boom sinks slightly due to gravity as the boom weight is compensated by the build up of pressure in the lower part of the cylinder. The boom motion excites the chassis which in turn excites the cab. The relative motion between chassis and cab makes the valve move due to the linkage kinematics. There is a small deadzone where valve motion does not create any flow area. Once this deadzone is exceeded, some valve flow area exists and hydraulic flow can commence.

As can be seen in Figs. 6–8, with the conventional cab mounts, the system is unstable. Figure 6 shows the valve motion nondimensionalized by the maximum valve opening. The valve motion quickly builds to the maximum and bangs continuously between maximum positive and negative opening. Figure 7 shows the unstable angular motion of the boom. This has the appearance of what was observed on the actual backhoe. Figure 8 shows the corresponding chassis angular motion. It is bouncing around at plus–minus 4°.

The mount stiffnesses at the front and rear of the cab were doubled from their nominal values and the simulation was run again. The results are also shown in Figs. 6–8. As can be seen, the entire system is stable and well behaved. This also corresponds to the trend observed on the actual backhoe.

Although the instability in the backhoe can be resolved by using stiffer cab mounts, this will degrade the ride motion at the idle speed. Another way to stabilize the backhoe is with automatic control. Here the response of the backhoe system with a feedback control is demonstrated. It is assumed that a displacement-type actuator is installed in the valve control rod as shown in Fig. 9. Here the actuator is proposed to be a DC motor with a ball-screwed to convert rotary to linear motion. For controller design purposes, this actuator is represented by the flow source, $S_f$, shown in the bond graph fragment of Fig. 9. Once the desired boom position is set by the operator, the error between the desired valve position and actual valve position is fed back to the control unit and the flow source is activated to reduce the error. Here it was found the Proportional plus Integral control worked fine. As shown in Figs. 6–8, the entire system is stable and the responses are similar to the stable passive system with the stiffer mounts.

While it is not possible to publish the measured motion-time histories of the system response, the model presented here is qualitatively accurate and was very useful in the design of a “fix” for the problem of the vehicle. The model allowed testing of many different approaches to solving the stability problem without requiring hardware realizations for each.

Conclusions
A complete pitch/plane model of a backhoe was developed that includes the hydraulic dynamics and kinematics of the control linkage. The model was developed in pieces using bond graph fragments, and the overall model was assembled by straightforwardly assembling the bond graph fragments. Equations were derived directly from the bond graph and programmed for simulation using a digital computer.

Simulations were run for an initial condition response from near equilibrium. The model predicts the instability observed on the actual backhoe, and is now ready to be used as a design tool for future backhoe development.

It was shown that the backhoe can be stabilized passively by using stiffer mounts between chassis and cab. This solution will cause an increase in cab acceleration during engine idle tests. An automatic control solution was also demonstrated. This consisted of an actuator in series with the valve control rod. By measuring and feeding back valve position, the controller was demonstrated to stabilize the system without requiring stiffer cab mounts.

References

Control Design for Relative Stability in a PWM-Controlled Pneumatic System

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This paper presents a control design methodology that provides a prescribed degree of stability robustness for plants characterized by discontinuous (i.e., switching) dynamics. The proposed control methodology transforms a discontinuous switching model into a linear continuous equivalent model, so that loop-shaping methods may be utilized to provide a prescribed degree of stability robustness. The approach is specifically targeted at pneumatically actuated servo systems that are controlled by solenoid valves and do not incorporate pressure sensors. Experimental demonstration of the approach validates model equivalence and demonstrates good tracking performance. [DOI: 10.1115/1.1591810]

Introduction
Many papers that treat the control of pneumatic systems offer some introductory claim about the low cost of pneumatic actuators. Such a claim is in part justified, since a typical pneumatic actuator costs on the order of tens of U.S. dollars. The claim of low cost, however, is also somewhat misleading, since the components required for the servo control of a pneumatic actuator, specifically the proportional valve and pressure sensors, have a combined cost typically an order of magnitude greater than the actuator itself. Specifically, servo control of pneumatic systems...
typically requires some type of proportionally controllable valve, which enables control of cylinder pressures via proportional con-
striction of the pneumatic flow (i.e., throttling). Most frequently, these valves are a proportional servovalve type, which is a 4-way
spool valve with a position-controllable spool. Control approaches
for pneumatic servo systems that utilize proportional servovalves are
described in papers by Liu and Bobrow [1], Bobrow and Jab-
bari [2], Bobrow and McDonell [3], and Drakunov et al. [4], and
Kimura et al. [5], among others. Other works treating the servo
control of pneumatic actuators incorporate (noncommercially-
available) alternative configurations of proportional valves, such
as jet pipes valves (Jacobsen et al. [6], Henri et al. [7]) or flapper
valves (Ben-Dov and Salcudean [8]). Regardless of the valve
type, the proportional valve required to implement a proportional
control approach is one of the most expensive components of a
pneumatic servo system, typically costing on the order of several
hundred U.S. dollars.

In addition to a proportional servovalve, pressure sensors are
typically incorporated in pneumatic servo controllers to measure
the pressure states for feedback control. Due to the compressibil-
ity of air, the pressures on each side of a pneumatic cylinder
constitute two states of the basic dynamic model of a pneumatic
servo system. Though pressure feedback is utilized for various
purposes, the most fundamental reason requiring its use in servo
control is to distinguish between the choked and unchoked flow
conditions that result from compressible flow through the control
valve. Such conditions are unobservable from measurement of
motion and/or force at the output of the actuator.

This paper presents a control methodology that enables servo
control of a pneumatic actuator without a proportional valve or
pressure sensors. Specifically, the methodology provides flow
control via binary-position solenoid valves rather than propor-
tional valves, and concomitantly enables a prescribed degree of
stability robustness in order to adequately compensate for the loss
of information that results from the absence of pressure sensors.
Since the cost of solenoid valves is on the order of ten U.S.
dollars, the use of solenoid instead of proportional valves, along
with the elimination of pressure sensors, can reduce the cost of pneu-
matic servo control implementation by an order of magnitude and
can therefore provide high-power actuation at a significantly
lower cost relative to a power-comparable DC motor-actuated
system.

Several prior works have demonstrated the viability of servo-
control of pneumatic actuators via solenoid on-off valves, includ-
ing the work of Ye et al. [9], Kunt and Singh [10], Lai et al. [11],
Royston and Singh [12], Paul et al. [13], Norigitsu [14–16], Shih
and Hwang [17], and van Varseveld and Bone [18]. None of these
prior works, however, enable a prescribed degree of stability
robustness. As previously mentioned, such stability robustness is
particularly important in the absence of pressure sensors, since
the controller cannot otherwise distinguish between choked and
unchoked flow through the valve, and thus the robustness of the
control approach must adequately and reliably compensate for the
loss of pressure information. This paper provides a method to
transform the nonanalytic, nonlinear description of pulse-width-
modulated (PWM) based control of a pneumatic system into an
analytic, linear model, which in turn enables the application of
frequency domain loop shaping to address issues of performance
and stability robustness. The switching aspect enables use of so-
lenoid (rather than proportional) valves, and the robustness en-
ables elimination of pressure sensors, both of which effectively
reduce the cost of implementation by an order of magnitude. This
approach is illustrated by way of example on a single degree-of-
freedom pneumatic servo system.

Modeling a Pneumatic Servo System Controlled by
Switching Valves

A schematic of a single degree-of-freedom pneumatic actuation
system is shown in Fig. 1. The general objective of this section is
to cast the dynamics of the PWM-controlled pneumatic actuator
into a linear continuous form, in order to apply control design
techniques that incorporate frequency domain concepts of stability
robustness, and more specifically that enable the implementation
of prescribed gain and phase margins. A lumped-parameter model
that treats all unmodeled forces, such as Coulomb and viscous
friction and external disturbance forces, as a single disturbance
term is given as

\[ M \ddot{x} = P_B A_B - P_A A_A + F_{\text{disturbance}} \]  

(1)

where \( P_A \) and \( P_B \) are the (gage) pressures inside chambers \( A \)
and \( B \) of the pneumatic actuator respectively and \( A_A \) and \( A_B \)
are the areas of the piston seen by each chamber. The pressure in
each chamber will be controlled by a two-position three-way solenoid
valve that serves to connect the chamber to either a high-pressure
supply or to atmospheric pressure.

The pressure response in each chamber should be first order in
character, assuming that the primary energetic behavior will result
from flow resistance of the valve and flow capacitance of the
cylinder volume. The pressure dynamic in each cylinder can there-
fore be reasonably modeled by

\[ \tau_{A,B} \dot{P}_{A,B} + P_{A,B} = v_{A,B}g(t - T_D) \]  

(2)

where the discrete control input \( v_{A,B} \in \{0, P_s\} \) is the pressure
boundary condition imposed by the valve state (either atmospheric
or supply pressure); the time constant \( \tau_{A,B} \) is a typically nonlinear
function of the upstream and downstream pressures, the cylinder
displacement \( x \), and various other geometric and thermody-
namic quantities; and \( T_D \) is the time delay exhibited between the
control command and the pressure dynamic as a byproduct of the
spool/sleeve overlap required to inhibit leakage flow in the spool
valves. In order to provide linear system dynamics and preclude
dependence on pressure sensors, the pressure dynamic time con-
stant was assumed for this treatment to be invariant (i.e., constant
so that \( \tau_{A,B} = \tau \)). Such an assumption captures the fundamental
dynamics of the system, but assumes that the robustness of the
control approach can adequately and reliably compensate for the
loss of information.

The controller can command one of four switched modes to the
pneumatic actuator, corresponding to one of the four valve state
permutations \((v_A, v_B)\) given by \( \{0, P_s\} \times \{0, P_s\} \):

Mode 1: \( v_A = P_s \) and \( v_B = 0 \)
Mode 2: \( v_A = 0 \) and \( v_B = P_s \)
Mode 3: \( v_A = P_s \) and \( v_B = P_s \)
Mode 4: \( v_A = 0 \) and \( v_B = 0 \)

Each mode results, respectively, in the following system dynamics
(assuming the disturbance force to be zero):

Mode 1: \( M (\tau \ddot{x}(t) + \dot{x}(t)) = P_s \dot{e}(t - T_D) A_A \)  

(3a)

Mode 2: \( M (\tau \ddot{x}(t) + \dot{x}(t)) = -P_s \dot{e}(t - T_D) A_B \)  

(3b)
Model 3: $M(\tau x(t) + \ddot{x}(t)) = P_s \dot{e}(t - T_D)A_A - P_s \dot{e}(t - T_D)A_B$ (3c)

Mode 4: $\tau x(t) + \ddot{x}(t) = 0$ (3d)

where $\dot{e}(t)$ is the unit step.

**Average Model**

State-space averaging can be utilized to convert the switching model given in Eqs. (3a-d) to a continuous average model [19], which can be utilized for implementing a robust control design. Denoting as $d_1$, $d_2$, $d_3$, and $d_4$ the fractions of a normalized switching period that each mode is active (i.e., the duty cycle of each mode), a state-space average representation of the system results in the following average model:

$$M \tau \ddot{x}(t) = -M \ddot{x}(t) + P_s \dot{e}(t - T_D)[A_A d_1 - A_B d_2 + (A_A - A_B) d_3]$$

(4)

where

$$d_1 + d_2 + d_3 + d_4 = 1.$$  

(5)

Note that the switching fraction $d_4$ drops out of Eq. (4), due to the constraint of Eq. (5) and a common term present in all modes of Eqs. (3a-d). Denoting the input term as

$$\dot{u}(t) = A_A d_1 - A_B d_2 + (A_A - A_B) d_3$$

(6)

the average model is given in the s-domain as

$$\hat{G}(s) = \frac{X(s)}{U(s)} = \frac{e^{-T_D P_s}}{M s^3 + M s^2}$$

(7)

The average model given by Eq. (7) assumes that the input $\dot{u}(t)$ can vary continuously in time, which is not the case. Specifically, once a given duty cycle is commanded, the control command cannot be changed until the next PWM period. The control command is therefore subjected to a sample-and-hold operation. Given the standard frequency domain approximation of a sample-and-hold, the transfer function of the average model from the continuous control command $u(t)$ to the motion of the output $x(t)$ can be given as:

$$\hat{G}(s) = \frac{X(s)}{U(s)} = \frac{1 - e^{-T_delta}}{T_delta} \frac{e^{-T_D P_s}}{M s^3 + M s^2}$$

(8)

It should be emphasized that the zero-order sample-and-hold effect is not the result of digital implementation but instead due to the PWM nature of the control signal. As an aside, electrical PWM amplifiers typically switch on the order of 10 kHz. In such applications, it may be assumed that the sample-and-hold effect is negligible. In pneumatic applications the sample-and-hold can have a non-negligible effect on the dynamics of the servo-positioning system due to its longer switching period, and so must be included.

**Control Design**

Though the switching system described by Eqs. (3a-d) has been described by the continuous-time frequency domain model of Eq. (8), the PWM-controlled system is still not of the form necessary to apply standard frequency domain techniques. Specifically, the control input, nominally given by Eq. (6), requires an additional constraint to uniquely specify the control. Though one could formulate various constraint equations, for purposes of this paper the third mode was simply eliminated, since it is not required for motion control of the actuator. Accordingly, positive control input values are specified by Mode 1 and negative values specified by Mode 2 as follows:

$$u(t) = A_A d_1 - A_B d_2$$  

(9a)

and

$$d_1 = \text{sat}(u/A_A), \quad d_2 = 0; \quad u \geq 0$$

(9b)

$$d_1 = 0, \quad d_2 = \text{sat}(-u/A_B); \quad u < 0$$

(9c)

where the saturation functions are implemented to ensure the condition given by Eq. (5). Note that this control specification also addresses the asymmetric influence of unequal piston areas on opposing sides of the pneumatic actuator (due to the single rod configuration).

The combination of Eqs. (8) and (9a-c) enable treatment of the PWM-controlled pneumatic actuator as a linear, analytical, continuous-time system, which in turn enables the use of relative stability notions. Though several approaches to the design of such systems are available, the use of loop shaping addresses directly two significant issues in the control design of PWM-controlled pneumatic actuators. The first is stability robustness, which must be present to compensate for the elimination of pressure sensors (and the resulting loss of information). The second is saturation, which is a fundamental aspect of PWM-based controllers. Specifically, as described by Eqs. (9a-c), the duty cycles cannot exceed 100%, and so the control command $u(t)$ is bounded by $[−A_B, A_A]$. The former issue, stability robustness, can be addressed directly in a loop-shaping approach by simply shaping the open loop frequency response so that it exhibits desired gain and phase margins (e.g., with a lead-lag form of compensator). The latter issue, saturation, can also be addressed fairly directly in a loop-shaping context. Rather than shape the open-loop transfer function, one can constrain the maximum gain on the loop-shaping compensator to avoid saturation, given bounds on the input to the compensator (i.e., bounds on the tracking error). For the case of a pneumatic cylinder, the maximum amplitude of the error signal will be the stroke length of the cylinder, $L$. In order to avoid the saturation limits $[−A_B, A_A]$ of the control command $u(t)$, the frequency response of the compensator $K(s)$ must be such that

$$|K(s)|_{\text{max}} = \frac{\min(|A_A|, |A_B|)}{L}$$

(10)

**Control of a Pneumatic Servo System Controlled by Switching Valves**

The proposed control methodology was implemented on a pneumatic system as depicted in Fig. 1 with a 1.9 cm (3/4 in) inner diameter, 10 cm (4 in) stroke single-rod double-acting pneumatic cylinder (Bimba 044-DXP) equipped with two pilot assisted
3-way solenoid-activated valves (SMC VQ2200H-5B). The cylinder rod is rigidly connected to a 10 kg brass block on a track with linear bearings (Thompson 1CBO8FAOL10). The setup is instrumented with a linear potentiometer (Midori LP-100F) for position feedback. Model parameters were estimated or measured to be: \( M = 10 \) kg, \( A_A = 2.8 \) cm\(^2\), and \( A_B = 2.5 \) cm\(^2\). Additionally, the pressure response described by Eq. (2) was measured at the mid-stroke position and the pressure time constant and valve time delay were determined to be \( \tau = 9 \) ms and \( T_D = 19 \) ms, respectively. The supply pressure was \( P_s = 586 \) kPa gage (85 psig) and the PWM switching period was \( T = 38.5 \) ms (26 Hz switching frequency). The open loop frequency response of this system, as described by Eq. (8), is shown in Fig. 2. Based on the frequency response shown in Fig. 2, a compensator was designed to provide an open-loop frequency response that would provide a desired degree of stability robustness and additionally avoid control saturation, while also maintaining a high low-frequency gain for purposes of command following and disturbance rejection. The resulting compensator was given by:

\[
K(s) = k \frac{s + a_3}{s} \left( \frac{s + a_1 + 1}{a_1\beta_2 s + 1} \right) \left( \frac{s + a_2 + 1}{a_2\beta_2 s + 1} \right)
\]  

where \( k = 3.98 \times 10^{-5} \) (98 dB), \( a_3 = 7.54 \), \( a_1 = 3.01 \), \( \beta_1 = 3.11 \times 10^{-2} \), \( a_2 = 1.80 \times 10^{-1} \), and \( \beta_2 = 4.91 \times 10^{-2} \). This is a double lead, single lag compensator where the first lead network is selected to add 70° of phase at 0.3 Hz, the second lead network is selected to add 65° of phase at 4.0 Hz, and the lag network is selected to add integral action with a break-point of 1.2 Hz. A bound of \( |K(s)| < -52 \) dB ensures that the control output should not saturate for errors in the frequency band of interest of approximately 0.02–2 Hz. The frequency response of this compensator is shown in Fig. 3. With this compensator, the frequency response of the open-loop transfer function \( K(s)G(s) \) is shown in Fig. 4 along with the uncompensated plant and the compensator. Stability robustness measures are 6.7 dB of gain margin and 33° of phase margin with a crossover frequency of 2.0 Hz.

**Experimental Results**

The compensator described by Eq. (11) and Fig. 3 was experimentally implemented on the previously described experimental setup. In order to ensure a reasonable approximation of the sample-and-hold used in Eq. (8), a prefilter \( F(s) \) was placed on the command to the control loop. The filter \( F(s) \) has the form of a critically damped second-order low-pass filter.

![Fig. 3](image3.png)  
*Fig. 3* Frequency response plots of the compensator \( K(s) \) obeying the saturation gain limit of \(-52\) dB imposed by PWM control near the targeted cross-over frequency of 2 Hz.

![Fig. 4](image4.png)  
*Fig. 4* Frequency response plots of the uncompensated open-loop system, the compensator, and the compensated open-loop system. The compensated open-loop response shows a phase margin of 33° and a gain margin of 6.7 dB at a cross-over frequency of 2.0 Hz.

![Fig. 5](image5.png)  
*Fig. 5* Step response. The filtered commanded step is shown as dashed and the measured system response is shown as solid.

![Fig. 6](image6.png)  
*Fig. 6* Sinusoidal response at 0.5 Hz
The figure shows both the control design model prediction as well as experimentally measured points overlaid.

\[ F(s) = \frac{1}{(\tau_f s + 1)^2} \]  

where \( \tau_f = 0.0159 \) seconds was chosen for a filter cut-off frequency of 10 Hz, well below the PWM switching frequency of 26 Hz. It should be noted that this filter does not decrease the stability robustness of the system (since it is outside the closed loop), and also does not adversely affect the closed-loop performance.

Specifically, the presence of a sample-and-hold in the closed-loop makes it impossible to achieve a significant open-loop gain at frequencies near the PWM switching frequency, and as a result, the closed-loop response will always have a tracking bandwidth less than the switching frequency (and in practice, well below it). Since the loop cannot track frequencies near or above the switching frequency, the input-shaping filter does not adversely affect closed-loop performance, but simply precludes frequencies that the loop cannot track.

Figure 5 shows the measured step response of the system. Figures 6 and 7 show sinusoidal responses at 0.5 and 1.0 Hz, respectively. Figure 8 shows a comparison of the predicted gain and phase characteristics of the closed-loop system to several measured points. Good agreement is exhibited as the phase falls off, indicating validation of the equivalent model formulation. Disagreement between predicted and measured phase at lower frequencies (0.1 and 0.5 Hz) is attributed primarily to unmodeled (Coulomb) friction effects. Apart from the well known fact that frictional effects typically influence lower frequencies, further evidence of this attributed unmodeled effect is taken from the fact that the open-loop frequency response of Fig. 2 shows little influence of the equivalent model at frequencies of 0.1 and 0.5 Hz. At higher frequencies, the effects of PWM switching as incorporated into the equivalent model dominate over frictional effects.

Conclusions

Solenoid-valve controlled pneumatic actuators without pressure sensors can provide low-cost, high-power servo actuation. Such a system requires a control methodology that can accommodate the switching nature of solenoid valve control, and concomitantly provide for a prescribed degree of stability robustness to compensate for the loss of pressure measurements. This paper presents an approach that transforms a discontinuous switching model into a linear continuous equivalent model, so that loop-shaping methods may be utilized to incorporate a prescribed degree of stability robustness into the system. The method is illustrated by way of example on a single degree-of-freedom pneumatic system. Experimental demonstration of the approach indicates good model equivalence and good tracking performance. The method is generalizable to any linearizable PWM-controlled process.

References