ABSTRACT

A dynamic model of a free liquid piston that exploits piston geometry to produce a high inertance was developed for use in a free piston engine compressor. It is shown that for the size scale targeted, advantageous piston dynamics can be achieved with a reduced piston mass compared to a rigid piston design. It is also shown that the viscous losses associated with the liquid piston are negligible for the application discussed. The slow dynamics achieved by the liquid piston also allow for reduced valve sizes for the compressor, creating a more energy-dense device on a systems level. Other advantages gained by this design compared to prior work are discussed, including the elimination of a separated combustion chamber, smaller (integrated) pump check valve, and the capability of more balanced operation for a single-piston compressor. A dynamic model of the proposed high inertance liquid piston is presented and simulation results are discussed.

INTRODUCTION

Energetic limitations have long plagued the development of untethered human-scale robotic systems. Typically, systems are actuated by DC servo motors powered by NIMH or Li-ion batteries. Given the low energy density of state-of-the-art rechargeable batteries, operational times of these systems in the 100W range are restrictive [1]. A second and related concern for small-scale untethered applications is the relatively low power density of electromagnetic actuators. One approach to address problems of low energy density batteries and low power density actuators is to avoid the electromechanical domain and utilize pneumatic actuation. Power supplies for pneumatic systems also need to be addressed, since portable tanks that could carry enough air for a useful operation time would be size and weight prohibitive. Traditional air compressors are also too heavy to be used effectively as on-board air supplies for the scale of interest.

Goldfarb, et al [2] have shown the viability of using catalytic decomposition of hydrogen peroxide to produce hot gas to directly drive pneumatic actuators.

Riofrio, et al [3] designed a free piston compressor specifically for a lightweight untethered air supply for actuation of traditional pneumatic cylinders and valves, using hydrocarbon fuels as an energy source. The piston, acting as an inertial load, converts the thermal energy on the combustion side of the engine into kinetic energy, which in turn compresses air into a reservoir to be used for a pneumatic actuation system.

A second device by Riofrio et al [4], a free liquid-piston compressor (FLPC), was designed using a liquid trapped between elastomeric diaphragms as a piston. The liquid piston eliminated the blow-by and friction losses of standard piston configurations [4]. This device incorporated a combustion chamber that was separated from an expansion chamber. Once the high pressure combustion gasses were vented into the expansion chamber, PV work was converted to inertial kinetic energy of the piston. The separated combustion chamber kept air/fuel injection pressure high prior to ignition for efficient combustion, and allowed for air/fuel injection that was decoupled from power and return strokes of the engine cycle. The separated combustion chamber and the high pressure injection of both air and fuel allowed for an engine devoid of intake and compression strokes.

This work continues investigation of a free liquid piston compressor power source, focusing on exploiting the geometry of the liquid piston to create a high inertance, which advantageously slows the dynamics of the system without the penalty of adding more mass. Modeling and simulation of the high inertance free liquid piston is conducted, and implications on the performance of a free-piston engine compressor utilizing this liquid piston concept are discussed.
LIQUID PISTON INERTANCE

Consider a fluid filled pipe approximated with three regions of effective lengths \( L_1, L_2, \) and \( L_3 \), with distinct cross sectional areas and liquid masses as shown in Fig. 1. This configuration represents the liquid chamber between two moving seals, such as solid pistons or elastomeric diaphragms. An external force acting on either of the moving seals will cause fluid flow through the chamber.

![Figure 1. Three regions of a generic high inertance liquid piston contained by diaphragms or sliding pistons on both ends.](image)

The power flowing through the fluid filled pipe of Fig. 1, in response to the left and right boundaries moving, can be represented as the time derivative of the kinetic energies in each of the flow regions:

\[
PQ = \frac{d}{dt} \left[ \frac{1}{2} m_1 \left( \frac{Q}{A_1} \right)^2 + \frac{1}{2} m_2 \left( \frac{Q}{A_2} \right)^2 + \frac{1}{2} m_3 \left( \frac{Q}{A_3} \right)^2 \right]
\]  

(1)

where \( P \) is the pressure difference across the left and right moving boundaries, and \( Q \) is the volumetric flow rate of the piston fluid.

Substituting \( m_i = \rho L_i A_i \) for the masses of liquid in each flow region, differentiating, substituting \( \dot{L}_1 = -\frac{Q}{A_1}, \dot{L}_2 = 0 \) and \( \dot{L}_3 = \frac{Q}{A_3} \), and solving for pressure, we obtain Eq. 2:

\[
P = \left[ \frac{\rho L_1}{A_1} + \frac{\rho L_2}{A_2} + \frac{\rho L_3}{A_3} \right] \dot{Q} + \rho \left[ \frac{1}{A_1} - \frac{1}{A_3} \right] Q^2
\]  

(2)

It is interesting to note that for steady-state flow, i.e. \( \dot{Q} = 0 \), Eq. 2 simplifies to

\[
P = \frac{\rho}{2} \left[ \frac{1}{A_1^2} - \frac{1}{A_3^2} \right] Q^2
\]  

(3)

Assuming that \( A_1 \gg A_3 \) and solving for \( Q \), the standard hydraulic flow equation is obtained:

\[
Q = A_1 \sqrt{\frac{2}{\rho} \sqrt{\Delta P}}
\]  

(4)

It follows that the relationship between pressure and flow rate of Eq. 2 consists of the steady-state term (Eq. 3) due to the area changes between regions, and the dynamic term relating \( P \) and \( \dot{Q} \) through the inertance of the fluid slug. The inertance, \( I \), of the liquid piston is therefore:

\[
I = \left[ \frac{\rho L_1}{A_1} + \frac{\rho L_2}{A_2} + \frac{\rho L_3}{A_3} \right]
\]  

(5)

For convenience, the steady-state term of Eq. 3 is denoted \( A_e \):

\[
A_e = \frac{\rho}{2} \left[ \frac{1}{A_1^2} - \frac{1}{A_3^2} \right]
\]  

(6)

It can be seen that the second region of this configuration, termed the high inertance (HI) section, can be given a large length to area ratio \( L_i / A_i \) to dominate the inertance in Eq. 2. Thus, the fluid’s dynamics can be made slower through piston geometry rather than by the mass of the liquid alone.

**Design Implications of Slower Piston Dynamics:**

The FLPC described by Riofrio, et al [4], showed the viability of using a free piston compressor for use as a portable pneumatic power source for human scale robotics. The design of the FLPC does, however, have some issues that lead to either compromised performance or compromised efficiency for a compact device. The high inertance free liquid piston presented here, within the context of being incorporated into an engine-compressor (HI-FLPC) has the ability of solving three such significant issues. These issues are: 1) valve sizing, 2) complications associated with the separated combustion chamber, and 3) a balanced engine.

**Valve Sizing.** In a free-piston engine compressor, the check valve responsible for pump flow between the pump chamber and the reservoir has to be large enough to prevent a pressure rise in the pump chamber appreciably above the reservoir pressure (valve needs a large flow area), yet fast enough to prevent a backflow from the reservoir to the pump chamber once the pressure difference reverses at the end of the stroke (valve needs to close quickly). The speed of the piston will require a certain mass flow rate, which can be achieved by either 1) a large flow orifice area and a small pressure difference across the valve, or 2) a small orifice area and a large pressure difference. The extreme of case 1 will cause a backflow through the valve due to the fact that a larger passive valve is slower to close. The extreme of case 2 will cause the piston to bounce against the pressure in the pump.
chamber before full pumping occurs. A solution that reduces the severity of this tradeoff is to reduce the required mass flow rate by slowing the overall piston motion while maintaining the same piston kinetic energy. Incorporating a liquid piston with high iner- tance will address this issue by achieving slower dynamics without the mass penalty of more fluid, which will allow for a smaller pump check valve, and thus a more compact and lighter weight device.

**Separated Combustion Chamber.** The separated combustion chamber of the FLPC [4] was necessary for holding injection pressure of the air and fuel before ignition. Flow across the combustion valve after ignition caused inefficiencies in the conversion of thermal energy to piston kinetic energy. The fixed-volume separated combustion chamber also led to scavenging problems due to the relatively large volume of spent fuel products that cannot evacuate. A piston with dynamics slow enough could allow air/fuel injection and ignition to occur before significant piston motion. This would allow a high pre-combustion pressure (equivalent to a high compression ratio in traditional 4 stroke engines) without the need for the sealed-off volume of the separated combustion chamber. The elimination of the separated combustion chamber results in a significant decrease in dead volume where spent fuel could cause scavenging problems. The lack of a separated combustion chamber also eliminates flow losses across the combustion valve.

**Engine Balance.** The linear configuration of the FLPC device is not self-balanced and could affect performance of an overall pneumatic system. The long, small-diameter inertance section of the HI-FLPC piston can be configured such that the combustion and compression chambers oppose each other, giving the device a more balanced operation. Coiling of the inertance tube around the compressor will also help retain a compact design, although care must be taken not to add significant pressure losses due to the configuration of the inertance section of the piston.

**Dynamic Model of HI-FLPC**

Whereas the previous FLPC developed dynamics for the piston using a rigid body mass-spring approach, this work will utilize Eq. 2 as the foundation of the piston model in the free-piston engine compressor. The inertial and steady-flow components can be summarized as

\[
\Delta P = I\dot{Q} + A_cQ^2 + RQ
\]  

(8)

A preliminary simulation of a liquid piston was conducted to investigate the magnitude of viscous losses. Equation (8) was implemented in MATLAB, with the resistance term of Eq. (9) derived from the Darcy-Weisbach equation:

\[
R = \frac{8\rho}{\pi^2d_2}Q \cdot f \frac{L_2}{d_2}
\]  

(9)

Where \( \rho \) is the density of the fluid (water), and \( L_2 \) and \( d_2 \) are the diameter and length of the high inertance tube, respectively. The friction factor \( f \) was taken from the Moody Chart to be 0.025, based on drawn tubing and a conservative Reynolds number calculated at the average velocity of fluid in the tube for a 40 millisecond pump stroke obtained from a dynamic simulation without losses for our scale of interest. This conservative calculation for \( f \) will help offset possible additional pressure losses associated with the oscillatory nature of the piston flow, which is not accounted for in the model. Given the chosen area ratios between region 2 to region 3 of the liquid piston, pressure losses due to the expansion of flow (Carnot-Borda losses) were estimated to be less than 5 kPa at simulated fluid velocities, and were therefore neglected.

Other physical piston parameters were chosen appropriately for the size and power range of the HI-FLPC. Most critically, the high inertance tube of the piston was modeled as 147.3 cm long \((L_2)\) with a cross-sectional area \( A_2 \) of 1.98 cm. The initial pressure differential acting on the piston was taken to be \(2.05 \times 10^6\) Pa, similar to pressures achieved from combustion in the FLPC [4]. The pressure-volume profile was similar to that used in [4]. If stiffness effects of the diaphragms are ignored, the average fluid velocity will be artificially high and therefore the viscous drag will be an upper bound.

Figure 2 shows results for this simulation. The total kinetic energy of the piston is seen to be more than one order of magnitude greater than the losses due to viscous effects. It is concluded that for the length and cross-sectional area used for the inertance tube in this simulation viscous losses are not significant in relation to the kinetic energy carried by the piston.
**Diaphragm Stiffness.** For the HI-FLPC design, regions 1 and 3 of the piston chamber (Fig. 1) are sealed by and mated to the combustion and pump chambers, respectively, by elastomeric diaphragms similar to those used in the FLPC, shown in Fig. 3. Mass of the diaphragms was neglected along with any damping effects, so that the diaphragms were modeled as stiffness only. Since the dynamic model of the piston relates pressure and volumetric flow rate, the spring stiffness derived relates pressure differential across the piston to volume displaced by the combustion chamber. Using a diaphragm 8.55 mm thick with a 38.1 mm diameter, characterization of this stiffness was achieved experimentally, with the resultant curve:

\[
\Delta P = 0.0043(V_c - V_o)^{3.44} \quad (10)
\]

Adding this stiffness effect to Eq. 8 yields

\[
\Delta P = I\dot{Q} + A_o\dot{Q}^2 + R\dot{Q} + K(V_c - V_o)^{2.44} \quad (11)
\]

where \((V_c - V_o)\) is the volume sweep of the combustion chamber.

**Total Dynamic Equation for the Liquid Piston.** Combining the expressions for liquid inertance, viscous fluid losses, and diaphragm stiffness, and expressing the flow rate \(\dot{Q}\) as the rate of change of combustion chamber volume, \(\dot{V}_c\), the following differential equation is obtained for the piston dynamics:

\[
\dot{V}_c = \frac{1}{I}[\Delta P - R\dot{V}_c - A_o\dot{V}_c^2 - K(V_c - V_o)^{2.44}] \quad (12)
\]

**SIMULATION**

Computer simulation of the High-Inertance FLPC model was carried out. Control volumes for the combustion chamber and pump chamber were modeled, with the high inertance liquid piston dynamics coupling their behavior, as shown in Fig. 4. A control volume representing the reservoir was also incorporated. Valve dynamics and mass flows for the air/fuel intake and exhaust valves of the combustion chamber were modeled, as well as the breathe-in and pump valve for the pump chamber.

The dynamic model presented by Yong, et al, in [5] was used as a basis for the modeled components other than the piston dynamics, including combustion rate dynamics. The following represents the power balance for each \(j^{th}\) control volume (specifically, the combustion chamber, the pump chamber, and the reservoir):
\[ U_j = H_j + \dot{Q}_j - \dot{W}_j \]  

(13)

where \( \dot{U} \) is the rate of change of internal energy, \( \dot{H} \) is the net enthalpy flowing into the CV, \( \dot{Q} \) is the rate of heat transfer into the CV and \( \dot{W} \) is the work rate of the gas in the control volume. Each term in Eq. (13) can be expanded as follows:

\[ \dot{H}_j = \sum_k \dot{m}_{j,k} \left( c_{p_{j,\text{in}}} \right) \left( T_{i_{j,\text{in}}} \right) \]  

(14)

and

\[ \dot{W}_j = P_j V_j \]  

(15)

\[ \dot{U}_j = \dot{m}_j \left( c_v \right) T_j + m_j \left( c_v \right) T_j = \frac{1}{\gamma_j - 1} \left( P_j V_j + P_j \dot{V}_j \right) \]  

(16)

where \( \dot{m} \) is the \( k \)th mass flow rate entering or leaving each \( j \)th CV with constant-pressure specific heat \( c_{p_{j,\text{in}}} \) and temperature \( T_{i_{j,\text{in}}} \), \( P \) and \( V \) are the pressure and volume in the CV, \( c_v \) is the constant volume specific heat and \( \gamma \) is the ratio of specific heats of the gas in the CV. Equations (14-16) can be used to form the following differential equations:

\[ \dot{P}_j = \left( \gamma_j - 1 \right) \sum \dot{m}_j \left( c_{p_{j,\text{in}}} \right) T_{i_{j,\text{in}}} + \left( \gamma_j - 1 \right) \dot{Q}_j - \gamma_j P_j \dot{V}_j \]  

(17)

\[ \dot{T}_j = \sum \dot{m}_j \left( c_v \right) T_{i_{j,\text{in}}} - \left( c_v \right) T_j - \frac{P_j \dot{V}_j + \dot{Q}_j}{m_j \left( c_v \right)} \]  

(18)

The mass flow rates \( \dot{m}_j \) for the valves are determined by the following equation [6]:

\[ \dot{m}_j = \psi_j \left( P_u, P_d \right) \]  

(19)

where \( \psi_j \) is a non-dimensional discharge coefficient of the valve, \( a_j \) is the area of the valve orifice, \( P_u \) and \( P_d \) are the upstream and downstream pressures, \( T_u \) is the upstream temperature, \( \gamma_u \) is the ratio of specific heats in the upstream gas, and \( C_1, C_2, \) and \( P_{cr} \) are determined by:

\[ C_1 = \left( \frac{2}{\gamma_u \left( \gamma_u + 1 \right)} \right)^{\gamma_u / \left( \gamma_u - 1 \right)} \]  

and

\[ C_2 = \sqrt{\frac{2 \gamma_u}{\gamma_u + 1}} \]  

(20)

where \( R_u \) is the gas constant of the upstream substance.

A model of the combustion process and its influence on the pressure and temperature in the combustion chamber was taken from Yong et al [5]. All valve operation dynamics influencing each \( a_j \) were modeled as second order and tuned by experimental data from the FLPC.

**Simulation Results.**

Two simulation models were compared to illustrate the effect of the high inertance liquid piston. The first model, representing the HI-FLPC, incorporated a high inertance piston design with an inertance tube length \( L_i \) of 1.473 m, and a cross-sectional area \( A_s \) of 1.98 cm\(^2\). A second simulation with no cross-sectional area change in the liquid piston was examined. All parameters excluding piston geometry and piston mass for the two models were kept the same.

Figure 5 shows simulation results for the pressures and volumes in the combustion, pump, and reservoir chambers for the injection, combustion, and pump phases of the HI-FLPC. Note that pumping begins at approximately 40 msec when pump chamber pressure rises above reservoir pressure (about 25 msec after combustion). The reservoir pressure increases by approximately 20 kPa but is not visible on the scale of the figure.

Figure 6 shows simulation results for the simulation with no cross-sectional area change, where the piston mass was adjusted to achieve the same cycle time as the HI-FLPC. The piston mass required to achieve this similar behavior was 12.5 kg of fluid. This represents a mass 30 times that of the HI-FLPC piston mass of 0.414 kg.
Another point of interest in the HI-FLPC simulation is the injection phase (occurring between 0 and 11 msec in Fig. 4). Given an air/fuel valve orifice area of 1.54 mm², which is based on a valve proposed for implementation, injection pressure of air/fuel in the combustion chamber pressure is dynamically “held” by the piston long enough for good combustion, supporting the idea that the HI-FLPC does not require a separated combustion chamber.

CONCLUSIONS

A dynamic model of a high inertance free liquid piston was developed and presented. Previous work on a free-piston engine compressor revealed certain complications associated with the fast dynamics of the piston motion. Following from this motivation, the concept of inertance was exploited to slow the dynamics of the piston motion while concomitantly reducing the mass of the piston. It was shown that a high inertance liquid piston with a mass of 0.414 kg has the equivalent dynamic response of a 12.5 kg liquid piston of uniform cross sectional area. It was also shown that the required “inertance tube” section of the high inertance liquid piston exhibits insignificant viscous losses for the geometries considered. Finally, the dynamic response of the high inertance liquid piston resolves significant issues when incorporated into a free-piston engine compressor device. These issues are: 1) valve sizing, 2) complications associated with a separated combustion chamber, and 3) a balanced engine.

REFERENCES


