therefore, these parameters are estimated. All parameter values, including the controller gains, are shown in Table I.

The experiment was conducted in two stages. In the first stage of the experiment, pressure sensor signals were used in the control law to control the end-effector with a mass of 3.6 kg. For this, the robotic arm was controlled to execute a sinusoidal motion at different frequencies. The same experiment was then repeated for step inputs. In another set of readings, disturbances were introduced in the system by applying external forces (using our hand) to the robotic arm to ensure the robustness of the observer in presence of disturbances and uncertainties (such as
Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a )</td>
<td>652 kPa</td>
</tr>
<tr>
<td>( P_{\text{ref}} )</td>
<td>101 kPa</td>
</tr>
<tr>
<td>( T )</td>
<td>1.4</td>
</tr>
<tr>
<td>( R )</td>
<td>287.1 J/kg/K</td>
</tr>
<tr>
<td>( C_f )</td>
<td>0.31</td>
</tr>
<tr>
<td>( A_{\text{val}} )</td>
<td>7.07 \times 10^{-6} \text{ m}^2</td>
</tr>
<tr>
<td>( A_s )</td>
<td>6.28 \times 10^{-6} \text{ m}^2</td>
</tr>
<tr>
<td>( A_d )</td>
<td>5.28 \times 10^{-6} \text{ m}^2</td>
</tr>
<tr>
<td>( M )</td>
<td>3.6 kg</td>
</tr>
<tr>
<td>( B )</td>
<td>9.29 N\text{ s/m}</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>50 rad/s</td>
</tr>
<tr>
<td>( \phi )</td>
<td>20 \text{ m/s}^2</td>
</tr>
<tr>
<td>( K )</td>
<td>2.0 \times 10^{-6} \text{ m}^2</td>
</tr>
</tbody>
</table>

friction) in the system. In all these cases, the actual pressures in both the chambers were recorded and compared with the corresponding observed pressures.

In the second stage of the experiment, to prove the effectiveness of the observer, the pressure sensors were disconnected from the system. The robotic arm was then controlled using the estimated pressures from the pressure observer. The end-effector was commanded for the same sinusoidal and step input as used for the first stage of the experiment. Subsequently, the tracking performance of the robotic arm was compared to the tracking obtained using pressure sensors.

VI. RESULTS AND DISCUSSION

Fig. 4 shows a comparison of the observed and actual pressures, while the system is tracking (using measured pressures as feedback—motion tracking not shown) a 0.5-Hz sinusoidal motion of amplitude 30 mm.

Figs. 5 and 6 show the convergence of the observed pressure at 2 and 3 Hz motion tracking, respectively. Again, the controller used the measured pressures in these cases. The initial conditions of the observed pressures were set different from the actual initial values to check the convergence rate. As can be seen in Fig. 5(a), the initial pressure of the observer for chamber “A” was set to atmosphere pressure (101 kPa) when the actual pressure in the chamber was 475 kPa. The observed value converges in nearly 0.3 s. For chamber “B,” the observed pressure converges in 0.2 s [see Fig. 5(b)]. Similarly, Fig. 7 shows the observer results for a step motion.

As shown in Figs. 4–7, the observed pressures quickly converge toward the actual pressures but display some steady-state error. A maximum multiplicative error of +0.4 and −0.9 atmospheric pressure was seen. This multiplicative error, or gain error, will be accommodated by the robustness properties of the sliding-mode controller. More significantly for purposes of control, the phase delay of the observer method was similar or smaller than the actual pressure sensor signal. The prime cause of the error between the observed and measured pressure signal is suspected to be a difference between the actual and calculated mass flow rates. This leads to a steady-state error in the observed pressures as discussed at the end of Section III. The error between the flow rates is higher at small area openings of the valve since the mass flow rate calculations are based on the average discharge coefficient. The discharge coefficient is actually a function of the valve opening area, among other parameters, and at small valve openings, frictional flow losses are more dominant, and hence, the value of the discharge coefficient is much lower than the average value used in the experiment. This effect, coupled with unmodeled leakage in the valve, is dominant at lower frequencies when the valve openings are small. Another potential contributing factor in the error is the frictional flow losses in the pipes between the valve and the cylinder, which is neglected in the design of the observers. The length of the air tubes used in the experiment were kept fairly short to help minimize this unmodeled effect.
The results of the case when external disturbances are added to the system are shown in Fig. 8. The disturbances were introduced in the system by applying external forces (by hand) to the robotic arm. The force was added between 1.2 and 2.8 s and between 5.2 and 6.5 s. In this case also, the observed response closely follows the actual response of the system. This shows the robustness of the observer in the presence of disturbances and uncertainties (such as friction) in the system.

The design of the observer is independent of the frictional forces between the payload and the surface—or indeed independent of any model of the load dynamics. The observer is therefore robust to all changes in the load, or alternatively, is load-independent. Furthermore, the convergence rate is unaffected if the payload varies, as might be the case with an industrial robotic manipulator.
The motion tracking results of the controller using measured versus observed pressures, with a mass of 3.6 kg at the end-effector, are demonstrated in Figs. 9 and 10. In all the figures shown, the solid line shows the desired trajectory and the dashed line shows the actual trajectory followed by the end-effector. Fig. 9(a) shows the tracking of the end-effector at a 0.25-Hz sinusoidal frequency when the controller uses pressure sensors present in the system. Fig. 9(b) shows the result of sinusoidal tracking when the controller uses the pressure observer developed in this paper. It can be seen that the results obtained using pressure sensors versus the pressure observer demonstrates essentially the same tracking performance. A small deviation in the tracking is observed in both cases when the velocity of the end-effector is zero. This is presumably because of the neglected Coulomb friction in the controller design.
Fig. 11. (a) Desired (solid) and actual (dashed) position at 0.5 Hz square-wave frequency tracking using pressure sensors. (b) Desired (solid) and actual (dashed) position at 0.5 Hz square-wave frequency tracking using pressure observers.

Fig. 10(a) and (b) demonstrates the results at a 2.5-Hz sinusoidal frequency. At this frequency a phase lag and attenuation in the amplitude is observed in the response. The results of step command tracking are shown in Fig. 11(a) and (b). The results are similar to the sinusoidal tracking where the response of the system is almost identical using pressure sensors [see Fig. 11(a)] or pressure observers [see Fig. 11(b)].

As commented earlier, a feature associated with this observer atypical of standard observer designs is that the convergence rate cannot be influenced. However, given that the observer equations (5) are indeed the expected isothermal pressure dynamics, the dynamic response and bandwidth of the observer is that of the actual pressure dynamics (limited only by the bandwidth of the position and velocity sensors). This, in turn, yields an observer that, unlike typical observers, does not present stability concerns when combined with a controller that accounts for the pressure dynamics, such as the one used here. Indeed, we can even state that the observer presents less of a concern regarding closed-loop stability than do limited bandwidth pressure sensors that often result from filtering the pressure signals. From the experimental tracking results, the convergence rate of the observer is verified as fast enough to provide motion control that appears indistinguishable from the motion control that utilizes pressure sensors. It is even arguable that the observed pressures result in tracking superior to that from using pressure sensors, which in this case, have some internal filtering.

Fig. 12 shows the measured closed-loop frequency response of the controlled system using the pressure observer. The bandwidth is observed to be about 5 Hz. It should be noted that

Fig. 12. Gain and phase plot of the closed-loop system with the controller using pressure observers. The plot is fitted with a linear sixth-order transfer function with relative order three, representative of the closed-loop controller/observer/plant dynamics.
the 5 Hz bandwidth is not a limitation of the controller. At this frequency, saturation in the valve output was observed, which limited the bandwidth. The bandwidth can be increased with the use of valves with higher mass flow rates (larger maximum orifice sizes) or by reducing the mass at the end-effector. An increase in bandwidth can also be obtained by increasing the supply pressure.

VII. CONCLUSION

In this paper, the design of a Lyapunov-based pressure observer for a pneumatic servo system was presented. The pressure observer was proven globally Lyapunov-stable. The effectiveness and load-independent nature of the proposed pressure observer was demonstrated using experimental results. It is also shown in the paper that the proposed observer, along with a robust controller, can be implemented in lieu of expensive pressure sensors. Although not strictly proven closed-loop stable, typical closed-loop stability concerns associated with the additional dynamics of an observer are assuaged in this case by the fact that the observer has the same dynamics as the modeled pressure dynamics utilized in the controller’s formulation. The results presented demonstrate that the tracking performance using the pressure observer versus using pressure sensors is, in essence, indistinguishable. This shows that the system can be accurately controlled using the pressure observer, resulting in a lower cost system, with no performance tradeoffs. Additionally, the use of pressure observers along with the controller developed results in a lower weight, more compact, and lower maintenance system.

REFERENCES


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