Dynamic Constraint-Based Energy-Saving Control of Pneumatic Servo Systems

This paper proposes a control approach that can provide significant energy savings for the control of pneumatic servo systems. The control methodology is formulated by decoupling the standard four-way spool valve used for pneumatic servo control into two three-way valves, then using the resulting two control degrees of freedom to simultaneously satisfy a performance constraint (for which this paper is based on the sliding mode sliding condition), and an energy-saving dynamic constraint that minimizes cylinder pressures. The control formulation is presented, followed by experimental results that indicate significant energy savings with essentially no compromise in tracking performance relative to control with a standard four-way spool valve. [DOI: 10.1115/1.2232688]

1 Introduction

A typical pneumatic servo system, which consists primarily of a proportionally controllable four-way spool valve and a pneumatic cylinder, is depicted in Fig. 1. In this system, the position of the valve spool controls the airflow into and out of each side of the cylinder, which in turn results in a pressure differential across the piston and thus imposes a force on the load. In such a system, feedback control is incorporated to command a valve spool motion that will result in a desired motion of the piston load. A considerable amount of work has been conducted in the modeling and feedback control of such systems, including the work by Shearer [1–3], Mannetje [4], Ben-Dov and Salcudean [5], Wang et al. [6], Maeda et al. [7], Ning and Bone [8], Bobrow and McDonell [9], and Richer and Hurmuzlu [10,11], among others. Despite this prior work on the control of pneumatic servo systems, relatively little work has focused on the energetic efficiency of such systems. This topic has presumably received little attention from the research community because most applications draw energy from an essentially limitless reservoir of power (i.e., from a power plant). In many applications, however, the available energy is considerably more limited (e.g., in the case of a mobile robot), and in such cases, the energetic efficiency of the controller is significant. Fluid-powered systems, in particular, offer intriguing possibilities with regard to the energetic efficiency of control. Specifically, the energetic role of an actuator at any given point in time is either to generate or dissipate power. In a fluid-powered system, the energetic role of power dissipation can be provided passively by controlling the resistance to fluid flow. Therefore, ideally, a fluid-powered system need only draw energy from the (high-pressure) fluid supply when the actuator is generating power, but need not use the supply when dissipating power. Design and open-loop control approaches related to this notion have been investigated by Sanville [12], Quaglia and Gastaldi [13,14], Pu et al. [15], Wang et al. [16], Kawakami et al. [17], and Arinaga et al. [18], while approaches that enable (closed-loop) servo control have been presented by Yao and Liu [19] and Margolis [20]. Specifically, Sanville utilized a secondary reservoir in an open-loop system to collect exhaust air rather than vent it to atmosphere, and then used the reservoir as an auxiliary low-pressure supply. Quaglia and Gastaldi proposed a nonconventional pneumatic cylinder that incorporates multiple cylinder chambers embedded into a single actuator with the intent of recycling compressed air. Pu et al. describe a pneumatic arrangement that incorporates a standard four-way spool valve controlled pneumatic servoactuator, with an additional two-way valve between the two sides of the cylinder. They demonstrated “preliminary” results, but since no experimental comparisons were presented, it is unclear what improvements in efficiency were achieved. Wang et al. studied the use of input shaping to choose a command profile for point-to-point motions that would result in energy savings for closed loop pneumatic servo actuators, and showed that some velocity profiles could reduce energy demand relative to other profiles. Kawakami et al. and Arinaga et al. utilized metering circuits to reduce the airflow requirements for open-loop point-to-point motions. Yao and Liu [19] and Margolis [20] separately developed controllers for energy regeneration or recovery in servo-controlled hydraulic systems. The approach of Yao and Liu leverages the capability of fluid-powered systems to passively impose dissipative forces via modulation of flow resistance. Specifically, they incorporate a combination of five two-way proportional valves arranged about a double-acting hydraulic cylinder and modulate the resistance in the appropriate flow paths during the dissipative portions of the motion control cycle. Margolis proposes a different approach that incorporates a switching four-way valve in combination with a gas accumulator, which in essence utilizes the accumulator for energy storage and minimizes the valve losses via a switching control based on sliding mode control. Like the work of Yao and Liu and Margolis, this paper treats the problem of energy saving in the context of a servo-controlled actuator. Specifically, the authors propose an approach different from the previously described servo-control approaches (i.e., different in both actuator configuration and servo-control approach) that enables significant energy savings in closed-loop controlled pneumatic servo actuation.

2 Control Approach

In a standard pneumatic servo system, the charging of one cylinder chamber is related to the discharging of the other by the kinematic constraints imposed by a standard four-way spool valve. Specifically, the area that connects one cylinder chamber to supply pressure is constrained to be the same as the area that connects the other chamber to exhaust. The proposed control approach decouples this relationship by utilizing two three-way valves in place of the single four-way valve, as shown in Fig. 2, such that each valve area can be controlled independently. As such, the single actuation degree of freedom is influenced by two control degrees of freedom (i.e., the modified servo actuator is a
two-input, single-output system). The extra control degree of freedom enables the satisfaction of an additional artificial constraint, which as proposed, can be constructed to minimize the airflow required to track a given trajectory. In other words, a standard pneumatic actuation system fundamentally has two control degrees of freedom (the signed orifice area for each chamber, where positive indicates charging and negative discharging). These areas are typically made equal and opposite for design convenience (i.e., use of a four-way spool valve). This paper proposes instead to use this extra control degree of freedom to satisfy an energy saving constraint, which dynamically relates the valve areas through the system states. Note that the proposed approach can be contrasted to a bimodal energy saving approach proposed by the authors [21], in which the signed valve areas are algebraically rather than dynamically related. Formulation and introduction of this dynamic constraint into the pneumatic system dynamics in effect reduces the system to a single-input, single-output system that has the added property of minimizing airflow for a given (predominately inertial) load and trajectory. Any suitable controller can be utilized on the resulting system dynamics. The authors demonstrate this approach by using a sliding mode controller, which is well suited to the control of pneumatic servo systems.

### 3 Modeling the Pneumatic Servo System

The texts [22,23] describe variations on modeling pneumatic servo actuators. The model used in the work presented herein is reasonably standard and presented briefly here so that the model-based control approach can be described. The load dynamics of the system shown in Fig. 1 can be written as

\[ M \ddot{x} + B \dot{x} = P_a A_a - P_b A_b - P_{\text{atm}} A_r, \]  

where \( M \) is the payload plus the piston and rod assembly mass, \( B \) is the viscous friction coefficient, \( P_a \) and \( P_b \) are the absolute pressures in chambers \( a \) and \( b \), respectively, \( P_{\text{atm}} \) is atmospheric pressure, \( A_a \) and \( A_b \) are the effective areas of each side of the piston, and \( A_r \) is the cross-sectional area of the piston rod. Note that for the work presented here, the Coulomb friction forces from the piston and rod seals were considered disturbances. It should be noted, however, that the proposed approach does not require this assumption, and as such, the piston and rod seal friction could be explicitly modeled if so desired, with the restriction that sliding mode control requires a continuously differentiable description of friction, such as that presented in [24]. Assuming air is a perfect gas undergoing an isothermal process, the rate of change of the pressure inside each chamber of the cylinder can be expressed as

\[ \dot{P}_{(a,b)} = \frac{RT}{V_{(a,b)}} \dot{m}_{(a,b)} - \frac{P_{(a,b)}}{V_{(a,b)}} \dot{V}_{(a,b)} \]  

where \( P_{(a,b)} \) is the pressure inside each side of the cylinder, \( m_{(a,b)} \) is the mass flow rates into or out of each side of the cylinder, \( R \) is the universal gas constant, \( T \) is the fluid temperature, and \( V_{(a,b)} \) is the volume of each cylinder chamber. The volume in each chamber, and the volume rate of change, is related to the rod position \( x \) by

\[ V_a = V_{\text{mid},a} + A_a x \]  
\[ V_b = V_{\text{mid},b} - A_b x \]  
\[ \dot{V}_a = A_a \dot{x} \]  
\[ \dot{V}_b = -A_b \dot{x} \]

where \( V_{\text{mid},a} \) and \( V_{\text{mid},b} \) are the volumes of chambers \( a \) and \( b \) respectively at \( x=0 \). Note that if the process were assumed adiabatic rather than isothermal (i.e., at the other extreme of the heat transfer assumption), the right-hand side of Eq. (2) would be multiplied by the ratio of specific heats (\( \sim 1.4 \) for air), but the pressure dynamics would otherwise remain the same (see [1] for details). As such, as long as the control approach is robust to limited parameter variation, the control problem is not sensitive to assumptions regarding the presence of heat transfer. Based on isentropic flow assumptions, the mass flow rate through a valve orifice with effective area \( A_v \) for a compressible substance can be stated functionally as

\[ m_v = A_v \Psi_a = \begin{cases} A_v \Psi_a(P_a, P_{\text{atm}}) & \text{for } A_{v,a} \geq 0 \text{ (charge)} \\ A_v \Psi_a(P_{\text{atm}}, P_a) & \text{for } A_{v,a} < 0 \text{ (discharge)} \end{cases} \]

\[ m_b = A_v \Psi_b = \begin{cases} A_v \Psi_b(P_b, P_{\text{atm}}) & \text{for } A_{v,b} \geq 0 \text{ (charge)} \\ A_v \Psi_b(P_{\text{atm}}, P_b) & \text{for } A_{v,b} < 0 \text{ (discharge)} \end{cases} \]

where the normalized mass flow rate \( \Psi_{(a,b)} \) will reside in either a sonic (choked) or subsonic (unchoked) flow regime

\[ \Psi(P_a, P_d) = \begin{cases} \frac{C_1 C_p P_a}{\sqrt{T}} & \text{if } \frac{P_d}{P_a} \leq C_r \text{ (choked)} \\ \frac{C_1 C_p P_a}{\sqrt{T}} \left[ \frac{P_d}{P_a} \right]^k \left[ 1 - \left( \frac{P_d}{P_a} \right)^{(q-1)/k} \right] & \text{otherwise (unchoked)} \end{cases} \]  

where \( C_r \) is the discharge coefficient of the valve, \( P_a \) and \( P_d \) are the upstream and downstream pressures, respectively, \( T \) is the air temperature (which given the isothermal assumption is constant), \( k \) is the ratio of specific heats, \( C_r \) is the pressure ratio that divides the flow regimes into unchoked and choked flow, and \( C_1 \) and \( C_2 \) are constants defined as...
Specifically, a positive valve area command corresponds to charging \( a \) and discharging \( b \) are described by

\[
Mx^{(3)} + B\ddot{x} = RTA\left(\frac{A}{V_a}\Psi(P_a, P_b) + \frac{A}{V_b}\Psi(P_b, P_{atm})\right)
\]

\[
- \left(\frac{P_a A_b V_a}{V_a} - \frac{P_b A_b V_b}{V_b}\right)
\]

and the system dynamics for a negative control valve command (charging \( b \) and discharging \( a \)) are described by

\[
Mx^{(3)} + B\ddot{x} = RTA\left(\frac{A}{V_a}\Psi(P_a, P_{atm}) + \frac{A}{V_b}\Psi(P_b, P_b)\right)
\]

\[
- \left(\frac{P_a A_b V_a}{V_a} - \frac{P_b A_b V_b}{V_b}\right)
\]

Though various approaches exist for the control of nonlinear systems, sliding mode control is generally well suited to the control of pneumatic servoactuators. For the system shown in Fig. 1, the plant output, which is the load position \( x \), must be differentiated three times to produce the control input, which is the valve area \( A_v \), and as such the system is characterized by third-order dynamics. Defining a sliding surface as

\[
s = \left(\frac{d}{dt} + \lambda\right)(x - \bar{x}) \tag{16}
\]

where \( \bar{x} \) is the tracking error of the piston position \( x \) compared to the desired piston position \( x_d \) (i.e., \( \bar{x} = x - x_d \)), \( \lambda \) is a strictly positive constant, and \( n \) is the number of times the output must be differentiated to recover the input (which as previously described for this system is three). In standard sliding mode control (see [24] for the development), the controller consists of two components, an equivalent control law, which utilizes model and error information to provide marginal stability in the sense of Lyapunov, and a switching component, which robustly enforces the condition \( \dot{V} < 0 \) (where \( V \) is the Lyapunov function), and thus provides for uniform asymptotic stability. Thus, the form of sliding mode control is given by

\[
A_v = A_{v,sat} - K \operatorname{sat}\left(\frac{s}{\Phi}\right) \tag{17}
\]

where \( K \) is a strictly positive gain, \( \Phi \) is the boundary layer thickness, and \( A_{v,sat} \) is the equivalent control component of the valve area command. The equivalent control component of Eq. (17) is formulated by taking \( \dot{s} = 0 \), which for this system yields

\[
x^{(3)} = x_d^{(3)} - 2\lambda(\dot{x} - \dot{x}_d) - \lambda^2(\ddot{x} - \ddot{x}_d) \tag{18}
\]

Combining Eq. (18) with the system dynamics described by Eqs. (9)–(15) yields the equivalent control term for standard sliding mode control

\[
A_{v,sat} = \begin{cases} 
M[x_d^{(3)} - 2\lambda(\dot{x} - \dot{x}_d) - \lambda^2(\ddot{x} - \ddot{x}_d)] + B\ddot{x} + \left(\frac{P_a A_b V_a}{V_a} - \frac{P_b A_b V_b}{V_b}\right) & \text{for } A_v \geq 0 \\
RT\left[\frac{A}{V_a}\Psi(P_a, P_a) + \frac{A}{V_b}\Psi(P_b, P_{atm})\right] & \text{otherwise}
\end{cases} \tag{19}
\]
computing the control effort. If the sign of corresponding to the dynamics of that case should be utilized in computing the control effort. If the sign of \( A_b \) is negative, the system will be charging chamber \( b \) and discharging \( a \), and so the corresponding equivalent control should be used.

5 Dynamic Constraint-Based Energy-Saving Control

As previously described, decoupling the single four-way spool valve into two three-way valves, as shown in Fig. 2, relaxes the simple geometric relationship that relates the signed valve areas, and enables its replacement with an alternative desired relationship. In the case of the proposed energy-saving controller, this replacement relationship is one that minimizes the airflow for the tracking of an inertial load. Thus, the essence of the problem is to formulate a relationship between the valve areas that will result in energy saving. Such a relationship can be formulated by observing the relationship between the mass flow rate input and average pressure for sinusoidal trajectory tracking of the pneumatic servo system described by Eqs. (1)–(6). For a sinusoidal motion trajectory given by

\[
x = X \sin(\omega t)
\]

the dynamic relationship of Eq. (1) will take the form

\[
-MX\omega^2 \sin(\omega t) + BX\omega \cos(\omega t) = \dot{P}_a A_a - \dot{P}_b A_b - \dot{P}_{\text{atm}} A_r
\]

The time derivative of this expression yields the required pressure rates, given by

\[
-MX\omega^3 \cos(\omega t) - BX\omega^2 \sin(\omega t) = \dot{P}_a A_a - \dot{P}_b A_b
\]

Assuming both cylinder chambers contribute to the time rate of change of force in some proportion, the time rate of change of pressure in each chamber can be written as

\[
\dot{P}_a = -\frac{\alpha}{A_a} \left[ MX\omega^3 \cos(\omega t) + BX\omega^2 \sin(\omega t) \right]
\]

\[
\dot{P}_b = \frac{1 - \alpha}{A_b} \left[ MX\omega^3 \cos(\omega t) + BX\omega^2 \sin(\omega t) \right]
\]

where \( 0 < \alpha < 1 \). The respective pressures in each chamber must therefore take the following forms:

\[
P_a = \frac{1}{A_a} \left[ MX\omega^3 \cos(\omega t) + BX\omega^2 \sin(\omega t) \right] + P_{\text{ave},a}
\]

\[
P_b = \frac{(1 - \alpha)}{A_b} \left[ MX\omega^3 \sin(\omega t) - BX\omega^2 \cos(\omega t) \right] + P_{\text{ave},b}
\]

where \( P_{\text{ave},a,b} \) are the respective average pressures in chambers \( a \) and \( b \). Considering initially the dynamics of chamber \( a \), substituting the pressure relationships of Eqs. (23a) and (24a) and the volume relationships given by Eqs. (3), (5), and (20) into the pressure dynamics described by Eq. (2), yields a description of the requisite mass flow rate for chamber \( a \). After simplification based on trigonometric identities, the requisite mass flow rate for chamber \( a \) is given by

\[
RT\dot{m}_a = \alpha \omega^3 X^2 \left[ B \cos(2\omega t) - M \omega ^2 \sin(2\omega t) \right] - \frac{\alpha \omega^3 \omega X B V_{\text{mid}} \sin(\omega t)}{A_a}
\]

\[
+ \left( P_{\text{ave},a} \right) \frac{\alpha^2 \omega^3 V_{\text{mid}}}{A_a} X \cos(\omega t)
\]

Since positive mass flow rate is supplied by the source and negative is exhausted to atmosphere, the objective of minimizing airflow from the supply is equivalent to minimizing the positive portion of Eq. (25). Since the respective terms on the right-hand side of Eq. (25) have different frequencies or phase, minimizing the positive portion of this expression over an even number of cycles requires minimization of the respective amplitudes of each term. Though the first two terms cannot be influenced by average pressure, the last term can be eliminated by choosing the average pressure as

\[
P_{\text{ave},a} = \frac{MV_{\text{mid}} \alpha^2\omega^2}{A_a}
\]

If one calculates the desired average pressure for typical values of mass, geometry, and tracking frequency, the resulting pressure is one to two orders of magnitude below atmospheric pressure. Since atmospheric pressure is generally the lowest achievable, the mass flow rate will effectively be minimized when the average pressure is atmospheric. An equivalent analysis of the chamber \( b \) mass flow yields essentially the same criteria for the average pressure in chamber \( b \), which is given by

\[
P_{\text{ave},b} = \frac{MV_{\text{mid}}(1 - \alpha)\omega^2}{A_b}
\]

Thus, one can assert that the objective of minimizing supply airflow for a given load and trajectory corresponds to the state-based objective of minimizing the average combined pressures in the cylinder. In order to enforce this objective, a first-order dynamic constraint of the following form can be formulated to drive the average pressure in the cylinder to a desired value

\[
\dot{J} = -\eta (J - J_{\text{target}})
\]

where \( J \) is a positive definite measure of the average pressure, given by

\[
J = \frac{1}{2} P_a^2 + \frac{1}{2} P_b^2
\]

\( J_{\text{target}} \) is the function \( J \) evaluated at the target chamber pressures, and \( \eta \) is a strictly positive constant that determines the rate of convergence of the dynamic constraint of Eq. (28). Utilizing the previously described argument, the target cylinder pressure is chosen as atmospheric, such that \( J_{\text{target}} = J(P_{\text{atm}}, P_{\text{atm}}) \). Thus, in the absence of tracking demands, the pressure in each chamber of the cylinder should converge to atmospheric (since it cannot be driven below atmospheric). Note that objective functions other than Eq. (29) could also be used, given that \( J(P_a, P_b) \) is convex in the region \( P_a, P_b \in [0, \infty) \). That is, Eq. (28) will attract \( J \) to \( J_{\text{target}} \) for any \( J(P_a, P_b) \), and the (presumed) minimization of \( J \) will correspond to a minimization of the cylinder pressures for a convex relationship. Though any convex function could work, Eq. (29) was selected due to its fairly standard form (i.e., it is the two-norm of the difference vector).

As previously described, the energy-saving constraint establishes a dynamic relationship between the (signed) valve areas and thus reduces the system to a single-input, single-output system, upon which a standard control approach can be utilized. Specifically, the chamber \( b \) valve area command can be related to the chamber \( a \) valve area command through the dynamic constraint by combining Eqs. (28) and (29) as follows:

\[
P_a \dot{P}_a + P_a \dot{P}_b + \eta \left( \frac{1}{2} P_a^2 + \frac{1}{2} P_b^2 \right) = J_{\text{target}} = P_{\text{atm}}^2
\]

Substituting the pressure dynamics described by Eq. (2) for \( \dot{P}_a \) and \( \dot{P}_b \) in Eq. (30) and rearranging yields the following constraint between the mass flow rates of the respective chambers:
\[ \dot{m}_b = \left( \frac{P_R}{V_R} \right) \frac{V_a}{2RT} \frac{V_b - V_a}{2} - \left( \frac{P_R}{V_R} \right) m_a + \eta V_b J_{target} \]

(31)

where \( P_R \) is chamber pressure ratio \( P_a/P_b \), and \( V_R \) is chamber volume ratio \( V_a/V_b \). Therefore, the static relation of a four-way spool valve imposed by Eq. (13) has been replaced by a state dependent dynamic relationship of the form

\[ A_{v,a} \dot{V}_b = - \left( \frac{P_R}{V_R} \right) A_{v,a} \Psi_a + g(P_a, P_b, \dot{x}, J_{target}) \]

(32)

where \( g(\cdot) \) follows from the combination of Eqs. (25) and (3)–(6). Substitution of Eq. (26) into the system dynamics defined by Eqs. (1) and (2) enables reformulation of the system dynamics as a single-input, single-output system as follows:

\[ \dot{x} = A_{v,a} \Psi_a + b \dot{x} + \frac{A_{v,b}}{MV_a} \left( 1 - \frac{\eta V_a}{2V_b} \right) - \frac{A_{v,b}}{MV_b} \left( 1 - \frac{\eta V_b}{2V_a} \right) \]

(33)

where \( A_{v,b} \) is the area ratio given by \( A_v/A_b \). Any appropriate control law can be applied to the energy-saving system dynamics of Eq. (33). In this paper, the authors incorporate a sliding mode control approach. Utilizing a sliding mode control approach and Eqs. (9), (31), and (32), the equivalent control commands for each respective valve are

\[ A_{v,a,eq} = \begin{cases} \frac{m_a}{\Psi(P_a, P_b)} & \text{for } A_{v,a} > 0 \text{ (charging chamber a)} \\ \dot{m}_a/\Psi(P_a, P_b) & \text{otherwise (discharging chamber a)} \end{cases} \]

(34)

\[ A_{v,b,eq} = \begin{cases} \frac{m_b}{\Psi(P_a, P_b)} & \text{for } A_{v,b} < 0 \text{ (discharging chamber b)} \\ \dot{m}_b/\Psi(P_a, P_b) & \text{otherwise (charging chamber b)} \end{cases} \]

(35)

where \( \dot{m}_b \) is given by Eq. (31) and \( A_{v,a,eq} \) and \( A_{v,b,eq} \) are the equivalent valve area commands of the valves connected to chambers \( a \) and \( b \), respectively. The complete (i.e., robust) control law is formed by adding a robustness component to each of the equivalent control laws, as described by Eq. (17). Note that the equivalent control requires selection of two gains, \( \eta \) and \( \lambda \). As in standard sliding mode control, \( \lambda \) determines the speed of tracking error convergence when the system is on the sliding surface. In the energy-saving approach, the relative magnitudes of \( \eta \) and \( \lambda \) determine the respective prioritization of energy saving (i.e., control of the pressure sum) versus trajectory tracking (i.e., control of the pressure difference).

6 Experiments

Experiments were conducted to compare the tracking performance and average required mass flow rate of the proposed dynamic constraint based control versus that of a standard sliding mode controller. A schematic for the system setup is illustrated in Fig. 2. The double acting cylinder (Bimba 314-DXP) used in the experiment has a stroke length of 10.2 cm (4.0 in.), inner diameter of 5.1 cm (2.0 in.), and piston rod diameter of 1.6 cm (0.62 in.). Two four-way proportional valves (PositionerX SVP-360) are attached to the chambers with two ports of each valve blocked to make the valves function as three-way valves. A brass block serves as a mass load of 10 kg (22 lb), which slides on a track with linear bearings (Thompson 1CB08FAOL10). Three pressure transducers (Omega PX202-200GV) are attached to the pressure supply tank and each cylinder chamber, respectively, and a linear potentiometer (Midori LP-100F) with 10 cm (3.94 in.) maximum travel measures the linear position of the inertial load. Control is provided by a Pentium 4 computer with an A/D card (National Instruments PCI-6031E), which drives the two proportional valves via a pair of KEPCO bipolar power supply/amplifiers. The control inputs are the valve areas, which are commanded indirectly by commanding the spool displacements.

Model parameters used for the model-based controller were \( M = 11.4 \ kg \) (25 lb), \( B = 13.1 \ kg/s \) (28.8 lb/s), \( A_a = 20.3 \ cm^2 \) (3.14 in.\(^2\)), \( A_b = 18.2 \ cm^2 \) (2.83 in.\(^2\)), \( C_p = 0.8 \), \( C_s = 0.528 \), \( k = 1.4 \), \( T = 298 \ K \), and \( R = 287 \ m^2/(s \ K) \). Note also that each chamber has a dead-space volume of 18 cm\(^3\) (0.8 in.\(^3\)), which is needed in relating each chamber volume to the measured piston position, and that the valve commands were saturated at the maximum valve openings \( A_{v,max} = 7.35 \ mm^2 \) (0.0114 in.\(^2\)). These parameters were obtained via direct measurement, when possible, or through calculation when experimental measurement was not possible.

The sliding mode control gains and boundary layer thicknesses used in the control experiments were selected based on the system model and desired tracking bandwidth, then tuned to provide good tracking performance, first via simulation and then by experiment. Improved performance for both approaches was achieved with variable robustness gains, where the gains were simply increased linearly with increased tracking error, such that

\[ K = k_{min} + k_{slope} \cdot |x| \]

(36)

Note that the absolute value operator ensures that \( K \) is always strictly positive (i.e., \( K \) always increases with the magnitude of the tracking error). Equation (36) was additionally saturated to limit the amount by which the gain could vary. Finally, note that the variable gain does not violate any sliding mode stability or performance robustness guarantees, since as formulated, it can be guaranteed to be both positive and greater than some minimum value, as required by the degree of model uncertainty. For the
standard sliding mode controller, the control gains were selected as follows:

\[ \lambda = 100 \text{ s}^{-1} \]
\[ \Phi = 38.5 \text{ m/s}^2 \ (1500 \text{ in.}/\text{s}^2) \]
\[ k_{\text{min}} = 3.0 \text{ mm}^2 \ (0.0045 \text{ in.}^2) \] (37)
\[ k_{\text{slope}} = 0.0018 \text{ mm}^2 \text{s} \ (2.7 \times 10^{-6} \text{ in.}^2 \text{s}) \]

\[ 3.0 \text{ mm}^2 \ (0.0045 \text{ in.}^2) \leq K \leq 5.5 \text{ mm}^2 \ (0.0085 \text{ in.}^2) \]

For the dynamic constraint based controller, the control gains were selected as follows:

\[ \lambda = 620 \text{ s}^{-1} \]
\[ \Phi = 256 \text{ m/s}^2 \ (10,000 \text{ in.}/\text{s}^2) \]
\[ k_{\text{min}} = 2.0 \text{ mm}^2 \ (0.003 \text{ in.}^2) \] (38)
\[ k_{\text{slope}} = 0.00022 \text{ mm}^2 \text{s} \ (3.3 \times 10^{-7} \text{ in.}^2 \text{s}) \]

\[ 2.0 \text{ mm}^2 \ (0.003 \text{ in.}^2) \leq K \leq 4.1 \text{ mm}^2 \ (0.0063 \text{ in.}^2) \]

The energy-saving dynamic constraint control parameters [i.e., parameters for Eq. (28)] were selected as follows:

\[ \eta = 700 \text{ s}^{-1} \]
\[ J_{\text{target}} = 10,200 \text{ kPa}^2 \ (213 \text{ psia}^2) \] (39)

Recall that \( J_{\text{target}} \), as given by Eq. (29), results from choosing both desired chamber pressures as atmospheric.

For each experiment, the average mass flow rate was found by charging a 5 gal pressure supply tank to \( \sim 600 \text{ kPag} \) (90 psig) before running each experiment and measuring the tank pressure as the experiment was performed. Assuming the air in the supply tank to be an ideal gas undergoing an isothermal process, the energy in the fixed-volume tank is proportional to the mass, which is in turn proportional to the tank pressure. Tracking experiments were conducted on the previously described experimental setup (i.e., a cylinder driving a primarily inertial load) to compare the performance and energy savings of the standard sliding mode control approach, which utilizes a typical four-way spool valve (i.e., kinematic dependence of inlet/exhaust valve areas) with the proposed dynamic constraint based approach, which is based on the use of two three-way valves (i.e., dynamic dependence of inlet/exhaust valve areas). Experiments were conducted for sinusoidal tracking frequencies of 0.25–1.5 Hz. The experimental results of tracking performance for 0.25 Hz sinusoidal command signal are shown in Figs. 3 and 4 or standard sliding mode control and dynamic constraint-based control, respectively. Both systems demonstrate similar tracking performance, though on close inspection the standard approach is slightly better. The discrepancy is presumably due to Coulomb friction, which was not explicitly modeled and the effects of which are typically diminished by a high actuator output impedance. Figure 5 shows the supply tank pressure during the initial 30 s tracking history for the sinusoidal trajectories shown in Figs. 3 and 4, demonstrating clearly the energy savings provided by the dynamic constraint-based approach. Figure 6 shows the pressure variations in both chambers for standard control (the two high-pressure traces) and dynamic constraint-based control (the two low-pressure traces). As indicated by the figure, the mean pressures in the dynamic constraint-based control hover just above atmospheric pressure, yielding a low actuator output impedance. In fact, in the absence of tracking demands, the chamber pressures will converge to atmospheric (or whatever mean pressures correspond to \( J_{\text{target}} \)).

Figures 7 and 8 show sinusoidal tracking of a 1.5 Hz sinusoid for standard sliding mode and dynamic constraint-based control,
respectively, demonstrating essentially the same tracking performance. Figure 9 shows the supply tank pressure during the initial 30 s tracking history for the sinusoidal trajectories shown in Figs. 7 and 8, demonstrating clearly the energy savings. The flow-rate savings observed during the 1.5 Hz tracking are less than those observed during the 0.5 Hz tracking because the actuator demands are greater for the higher-frequency tracking and, thus, the actuator output impedance must be higher to provide the desired degree of tracking performance. The increase in required actuator output impedance is reflected in the data shown in Fig. 10, which compared to Fig. 6, indicates noticeably higher average chamber pressures for the dynamic constraint-based control case.

A summary of the average energy (i.e., mass flow rate) savings for various sinusoidal tracking frequencies is listed in Table 1. As illustrated in the table, the maximum energy savings occur at 0.5 Hz, somewhere between the lowest and highest tracking frequencies. The reduction in energy saving at very low frequencies is presumably because the dynamics of the pneumatic servoactua-
tor is largely influenced by Coulomb friction, which violates the inertially dominated case described by Eq. (20). The reduction in energy savings at high frequency is due to the fact that the average pressure must increase at high frequencies in order to satisfy the tracking objective. As shown in Table 1, the maximum energy savings for this system occurs at $0.5 \text{ Hz}$, which lies somewhere between the two extremes of friction-influenced dynamics and high actuator demand.

## 7 Conclusion

This paper presents an energy-saving approach to the control of pneumatic servo-actuation systems. The control approach is, in essence, a variable impedance controller, which maintains only the output impedance required to track the desired trajectory, and thus minimizes the required mass flow rate of air. Experiments demonstrate that the power consumption of a pneumatic servo system is reduced by as much as 45%, with essentially no sacrifice in the output impedance required to track the desired trajectory, and the large actuator demands required to overcome inertial forces at higher frequencies.

## References


