A METHOD FOR THE FREQUENCY DOMAIN DESIGN OF PWM-CONTROLLED PNEUMATIC SYSTEMS

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ABSTRACT
This paper presents a rigorous analysis and design method for PWM-based control of pneumatic systems. An equivalent analytical model incorporating the effects of a finite PWM switching period is formulated. This equivalent model was motivated by a lack of control design and analysis techniques needed to treat the inherently non-analytical switching models associated with PWM-based systems. The equivalent model enables the design of a loop compensator that rigorously addresses control design issues of stability robustness, disturbance rejection, insensitivity to sensor noise, performance bandwidth and actuator saturation. Simulation of this compensator with both the equivalent design model and a full nonlinear switching model for a particular pneumatic robot application is presented which demonstrates and validates the proposed method.

INTRODUCTION
Pneumatic actuators can offer significantly higher mass-specific power densities than can DC motors, especially given the fact that a pneumatic actuator can be appropriately load-matched for most robotic applications without the use of a mechanical transmission (e.g., they do not require a gearhead). Unlike hydraulic actuation systems, pneumatic systems do not require a return path for the working fluid, and therefore the actuation system can be simpler, lighter, and more compact than hydraulic counterparts. Unlike DC motors, the control of pneumatic systems is more complex and more difficult, due in part to the fact that the fluid transmission is typified by fluid mechanics that are nonlinear and generally difficult to characterize accurately, and in part to the fact that the supply and control components (i.e., servovalves) are not as readily available or well-packaged as their electrical counterparts.

Control of pneumatic systems is generally implemented via the proportional control of servovalves, in which pneumatic fluid flow is generally proportionally constricted (i.e., throttled) by electrically-controlled valves. This proportional control approach has been studied by several researchers, including Shearer [1], Liu and Bobrow [2], Kunt and Singh [3], Ye et al. [4], and Bobrow and McDonell [5]. The control of such systems is complicated by the need to accurately describe the flow characteristics of a proportional servovalve, which is typically accomplished empirically with a set of pressure/flow relationships. Like a linear servoamplifier for DC motors, the servovalve control approach for pneumatic systems controls the power delivered to the actuator by power dissipation. Such an approach is particularly undesirable in systems for which efficiency is paramount. Pulse-width modulated (PWM) control offers an alternative to the servovalve control approach that both circumvents the accurate dynamic characterization of throttling a fluid flow, and more significantly, offers appreciably higher control efficiency. The primary problem with the control of PWM pneumatic systems is that the introduction of switching results in a non-analytic system model, and thus excludes the direct application of well-developed analytic control approaches. This paper provides a method to transform the non-analytic, nonlinear description of PWM-based control of a pneumatic system into an analytic, linear model, which in turn enables the application of frequency domain methods to address issues of performance and stability robustness.

PWM-BASED CONTROL
In a continuously-controlled pneumatic system, the power delivered to an actuator is metered by varying flow resistance continuously through a proportional valve. As a result, the
flow is metered by dissipating power, and thus the power delivered to the actuator is controlled by dissipation. This, for example, is the manner in which a linear servoamplifier controls the electrical power delivered to a DC motor. Depending on operating conditions, such a continuously-controlled system can dissipate significantly more power in the control system than it delivers to a load (thus the large heat sinks and cooling fans on a linear servoamplifier). In a PWM-controlled system, the power delivered to an actuator is metered by delivering packets or quanta of energy via a valve that is either completely on or completely off. If delivery of these packets of energy occur on a time scale that is significantly faster than the system dynamics (i.e., dynamics of the actuator and load), then the system will respond in essence to the average power delivered to the actuator, in a manner similar to the continuous case. Unlike the continuous case, little to no power is dissipated in the process. Specifically, in the ideal on-state, no flow resistance is offered, and thus no power is dissipated. In the ideal off-state, no flow occurs, and thus no power is dissipated. A PWM-controlled system is therefore capable of delivering a significant amount of controlled power to an actuator while dissipating essentially none. Electrical PWM-mode amplifiers, for example, are recognized as dissipating negligible amounts of power [6]. In addition to the paramount issue of efficiency, in the case of PWM-controlled pneumatic systems, the use of on-off valves also reduces the modeling task to that of characterizing the flow through a fixed restriction and/or supply line. Within the context of control design, the non-linearity of the flow through a servovalve’s variable orifice can thus be largely avoided by operating in saturation (i.e., PWM mode). Finally, the use of PWM valves also offers the advantage of sidestepping the issue of “stiction” that often plagues the modeling and control of proportional servovalves.

Though previous research has demonstrated the viability of a PWM-based approach to the control of pneumatic actuators [7-11], prior literature on the subject has been marked by a lack of a rigorous analytical approach with which to design and analyze such a system. The methods described in this paper address this void by presenting a formulation that enables the direct consideration of issues such as stability, robustness, disturbance rejection, and performance bandwidth.

**MODELING A PWM-CONTROLLED PNEUMATIC ACTUATOR**

Figure 1 depicts a generalized pneumatic actuation system, which consists of a four-way solenoid valve controlling the flow to a double acting pneumatic cylinder, which in turn drives a mechanical load (in this case an inertia). PWM control of this system is realized by switching the valve such that one side of the cylinder is connected completely to the supply pressure and the other side completely to exhaust, or vice-versa.

A power balance of a control volume inside one of the piston’s chambers may be written as,

\[ \frac{dU}{dt} + \frac{dH}{dt} + \frac{dQ}{dt} + \frac{dW}{dt} = 0 \]  

where \( U \) is the internal energy contained within the (assumed pure) substance of the control volume, and \( H, Q, \) and \( W \) are the enthalpy, heat, and mechanical work fluxes, respectively. Assuming adiabatic conditions in the chamber (i.e., \( dQ/dt = 0 \) relative to the other terms) and assuming that the constitutive behavior of the pure substance can be described as an ideal gas, the pressure dynamics in each chamber can be described by,

\[ \dot{P}_{a,b} = \frac{c_p (T_{in} \dot{m})_{a,b} - \left( \frac{c_v}{R} + 1 \right)(P \dot{V})_{a,b}}{c_v V_{a,b}} \]  

where the \( a \) and \( b \) subscripts denote the quantities associated with either chamber A or chamber B, \( c_p \) and \( c_v \) are the specific heats of the fluid substance at constant pressure and volume, respectively, \( R \) is the gas constant of the substance, \( T_{in} \) is the inlet temperature, \( \dot{m} \) is the mass flow rate at the inlet, and \( P \) and \( V \) are the pressure and volume in the chamber, respectively. Further assuming Hagen-Poiseuille flow through a supply line of length \( L \) and an open valve orifice of the same hydraulic diameter as the supply line, \( D \), and assuming fully developed, laminar, incompressible, viscous flow in the supply line, the mass flow may be written as,

\[ \dot{m}_{a,b} = \left( \frac{\pi D^4}{128 \mu L} \right) \Delta P_{a,b} \rho = \alpha \Delta P_{a,b} \rho \]  

where \( \rho \) is the density, \( \mu \) is the dynamic viscosity of the fluid, and \( \Delta P \) is the difference between the inlet pressure and the

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**Figure 1. Schematic Diagram of Double Acting Pneumatic Cylinder Controlled with a Four-Way Servovalve Operating in Saturation (i.e., PWM Mode).**
chamber pressure. Regarding these assumptions, simulation results verify that the maximum flow velocities are low enough to ensure the flow is within the laminar region. The incompressible flow assumption is justified by noting that the energy storage due to fluid compressibility in the supply line is small relative to that in the piston chamber, and compressibility effects are therefore lumped into the piston chamber. Finally, the fully developed flow assumption is reasonable given a high aspect ratio of the supply line.

Defining switching mode 1 as connecting the pressure supply $P_s$ to chamber A and porting chamber B to the atmosphere $P_{atm}$, and assuming that the conditions where $P_s < P_a$ and $P_b < P_{atm}$ are sufficiently rare such that they can be neglected, the pressure dynamics of each chamber during switching mode 1 is given by,

$$
\dot{P_a} = \frac{c_p \alpha (P_s - P_a) P_a - (c_v + R) P_a \dot{V}}{c_v V}
$$

$$
\dot{P_b} = \frac{c_p \alpha (P_{atm} - P_b) P_b + (c_v + R) P_b \dot{V}}{c_v (C - V)}
$$

where the volume of chamber A is given by $V$ and the total volume of the piston (chamber A + chamber B) is given by $C$. Defining switching mode 2 as connecting the supply and exhaust to the opposite chambers as in switching mode 1, and again assuming that a condition where $P_a < P_{atm}$ and $P_b < P_s$ is sufficiently rare that it can be neglected, the pressure dynamics of each chamber during switching mode 2 is given by:

$$
\dot{P_a} = \frac{c_p \alpha (P_{atm} - P_a) P_a - (c_v + R) P_a \dot{V}}{c_v V}
$$

$$
\dot{P_b} = \frac{c_p \alpha (P_s - P_b) P_b + (c_v + R) P_b \dot{V}}{c_v (C - V)}
$$

Finally, the motion of the mass $M$ attached to the cylinder is given by,

$$
\dot{V} = \frac{1}{M} \left( P_a A^2 - P_b A^2 + AF \right)
$$

where $A$ is the cross sectional area of the piston and $F$ is an external disturbance force, and the output motion is $x = V/A$. The resulting model is thus fourth order, nonlinear and non-analytical and can be written in state-space form as,

$$
\dot{x}(t) = \begin{cases}
    f_1(x(t), u(t)), & \frac{n}{f_s} \leq t < \frac{n + d_n}{f_s} \\
    f_2(x(t), u(t)), & \frac{n + d_n}{f_s} \leq t < \frac{n + 1}{f_s}
\end{cases}
$$

where $1/f_s$ is the PWM period, $d_n$ is the duty cycle during period $n$, and $f_1$ and $f_2$ represent the dynamics during switching mode 1 and 2 respectively.

For valves where the parameter $\alpha$ in Equation (3) is sufficiently large, the pressure dynamics will be significantly faster than either the dynamics of motion or the PWM switching frequency. This can be verified via simulation and results in reducing the model to one which is second order, linear and non-analytic, where $P_s = P_a$ and $P_b = P_{atm}$ during switching mode 1, and $P_s = P_b$ and $P_a = P_{atm}$ during switching mode 2. This was the case for the parameters of interest to the authors, where settling times for the pressure dynamics were found to be about 50 microseconds. Therefore the example given for purposes of illustration in this paper assumes negligible pressure dynamics and treats the corresponding second order, linear, non-analytic model. The validity of the use of such a reduced order model for control design purposes is subsequently verified by applying the closed-loop control system to the full fourth order nonlinear system given by Equations (4-9). The inclusion of non-negligible pressure dynamics for pneumatic systems with smaller flow orifices is also straightforward.

**EQUIVALENT DESIGN MODEL**

A method that has been successfully applied to the analysis of switching systems is state space averaging, which seeks to represent the average behavior of the states of a switching model over each switching period. Given a switching model of the form,

$$
\dot{x}(t) = \begin{cases}
    A_1 x + B_1 u, & \frac{n}{f_s} \leq t < \frac{n + d_n}{f_s} \\
    A_2 x + B_2 u, & \frac{n + d_n}{f_s} \leq t < \frac{n + 1}{f_s}
\end{cases}
$$

the state space average representation is given by,

$$
\dot{x} = [A_1 d + A_2 (1 - d)]x + [B_1 d + B_2 (1 - d)]u
$$

where $0 \leq d \leq 1$, is the duty cycle. The resulting average model is analytical but non-linear. Linearization of such a model about an equilibrium point enables the use of linear control design tools. For the present pneumatic system under consideration, linearizing about an equilibrium position of...
that the sample-and-hold approximation attenuates the magnitude and decreases the phase near the PWM switching frequency of 50Hz. The implications agree with those given by intuition: no compensator can be designed such that the closed-loop bandwidth of the system is greater than the PWM switching frequency \( f_s = 1/T \). Therefore, the control design objective is to formulate a compensator resulting in a robustly stable closed-loop system with a performance bandwidth below the PWM switching frequency.

**CONTROL DESIGN**

In order to directly address issues of stability, robustness, disturbance rejection and performance bandwidth, consider the system block diagram in Figure 3. To design a compensator \( K(s) \) and input-shaping filter \( F(s) \), consider first the case where \( F(s) \) is unity. Define \( \Omega_r, \Omega_d \) and \( \Omega_n \) as the frequency sets in which the reference, disturbance and noise signals, respectively, have frequency components of non-negligible power. Further, define the set \( \Omega_p = \Omega_r \cup \Omega_d \) and assume that \( \Omega_p \) and \( \Omega_n \) are mutually exclusive. The performance equation of the closed-loop system is given as,

\[
e(s) = S(s)[r(s) - d(s)] + C(s)m(s) \quad (15)
\]

where \( S(s) = 1/[1 + K(s)G(s)] \) and \( C(s) = K(s)G(s)/[1 + K(s)G(s)] \) are the sensitivity and complementary sensitivity transfer functions respectively.
In designing the compensator $K(s)$, this performance equation can be used to address performance specifications and stability robustness measures. Good command following and disturbance rejection is achieved as $S(s) \rightarrow 0$ for frequencies $\omega \in \Omega_p$. This implies $|K(s)G(s)| > 1$ for $\omega \in \Omega_p$. Insensitivity to sensor noise is achieved as $C(s) \rightarrow 0$ for frequencies $\omega \in \Omega_n$. This implies $|K(s)G(s)| < 1$ for $\omega \in \Omega_n$. Stability robustness can be specified with the gain and phase margin of the open-loop transfer function $K(s)G(s)$.

Saturation issues must also be addressed in designing the compensator due to the fact that the duty cycle is only defined from 0% to 100%. Given the specific system being designed here, where the total length of the cylinder is 0.1 m, the maximum amplitude of the error signal into the compensator is 0.2 m. Recalling that $u = \Delta d \in [-\frac{1}{2}, \frac{1}{2}]$, the maximum amplitude of the signal out of the compensator is 1. Therefore a least upper bound on the gain of the compensator is $5 \text{ m}^{-1}$, or about 14 dB, for all frequencies. In reality, the error signal might never exceed half or even a quarter of the maximum possible value. In either case, saturation due to bounds on the duty cycle imposes only mild bounds on the gain of the compensator. Using a quarter of the maximum error amplitude for this particular system, a bound of $|K(s)| < 26 \text{ dB}$ in regions critical to shaping the frequency response of the open-loop system should ensure that saturation is seldom encountered.

In order to shape the open-loop frequency response of the system and meet the design specifications above, the compensator is selected to be of the form,

$$K(s) = \left( \frac{s + 2}{s} \right)^2 \left( \frac{\alpha s + 1}{\alpha \tau s + 1} \right)^2$$  \hspace{1cm} (16)
margin and 44° of phase margin. The frequency response of the closed-loop system is shown in Figure 6 and exhibits desirable bandwidth characteristics. It should be noted that, as demonstrated in the open-loop frequency response of Figure 2, the open-loop gain crossover frequency cannot be greater than the switching frequency, since the magnitude of the sample-and-hold approaches zero at the switching frequency. From the arguments based on Equation (15), the closed-loop response will therefore always have a tracking bandwidth less than the switching frequency, and in practice, well below it. Since the loop cannot track frequencies around and above the switching frequency, an input-shaping filter \( F(s) \) is selected to attenuate frequencies near and above the PWM switching frequency. This filter does not decrease the closed-loop performance or degrade the stability robustness, but rather prevents the control loop from acting on frequencies that it cannot track. The filter \( F(s) \) is therefore chosen as a critically-damped second-order low-pass filter of the form

\[
F(s) = \frac{1}{(\tau_f s + 1)^2}
\]  

where \( \tau_f = 0.0159 \) such that the breakpoint is at 10 Hz.

Figure 7 shows the closed-loop step response of the loop defined by \( G(s) \), \( K(s) \), and \( F(s) \) as described by Equations (14), (16), and (17) respectively and as illustrated in Figure 3 (with \( d(s) \) and \( n(s) \) as zero). Also shown in Figure 7 is the closed-loop response for which the equivalent model \( G(s) \) has been replaced by the full fourth order nonlinear switching model as described by Equations (4-9) and as illustrated in Figure 8. The step command was applied at 0.5 seconds and the magnitude was selected to be 40% of the achievable stroke of the system. The agreement between these responses serves to validate the proposed control design method. The control signal generated by the compensator, which is the duty cycle shifted by an offset of \( d_0 \), verifies that saturation limits are obeyed. It should be noted that the response of the nonlinear switching model contains about a 4 mm ripple as a product of the pulse-width modulation. This ripple cannot be decreased by controller design, but rather is a characteristic of PWM systems with the assumed switching structure (i.e., with a four-way servovalve), and in such control structures is purely a function of the switching frequency of the PWM valve relative to the system dynamics of the actuator and load. This ripple can however be eliminated by decoupling the single four-way valve into two three-way valves. Such a PWM switching configuration allows an additional degree of control freedom whereby the proposed PWM modeling and control method can likewise be applied. Issues associated with this configuration were avoided here in
order to more clearly demonstrate the modeling and control issues associated with the simpler case of PWM control of a four-way valve.

CONCLUSION

A rigorous analysis and design method for PWM-based control of pneumatic systems has been developed and presented. The method proposes the use of a linearized state-space average model, which in conjunction with a frequency-domain representation of a sample-and-hold, accurately represents the gain and phase behavior of a PWM-controlled pneumatic system. Frequency domain control design based on the resulting equivalent model enables issues of stability robustness, disturbance rejection, insensitivity to sensor noise, performance bandwidth and actuator saturation to be addressed directly. Simulation results for a particular pneumatic robot application have been presented which demonstrate and validate the proposed approach.

REFERENCES


