

RECURSIVELY-APPLIED SCANNING ALGORITHMS FOR INVERSE ANALYSES OF GASTROINTESTINAL BIOMAGNETIC FIELDS

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Abstract

The process of identifying dipolar sources of bioelectric current in the gastrointestinal (GI) tract is of great importance to the future of medical diagnosis. Superconducting QUantum Interference Device (SQUID) magnetometers measure the minute magnetic fields associated with electrical control activity (ECA) from GI smooth muscle. The phenomenology of abnormal ECA current propagation can offer important insights into the early identification and treatment of many diseases, including gastroparesis and ischemia. In order to identify these gastrointestinal sources of current, a biomagnetic field reconstruction program has been developed in which the abdomen is modeled using a set of dipoles that are distributed so as to produce an anatomically realistic simulation. A numerical algorithm for solving the inverse biomagnetic problem was developed, in which a novel dipole localization procedure was implemented in order to identify dipole sources of current based on magnetic field values recorded using SQUID magnetometers. This computational inverse reconstruction was tested with excellent results on simulated data sets containing 1, 3, and 190 dipoles. In all cases, dipoles were identified at correct locations, consistent with propagating GI electrical activity, and with dipole moments that oscillated with frequencies accurately corresponding to the known ECA frequencies. These results are very promising for future studies of abnormal gastrointestinal propagation phenomena, and for the development of noninvasive medical diagnosis procedures for the stomach and intestine.

Key Words: Biomagnetic fields, gastric propagation, inverse algorithms.

1. Introduction

The scientific field of noninvasive biomagnetic field measurements has witnessed considerable progress in the latest years especially in light of its applications to gastrointestinal research. In spite of this, however, a large number of theoretical and experimental problems remain

unsolved due to a number of causes. One of these is the difficulty in formulating accurate theoretical models based on which bionumerical algorithms can be designed in this area of scientific interest. As far as the biomagnetic inverse problem is concerned, many analytical and numerical methods have been proposed.

In a relatively simple model of the human stomach, the electric current of the stomach smooth muscle can be modeled through the use of one current dipole. For an unbounded homogeneous medium, the magnetic field due to this dipole can be computed according to the Biot-Savart law by making use of the formula

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \iiint_{\gamma} \mathbf{J}(\mathbf{r}') \times \frac{(\mathbf{r} - \mathbf{r}_0)}{|\mathbf{r} - \mathbf{r}_0|^3} d^3\mathbf{r}' \quad (1)$$

where μ_0 represents the magnetic permeability of free space and γ is a small volume over which integration is performed. $\mathbf{J}(\mathbf{r})$ is the current density and $(\mathbf{r} - \mathbf{r}_0)$ represents the distance between the magnetic field source and the measurement point of the field. If the assumption is made that the magnetic field \mathbf{B} is due to a dipolar point source, the current dipole has a moment \mathbf{Q} such that the current is concentrated at a unique location \mathbf{r}_0 . Following the principles of electromagnetic theory elaborated in [1], [2] and [3], the magnetic field equation can then be rewritten as

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \mathbf{Q} \times \frac{(\mathbf{r} - \mathbf{r}_0)}{|\mathbf{r} - \mathbf{r}_0|^3} \quad (2)$$

where the vector \mathbf{Q} can in fact be written as a linear combination in the form

$$\mathbf{Q} = (Q_x \ 0 \ 0)^t + (0 \ Q_y \ 0)^t + (0 \ 0 \ Q_z)^t \quad (3)$$

In the expression above, each of the individual vectors on the right forms a basis and is sometimes called an elemental dipole. Equation 2 is a useful expression because it allows for each component of \mathbf{B} to be evaluated

separately by computing the cross product of \mathbf{Q} and $(\mathbf{r} - \mathbf{r}_0)$. For example, one obtains

$$B_z = \frac{\mu_0}{4\pi} \frac{1}{|\mathbf{r} - \mathbf{r}_0|^3} [Q_x(y - y_0) - Q_y(x - x_0)] \quad (4)$$

for the component of the magnetic field in the z direction.

2. GI Simulation of Magnetic Fields

The calculation of magnetic field values based on known positions and orientations of current dipoles is called the forward biomagnetic problem, while the inverse process for determining the location and orientation of \mathbf{Q} based on recordings of \mathbf{B} is naturally known as the inverse problem [4]. Solving this problem can be very difficult because solutions are not unique and it is often difficult to identify realistic dipoles [5, 6]. The approach adopted here was to perform a least-squares approximation of biomagnetic field data simulated for the stomach and intestine using the theoretical expression for \mathbf{B} in equations (3) and (4). While the least-squares procedure itself has been used before in other situations, there are no previous studies in scientific literature for applications of this method to gastrointestinal biomagnetic fields, where the magnetic field is produced as a result of a very large number of dipoles that are located at a variety of positions in the abdominal volume.

In order to solve the inverse problem for the abdomen, biomagnetic field input was assumed to be collected by a SQUID magnetometer with 176 coils, whose measurements were simulated to be taken in a horizontal plane above the abdomen. In this model, the vector $(\mathbf{r} - \mathbf{r}_0)$ in eq. (4) thus represents the distance from the dipole location (\mathbf{r}) to the measurement point (\mathbf{r}_0) . Recordings were simulated by computing magnetic field at the locations of the coils, while the magnetic field recorded was assumed to be due to 190 current dipoles. The simulated locations of the latter were generated for the stomach and intestine at locations are shown in Figure 1, while current propagation was included into the model for three distinct areas: the stomach, duodenum, and ileum. In the stomach, 20 pairs of dipoles were simulated to be propagating at a frequency of 3 cpm while 5 duodenum dipoles with a propagation frequency of 12 cpm were also included in the model. Finally, a region of the ileum was assumed to contain 4 dipoles propagating at a frequency of 8 cpm. The rest of the dipoles in the model were randomly assigned locations within the abdomen and their propagation frequencies ranged from 7 to 12 cpm. The output of the simulation consisted of \mathbf{B} values due to the 190 dipoles and computed for all 176 coils according to the rules of magnetic field vector addition. GI electrical activity recordings were simulated for a 60 s time interval, recorded at a simulated frequency of 200 Hz and then used as input to a second program, whose purpose was to use an inverse-problem algorithm to determine the best-

fitting dipole based on forward model data, as described above.

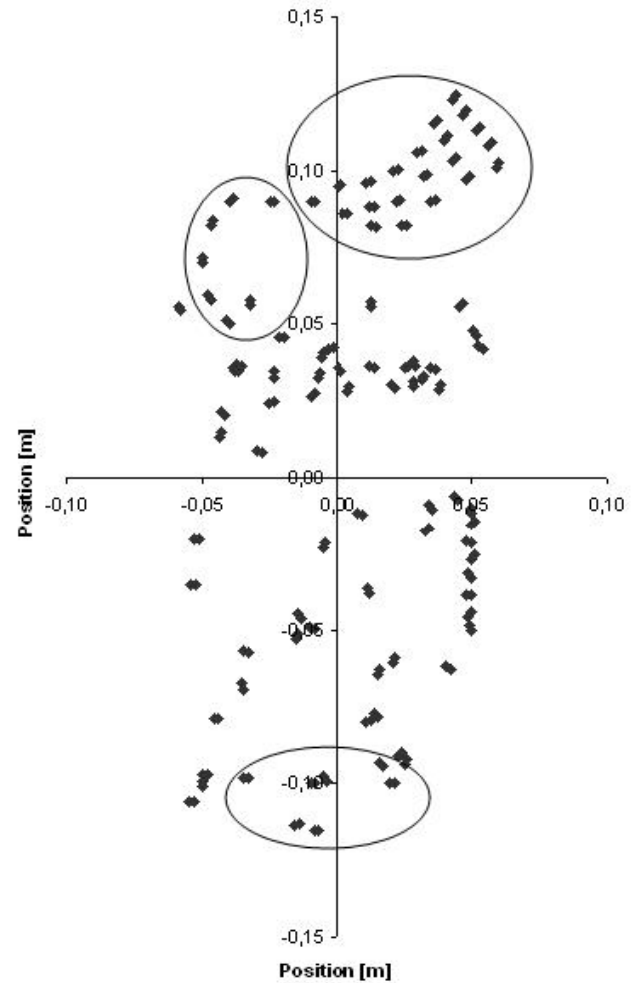


Figure 1. Spatial distribution of 190 GI dipoles for which resulting magnetic fields were simulated. The areas inside the three ellipses represent the regions for which dipole propagation was included in the forward calculations of the magnetic field. These regions roughly correspond to the stomach (upper right), duodenum (upper left) and ileum (lower center), for which propagation studies are of particular interest.

3. Dipole Localization Procedure

The computational method used for determining dipole locations was designed to make use of the assumed positions of the 176 SQUID coils described above. The perimeter determined by these in the measurement plane was divided into an $n \times n$ planar grid, where n was chosen so as to insure that the resulting grid was sufficiently dense. In order to simulate realistic experiment conditions, the surface of interest for identifying dipoles was reduced to the grid nodes encompassed by an ellipse bounded by the grid. This was performed primarily because marginal nodes usually correspond to SQUID channels (coils) whose recordings are most dramatically affected by noise.

The algorithm for determining dipole locations and orientations based on the arrangement above was designed with the purpose of reducing the high amount of computational time that is typically required for solving the inverse problem. Some inverse problem methods make use of an algorithm in which values of \mathbf{B} are computed for a randomly chosen set of data points in the grid, after which the best fitting location in a least squares sense is found based on the resulting calculation. The algorithm presented here, however, makes use of a different and novel procedure for performing the least squares approximation, resulting in considerable reduction of required computational time. Instead of computing magnetic field values at randomly selected locations, the grid section described by the ellipse mentioned above was divided into eight regions of equal area that were symmetric with respect to both axes of the measurement plane.

The coordinates of the centroids for each region were computed analytically using the methods of mathematical analysis, although numerical values can be obtained for these if the situation demands it. In the next step, a least-squares approximation of dipole orientations was calculated for dipoles located at each region centroid. The best-fitting dipole was then selected based on a minimization of the χ^2 goodness of fit value for the approximations performed using each centroid. The following steps of the algorithm follow the same principles as above because they are based on the principle of recursion. The grid region with the best-fitting dipole location is thus divided again into two sub-regions of equal area, for which centroid positions are computed and the process of determining the best-fitting dipole is repeated. The calculation continues with successive divisions of the grid sub-regions until the search area is reduced to a single grid node, which is then accepted as the reconstructed dipole location.

The process described above was applied to the forward model input data generated by the GI electrical activity simulation. Because the stomach and each portion of the intestine are both known to have different frequencies of current propagation, a Butterworth filter of second order was applied in order to analyze only the input data containing one frequency range at a time. The filter and inverse analysis were applied for frequency limits ranging from 7-8 cpm to 12-13 cpm. Every time frame for which magnetic field data existed was analyzed in order to determine the best fitting dipole location and orientation using the algorithm described above. Due to the fact that filtering had been applied to the input data, the presence of a single current dipole was assumed to be sufficient in order to account for the resulting magnetic field. The dipole locations determined by the analysis were then plotted against the true locations specified by the forward-model solutions. The results of this analysis are shown in Figure 2.

4. Results and Discussion

The results of applying the inverse-solution scanning algorithm to the forward model simulation are shown in Figure 2. The most important criterion that was found to be appropriate for identifying propagating bioelectric dipoles is the requirement that the best-fitting ones be concentrated in a specific region of the reconstruction grid for a consistent period of time.

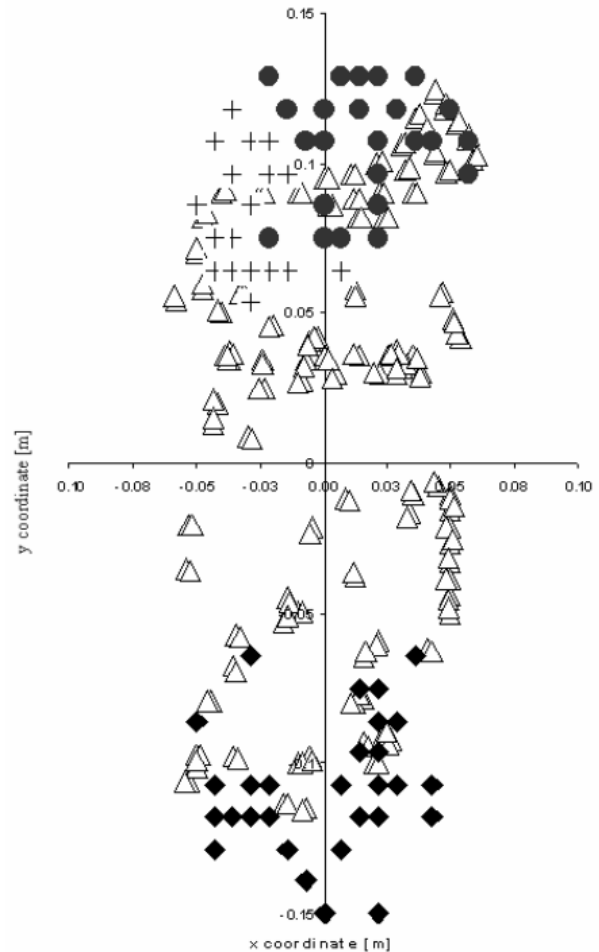


Figure 2. Reconstruction results using the inverse algorithm for the forward model simulation. Forward model dipoles are shown in white triangles while reconstructed solutions are represented by three different symbols: stomach dipoles are represented by black circles, duodenum dipoles by small crosses and ileum dipoles by black rhombi. The three areas where current propagation was identified are identical to those in Figure 1, i.e. the stomach (upper right), the duodenum (mid-left), and ileum (lower center).

Due to the fact that non-propagating dipoles may also be identified by the algorithm for each frequency analyzed, this condition is essential because the approximate location of identified dipoles was found to be preserved and restricted to specific areas only in the case where propagating sources of current existed. This finding is made obvious in Figure 2, where regions of propagation are correctly identified with the areas of dipolar clustering. The latter phenomenon did not occur,

however, for the forward model data sets that had been filtered to isolate dipoles with frequencies of 9-10 and 10-11 cpm. This is again consistent with the information provided by the forward model simulation because no propagating dipoles had been simulated with those particular frequencies. This is shown in Figure 2, where it becomes clear that the dipoles identified by the inverse algorithm do not occur at specific positions on the grid, but rather at random locations, which indicates that accurate identification of periodicity and locations is correctly provided by the inverse algorithm developed. This conclusion is strengthened by the results of the same analysis when applied to the filtered data for the frequency ranges of 11-12 and 12-13 cpm. For the latter two cases, the forward model had been programmed to simulate propagation in the region of the duodenum,

which is correctly identified in Figure 2 as a portion of the abdomen containing propagating dipoles. In the stomach, periodicity was simulated for a group of 20 dipoles with frequencies of 3 cpm. The reconstructed dipoles obtained in this case are shown in the upper region of Figure 2, at and around the location of the forward model dipoles.

The periodicities of the propagating dipoles given by the inverse solution were verified by plotting the values of their x and y coordinates as functions of time. These plots are shown separately for the stomach, duodenum and ileum signals in Figure 3 for selected time intervals of the simulation. The pattern exhibited by the movements of the three dipoles are in agreement with the specifications of the forward model.

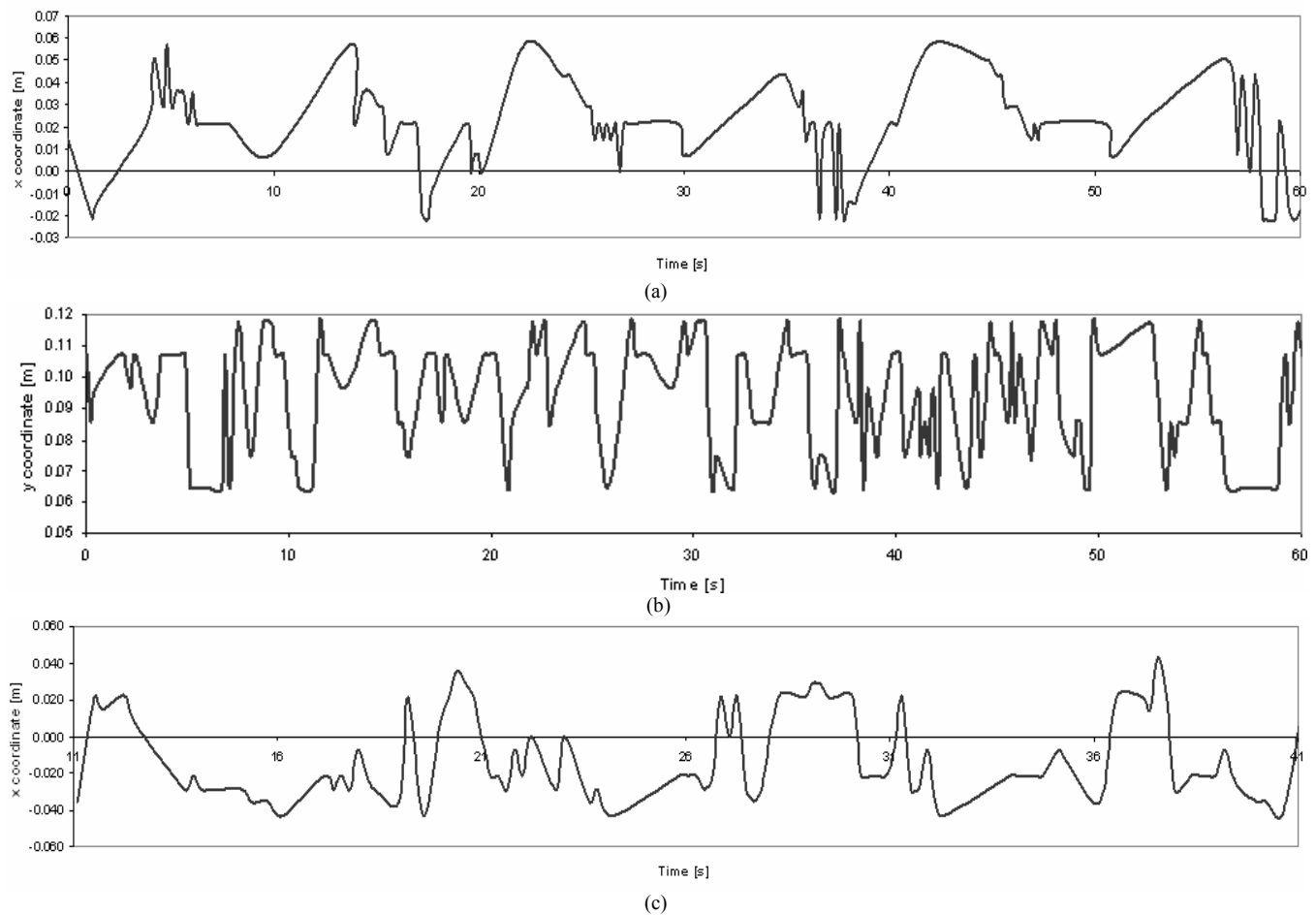


Figure 3. Periodicity of current propagation as reconstructed by the inverse problem dipole localization algorithm. In (a), the evolution of the x coordinate for the gastric dipole is shown throughout a 60 s period. In (b), propagation in the y direction is presented for the duodenum, where a 12 cpm pattern is expected. The oscillation period for the x component of the ileum dipole is presented in (c) for a 30 s interval in which four oscillations are observed according to the inverse solution. Propagation is identified and preserved in all three cases of the reconstruction presented above in a manner that is consistent with forward model specifications.

Conclusion and Future Research

The challenge of solving the inverse biomagnetic problem for a large number of current dipoles with irregular spatial distributions is a very complex and time-consuming task.

This is especially due to the difficulty of manipulating large numbers of multidimensional matrices, which is necessary for program operations. Moreover, inverseproblem solutions are usually quite sensitive to noise and restrictive filtering must therefore be applied very carefully in order to isolate the signal of interest

without destroying the relevant characteristics of the magnetic field recordings.

The forward model simulation and inverse algorithm described in this article were both developed with full knowledge of these important factors, which are crucial for a successful and accurate analysis of any set of experimental biomagnetic data. The practical applications of our inverse algorithm are manifold. A thorough study of abnormal current propagation in the GI tract is under way and we expect to obtain conclusive information concerning the physiological mechanisms of the stomach and intestine that could provide helpful information for the diagnosis of ischemia and gastroparesis.

Acknowledgements

The authors wish to thank reviewers for their insightful remarks and comments concerning the manuscript. The Veterans' Affairs Research Service and the Living State Physics Laboratories at Vanderbilt University provided funding for this research project through NIH Grant R01 DK 58697-01A. Financial support for travel expenses were provided by the College of Natural and Applied Sciences at Lipscomb University.

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