Modeling and Control of a Monopropellant-Based Pneumatic Actuation System

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Abstract
This work describes the modeling and control of a proposed actuation system that is capable of pressurizing a chamber volume via the catalytic decomposition of a liquid monopropellant controlled by a binary on/off propellant valve. Two design configurations of the actuation system are presented and common portions of both are energetically modeled. The parameters of the resulting dynamic model are meaningful physical properties of either the propellant or the system. A model-based switching controller is then applied to the task of pressure tracking. Model validation and controller performance are shown experimentally.

1 Introduction
One of the most significant challenges in the development of an autonomous human-scale robot is the issue of power supply. Perhaps the most likely power supply/actuator candidate system for such a robot would be a lithium variant battery and DC motor combination. This type of system, however, would have to carry an inordinate amount of battery weight in order to perform a significant amount of work for a significant period of time. A state-of-the-art example of a human-scale robot that utilizes electrochemical batteries combined with DC motor/harmonic drive actuators is the Honda Motor Corporation humanoid robot model P3, which weighs 130 kg, carries 11.5 kg of batteries and provides approximately 15-25 minutes of operation, depending on its workload. Operation times of this magnitude or smaller are not uncommon and represent a major technological roadblock for designing mobile robots that can operate power-autonomously for extended periods of time.

One alternative to using batteries and DC motors is to directly convert stored chemical potential energy into mechanical work through the pneumatic domain. In particular, liquid chemical fuels have high thermodynamic energy densities, and this thermal energy can be extracted as mechanical energy through the expansion of gaseous products. Control of power across a required bandwidth can be achieved through the use of pneumatic servo valves that control the flow of the hot gaseous products into a pneumatic piston. Though several possibilities exist for the generation of gases, monopropellant liquid fuels are well suited for this type of system. Monopropellants are a class of fuels that rapidly decompose (or chemically react) in the presence of a catalytic material. Unlike combustion or hypergolic reactions, no mixing or ignition is required. This results in a simple, low weight energy converter system. A monopropellant-based system provides a good solution to the design trade-offs between fuel energy density and system weight for the scale of interest. As such, the authors have been developing monopropellant-powered actuators for use in self-powered human-scale robots, and in particular, have been pursuing two configurations for converting the stored chemical energy of a monopropellant into controlled mechanical work. The first configuration controls the flow of the liquid monopropellant through a catalyst pack and into a centralized high-pressure hot gas reservoir, then controls the piston output by controlling the flow of hot gas from the reservoir into and out of a pneumatic cylinder. This approach, called a centralized configuration, is depicted schematically in Fig. 1. The authors have reported preliminary experimental findings characterizing the energetic capability of this system in [1, 2]. The second configuration controls the piston output by precisely controlling the direct injection of liquid monopropellant through the catalyst pack and directly into each chamber of the cylinder. This approach, called direct injection, is depicted schematically in Fig. 2. Both the centralized and direct injection configurations entail the control of two fluids. For the centralized configuration, a liquid monopropellant controller maintains a desired reservoir pressure via the control of a binary fuel valve, and a hot gas flow controller utilizes a proportional spool valve to control the piston output via control of the gas flow into and out of the cylinder. For the direct injection configuration, a liquid monopropellant controller controls the pressurization of each cylinder chamber via a binary fuel valve, and a hot gas flow controller controls the depressurization of each cylinder chamber via the hot gas exhaust flow.

This paper treats the development of the controllers for the monopropellant control loops. Specifically, both configurations require pressure control via a binary control valve with the chemical reaction dynamics in the control loop. Therefore, both approaches require the development of a dynamic model of the chamber pressurization through the monopropellant reaction dynamics, and additionally require a methodology that can control a pressure across those dynamics with a binary (i.e., non-proportional) control input. Specifically, this paper describes such a model and controller, and shows experimental data of pressure tracking that demonstrates both the validity of the model and the effectiveness of the control method.
2 System Modeling

As mentioned, the system under consideration for modeling is that of the catalyst pack and chamber volume. The chamber volume represents either the pressure reservoir in the case of the centralized system, or the actuator chamber in the case of the direct injection system. The general approach taken here is to model these two components using an energy balance method based on first principles. Given that these two components taken together are at a location where the pneumatic supply energy is generated and stored, it is imperative that such an energy-based modeling approach is taken. The resulting model may be subsequently utilized for system design issues regarding control and efficiency. The task of modeling these two components is accomplished by drawing a control volume around each, performing an energy (or power) balance regarding the flux and storage rate of energy in each, and then matching energy flux rate conditions at the boundary between the two components.

2.1 Modeling the Catalyst Pack

Figure 3 shows a schematic with a control volume drawn around the catalyst pack. With regard to this control volume (CV), a power balance relating the rate of energy storage to the energy flux rate across the boundary may be written as,

\[ \dot{U}_{cat} = \dot{H}_{cat} + \dot{Q}_{cat} - \dot{W}_{cat} \tag{1} \]

where \( \dot{U}_{cat} \) is the rate of internal energy stored inside the CV, \( \dot{H}_{cat} \) is the enthalpy rate across the CV, \( \dot{Q}_{cat} \) is the heat flux rate into the CV, and \( \dot{W}_{cat} \) is the rate of work done to the external environment. Assuming the volume of the catalyst pack is constant results in \( \dot{W}_{cat} = 0 \). In seeking a lumped parameter model of the combined catalyst pack / chamber system, and assuming that the flow restriction between the catalyst pack and the chamber is small, it is also assumed that \( \dot{U}_{cat} \) is small compared to that of the chamber. Therefore, this term is neglected in the power balance equation of the catalyst pack, and hence \( \dot{U}_{cat} = 0 \). This agrees with intuition in that the pneumatic energy is stored almost exclusively in the larger volume of the chamber. The net enthalpy rate across the boundary may be written as the flow power into the CV minus the flow power out of the CV:

\[ \dot{H}_{cat} = P \dot{\bar{Q}} - \dot{m}_{cat} c_{p} T_{ADT} \tag{2} \]

The term \( P \dot{\bar{Q}} \) represents the hydraulic flow power associated with the supply pressure of the monopropellant, \( P_{s} \), and the volumetric flow rate of liquid monopropellant, \( \dot{\bar{Q}} \). The term \( \dot{m}_{cat} c_{p} T_{ADT} \) represents the pneumatic (i.e. compressible gas) flow power, where \( \dot{m}_{cat} \) is the mass flow rate leaving the catalyst pack, \( c_{p} \) is the specific heat at constant pressure, and \( T_{ADT} \) is the adiabatic decomposition temperature of the monopropellant. The relative magnitude of the hydraulic flow power is assumed to be negligible compared to the flow power of the gas leaving the catalyst pack, and therefore \( P \dot{\bar{Q}} = 0 \). This assumption is contingent upon the mass specific energy stored in the monopropellant and becomes increasingly accurate as a higher energy density monopropellant is considered. For 70% \( \text{H}_{2} \text{O}_{2} \) in steady-state operation with a 4.9 MPa (700 psig) supply pressure, the ratio of \( \dot{m}_{cat} c_{p} T_{ADT} \) to \( P \dot{\bar{Q}} \) is about 220, thereby legitimizing the assumption for one particular monopropellant at one particular supply pressure.

The catalytic decomposition of a monopropellant results in the liberation of heat. To model the heat released, a first order dynamic is assumed to be associated with the monopropellant finding a catalyst site,

\[ r \dot{\bar{Q}}_{r} + \bar{Q}_{r} = km_{in} \tag{3} \]

where \( \bar{Q}_{r} \) is the heat released by the monopropellant, \( r \) is the time constant, and \( k \) is the heat of decomposition.
minus the heat of vaporization of product liquids. Note that the steady-state heat release rate for a constant mass flow rate agrees with a basic thermodynamic property \( k \) that can be measured (or looked up) for a given monopropellant. Recognizing this heat released as heat entering the CV, and further assuming that the catalyst pack is well enough insulted such that the heat loss in the form of conduction or convection is small in comparison, it can be stated:

\[
\dot{Q}_{cat} = \dot{Q}_r
\]

This assumption is also based on a lumped parameter philosophy in that the heat loss from the catalyst pack is dwarfed by the heat loss from the much larger surface area of the chamber.

The mass flow rate of monopropellant into the catalyst pack can be modeled by the standard hydraulic flow equation,

\[
\dot{m}_l = \rho_l \dot{Q} = \rho_l c A \sqrt{P_s - P}
\]

where \( \rho_l \) is the density of the monopropellant, \( c \) is the valve discharge coefficient, \( A \) is the valve orifice area, and \( P \) is the downstream pressure (i.e. the pressure inside the chamber). Strictly speaking, the flow restriction represented by the discharge coefficient is a function of both the valve geometry as well as the restriction presented by the material in the catalyst pack. It is typically acceptable for discharge coefficients to be determined experimentally.

2.2 Modeling the Chamber

Figure 4 shows a schematic with a control volume associated with the chamber. It should be noted that the chamber is initially modeled here as sealed (i.e. the use of the high pressure gaseous products from the reservoir for actuation, or the exhaust and volumetric changes in the actuator chamber, will require subsequent modification to the model). A power balance approach may again be taken resulting in:

\[
\dot{U}_{ch} = \dot{H}_{ch} + \dot{Q}_{ch} - \dot{W}_{ch}
\]

Given a static chamber volume, \( \dot{W}_{ch} = 0 \). Express \( \dot{U}_{ch} \) by integrating \( dU = mc_c dT \) from a reference internal energy, \( U_{ref} \), and reference temperature, \( T_{ref} \):

\[
\int_{U_{ref}}^{U} dU = \int_{T_{ref}}^{T} mc_c dT
\]

The resulting relationship, \( \dot{U}_{ch} = \dot{U}_{ch} - U_{ref} = mc_c (T - T_{ref}) \), can then be differentiated whereupon \( \dot{U}_{ch} \), \( \dot{T}_{ref} \), and \( \dot{T}_{ref} \) may be set at zero. This results in the following relation for the chamber,

\[
\dot{U}_{ch} = \dot{m}_c c_v T + \dot{m}_c c_v \dot{T}
\]

Fig. 3. A schematic of the control volume (CV) used to model the catalyst pack.

and \( \dot{m}_c = \dot{m}_{cat} \) by matching the boundaries of the two control volumes. Following a similar derivation for \( \dot{H}_{ch} \) as for \( \dot{U}_{ch} \) above and assuming that \( \dot{T}_{AIR} = 0 \), the enthalpy rate entering the control volume may be expressed as:

\[
\dot{H}_{ch} = \dot{m}_c c_p T_{AIR}
\]

It is assumed that the heat loss rate to the walls of the chamber (considered to be outside the control volume) are governed by the linear heat transfer law,

\[
\dot{Q}_{ch} = -\frac{1}{R_w} (T - T_w)
\]

where \( T_w \) is the temperature of the chamber wall. The wall temperature has an assumed first order dynamic with respect to the temperature inside the chamber,

\[
\tau_s \dot{T}_w + T_w = T
\]

where \( \tau_s \) denotes the time constant associated with heat loss. Except for cases where start-up is important or a long duration passes between valve openings, (12) can be adequately approximated by setting \( T_w \) at a constant value lower than \( T \).

Using relationships (8) and (10), the power balance given by (6) can be expressed as:
Alternatively, (8) may be expressed in terms of pressure inside the chamber by representing the internal energy rate in the following form:

\[
\dot{U}_{ch} = \frac{d}{dt} (m_{ch} c_v T) = \frac{d}{dt} \left( m_{ch} \frac{R}{\gamma - 1} T \right)
= \frac{d}{dt} \left( \frac{PV}{\gamma - 1} \right) - \frac{1}{\gamma - 1} (PV - P \dot{V})
\]

where \( \gamma \) is the ratio of specific heats, \( V \) is the volume of the chamber, and as previously noted \( P \) is the pressure inside the chamber. Use of this relationship (14) and (10) substituted into (6) leads to the following dynamic equation in terms of chamber pressure:

\[
\dot{P} = \frac{(\gamma - 1)(m_{ch} c_v T_{ADT} + \dot{Q}_{ch})}{V}
\]

### 2.3 Experimental Results

The modeled response of the catalyst pack / chamber system was compared to that of an experimental setup. The setup consisted of a single binary on/off propellant valve controlling the flow of propellant from a tank containing 70% H\(_2\)O\(_2\) pressurized to 4.9 MPa (700 psig) with nitrogen gas. A small catalyst pack immediately following the propellant valve was connected to a fixed chamber volume of 29 cm\(^3\). To validate the model, the valve was given several pulses of differing durations turning on the propellant valve. The pressure in the chamber was subsequently measured via a pressure sensor. The measured dynamic pressure response was then compared to a simulation model constructed using the modeling equations of Sections 2.1 and 2.2. Good agreement between experiment and simulation was seen in both the rise time and steady state value of the pressure response. Figure 5 shows the response from the model compared to the experimentally measured response, for three pulse durations. All parameters of the model were set as their physical properties dictated with the exception of the empirical valve discharge coefficient, \( c \), the heat release time constant, \( r_r \), and the heat loss time constant, \( r_h \).

### 3 Control

The control of the catalyst pack / chamber system is unconventional in that the control input to the system (the binary monopropellant valve) is non-proportional in nature. Though previous research has demonstrated the viability of accurate control via solenoid on/off valves [3-7], they have in general been marked by a lack of a rigorous analytical approach with which to design and analyze such a control system. Exceptions include the work by van Varseveld and Bone [8], Barth et. al. [9, 10] and Paul et. al. [11]. The work by van Varseveld and Bone [8] experimentally develops a discrete-time model of a PWM-controlled pneumatic servo system with an autoregressive system identification approach, then applies discrete-time control methods to develop a controller. Barth et. al. [9, 10] provides a method to transform the non-analytic description of PWM-based control of pneumatic systems into an analytic model, which in turn enables the use of conventional analytical control approaches such as frequency domain design and sliding mode control. The work by Paul et. al. [11] formulates a controller that directly switches valves by evaluating the derivative of the chosen Lyapunov function. This method was extended and formalized in Barth et. al. [12] and is hereby applied to the catalyst pack / chamber system under consideration. The general approach of this model-based control method is to evaluate the effect of each discrete input (in this case “on” or “off” for the propellant valve) on the derivative of
a Lyapunov function. The input that enforces a negative derivative of the Lyapunov function is chosen.

### 3.1 Switching Control Design

Consider first the case of a general nonlinear plant with a discrete-valued control input. Assume the plant dynamics are given in the following form,

\[
d^n x = f(x) + b(x)u
\]

where the state vector is \( x^T = [x, \dot{x}, \ldots, x^{(n-1)}] \). Define the following continuous positive definite Lyapunov function:

\[
V = \frac{1}{2} s^2
\]

Choose the typical integral sliding surface as,

\[
s = (d/dt + \lambda) \int e dt = \lambda_P \int e dt + \sum_{r=1}^{n} \frac{n!}{r!} \lambda^r \frac{d^r e}{dt^r} + \lambda \frac{d^n e}{dt^n} \]

where \( e = x - x_d \). Note that \( s \) is a measurable quantity if \( d^i x_d \) for \( i = 0, 1, \ldots, n \) is known. Stability in the Lyapunov sense is guaranteed and the error is driven to zero when the following condition is enforced:

\[
\dot{V} = \dot{s} s \leq 0
\]

Substituting in the plant dynamics (16), \( \dot{s} \) may be expressed as the summation of a candidate term dependent upon the selection of the input, and a measurable non-candidate term independent of the input:

\[
\dot{s} = b(x)u + f(x) - d^i x_d + \sum_{r=1}^{n} \frac{n!}{r!} \lambda^r \frac{d^r e}{dt^r} + \lambda \frac{d^n e}{dt^n} \]

If the input to the plant is only allowed to be a finite set of \( p \) discrete values,

\[
u \in \{u_1, u_2, \ldots, u_p\}
\]

each input value \( u_i \) will have an associated \( V_i \) term:

\[
\dot{V}_i = s(e^{(p+1)}, \ldots, e) \dot{s}(u_i, x, x_d^{(n)}, e^{(n-1)}, \ldots, e)
\]

for \( i = 1, 2, \ldots, p \)

The stability and error convergence condition given by Equation (19) may be enforced by selecting an input value \( u_i \) with an associated \( V_i \leq 0 \). Given a finite set of allowable input values, each \( V_i \) term can be computed online in real-time and \( u \) may be selected according to the following control law:

\[
u = u_i \text{ such that } \dot{V}_i = \max_{j=1,2,\ldots,p} \dot{V}_j \text{ and } \dot{V}_i \leq 0
\]

That is, the \( u_i \) corresponding to the least negative \( \dot{V}_i \) is selected. In one sense, this control law can be viewed as traditional sliding mode control without the equivalent control term. In a more general sense, it is a method that directly enforces the stability and error convergence condition by selecting trial inputs and evaluating the expected \( \dot{V} \).

For the system considered here, the pressure inside the chamber is the variable of interest. Therefore the error is \( e = P - P_d \), where \( P_d \) is the desired pressure. Using the results of Section 2, the pressure dynamics may be put in the following form,

\[
\dot{P} = f(x) + b(x)u
\]

where:

\[
f(x) = -\frac{(y-1)}{\tau V} \dot{Q} - \frac{(y-1)}{V\tau R_d} (\dot{T} - T + 2)
\]

\[
b(x) = \frac{(y-1)}{\tau V} - k_P c A \sqrt{P_d - P}
\]

The input takes on values of \( u_1 = 0 \) or \( u_2 = 1 \) for closed or open valve positions respectively. Note that although (25) is not shown explicitly as a function of the states \( \dot{P} \) and \( P \), it may be written as such through appropriate substitutions of the modeling equations of Section 2. Alternatively, a model-based observer may be constructed to retrieve \( \dot{Q}, \dot{T}, T \), and \( T_c \) based on the error of measured and modeled pressure. Given that the order of the system is \( n = 2 \), the following sliding surface can be chosen:

\[
s = \dot{e} + 2\lambda e + \lambda^2 \int e dt
\]

The derivative of (27) is therefore:

\[
\dot{s} = \dot{P} + 2\lambda \dot{e} + \lambda^2 e
\]

Substituting (24), (25) and (26) into (28), and then subsequently substituting (27) and (28) into (22) for each control input value \( u_i \) and \( u_j \), condition (19) is enforced via the control law (23).

### 3.2 Experimental Results

Using the switching controller developed in Section 3.1 applied to the model derived in Sections 2.1 and 2.2, the experimental setup of Section 2.3 was used to assess the performance of the closed-loop pressure control system. The experimental setup was made to track a rising pressure profile (as the controller has no way to exhaust or decrease the pressure). Shown in Figure 6 is the experimentally implemented controller as it tracks three different rising pressure profiles.

### 4 Conclusions and Future Work

This work has presented the dynamic modeling of a propellant valve, catalyst pack and hot gas pressure chamber associated with a proposed monopropellant-based actuation system. This modeling effort was pursued using fundamental energetic principles in an effort to obtain a
model invariant to design variations such as geometry or size. The motivation for obtaining the model was to describe the dynamics associated with either the centralized or direct injection configuration, in part to aide in the design of such monopropellant-based actuation systems. An experimental verification of the model revealed good agreement with both dynamic and steady-state characteristics of the system. To address the unique issues regarding the control of this system, a model-based switching controller was applied and experimentally implemented. The tracking performance of the closed-loop system was shown to effectively control chamber pressure.

The work present here was developed mainly for use with the centralized configuration. It serves as the basis for a more inclusive model needed for the direct injection configuration. With regard to the direct injection configuration, future work includes modeling the effects of a varying volume with an added exhaust component where work is either being extracted to added to the gas inside the piston chamber. Future control design issues for the direct injection configuration will by necessity include the development of a control law capable of switching the propellant valve while also proportionally controlling an exhaust valve.

References