Patent Breadth in an International Setting[1]

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ABSTRACT: This paper shows how patent breadth can vary across countries (in asymmetric equilibrium) even if countries are identical. Our two-country model has governments setting patent breadths and firms competing to develop and produce a new product. If a firm does not innovate then it is allowed to produce an imitation providing this does not infringe on a patent. It has long been recognized that patent setting varies between North and South to reflect levels of development. The present paper addresses this issue, but also the more puzzling question of why countries of similar levels of development should set patents differently.

KEYWORDS: Asymmetric equilibrium, innovation, patent breadth, R&D, strategic interaction.

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1. Introduction

There is a significant amount of variation across countries in the protection of intellectual property rights (IPRs). The inclusion of the Trade-Related Aspects of Intellectual Property Rights (TRIPs) Agreement into the core set of agreements that defines the World Trade Organization (WTO) prompted a great deal of discussion about this. Not surprisingly, the greatest degree of variation occurs between countries in the North and the South. This variation is taken to reflect differences in the extent to which economic growth in the respective regions depends on innovation. Yet there is a surprising degree of variation in IPR protection across Northern countries at a similar level of development as well (Maskus 2000). At one end of the spectrum, exhibiting a relatively high degree of IPR protection, US law embodies the doctrine of equivalents. This allows a patentee to claim infringement even though the patent has not literally been infringed, if the competing product provides essentially the same service or outcome.[4] The United Kingdom has no such doctrine. Providing a more moderate level of protection it follows the European Patent Code, which calls for a balance between literal interpretations and more liberal interpretations that use the claims as a guideline. At the other end of the spectrum, in Japan patents have historically been interpreted in an extremely narrow manner. (See Klemperer 1990 for a more detailed discussion). The existence of these differences raises the question of why there is such variation in patent breadths across apparently similar countries and whether it is desirable for international agreements to harmonize these rules.

The purpose of this paper is to put forward one possible explanation for why IPR protection may vary across similar countries. As far as we are aware, our paper is among the first to analyze the strategic incentives of governments in setting the breadth of patent protection. We examine a duopoly model, one firm in each country, in which a firm that successfully develops a new product is given a monopoly right to produce that good. The other (non-innovating) firm is assumed to learn enough from the innovator's product to be able to produce a good that imitates and hence competes with the patented product.

[4] "If two devices do the same work in substantially the same way, and accomplish substantially the same result, they are the same, even though they differ in name, form or shape."Union Paper-Bag Machine Co. v. Murphy, 97 U. S. 120, 97 U. S. 125.

O'Donoghue, Scotchmer and Thisse (1998) define the type of patent breadth that we examine as *lagging patent breadth.* The role of lagging patent breadth, which is specified in the patent protection given to the innovator, is to determine how close a substitute the imitating firm's product is allowed to be without infringing on the patent of the innovator. The narrowest possible patent will allow the imitating firm to produce a product that is a perfect substitute for the product of the innovator, whereas a very broad patent will preclude entry by the imitator because the imitation is not valued at all by consumers.[5]

The choice of patent breadth has implications both for the profitability of future innovation and for the social payoff to existing innovations. Allowing narrower patents results in more intense competition between the original innovator and the imitator, which will raise social welfare by reducing the markup on the new product and expanding the fraction of potential consumers that are served. However, the incentive to develop new products will also be reduced because of the reduced level of profits earned by the innovator when competition is more intense.

Our approach to the analysis of patent breadth draws together elements from three different literatures on intellectual property rights: Our model features strategic interaction between governments in the protection of IPRs as in Grossman and Lai (2003); the process of R&D is stochastic and firms can compete for the discovery of a new product as in the literature on patent races initiated by Loury (1979); our specification of patent breadths is similar in spirit to that of Klemperer (1990), who posits a spatial model of product differentiation in which the breadth of the patent determines how closely imitators can locate to the patented product without infringement.

We first derive the equilibrium for firms' choices of whether or not to engage in a patent race, taking as given patent breadths in the two countries. Of course, firm decisions in this regard are affected in an important way by their respective probabilities of making an innovation. Two cases are considered: a North-South model in which the Southern firm can imitate but not innovate and a North-North model in which firms are symmetric

[5] Chor and Lai (2009) is the only other paper that we are aware of to study the strategic incentives of governments to set patent breadth. In contrast to our approach, they study *leading patent breadth*, which protects against competition from products of higher quality. See the review of the literature below for further discussion. O'Donoghue et al develop a sequential model in which there is "leading breadth" and "lagging breadth". They do not explore the strategic interaction of patent-setting between national governments.

and have the same probability of success at an innovation. In the North-South model, profits of the innovating (imitating) firm are monotonically increasing (decreasing) in the breadth parameters adopted in each country.[6] Entry into the patent race requires that the worldwide level of patent protection is sufficiently high. For the North-North model, each firm must decide whether to try to innovate or simply to imitate. If both firms try to innovate then they engage in a patent race. Profits from innovation are increasing in the breadth of patent protection in each country, so the model yields a threshold locus of worldwide patent protection such that only one firm enters the patent race and a (higher) threshold locus such that both firms do so. In the North-South model, the number of firms entering the patent race is less than or equal to the socially efficient level. However, in the North-North model the number of firm entering the patent race can exceed the socially optimal level when the probability of success is high.

Correctly anticipating firms' subsequent actions, the governments are assumed to choose their patent policies to maximize national welfare.[7] For the Northern country in the North-South model, the profit-shifting effect of an increase in patent breadth dominates the loss in consumer surplus resulting from higher prices. Therefore, it is a dominant strategy for the Northern government to choose the broadest possible patent. For the Southern country whose firm is an imitator, welfare is monotonically decreasing in the breadth of the patent as long as the Northern firm chooses to undertake R&D. However, circumstances may arise in which the Northern firm cannot cover the costs of R&D even if it successfully innovates. Thus, the North-South model generates an outcome in which the North chooses the broadest protection and the South chooses the narrowest protection that is consistent with innovative effort by the Northern firm.

Equilibrium in the North-South model is highly asymmetric in the sense that the North sets the broadest possible patent while the South sets the narrowest (consistent with the Northern firm undertaking R&D). A natural question is whether an agreement to harmonize patent breadths across North and South would increase the level of world welfare. Our analysis shows that in fact harmonization reduces world welfare because of

[6]We assume national treatment of patent breadth; within a country, patent breadth applies equally to both domestic and foreign firms.

[7]Concerns over the distribution of gains to patent protection could easily be introduced to the model, although these are beyond the scope of the present paper.

the strict convexity of world welfare in patent breadth, which means that the marginal deadweight loss from increasing patent breadth declines as patent breadth is increased. Thus, world welfare is maximized at the non-cooperative Nash equilibrium in this case.

In the North-North model the incentives of a Northern government are slightly different. Since the countries and firms are symmetric, each of the firms has the same probability of being the innovator and each has the same probability of being the imitator (when the other is successful in innovation). As a result, the broadening of patent rights does not have a profit shifting effect between countries on average because the country gains from broader patent rights when it is the innovator and loses when it is the imitator.[8] For a given number of firms entering the patent race, the effects of patent breadth on consumer surplus dominate those on producer profits and so national welfare is decreasing in the breadth of the patent. There are two types of multiplicity of equilibria that arise in the North-North model. The first is that there may be a continuum of Nash equilibria that differ in the pattern of patent breadth across countries, but not in the number of firms that engage in R&D. That raises the possibility that identical countries can have different choices of patent breadth in equilibrium, but neither country will want to change its protection because its choice is essential to keeping the desired number of firms engaged in R&D. The second type of multiplicity arises when there are equilibria in which there is a patent race between two firms as well as equilibria where only one firm is engaged in R&D. In this case, international coordination on patent breadth could raise the welfare of both countries by moving to a more socially desirable level of innovative activity. Interestingly, the non-cooperative equilibrium could involve either too much or too little innovative activity, so coordination might either raise or lower patent breadth. Either way, in the North-North model harmonization does allow scope to increase efficiency.

The contribution that we obtain from analyzing the North-South model and the North-North model is to highlight the policy asymmetry between the outcomes across the two frameworks. In the North-South model there is ex ante asymmetry in the outcome of patent breadths across countries generated by the assumption that the Northern firm can innovate while the Southern firm cannot. In the North-North model there is an

[8]This result is based on the assumption that when the equilibrium is one in which only one of the firms enters the patent race, in expectation each of the two firms has an equal probability of being the innovator. We discuss this in more detail below.

ex post asymmetry in patent breadths generated by the multiplicity of equilibria even though there is ex ante symmetry regarding the probability of winning the patent race. This in turn provides an interesting explanation as to why apparently similar (developed) countries might have an incentive to set differing levels of patent breadth.

Our focus on patent breadth as the instrument of government policy in a two country model differs from previous work in international trade, which has primarily focused on the impact of extensions of patent life. Deardorff (1991) addresses the effect of extending patent protection to countries that are importers of patented products, and argues that this extension would probably reduce the welfare of the importing countries. Although the extension of patent protection is likely to have a favorable impact on the rate of innovation, the terms of trade losses to the importing countries from higher prices on patented products are likely to more than offset benefits received from higher innovation rates. This approach follows the classic approach of Nordhaus (1969) by analyzing the trade-off between the increased period of deadweight loss resulting from an increase in the life of a patent against the increased rate of innovation.

Grossman and Lai (2004) do analyze the non-cooperative equilibrium and the gains from international cooperation of a patent setting game in a steady state model with ongoing innovation. They characterize the Nash equilibrium when governments set the length of patent protection (or equivalently the strictness of enforcement), and show that the non-cooperative equilibrium will involve patent lives that are too short relative to the optimal level. Patent lives are inefficiently short because countries ignore the positive spillover from the rate of innovation on the rest of the world and because some of the benefits of increased protection will accrue to foreign innovators.

As mentioned above, Chor and Lai's (2009) paper is perhaps closest to ours in that it too considers international patent breadth, although the specific focus is different because they study leading patent breadth. In Chor and Lai's model, through the variation of patent breadth, policy-makers trade off the benefits of a higher rate of innovation against the cost of higher prices as patent holders enjoy a longer period of monopoly pricing for a longer period on average.

Our model follows a strand of the literature on IPRs that to our knowledge has not been considered in an international context. It turns out that when the payoffs to R&D are stochastic this can significantly alter outcomes relative to a situation where they are deterministic. Reinganum (1982) identifies such a situation by considering a case in which the rewards to imitation are exactly the same as the rewards to innovation. She points out that in the deterministic game, if imitation and innovation are equally rewarded, then a Nash equilibrium fails to exist with each firm preferring to imitate rather than innovate. But she shows that this is not the case in the presence of technological uncertainty. A firm cannot simply wait for its rival to innovate, since given the rivals' strategies there remains a positive probability that no rival will succeed within the planning horizon. If a firm wants any payoff, its optimal strategy is to pursue the payoff actively, rather than simply to wait for a rival to succeed in making the discovery and then reap the reward to imitation. This underlying set of incentives is at work in our model as well.

Klemperer (1990) and Gilbert and Shapiro (1990) have addressed the issue of patent breadth by examining the trade-off between the life of a patent and patent breadth; the life of the patent affects how long the patent holder receives benefits and patent breadth affects the level of payoffs received by the innovator in a given period. We assume that the measure of patent breadth determines how close the imitator's product can be to that of the innovator, but we differ in that we treat the imitators as also having a degree of market power. We also assume that the patent life is fixed, so that the only instrument the government has to influence innovation is the breadth of the patent. The exogeneity of patent life in our framework can be interpreted as resulting from the fact that the product will have a finite life during which it is in demand, so that extensions of the patent life beyond that point will be of no value to the firm.[9] Klemperer highlights an additional benefit of studying patent breadth in that patent breadth can be thought of as capturing the breadth of copyright as well. This will be especially relevant when we consider international agreements over patent breadth, since copyrights as well as patents are harmonized under the TRIPS agreement.

The paper proceeds as follows. Section 2 sets out the basic model and derives the efficient solution. Section 3 defines the patent-setting game and characterizes equilibrium

[9] In practice, the useful life of patents is less than the statutory life, as discussed by O'Donoghue et al (1998).

of the game. It is in this section that the scope for international coordination over patent breadth is considered. Conclusions are drawn in Section 4.

2. The Basic Model

We consider a model in which two governments, a home government and a foreign government, set the breadth of their patents on a newly developed product. Two firms, one in the home country and the other in the foreign country, compete to develop the new product then compete in production and sales. Home variables are denoted with "no-$*$" while foreign variables are denoted with a "$*$". Since we will use backwards induction to study patent setting, we will first specify the behavior of firms after which we will turn to the governments.

2.1. Product Innovation and Imitation

Firms take patent breadth as given. In order to participate in the competition to develop a new product, a firm must pay a *fixed cost of R&D*, r (assumed to be the same for both firms), which results in the successful development of a new product with probability $\theta \in [0,1]$ for the home firm and $\theta^* \in [0,1]$ for the foreign firm.[10] We will assume, without loss of generality, that if the probabilities of innovation across firms differ then the home firm has a higher probability of innovation than the foreign firm; $\theta \geq \theta^*$. If both firms engage in R&D then the event that one firm innovates successfully is statistically independent of that of the other. The probability that a new product is discovered is then given by $\Phi \equiv 1 - (1-\theta)(1-\theta^*) = \theta + \theta^* - \theta\theta^*$. In the event that only one firm makes a discovery, it receives the patent on the product. If both make a discovery there is a probability of 1/2 that the patent will be received by the home firm. So if both firms engage in R&D then the probability that the home firm is awarded the patent is $\theta(1-\theta^*/2)$. We will denote the newly discovered good with n to reflect the fact that it is an innovation and we will refer to the firm that is awarded the patent as the *innovator*.

[10] For analytical simplicity we assume that r, θ and θ^* are parametric . Essentially the same model characteristics would be obtained, at the cost of greater complexity, if θ and θ^* were made to depend on r.

Once the innovating firm (denoted by n) has been awarded the patent, the other firm (denoted by m) can develop a competing product that imitates the innovator's good as long as it does not infringe on the patent. We will assume that the fixed cost of developing the imitation is arbitrarily small, and that a firm can imitate whether or not it chose to engage in R&D. If the imitator does not engage in the initial stage R&D, then it free-rides on the R&D activity of the innovator.

We follow Klemperer (1990) by treating the breadth of the patent as representing the distance the imitator's product must be located in product space from the innovator's product in order to avoid infringing on the patent. We formalize this notion by expressing the preferences of the home consumer as

$$u = e\left(q_n + q_m\right) - \frac{1}{2}\left(q_n + q_m\right)^2 - wq_m + x, \tag{2.1}$$

where q_n (q_m) is the quantity consumed of the innovator's (imitator's) product, x is the quantity of other goods consumed and w as the breadth of the home country patent. This formalization captures the idea of lagging patent breadth in the sense that the product characteristic protected by the patent must be valued less highly in the imitation than in the innovation. A broader patent will thus impose a greater cost on the imitator, since the imitation must be located further from the innovation and thus be less attractive to the consumer. If $w = 0$ then patent protection is so narrow that the imitator can introduce a product that consumers view as equally valuable without violating the patent. If $w = e$, the protection is so broad that the imitation would be worthless to the consumers if it were produced. Good x is the numeraire in the model, and each country is endowed with a sufficient quantity of x to balance trade. An analogous expression is used to represent foreign preferences.[11]

The innovator produces q_n for the home market and q_n^* for the foreign market while the imitator produces q_m for the home market and q_m^* for the foreign market. For later use, $\mathbf{q}$ will refer to the vector of all outputs; $\mathbf{q} = (q_n, q_n^*, q_m, q_m^*)$.

[11] Klemperer (1990) assumes that the imitations are provided by a competitive fringe of suppliers, and that the reduction of welfare from consuming an imitation relative to the innovation differs across consumers. We assume that the innovator and imitator are Cournot duopolists, and that the relative reduction in welfare from consuming the imitation is the same for all consumers. This simplification enables us to focus on strategic interactions over patent breadths by governments, which Klemperer does not consider.

2.2. Patent Breadth and Firm Profits

In production, each firm perceives the home market and the foreign market as being segmented. Based on this perception, each firm chooses its level of output in each market under the (Cournot) conjecture that the other firm will hold its output in that market constant. The marginal cost of producing an additional unit is c for each firm in either market. From (2.1), the inverse demand function in the home market for n and m will be $p_n = e - q_n - q_m$ and $p_m = e - q_n - q_m - w$ respectively. Although we assume that the innovator and imitator have the same marginal cost, production of the copy is less profitable in the home market when $w > 0$ because the imitator must produce a good that is less valuable to consumers. If w is so large that the imitator cannot make non-negative profits then we assume that its output and profits are zero, yielding the entire market to the innovating firm. Analogous expressions give inverse demands in the foreign market.[12]

Profits made by the innovator in the home and foreign markets are calculated in the usual way as $\pi_n = (p_n - c)\, q_n$ and $\pi_n^* = (p_n^* - c)\, q_n^*$ respectively, while profits made by the imitator at home and abroad are calculated as $\pi_m = (p_m - c)\, q_m$ and $\pi_m^* = (p_m^* - c)\, q_m^*$. Total profits for n and m are then calculated as follows:

$$\begin{aligned}\Pi_n(\mathbf{w}, \mathbf{q}) &= \pi_n(w, q_n, q_m) + \pi_n^*(w^*, q_n^*, q_m^*)\,;\\ \Pi_m(\mathbf{w}, \mathbf{q}) &= \pi_m(w, q_n, q_m) + \pi_m^*(w^*, q_n^*, q_m^*)\,.\end{aligned}$$

where $\mathbf{w} = (w, w^*)$ is the vector of patent breadths.

We are now able to evaluate the payoffs to firms of investment in R&D or otherwise. There are four cases to consider: only the home firm invests in R&D; only the foreign firm invests in R&D; both firms invest in R&D, neither firm invests in R&D. Formally, let the home firm have an action $a \in \{d, f\}$, where d represents 'development' through R&D of a new product (whether successful or not) and f stands for 'free riding' on the R&D activities of the other firm.[13] The foreign firm has analogous actions $a^* \in \{d^*, f^*\}$. Letting $\psi(a, a^*, w, w^*, \mathbf{q})$ be the *expected payoff to the home firm* when it chooses a and

[12] We simply assume that an imitator produces a good that is as close as allowed by the patent to the innovation. This could be derived as the outcome of a profit maximizing decision.

[13] Equivalently, a firm plays d in order to enter the patent race and f in order not to.

the foreign firm chooses a^*, we have

$$\begin{aligned}
\psi\left(d, f^*, \mathbf{w}, \mathbf{q}; r\right) &= \theta \Pi_n\left(\mathbf{w}, \mathbf{q}\right) - r \\
\psi\left(f, d^*, \mathbf{w}, \mathbf{q}; r\right) &= \theta^* \Pi_m\left(\mathbf{w}, \mathbf{q}\right) \\
\psi\left(d, d^*, \mathbf{w}, \mathbf{q}; r\right) &= \theta\left(1 - \theta^*/2\right) \Pi_n\left(\mathbf{w}, \mathbf{q}\right) + \theta^*\left(1 - \theta/2\right) \Pi_m\left(\mathbf{w}, \mathbf{q}\right) - r \\
\psi\left(f, f^*, \mathbf{w}, \mathbf{q}; r\right) &= 0.
\end{aligned} \tag{2.2}$$

The expected payoff to the foreign firm of its action a^*, denoted by $\psi^*\left(a, a^*, \mathbf{w}, \mathbf{q}; r\right)$, is defined in a similar way.

2.3. Patent Breadth and National Welfare

In the patent-setting game, we treat w as the choice variable of the home government and w^* as the choice variable of the foreign government. Each government chooses its patent breadth to maximize its own nation's social welfare where social welfare is, as standard, the sum of consumer surplus and profits made by the domestic firm. Next we will consider how patent breadth affects national welfare.

National welfare at home is calculated (in terms of expectations) as the sum of two components: the home firm's expected profits and the home consumers' expected surplus. The home firm's expected profits are given by $\psi\left(a, a^*, \mathbf{w}, \mathbf{q}\right)$ from (2.2). Since $p_n = p_m + w$ in any equilibrium where imitated goods are consumed, consumer surplus can be expressed as

$$S(Q) = \frac{1}{2} Q^2 \qquad \text{where } Q \equiv q_n + q_m \tag{2.3}$$

Letting $\Phi(a, a^*) = \Pr[innovation|(a, a^*)]$, expected home welfare takes the form

$$v\left(a, a^*, \mathbf{w}, \mathbf{q}; r\right) = \psi\left(a, a^*, \mathbf{w}, \mathbf{q}; r\right) + \Phi(a, a^*) S(Q). \tag{2.4}$$

This function will be taken to represent the home government's objective function in the patent-setting game. An analogous function represents the foreign government's objective function.[14]

[14] From now on, unless stated otherwise, it will be understood that when we refer to profits and consumer surplus these are calculated in terms of their expectations.

From the point of view of the representative consumer at home, R&D by the home firm is always beneficial whether the foreign firm engages in R&D or not. If the foreign firm does not engage in R&D then the domestic consumer benefit from the home firm's R&D is $[\Phi(d, f^*) - \Phi(f, f^*)]\, S(Q) = \frac{\theta}{2}Q^2$, while if the foreign firm does engage in R&D then the domestic consumer benefit from the home firm's R&D is $[\Phi(d, d^*) - \Phi(f, d^*)]\, S(Q) = (\theta - \theta\theta^*)\,(Q)^2$. Note that citizens favor the narrowest possible patent; $s\,(a, a^*, w, q_n, q_m)$ is unambiguously decreasing in w. They would like their government to allow the imitation to be as close to the innovation as possible.

2.4. Efficiency

In this section we provide a benchmark by solving for the policies that would be chosen by a planner whose objective is to maximize the sum of home and foreign welfare,

$$\Omega\,(a, a^*, \mathbf{w}, \mathbf{q}; r) \equiv v\,(a, a^*, \mathbf{w}, \mathbf{q}; r) + v^*\,(a, a^*, \mathbf{w}, \mathbf{q}; r)$$

The planner is assumed to be able to choose the actions of firms concerning R&D, and the output levels of firms in the event that R&D is successful, with lump sum taxes available to finance R&D and to make transfers between countries.

Proposition 1. *World welfare, $\Omega\,(a, a^*, \mathbf{w}, \mathbf{q}; r)$, is maximized by choosing an R&D policy such that*

(a) $a = d, a^* = d^*$ *for* $r \leq \overline{r}_2 \equiv (1-\theta)\,\theta^*\,(e-c)^2$.

(b) $a = d, a^* = f^*$ *for* $r \in (\overline{r}_2, \overline{r}_1]$, *where* $\overline{r}_1 \equiv \theta\,(e-c)^2$

(c) $a = f, a^* = f^*$ *for* $r > \overline{r}_1$.

In the event of a successful innovation, any set of outputs satisfying $p_n = p_m = c$ in home and foreign markets with $w = w^* = 0$ will yield the maximum surplus.

Since the planner has access to lump sum taxes, distortions in output markets (either by choosing a price exceeding marginal cost or choosing $w, w^* > 0$ when the imitator is producing) are not needed to finance R&D. Therefore, marginal cost pricing is optimal and any allocation of output between the two firms that yields the perfectly competitive output level in each market will yield a consumer surplus of $\frac{1}{2}\,(e-c)^2$ in each country.

Expected world surplus is $\Omega(d, f^*, \mathbf{w}, \mathbf{q}; r) = \theta(e-c)^2 - r$ when only the home firm innovates and $\Omega(d, d^*, \mathbf{w}, \mathbf{q}; r) = (\theta + \theta^* - \theta\theta^*)(e-c)^2 - 2r$ when both innovate. The optimal R&D policy then follows immediately from a comparison of these payoffs.

3. The Patent-Setting Game

In this section we examine the outcome of the patent-setting game. We model the patent-setting game as having three stages. In the first stage, the home and foreign governments, simultaneously and without communicating, set w and w^* respectively. In the second stage, the home and foreign firms choose their R&D actions, a and a^*. In the third stage, the firms compete as Cournot duopolists if at least one of the firms has been successful in R&D. If only one of the firms is successful, then the unsuccessful firm competes as the imitator. If both firms are successful in R&D, we assume that each firm has an equal probability of obtaining the patent and competing as the innovator (with the loser competing as the imitator). We refer to this whole process as the *patent-setting game.* Using backwards induction, we can solve for an equilibrium in patent breadths, actions and quantities.

3.1. Production

We begin by solving for the equilibrium in the production game in the home country market, with the identity of the firms as innovator and imitator already having been determined by the outcome of the previous stage. Since firms are symmetric in terms of production cost, the equilibrium does not depend on the identity of the innovator. The equilibrium in the foreign market will be symmetric The breadth of the respective patents, $\mathbf{w}$, is known to firms.

For given patent breadth, w, a production level by the innovator is a *best response quantity* against a production level q_m when it maximizes $\pi_n(w, q_n, q_m)$. Firm n's *best response function* is written as $R_n(q_m; w)$ and firm m's best response function is written as $R_m(q_n; w)$. A *Nash equilibrium in quantities* is a pair $(\hat{q}_n, \hat{q}_m)$ for which $\hat{q}_n$ is a best response to $\hat{q}_m$ and vice versa. We are now able to obtain the following result, which gives Nash equilibrium quantities and profit levels for the home market.

Proposition 2. *For $w \leq w_{\max} = (e-c)/2$, the Nash equilibrium output and price levels are given by*:

$$\begin{aligned}\hat{q}_n(w) &= \frac{e-c+w}{3};\ \hat{q}_m(w) = \frac{e-c-2w}{3}; \hat{Q}(w) = \frac{2(e-c)-w}{3} \\ \hat{p}_n(w) &= \frac{ew+(w+1)c}{3};\ \hat{p}_m(w) = \hat{p}_n - w.\end{aligned}$$

For $w > w_{\max}$, output levels are:

$$\hat{Q} = \hat{q}_n = \frac{e-c}{2};\ \hat{q}_m = 0.$$

Profits in equilibrium are given by:

$$\hat{\pi}_i(w) = (\hat{q}_i(w))^2 \text{ for } i \in \{n, m\}.$$

Proof: See appendix.

For $0 \leq w < w_{\max}$, the best-response functions are illustrated in Figure 1. From Proposition 2 and from the figure we see that an increase in patent breadth has the effect of increasing $\hat{q}_n$ and decreasing $\hat{q}_m$. These effects on quantities are reflected in turn in an increase of $\hat{\pi}_n$ and decrease of $\hat{\pi}_m$. This stands to reason. As the patent is broadened, the imitator is forced to produce a product that is less valuable to consumers. This induces consumers to increase their demand for the innovation at the expense of the imitation, with reciprocal impact on quantities produced and profits. The overall effect on consumer welfare of an increase in w is negative, as reflected by the decrease in $\hat{Q}$. Characterization of the effect of changes in w^* is analogous.

3.2. Competition for the Patent

We will now examine the conditions under which, in equilibrium, one or both firms enter the patent race. Our goal is to characterize the industry structure. We will achieve this by studying the game of actions (still taking w and w^* as given). Note from Proposition 2 that, for $w \geq w_{\max}$, the innovator has the entire home market and the imitator does not produce; moreover, quantities produced are invariant over this range of policies. Therefore, without loss of generality, we may restrict attention to $(w, w^*) \in W \equiv [0, w_{\max}] \times [0, w_{\max}]$.

In order to simplify the discussion, we will focus on two extreme cases concerning the success probability of the foreign firm in innovating. In the first case, which we will refer to as the *North-South model*, we assume $\theta^* = 0$; the foreign firm can be an imitator but not an innovator. The second one, which we will refer to as the *North-North model*, is one in which the home and foreign firms have equal probabilities of success in innovation, $\theta^* = \theta$, and are equally adept at imitating if the other firm receives the patent.

3.2.1. The North-South Model

In the North-South model, the foreign firm never chooses to innovate since it has zero probability of success. Since $\psi(f, f^*, \mathbf{w}; r) = 0$, the optimal action of the home firm is to undertake R&D if and only if $\psi(d, f^*, \mathbf{w}; r) \geq 0$.[15] Recall that $\psi(d, f^*, \mathbf{w}; r)$ is increasing in both w and w^* on the interior of W, because the broadening of patent rights in either market favors the innovator at the expense of the imitator. Figure 2 illustrates the $\psi(d, f^*, \mathbf{w}; r) = 0$ locus, which will be negatively sloped and concave to the origin due to the properties of the profit functions established in Proposition 2. We refer to this locus as the *just profitable protection (jp) locus*, because firms will enter the patent race for any values of $\mathbf{w}$ that lie on or above that locus.

Since $\psi(d, f^*, \mathbf{w}; 0) > 0$ and ψ is decreasing in r, there will be a unique threshold value of r, denoted by $\rho_1(\mathbf{w})$, such that $\psi(d, f^*, \mathbf{w}; \rho_1) = 0$ for any $\mathbf{w} \in W$. Assuming that the firm chooses to engage in R&D if $\psi(d, f^*, \mathbf{w}; r) = 0$, it follows that the home firm will engage in R&D iff $r \leq \rho_1(\mathbf{w})$. This threshold value $\rho_1(\mathbf{w})$ must be increasing in w and w^* because broader protection makes entry more attractive. Using the payoff functions of Proposition 2, we can determine how the action of the home firm at stage 2 varies with government policy and the technology parameters.

[15] In the North-South model we refer to the Northern firm as 'undertake R&D' rather than 'entering the patent race' since there is no other firm to race against.

Proposition 3. *Assume the North-South model.*

(a) *If* $r > \rho_1(w_{\max}, w_{\max}) = \frac{\theta}{2}(e-c)^2$ *then in equilibrium the home firm does not undertake R&D for any feasible choices of* w *and* w^**.*

(b) *If* $\rho_1(0,0) = \frac{2\theta}{9}(e-c)^2 \geq r$ *then in equilibrium the home firm undertakes R&D for all feasible values of* w *and* w^**.*

(c) *If* $\rho_1(w_{\max}, w_{\max}) \geq r \geq \rho_1(0,0)$ *then there exists a non-empty set of patent protect* $\{\mathbf{w}\}$ *for which, in equilibrium, the home firm undertakes R&D if and only if* $\mathbf{w}$ *lies on or above the jp-locus.*

Note that for the value $r = \frac{13\theta}{36}(e-c)^2$ illustrated in Figure 2, the jp-locus goes through the points $(w, w^*) = (0, w_{\max})$ (as well as $(w, w^*) = (0, w_{\max}1))$. At this level of r, providing that the home government sets maximum patent breadth $w = w_{\max}$, it is profitable for the home firm to undertake R&D whatever value the foreign government chooses for w^*. It follows that, for $r \leq \frac{13\theta}{36}(e-c)^2$, the home government can ensure unilaterally that it is profitable for the home firm to undertake R&D. On the other hand, for $r > \frac{13\theta}{36}(e-c)^2$, the question of whether R&D by the home firm is profitable depends on both w and w^*. For $r \in (\frac{13\theta}{36}(e-c)^2, \frac{\theta}{2}(e-c)^2]$, the jp-locus would lie everywhere above the one pictured in Figure 2. In such cases, even if $w = w_{\max}$, the profitability of R&D is not assured; the foreign government must set w^* sufficiently high in order for the home firm to find it profitable to undertake R&D.

Proposition 2 also reveals that patent protection gives the home firm less than the socially optimal incentive to undertake R&D. To see this, observe that $\rho_1(w_{\max}, w_{\max}) = \frac{1}{2}\overline{r}_1 = \frac{\theta}{2}(e-c)^2$. This means that, for $\rho_1(w_{\max}, w_{\max}) < r \leq \overline{r}_1 = \theta(e-c)^2$, R&D will not be undertaken even when both governments set patent breadth as broad as possible, the fact that R&D would be socially valuable notwithstanding. Moreover, the level of r at which investment in R&D will cease to be profitable even though it would be socially valuable is further reduced if patent breadth is less than the maximum level in either country.

3.2.2. The North-North Model

In the North-North model, each firm has the same probability of being successful in research and obtaining the patent. This leads to the possibility that there are Nash equilibria in which both firms enter the patent race, as well as equilibria in which only one firm finds it profitable to undertake R&D. In the latter case, symmetry ensures that there be two Nash equilibria, (d, f^*) and (f, d^*). In order to provide a unique outcome for the game we will assume that at the beginning of the second stage, nature randomly selects one of the firms (with equal probability) to move first in the R&D decision. This assumption is made to provide a unique equilibrium to the game for all r values, as well as to maintain the symmetry between the home and foreign firms.[16] Since the home and foreign firms are symmetric, we can derive the best-response functions by considering the decision of the home firm only.

If the foreign firm does not enter the patent race then the best-response criteria for undertaking R&D are the same as in Proposition 3. If the foreign firm does undertake R&D but the home firm does not then the home firm earns the return of an imitator: $\psi(f, d^*, \mathbf{w}; r) \geq 0$. If the home firm chooses to enter the patent race, it earns $\psi(d, d^*, \mathbf{w}; r)$. Entry by the home firm will be a best response to entry by the foreign firm if and only if

$$\begin{aligned} \beta(\mathbf{w}; r) &\equiv \psi(d, d^*, \mathbf{w}; r) - \psi(f, d^*, \mathbf{w}; r) \\ &= \theta\left(1 - \frac{\theta}{2}\right)\Pi_n(\mathbf{w}) - \frac{\theta^2}{2}\Pi_m(\mathbf{w}) - r \geq 0. \end{aligned} \tag{3.1}$$

Equation (3.1) is increasing in w and w^* for all $\mathbf{w} \in W$, because broadening the patent makes innovation more attractive and imitation less attractive.

Since $\beta(\mathbf{w}; r) < \psi(d, f^*, \mathbf{w}; r)$, for all r and all $\mathbf{w} \in W$, the best responses of the home firm can be divided into three cases. If protection of intellectual property is sufficiently broad that $\psi(d, f^*, \mathbf{w}; r) > \beta(\mathbf{w}; r) > 0$, then it is a dominant strategy for the home firm to enter the patent race. If $\beta(\mathbf{w}; r) < 0 < \psi(d, f^*, \mathbf{w}; r)$, then the home firm enters iff the foreign firm has not entered. Finally, if $\psi(d, f^*, \mathbf{w}; r) < 0$ the home firm does not find it profitable to undertake R&D. These three regions for home best response, and the resulting implications for the Nash equilibrium, are illustrated in Figure 2. The

[16] This can be thought of as reflecting randomness in the R&D process that allows one of the firms to be slightly ahead at the point where a significant resource commitment must be made.

$\beta(\mathbf{w}; r) = 0$ locus, which will be referred to as the *both profitable protection (bp) locus*, must lie to the right of the jp-locus. For values of $\mathbf{w}$ above the bp-locus, entry is a dominant strategy for the home firm. Since the foreign firm is symmetric, the Nash equilibrium will involve a patent race between the home and foreign firms. For values of $\mathbf{w}$ between the jp- and bp-loci, patent protection is only broad enough to support one firm. Therefore, the firm that is randomly selected by nature to move first will enter the patent race, but the second mover will stay out. Finally, for $\mathbf{w}$ lying below the jp-locus neither firm will choose to enter.

It should be noted that which of these three outcomes can arise as equilibria will depend on the value of r. Defining $\rho_2(\mathbf{w})$ as the value of r that solves $\beta(\mathbf{w}; r) = 0$, it is clear from the above discussion that $\rho_2(\mathbf{w}) < \rho_1(\mathbf{w})$. We can then divide the values of r into five intervals, each differing in which of the possible equilibria that can arise.

Proposition 4. *Assume the North-North model. The relationship between R&D cost and the number of firms undertaking R&D in equilibrium is characterized as follows.*

(a) *If* $r > \rho_1(w_{\max}, w_{\max}) = \frac{\theta}{2}(e-c)^2$ *neither firm undertakes R&D for any feasible choices of* w *and* w^*.

(b) *If* $\rho_1(w_{\max}, w_{\max}) \geq r > \rho_2(w_{\max}, w_{\max}) = \theta(2-\theta)(e-c)^2/4$, *one firm undertakes R&D iff* $\mathbf{w}$ *lies above the jp-locus and no firms otherwise.*

(c) *If* $\rho_2(w_{\max}, w_{\max}) \geq r > \rho_1(0,0)$, *no firms undertake R&D iff* $\mathbf{w}$ *lies below the jp-locus, one firm undertakes R&D if and only if* $\mathbf{w}$ *lies on or above the jp-locus but below the bp-locus, and both firms enter into a patent race if and only if* $\mathbf{w}$ *lies on or above the bp-locus.*

(d) *If* $\rho_1(0,0) \geq r > \rho_2(0,0) = \frac{2\theta(1-\theta)}{9}(e-c)^2 \geq r$, *one firm undertakes R&D iff* $\mathbf{w}$ *lies below the bp-locus, and both firms enter into a patent race if and only if* $\mathbf{w}$ *lies on or above the bp-locus.*

(e) *If* $\rho_2(0,0) \geq r$ *then both firms enter into a patent race for all feasible values of* w *and* w^*.

Parts (a) and (e) cover the parameter range of r over which w and w^* do no affect the number of firms that undertake R&D; the former because no firms ever undertake R&D and the latter because they always do. For values of r in between, as illustrated in Figure 2, the value of r fixes the location of the jp-locus and the bp-locus in (w, w^*)-space and then the specific values of w and w^* determine how many firms undertake R&D. It is worth clarifying that for values of r approaching $\rho_1(w_{\max}, w_{\max})$, the jp-locus is located towards the upper bound $(w, w^*) = (w_{\max}, w_{\max})$ and there is no bp-locus. Conversely, for values of r approaching $\rho_2(0, 0)$, the bp-locus is located towards the lower bound of the parameter space $(w, w^*) = (0, 0)$ and there is no jp-locus. The critical range of r over which both the jp-locus and the bp-locus appear in Figure 2 is $r \in [\rho_1(0,0), \rho_2(w_{\max}, w_{\max})]$. (Note that $\rho_2(w_{\max}, w_{\max}) > \rho_1(0,0)$ for all $\theta \in [0,1]$.)

From Proposition 4 we can also see that, in the North-North model, patent protection can give firms more than the socially optimal incentive to undertake R&D; that is, firms may enter into a patent race when it would be socially optimal for only one firm to undertake R&D. To check this possibility, we need to work out the highest value of r at which both firms undertake R&D in equilibrium, $\rho_2(w_{\max}, w_{\max})$, which is the level of r at which R&D is profitable by both firms given the strictest level of patent protection. Comparing $\overline{\rho}_2$ with the maximum value of r at which it is socially efficient for both firms to undertake R&D, $\overline{r}_2 = (1-\theta)\theta(e-c)^2$, we have that

$$\rho_2(w_{\max}, w_{\max}) \lesseqqgtr \overline{r}_2 \text{ iff } \theta \lesseqqgtr \frac{2}{3}.$$

If $\theta > \frac{2}{3}$ then, for $w = w^* = w_{\max}$, in equilibrium firms enter into a patent race where it would be socially optimal for only one firm to undertake R&D. This contrasts markedly with the North-South model where over-investment in R&D is not a possibility. Our interpretation of this result is as follows. In the North-North model, when the probability of success is high, the social return to a patent race adds relatively little to the aggregate probability of obtaining the innovation. However, the private incentive to enter into a patent race is still high because, in the (likely) event that both firms are successful, each firm has an equal chance of getting the patent. Thus, part of the private benefit to the innovator is the possibility of obtaining the market at the other firm's expense. The possibility of undertaking too much R&D becomes less likely as the patent breadth is reduced but it does not disappear completely for $w > 0$.

3.3. Patent Breadth

Our aim now is to characterize the choice of patent breadth at home and abroad through strategic interaction between the governments. In contrast to the efficient solution in Proposition 1, we assume that the government is unable to control quantity choices of firms or to use lump sum taxes to finance R&D investments. Therefore, for $\mathbf{w} \in W$, welfare of the home government can be expressed as $\tilde{v}(\mathbf{w};r) = w\left(a(\mathbf{w}), a^*(\mathbf{w}), \mathbf{w}, \mathbf{q}(\mathbf{w}); r\right)$. A patent breadth $\hat{w} \in [0, w_{\max}]$ for the home government is a *best response against a patent breadth* w^* when it maximizes $\tilde{v}(\mathbf{w}; r)$. A *Nash equilibrium in patent breadths* is a pair $(\hat{w}, \hat{w}^*)$ where $\hat{w}$ is a best response against $\hat{w}^*$ and vice-versa. We will follow the format of the previous subsection by considering equilibrium first in the North-South model and then in the North-North model.

3.3.1. The North-South Model

The fact that only the home (Northern) firm can innovate creates an asymmetry between the welfare functions of the respective countries. For the home country, welfare is given by the sum of the innovating firm's profits in the two markets and consumer surplus at home when evaluated at the Nash equilibrium output levels,

$$\tilde{v}(\mathbf{w}, ; r) = \begin{cases} \theta\left[\Pi_n(\mathbf{w}, \hat{\mathbf{q}}(\mathbf{w})) + S(\hat{Q}(w))\right] - r. & \psi(d, f^*, \mathbf{w}, r) \geq 0 \\ 0 & \psi(d, f^*, \mathbf{w}, r) < 0 \end{cases} \tag{3.2}$$

Broader patent protection will shift profits from the foreign imitator to the home innovator, while reducing consumer surplus at home. Differentiating (3.2) with respect to w and w^* yields

$$\frac{d\tilde{v}(\mathbf{w}; r)}{dw} = \frac{w\theta}{3} \geq 0, \ \frac{d\tilde{v}(\mathbf{w}; r)}{dw^*} = \frac{2\theta(e - c + w^*)}{9} > 0 \text{ for } \psi(d, f^*, \mathbf{w}, r) \geq 0 \tag{3.3}$$

The gain in profits from making patent protection broader will dominate the loss of consumer surplus for all $w \in [0, w_{\max})$ and $\psi(d, f^*, \mathbf{w}, r) \geq 0$, so $\hat{w}(w^*) = w_{\max}$ in this region. Since welfare is independent of w for $\psi(d, f^*, \mathbf{w}, r) < 0$, setting $w = w_{\max}$ is a weakly dominant strategy for the home country. Increases in the breadth of foreign patents has a favorable spillover effect on the home country, because it raises the return from innovation.

For the foreign country, welfare is the sum of imitator profits and consumer surplus in the domestic market when worldwide protection is sufficiently broad that the home firm undertakes R&D.

$$\tilde{v}^*(\mathbf{w},;r) = \begin{array}{cc} \theta\left(\Pi_m(\mathbf{w},\hat{\mathbf{q}}(\mathbf{w})) + S\left(\hat{Q}^*(w^*)\right)\right) & if\ \psi(d,f^*,\mathbf{w},r) \geq 0 \\ 0 & if\ \psi(d,f^*,\mathbf{w},r) < 0 \end{array}$$

Differentiating with respect to w and w^* yields

$$\frac{d\tilde{v}^*(\mathbf{w};r)}{dw^*} = -\frac{\theta(2(e-c)-3w)}{3} < 0, \ \frac{d\tilde{v}^*(\mathbf{w};r)}{dw^*} = -\frac{4\theta((e-c-2w)}{9} < (0.4)$$

$$\text{for } \psi(d,f^*,\mathbf{w},r) \geq 0 \text{ and } \mathbf{w}\in W.$$

Making patent protection broader in the foreign country will transfer surplus from foreign consumers and the imitating foreign firm to the innovating home firm, which reduces foreign welfare. Increases in the home patent breadth will also reduce foreign welfare, since it reduces the profit of the imitating firm. Foreign welfare is independent of w and w^* if $\psi(d,f^*,\mathbf{w},r) < 0$, since the home firm does not undertake R&D. If $\psi(d,f^*,w,w_{\max},r) \geq 0$, then there exists an interval $J^*(w) = [j^*(w), w_{\max}] \in [0, w_{\max}]$ such that $\psi(d,f^*,w,w^*,r) \geq 0$ iff $w^* \in J^*(w)$. The best response correspondence for the foreign country can then be expressed as follows:

$$\hat{w}^*(w) = \begin{cases} j^*(w) & \text{if } \psi(d,f^*,w,w_{\max};r) \geq 0 \\ w^* \in [0,w_{\max}] & \text{if } \psi(d,f^*,w,w_{\max};r) < 0 \end{cases}$$

The foreign country will always choose the lowest value of w^* that is consistent with the home firm undertaking R&D, so $w^* = 0$ if foreign patent protection is not essential for the home firm to undertake R&D. If $\psi(d,f^*,w,w_{\max},r) < 0$, then the foreign country cannot influence the R&D decision and the foreign country is indifferent over all feasible value of w^*.

Since the home country has a weakly dominant strategy of setting $w = w_{\max}$, the pair $\{w_{\max}, \hat{w}^*(w_{\max}))\}$ will be a Nash equilibrium in which the home firm enters if $\psi(d,f^*,w_{\max}.w_{\max};r) \geq 0$[17]. If $\psi(d,f^*,w_{\max},w_{\max};r) < 0$, the home firm will not enter in the Nash equilibrium. We can use this characterization of the Nash equilibrium

[17]If we assume that the home country government does not play weakly dominated strategies, then this is the unique equilibrium. If the home country is allowed to play weakly dominated strategies, then there will also be Nash equilibria in which entry does not occur if $\psi(d,f^*,\mu_{\min},1) < 0$. For example, $(1,1)$ will be an equilibrium in this case.

to identify how the equilibrium government behavior varies with the value of r. Since Proposition 3 established that government choices of patent breadth can have no effect on firm decisions for $r > \rho_1(0,0)$, we restrict attention to $r \leq \rho_1(0,0)$.

Proposition 5. *Assume the North-South model.*

(a) *If* $\rho_1(w_{\max}, w_{\max}) \geq r > \rho_1(w_{\max}, 0) = \frac{13\theta}{36}(e-c)^2$ *then in Nash equilibrium the home government sets patent breadth* $w = w_{\max}$ *and the foreign government sets patent breadth* $w^* = j^*(w_{\max}) > 0$.

(b) *If* $\rho_1(w_{\max}, 0) \geq r$ *then in Nash equilibrium the home government sets patent breadth* $w = w_{\max}$ *and the foreign government sets patent breadth* $w^* = 0$.

Since the foreign firm never enters the patent race in the North South model, the foreign country government will only provide patent protection if its market is essential to ensure that the home firm undertakes R&D. In that case, it will provide the minimum amount of patent breadth required to make R&D by the Northern firm profitable.

The non-cooperative equilibrium is highly asymmetric with regard to patent breadth, because the South always chooses a level that is the minimum required to ensure entry of the firm. A natural question to ask is whether world welfare could be raised by the harmonization of patent breadth across countries. A movement in the direction of harmonization involves an increase in w^*, accompanied by a reduction in w to maintain $\psi(d, f^*, w, w^*; r) = 0$, which requires $dw/dw^* = -(e-c+w^*)/(e-c+w)$. Using (3.3) and (3.4), the effect of this change on world welfare is

$$\frac{d(\tilde{v}+\tilde{v}^*)}{dw^*} = \frac{5\theta\,(e-c)\,(w^*-w)}{3\,(e-c+w)}$$

An increase in w^* will reduce world welfare if $w^* < w$, which means that the harmonization of patent breadth in North and South will reduce world welfare. This yields the following result

Proposition 6. *An increase in* w^**, with a corresponding reduction in* w *to maintain zero profits, will reduce world welfare iff* $w^* < w$.

Harmonization reduces world welfare because of the strict convexity of world welfare in patent breadth, which means that the marginal deadweight loss from increasing patent breadth declines as patent breadth is increased. Thus, world welfare is maximized at the non-cooperative Nash equilibrium in this case.

3.3.2. The North-North Model

Let us return now to the situation where both firms have an equal probability of innovation; both countries are in the North. In this setting, Proposition 3 established that there are three possible outcomes: $\{d, d^*\}$ if $\psi(d, f^*, \mathbf{w}, r) > \beta(\mathbf{w}; \mathbf{r}) \geq 0$; $\{d, f^*\}$ or $\{f, d^*\}$.if $\psi(d, f^*, \mathbf{w}, r) \geq 0 > \beta(\mathbf{w}; \mathbf{r})$; and $\{f, f^*\}$.if $0 > \psi(d, f^*, \mathbf{w}, r)$. Under our assumption that $\{d, f^*\}$ and $\{f, d^*\}$ occur with equal probability when entry is profitable for only one firm, the payoff function of the home country will be given by [18]

$$\tilde{v}^{NN}(\mathbf{w}, ; r) = \begin{cases} \theta(2-\theta)V(\mathbf{w}) - r & if \quad \psi(d, f^*, \mathbf{w}, r) > \beta(w; \mathbf{r}) \geq 0 \\ \theta V(\mathbf{w}) - r/2 & if \quad \psi(d, f^*, \mathbf{w}, r) \geq 0 > \beta(\mathbf{w}; \mathbf{r}) \\ 0 & if \quad 0 > \psi(d, f^*, \mathbf{w}, r) \end{cases} \tag{3.5}$$

where $V(\mathbf{w}) = \frac{1}{2}\Pi_n(\mathbf{w}, \mathbf{q}(\mathbf{w})) + \frac{1}{2}\Pi_m(\mathbf{w}, \mathbf{q}(\mathbf{w})) + S(\mathbf{Q}(\mathbf{w}))$ is the sum of (expected) consumer and producer surplus in the event an innovation is made. The home country receives a higher surplus from an innovation when both firms enter the patent race, but must pay a higher expected R&D cost. The symmetry of the innovative ability of the firms means that each country's firm is equally likely to serve in the role of imitator and innovator in both the one firm and two firm states. Differentiating the expected surplus expression, $V(\mathbf{w})$, with respect to w yields

$$\frac{\partial V}{\partial w} = -\hat{q}_m(w) > 0 \tag{3.6}$$

Expected surplus is decreasing in w because the gains to the innovator from broader patent protection are more than offset by gains to the imitator and consumers.

In order to derive the best response function of the home government, we can define intervals $J(w^*; r) = [j(w^*; r), w_{\max}] \in [0, w_{\max}]$ and $B(w^*; r) = [b(w^*; r), w_{\max}] \in [0, w_{\max}]$ such that $\psi(d, f^*, w, w^*, r) \geq 0$ iff $w \in J(w^*; r)$ and $\beta(w, w^*, r) \geq 0$ iff $w \in B(w^*; r)$. Figure 3 illustrates an example of the home welfare function for a case

[18]The reason that, in $w(d, f^*, \boldsymbol{\mu}, \mathbf{q}; r)$, r is divided by 2 is because under the strategy profile $\{d, f^*\}$, ex ante, there is a fifty percent probability of the home firm being selected to choose first whether or not it wants to undertake R&D.

in which $J(w^*;r), B(w^*;r) \neq \emptyset$. These sets must be connected because the functions $\psi(d, f^*, w, w^*, r)$ and $\beta(w, w^*, r)$ are both increasing in w and w^*. Home welfare is decreasing on the intervals $[b(w^*;r), w_{\max}]$ and $[j(w^*;r), b(w^*;r))$ as a result of (3.6), with a discontinuity occurring where the change in w causes a change in the number of firms that invest in R&D. The maximum welfare point must be either at $b(w^*;r)$ or $j(w^*;r)$, with the former being preferred if the gain in expected surplus from having a second firm in the patent race exceeds the additional R&D cost that results. Applying this same reasoning to the remaining cases yields the following characterization of the home best response:

Lemma 1. *In the North/North model, the best response correspondence of the home country, $\hat{w}^{NN}(w^*;r)$, has the following properties*:

(a) *If $J(w^*;r), B(w^*;r) \neq \emptyset$, then $\hat{w}^{NN}(w^*;r) = b(w^*;r)$ if $\tilde{v}^{NN}(b(w^*;r), w^*;r) \geq \tilde{v}^{NN}(j(w^*;r), w^*;r)$ and $\tilde{v}^{NN}(w^*;r) = j(w^*;r)$ if $\tilde{v}^{NN}(j(w^*;r), w^*;r) > \tilde{v}^{NN}(b(w^*;r), w^*;r)$*

(b) *If $B(w^*;r) = \emptyset$ and $J(w^*;r) \neq \emptyset$, then $\hat{w}^{NN}(w^*;r) = j(w^*,r)$.*

(c) *If $B(w^*;r), J(w^*;r) = \emptyset$, then $\hat{w}^{NN}(w^*;r) = w \in [0, w_{\max}]$*

Lemma 1 shows that the best response of the home country will always be at a boundary value of patent breadth if R&D activity takes place, in the sense that it is the narrowest patent protection that will support the equilibrium number of firms engaging in R&D. If it is not possible to sustain R&D activity, then any value of w will yield a payoff of 0. The best response function of the foreign government will take the same form.

Lemma 1 can be used to characterize the Nash equilibrium in patent breadth. Proposition 4 identified 5 possible configurations of the bp- and jp-loci that can be observed, depending on the value of r. In two of those cases, (a) and (e), the R&D costs are at such extreme values that government policies have no impact on R&D decisions. Therefore, we will focus our discussion on the remaining cases. The simplest case to analyze is case (b), where the equilibrium outcome must thus be one with either 1 firm or no firms undertaking R&D. Since $B(w^*;r) = \emptyset$ for all w^* when $r \in (\rho_2(w_{\max}, w_{\max}), \rho_1(w_{\max}, w_{\max})]$, Lemma 1 establishes that the the best response functions of the home country will be to choose $j(w^*,r)$ if $J(w^*;r) \neq \emptyset$ and $w \in [0, w_{\max}]$. If the foreign government chooses w^* such that $J(w^*;r) \neq \emptyset$, then by Lemma 1b the home best response is $j(w^*;r)$. The symmetry of the home and foreign countries will ensure that this is a Nash equilibrium, since it is the maximum value of w^* consistent with $\psi(d, f^*, w, w^*, r) \geq 0$. Since this holds for any w^* such that $J(w^*; r) \neq \emptyset$, we obtain the following result:

Proposition 7. *Assume the North-North model and $r \in (\rho_2(w_{\max}, w_{\max}), \rho_1(w_{\max}, w_{\max})]$. Any (w, w^*) satisfying $\psi(d, f^*, w, w^*, r) = 0$ will be a Nash equilibrium. If governments do not play dominated strategies, these are the only Nash equilibria.*

Proposition 7 shows that in this range of r, there is a continuum of Nash equilibria in which identical countries choose different levels of patent breadth. Each government chooses its level of patent breadth to ensure that expected profits are zero, given the patent breadth set by the other country. It follows that any (w, w^*) satisfying $\psi(d, f^*, w, w^*, r) = 0$ will be a Nash equilibrium. For all of these values except the symmetric one, identical Northern countries choose different levels of patent breadth in equilibrium (and thus have different equilibrium payoffs). Although the countries set different patent breadths neither has an incentive to change its policy, given that of the other firm, because its protection level is required to sustain the desired level of R&D. Note the contrast with Proposition 5 for the North-South model, where the equilibrium is unique. In that case the home country has the incentive to provide maximum patent breadth because it knows its firm will be the innovator.

Next consider case (d) of Proposition 4, where the possible equilibria are one with a patent race involving both firms or an equilibrium with only one firm undertaking R&D. This case is associated with innovation costs of $r \in (\rho_2(0,0), \rho_1(0,0)]$, which are sufficiently low that at least one firm will find it profitable to enter regardless of the countries' choices of patent breadth (i.e. $J(w^*) = [0, w_{\max}]$ for all w^*). The range of (r, θ) values consistent with case (d) is illustrated in Figure 4a as the area between the $\rho_1(0,0)$ line and the $\rho_2(0,0)$ locus. This raises the possibility that there could be Nash equilibria in which patent protection is sufficiently broad that both firms enter as well as patent protection that is so narrow that only one firm enters. We begin by examining whether there are Nash equilibria in which patent protection is such that only one firm enters. Since $j(w^*) = 1$ for all w^* in this region, the only possible equilibrium with one firm investing in R&D is the symmetric equilibrium $(0, 0)$. This will be a Nash equilibrium if it is not profitable for one country to gain by broadening its patent protection to induce a patent race among the two firms. Lemma 1b establishes that a sufficient condition for $(0,0)$ to be an equilibrium is $B(1;r) = \emptyset$, since there is no feasible home patent breadth that will result in a patent race. This case is shown by the bp'-locus in Figure 4b, for which $B(0;r) = \emptyset$ because $\beta(w_{\max}, 0; r) < 0$. This case will arise if $r \geq \rho_2(w_{\max}, 0) = \theta(e-c)^2(26-17\theta)/72$, which is shown in Figure 4a as the shaded region **A** above the $\rho_2(w_{\max}, 0)$ locus and below the $\rho_1\,(0,0)$ line. Therefore, $\mathbf{w} = (0,0)$ is a Nash equilibrium for all (r,θ) values in region A. Note that $\rho_1(1,1) < \rho_2(w_{\max}, 0)$ if $\theta < 10/17$, so $\theta = 10/17$ is the lowest value of θ for which this condition holds for all choices of $e > c$.

For (r,θ) satisfying $r \in [\rho_2(0,0), \min(\rho_1(0,0), \rho_2(w_{\max}, 0)$ (i.e. values outside region **A** in Figure 4a), we have $B(0;r) \neq \emptyset$. It then follows from Lemma 1a that $\mathbf{w} = (0,0)$ is an equilibrium if $H(r) \equiv \tilde{v}^{NN}\,(0,0;r) - \tilde{v}^{NN}\,(b(0;r),0;r) \geq 0$. This case is shown by the bp-locus in Figure 4a, where $(0,0)$ will be a Nash equilibrium if it yields a higher payoff than is obtained at the pair $(b(0;r), w)$. It is shown in the Appendix that $H(r)$ is increasing in r, because home welfare is decreasing in w and higher values of r require the home country to choose broader patent protection in order to induce a patent race. For $\theta > 10/17$ and $H(\rho_2(w_{\max}, 0)) > 0$ there will exist a value $h(\theta) \in (\rho_2(0,0), \rho_2(w_{\max}, 0))$ such that $J(h(\theta)) = 0$ and $\mathbf{w} = (0,0)$ is an equilibrium for $r > h(\theta)$. For $\theta < 10/17$, there are two possibilities. If $J(\rho_1(0,0)) > 0$, then there exists $h(\theta) \in (\rho_2(0,0), \rho_1(0,0))$ such that $H(h(\theta)) = 0$ and $\mathbf{w} = (0,0)$ is an equilibrium for $r > h(\theta)$. If $H(\rho_1(0,0)) \leq 0$, then $\mathbf{w} = (0,0)$ will not be an equilibrium for any $r \in (\rho_2(0,0), \rho_1(0,0))$. It is shown that the latter case must hold for θ sufficiently close to 0.

These arguments have shown that an equilibrium with the narrowest patent breadth and one firm undertaking R&D will be an equilibrium when values of r and θ are relatively high, since in these cases the high cost of bringing in a second firm is large relative to the benefit it provides in the form of an increased likelihood of innovation. Figure 5 illustrates the set of (r,θ) for which $(0,0)$ is an equilibrium for the specific values $e = 2$ and $c = 1$. The **FG** locus is the locus of values at which the patent race and the equilibrium with 1 firm yield the same payoff (i.e. $H(r) = 0$). The policy pair $\mathbf{w} = (0,0)$ will be an equilibrium for values of (r,θ) above the **FG** locus.

We next consider the conditions under which there can be an equilibrium with a patent race. Note that if $(0,0)$ fails to be an equilibrium, then the asymmetric equilibrium with 2 firms and patent breadth of $(b(0,r),0)$ will be an equilibrium. For the home government, the previous argument established that $b(0,r)$ satisfies the condition of Lemma 1a for a best response. For the foreign government, all of its policy choices result in having a patent race with both firms investing in R&D, so $w^* = 0$. This also establishes that an equilibrium in the patent breadth game will exist for all $r \in (\rho_2(0,0), \rho_1(0,0)]$.

As in the case (b) considered above, there is a potential for a continuum of equilibria in which there is a patent race. For example, any of the points on the bp-locus in Figure 4b are candidates for patent race equilibria. Rather than provide a full characterization of the range of equilibria that can exist with a patent race, we focus on the conditions under which there will be a symmetric equilibrium in which $w = w^* > 0$ and both firms invest in R&D. This point is illustrated by the point **C** in Figure 4b where $w = b(w;r)$. Applying Lemma 1a, this symmetric pair will be an equilibrium if it yields higher welfare to the home country than can be obtained by choosing $j(w;r) = 0$, which involves choosing the narrowest patent protection and having only one firm enter. This involves a trade-off of the loss from a lower probability of a successful innovation against the gain of shifting the cost of patent protection to support innovation onto the foreign consumers. This alternative response is illustrated by point **B,** so $w = b(w;r)$ will be an equilibrium if the payoff at point **C** exceeds that at point **B**. It is shown in the Appendix that point **B** becomes more attractive relative to **C** as r increases, which is due to the fact that higher r makes having two firms in the industry less attractive and the corresponding rise in required patent breadth makes shifting the burden of paying for innovations onto the other country's consumers more attractive. For r sufficiently close to $\rho_2(0,0)$, the same argument as used above guarantees that point **C** must be preferred to **B**. Thus, we have two possibilities. Either the patent race equilibrium with symmetric patent breadth exists for all $r \in [\rho_2(0,0), \rho_1(0,0)]$ or there is a critical value $k(\theta)$ such that the patent race equilibrium exists for $r \leq k(\theta)$. It is shown in the Appendix that the symmetric patent race equilibrium will fail to be an equilibrium for values of θ on the $\rho_1(0,0)$ locus sufficiently close to 1. However, the patent race will be an equilibrium with symmetric patent breadth for values of θ on the $\rho_1(1,1)$ locus that are sufficiently low. Thus, low values of θ and ρ will be more favorable to the patent race equilibrium with symmetric patent breadth. The dotted **EG** locus in Figure 5 illustrates the value of r and θ for which the patent race with two firms and patent breadth (w,w) yields the same payoff as $(0,w)$

(under the assumptions that $c = 1$ and $e = 2$). For values below (above) the **EG** locus the symmetric pair (w, w) yields a higher (lower) payoff and is (is not) an equilibrium to the patent breadth game. We can summarize this discussion as follows.

Proposition 8. *Assume the North-North model and* $r \in (\rho_2(0,0), \rho_1(0,0)]$.

(a) There exists a $h(\theta) \in [\rho_2(0,0), \rho_1(0,0)]$ *such that patent races with* $\mathbf{w} = (0,1)$ *and* $\mathbf{w} = (1,0)$ *are equilibria for* $r \in [\rho_2(0,0), h(\theta)]$. *If* $h(\theta) < \rho_1(0,0)$, *which must hold for* $\theta > 10/17$, *then one firm entering with* $\mathbf{w} = (0,0)$ *is an equilibrium for* $r \in [h(\theta), \rho_1(0,0)]$

(b) There exists a $k(\theta) \in (\rho_2(0,0), \rho_1(0,0)]$ *such that patent races with* $w = b(w; r)$ *will be an equilibrium for* $r \leq k(\theta)$.

Proof: See appendix.

The comparison of the regions in Figure 5 for which the symmetric patent race is an equilibrium and for which there is an equilibrium with only one firm undertaking R&D yields several interesting points. First, the region between the **EG** and **FG** loci is the area in which there will be multiple symmetric equilibria. This indicates a potential coordination failure in the setting of patent breadth, since narrow (broad) patent protection by one country will induce a narrow (broad) patent protection response by the other country. Second, the range of values for which there is a symmetric equilibrium with a patent race (the area below **EG**) contains the region in which there is an extreme asymmetric equilibrium with a patent race in which one country offers the narrowest possible protection (the area below **FG**). This suggests that the symmetric equilibrium of a patent race is more likely to be an equilibrium than the asymmetric equilibrium, in the sense that it is an equilibrium for a larger set of parameter values. Although we have not examined the conditions under which the various points on the bp-locus will be equilibria, these results lead us to conjecture that the more symmetric patent breadth pairs (i.e. those closer to the diagonal in Figure 4b) will be more likely to be equilibria than the more asymmetric patent breadth pairs.

A final question concerning the non-cooperative equilibrium has to do with whether the competition between governments in setting of patent breadth results in choice of the outcome (i.e. patent race or one firm entry) that yields higher social welfare. In the case illustrated in Figure 5, the region for which the $\mathbf{w} = (0, 0)$ equilibrium with only one firm entering yields higher welfare than the patent race outcome with $w = w^*$ is a proper subset of the region for which $\mathbf{w} = (0, 0)$ is an equilibrium. Specifically, the range of (r, θ) values at which the two outcomes yield the same world welfare (not shown in the Figure) lies between the **EF** and **FG** loci. Thus, the symmetric pair that yields higher world welfare will always be a Nash equilibrium, but the symmetric pair that yields lower world welfare could also be a Nash equilibrium. This shows that the symmetric Nash equilibria could yield either more or less innovation than is optimal. International agreements that coordinate the choices of patent breadth across countries will have the potential to raise world welfare.

The remaining range of values of r from Proposition 4, which we have not analyzed above, is where equilibria with two, one, or zero firms investing in R&D are all possible. The analysis for this case will be similar to that in the previous two cases, in that it involves the possibility of a continuum of equilibria along both the jp- and bp-loci, and therefore we will not discuss it in detail here.

These results indicate that a multiplicity of equilibria can arise in two ways in the North-North model. One is that for a given number of firms in the industry, there can be a continuum of equilibria that just support that number of firms, but differ in which country offers the broader patent protection. A second type of multiplicity, in which there are equilibria that involve different numbers of firms, can arise as well.

4. Conclusions

We considered two cases: a North-South model in which the Southern firm could imitate but not innovate and a North-North model in which firms in each country had the same probability of success at an innovation. In the North-South model the equilibrium level of innovative activity had to be at less than the socially optimal level. However, in the North-North model the level of innovative activity could exceed the socially optimal level when the probability of success was high. There was no scope for agreement over patent breadths in the North-South model while agreement could be reached in the North-North model providing the market in each country was sufficiently large.

A promising direction for future work would be, following Bessen and Maskin (forthcoming), to consider the possibility that imitation is a productive activity for the imitating firm and thus raises the likelihood of future innovation by the firm. Bessen and Maskin (forthcoming) show that the equilibrium with patent protection might yield lower social welfare than an equilibrium without patent protection when the act of imitation produces knowledge that may affect the future rate of innovation. They analyze this in a model of sequential innovation where firms produce differentiated products, which results in a complementarity in innovative activity between firms. However, their setting is essentially domestic and does not allow for strategic interaction in the breadth of patent setting across countries. In a model that combined the Bessen-Maskin framework with ours, very broad patent protection in the North could have the adverse effect of reducing the degree of accumulation of knowledge for future innovation.

A. Appendix

Proof of Proposition 2. Fix $w \in [0, (e-c)/2]$. Then a standard solution for Cournot-Nash equilibrium yields the first part of the result. Solving for n's best response function in the home market, we obtain

$$R_n\left(q_m; w\right) = \frac{e-c-q_m}{2}.$$

Solving in the same way for m's best response in the home market, we obtain

$$R_m\left(q_n; w\right) = \frac{e-c-w-q_n}{2}.$$

Using these functions to solve for mutual best responses obtains the equilibrium result.

Using $w = (e-c)/2$ in the equilibrium solutions, we obtain $\hat{q}_n = (e-c)/2$ and $\hat{q}_m = 0$. The second part of the result follows. □

Proof of Proposition 8:

a) Calculating the critical values yields $\rho_2(w_{\max}, 0) = \theta(e-c)^2\left(17\theta -)26\right)/72 \leq \rho_1(0,0) = 2\theta(e-c)^2/9$ iff $\theta \geq 10/17$. Consider first the case with $\theta \geq 10/17$. If $r > \rho_2(w_{\max}, 0)$ we have $B(0;r) = \emptyset$ and $w = 0$ is a best response for the home country by Lemma 1b. Therefore, $w = (0,0)$ is a best response in this interval. For $r \in$

$(\rho_2(0,0), \rho_2(w_{\max}, 0)]$, we define $\tilde{H}(w) = \tilde{v}^{NN}(0,0;\rho_2(w,0)) - \tilde{v}^{NN}(w,0;\rho_2(w,0))$. Since $w = b(0;\rho_2(w,0))$, $\hat{w}(0) = 0$ if $\tilde{H}(w) > 0$ and $\hat{w}(0) = w$ if $\tilde{H}(w) \leq 0$ by Lemma 1a. Since $\partial\left(\tilde{v}^{NN}(0,0;\rho_2(w,0)) - \tilde{v}^{NN}(w,0;\rho_2(w,0))\right)/\partial r = 1/2$, $\partial\rho_2(w,0)/\partial w = \partial\beta(w,0,r)/\partial w > 0$, and $\partial\tilde{v}^{NN}(w,0;\rho_2(w,0))/\partial w < 0$,we have $\tilde{H}'(w) > 0$. Evaluation of the objective function at the endpoints yields $\tilde{H}(0) = -\theta(1-\theta)(e-c)^2/3 \leq 0$ and $\tilde{H}(w_{\max}) = 7\theta(5\theta - 2)(e-c)/144$. For $\theta \geq 10/17$, the latter term must be positive and there will exist a unique $\omega(\theta) \in (0, w_{\max})$ such that $\tilde{H}(\omega(\theta)) = 0$. and $\rho_2(\omega(\theta), 0) \in [0, \rho_2(w_{\max}, 0))$. Solving yields

$$\omega(\theta) = (e-c)\left(\frac{14 - 5\theta - \sqrt{208\theta - 59\theta^2 - 65}}{22 - 7\theta}\right) \tag{A.1}$$

which satisfies $\omega(\theta) < w_{\max}$ for $\theta \geq 10/17$. It then follows that $\mathbf{w} = (0,0)$ is an equilibrium for $r > \rho_2(\omega(\theta), 0)$.

For $\theta < 10/17$, let $g(\theta)$ solve $\rho_2(g,0) - \rho_1(0,0) = 0$, where $g(\theta) \in [0, w_{\max}]$. The above arguments also ensure that $\tilde{H}(w)$ is decreasing in w for $w \in [0, g(\theta)]$ with $H(0) < 0$. There are two possibilities. If $H(g(\theta)) > 0$, then there will exist a unique $\omega(\theta) \in (0, g(\theta))$ such that $\tilde{H}(\varpi(\theta)) = 0$. and $\rho_2(\omega(\theta), 0) \in [\rho_2(0,0), \rho_1(0,0))$. In this case $\mathbf{w} = (0,0)$ is an equilibrium for $r > \rho_2(\varpi(\theta), 0)$. Equation (A.1) will be the solution for $\omega(\theta)$ in this case as well. If $H(g(\theta)) \leq 0$, then $\mathbf{w} = (0,0)$ is not an equilibrium for any $r \in [\rho_2(0,0), \rho_1(0,0)]$.

b) Define $\tilde{K}(w) = \tilde{v}^{NN}(w,w;\rho_2(w,w)) - \tilde{v}^{NN}(0,w;\rho_2(w,w))$. It follows from Lemma 1a that (w,w) is an equilibrium for all $r = \rho(w,w)$ such that $\tilde{K}(w) \geq 0$. Differentiating this expression yields the fact that $\tilde{K}$ is strictly convex in θ for $\theta \in [0,1]$ with

$$\frac{d\tilde{K}}{dw} = -\frac{(e-c)(3-\theta) - w(5-2\theta)}{3},$$

Since this expression is equal to $-(e-c)\theta/6 < 0$ when evaluated at $w = w_{\max,}$, it follows that $\tilde{K}(w)$ is decreasing in w. Note also that $\tilde{K}(0) = -\tilde{H}(0) > 0$, so (w,w) must be an equilibrium for w sufficiently small. This yields two possibilities. If $\tilde{K}(w)$ is non-negative for all w such that $\rho_2(w,w) < \rho_1(0,0)$, then (w,w) is always an equilibrium. If there exists w such that $\rho_2(w,w) < \rho_1(0,0)$ and $\tilde{K}(w) = 0$, then (w,w) fails to be an equilibrium for all higher values of w. Solving $\tilde{K}(w) = 0$ yields the critical value for w to be

$$\gamma(\theta) = (e-c)\left(\frac{3 - \theta - \sqrt{8\theta - 3\theta^2 - 1}}{5 - 2\theta}\right)$$

Setting $k(\theta) = \rho_2(\gamma(\theta), \gamma(\theta))$ if $\rho_2(\gamma(\theta), \gamma(\theta)) < \rho_1(0, 0)$ and $k(\theta) = \rho_1(0, 0)$ otherwise yields the desired cutoff for Proposition 8.

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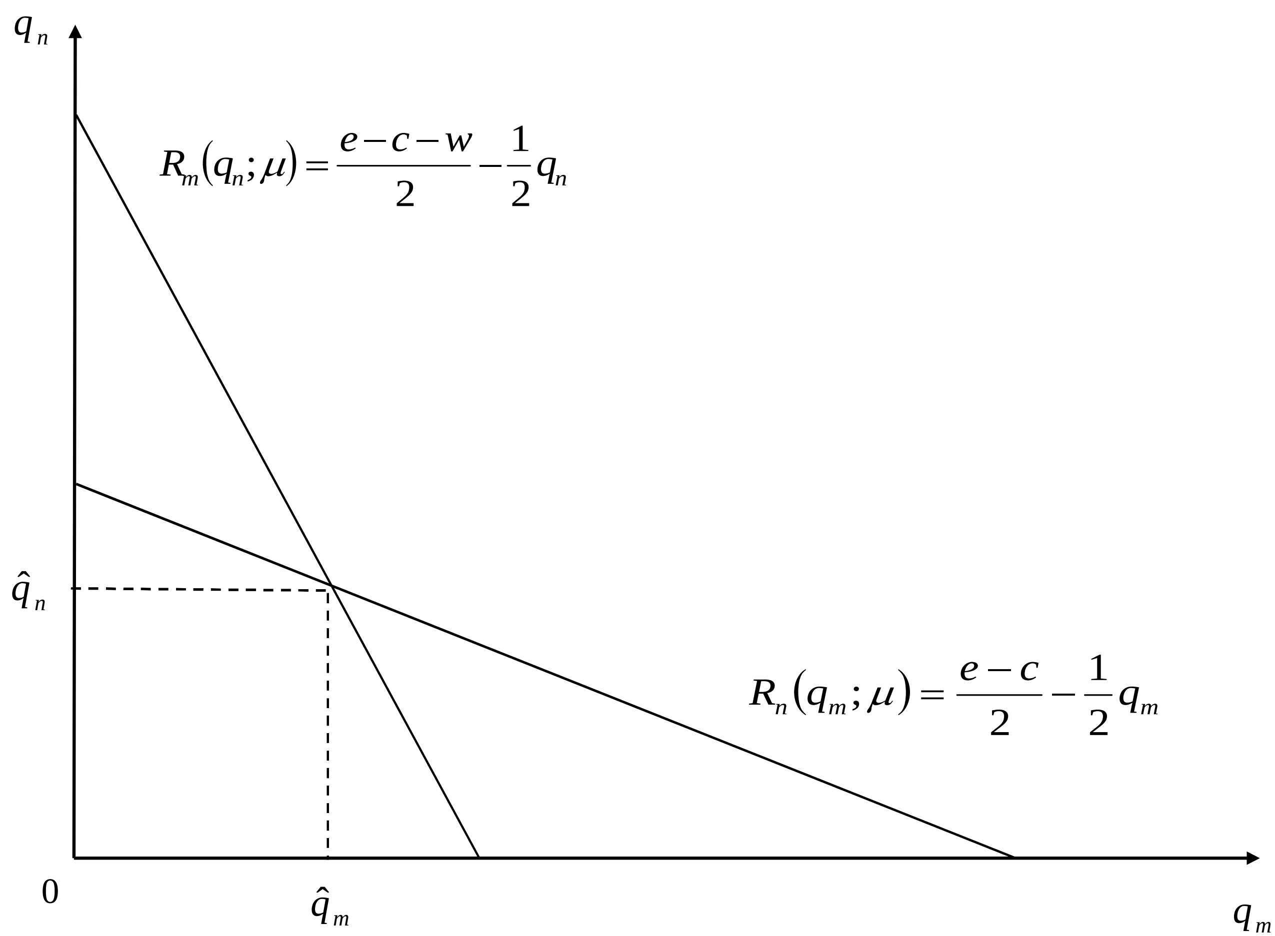

Figure 1: The effect of patent breadth on equilibrium quantities in the home market

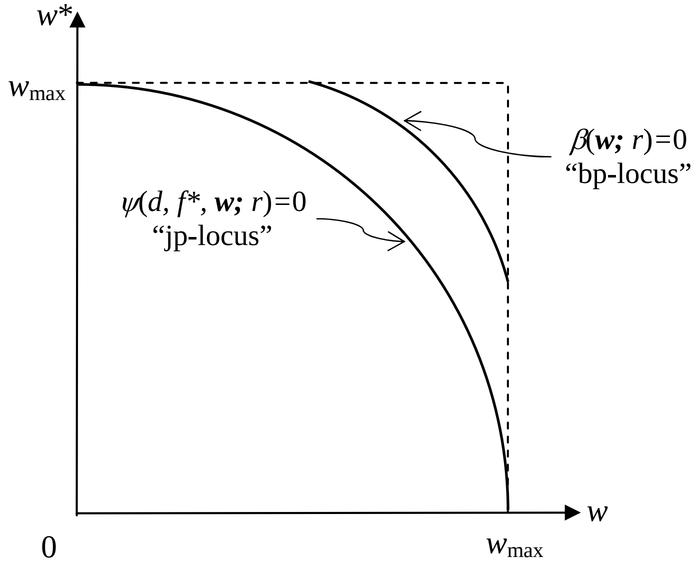

Figure 2: The jp-locus and the bp-locus

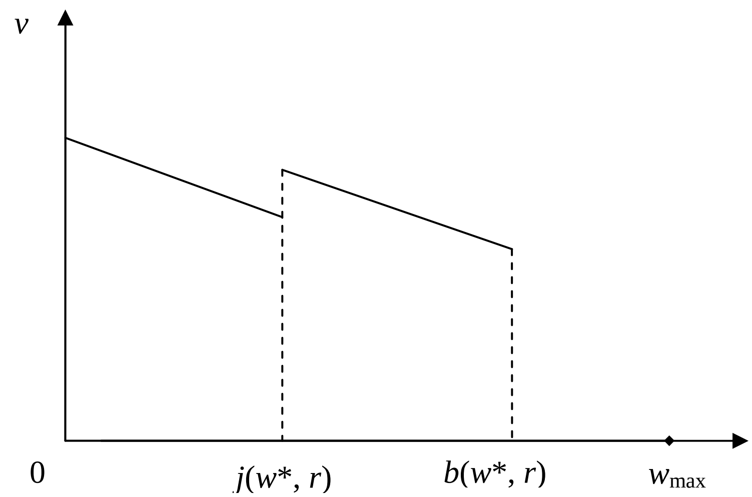

Figure 3: Home Welfare Function in the North-North Case (given foreign policy)

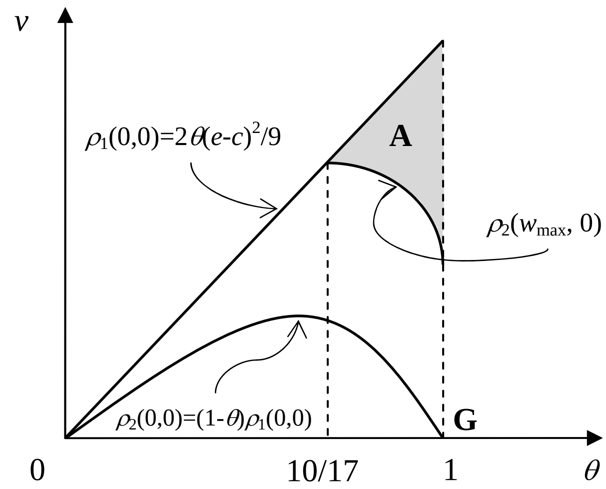

Figure 4a: Feasible (r,θ) values for case (d)

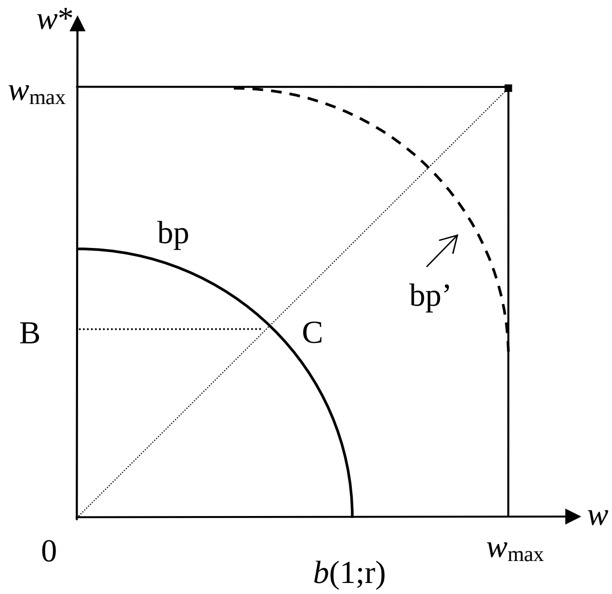

Figure 4b: bc loci for case (d)

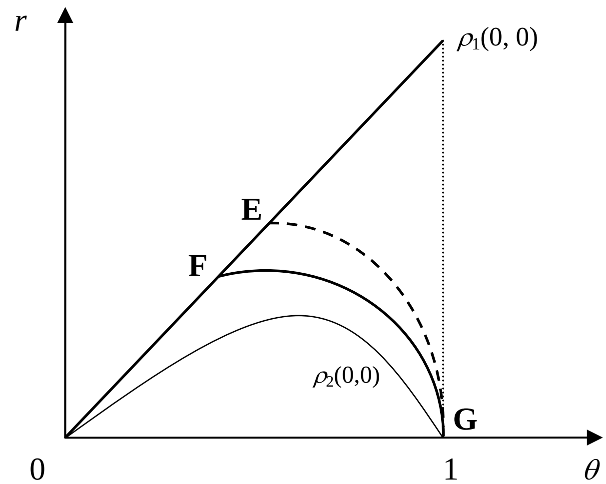

Figure 5: Symmetric Patent Race (below EG) and One Firm (above FG) Equilibria (e=2, c=1)