

THE WELFARE COST OF INFLATION, WEALTH REDISTRIBUTION AND THE LUCAS-
STOKEY LOANS PROGRAM

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The welfare gains from adopting a zero nominal interest policy depend on the implementation details. Here we focus on a government loan program that is similar to the one suggested by Lucas and Stokey (1983). This loan program will crowd out lending and borrowing and other activities. Lending and borrowing is viewed here as one form of money substitutes. Since money can be costlessly created the resources spent on creating money substitutes are a waste from the social point of view. The welfare gains from adopting a zero nominal interest rate policy are greater than the resources spent on creating money substitutes.

1. INTRODUCTION

The nominal interest rate is often viewed as a tax on real balances. Understanding the working of an economy with zero nominal interest is therefore crucial for estimating the welfare cost of this tax. Yet there is little agreement on exactly how a zero nominal interest - Friedman rule economy will operate. Will there be lending and borrowing at zero nominal interest? What will happen to the many substitutes for money that are currently used?

Aiyagari, Braun and Eckstein (ABE, 1998) is a good starting point. They use a cash-in-advance model of the type studied by Lucas and Stokey (1983). In their model there are cash and credit goods: cash goods are sold only for cash while credit goods are sold for costly credit. The distinction between credit good and cash good is endogenous and depends on the nominal interest rate. When the nominal interest rate declines more goods are bought for cash and resources used for the production of credit are saved. When the nominal interest rate is zero, all goods are "cash goods" and no resources are spent for creating credit goods. ABE show that in their model the welfare cost of inflation is the area under the demand for money curve as in Baily (1956) and Lucas (2000).

Following Lucas and Stokey (1983) ABE assume that transactions in the bonds market are costless. Thus a reduction in the nominal interest reduces the role of credit within the period but does not reduce the role of credit in smoothing consumption between periods. Here I allow money to replace between periods credit, in the same way that it replaces within period credit.

The implementation details of the zero interest rate policy are important. Friedman (1969) assumed that the government creates deflation by burning the receipts of lump sum taxes. Lucas and Stokey (1983) assumed that the central bank offers loans at the initial date and then burns the interest payments it receives. Both models assume an infinite horizon economy. And in both models agents are satiated with money in the steady state equilibrium.

Here I assume a Baumol-Tobin type model with two assets: Indexed bonds and real balances (money). A trip to the bank is required for changing the evolution of bonds. Changes in the evolution of real balances do not require a trip to the bank. Since smoothing consumption requires changes in asset holdings, money has an advantage in smoothing consumption.

Will money crowd out costly lending and borrowing? The answer is in the positive when restricting attention to a steady-state equilibrium in an infinite horizon non-stochastic economy. In such a setting it does not matter whether we introduced money in the initial period by loans or by a lump sum transfer.

In a more realistic overlapping generations model there is a difference between the two alternative implementation procedures. If we reduce the money supply by levying taxes, young agents who reach adulthood with no money will have to borrow money to pay the taxes and to smooth consumption. This requires trips to the bank (both by lenders and borrowers) and such trips are waste of resources from the social point of view. I argue that implementing the Friedman rule by instituting Lucas-Stokey type loans may solve the need of young agents to borrow money from private individuals.

The question is whether the government has an advantage over the banking sector in making loans. This important question will be discussed later. I start by examining Baily's original article that focuses on a single period analysis.

2. BAILY (1956) WITH MONEY SUBSTITUTES

The literature distinguishes between the "transaction" and the "store of value" function of money. Here I argue that when the nominal interest declines, money will crowd out both substitutes for the "transaction" function of money and substitutes for the "store of value" function. It may be useful to start with the original Baily (1956) article that focus on the "transaction" function.¹

Baily argued that the welfare cost of inflation is "a reasonably straightforward extension of the welfare cost of an excise tax" and treated money as a "good" with zero cost of production. He explicitly ruled out substitutes for money by assuming that "Bank deposits are not

¹ Baily analyzed the welfare gains from reducing the steady state inflation rate to zero and the steady state nominal interest rate to the level of the real interest rate. Lucas (2000) argued that much of the welfare gains are obtained when reducing the nominal interest rate from the real interest rate level to zero (and reducing the inflation rate from zero to the appropriate negative level). There are important differences between the two and it may be useful distinguish between the "welfare cost of inflation" and "the welfare cost of the nominal interest rate". Here I focus on the nominal interest concept.

used as money or are negligible in amount" (page 94). Here I argue that once we allow for substitutes, there is no clear relationship between the welfare cost and the area under the demand for money curve.² To simplify, I use an analogous single period "real" (non-monetary) economy. A monetary version is in Eden (2005, chapter 3.3).

There are two goods: X and Y . The representative agent has a large endowment of Y . His utility function is: $U(x)+y$, where lower case letters denote quantities consumed. It is assumed that there is a satiation level z such that $U'(z)=0$, $U'(x)>0$ for $x<z$, $U'(x)<0$ for $x>z$ and $U''<0$ everywhere.

The representative agent can produce X legally at no cost but has to pay a tax of τ units of Y per legally produced unit of X . He can also produce X illegally at a cost. The cost of producing x units illegally is $C(x)$ units of Y where the cost function $C(x)$ has standard properties: $C(0)=0$, $C'>0$ and $C''>0$.

Let x_1 (x_2) denote the quantity of X produced legally (illegally) and let $x=x_1+x_2$ denote total production. The representative agent gets a lump sum transfer of g units of Y . This lump sum tax is equal to the per agent tax revenues. Since there are many identical agents in the economy, the individual agent cannot influence the size of g .

The representative agent solves the following maximization problem:

$$(1) \quad \max_{x_1, x_2} U(x_1 + x_2) - C(x_2) - \tau x_1 + g$$

² ABE assume that there is no substitution between goods and this may explain the difference in results.

Assuming $x_2 > 0$, the first order conditions for an interior solution to this problem are:

$$(2) \quad C'(x_2) = U'(x = x_1 + x_2) \leq \tau \text{ with equality if } x_1 > 0.$$

Figure 1 illustrates using $x(\tau)$ and $x_2(\tau)$ to denote the solution to (2) as a function of τ . The Figure can be used to show that the total amount produced $x(\tau)$ is a decreasing function but the amount produced illegally $x_2(\tau)$ is an increasing function. It follows that the amount produced legally $x_1(\tau) = x(\tau) - x_2(\tau)$ is a decreasing function. When $\tau \geq \tau^*$ and the tax is above the intersection of the two curves, the amount produced is $x(\tau^*) = x_2(\tau^*)$ and the amount produced legally is zero.

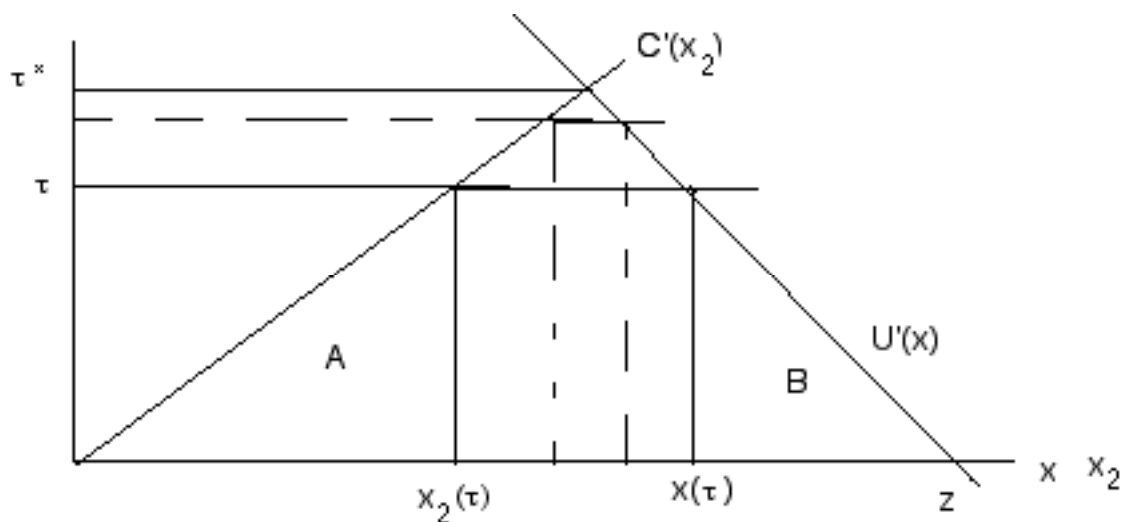


Figure 1

The welfare cost of the tax is:

$$(3) \quad U(z) - U(x(\tau)) + C(x_2(\tau))$$

The first component, $U(z) - U(x(\tau))$, is the area B in Figure 1. The second cost saving component, $C(x_2(\tau))$, is area A . It follows that:

Claim 1: The welfare cost of the tax is greater than A (= the cost of illegal production).

The analogy to a monetary economy is as follows. Legal production plays the role of "liquidity services" produced by using money. Illegal production plays the role of money substitutes and the tax rate plays the role of the nominal interest rate. Claim 1 says that the welfare cost of a positive nominal interest rate is greater than the cost of creating money substitutes.

How is the welfare cost related to the area under the demand for money curve? To answer this question we add to Figure 1 the demand for "legal production" $U'(x_1 + x_2(\tau))$ which treats the function $x_2(\tau)$ as given. The area under the demand for money (legal production) curve is: $b+c+e+f$. The true welfare cost is the area: $a+c+d+g+f$. I could not sign the difference: $b+e-a-d-g$.

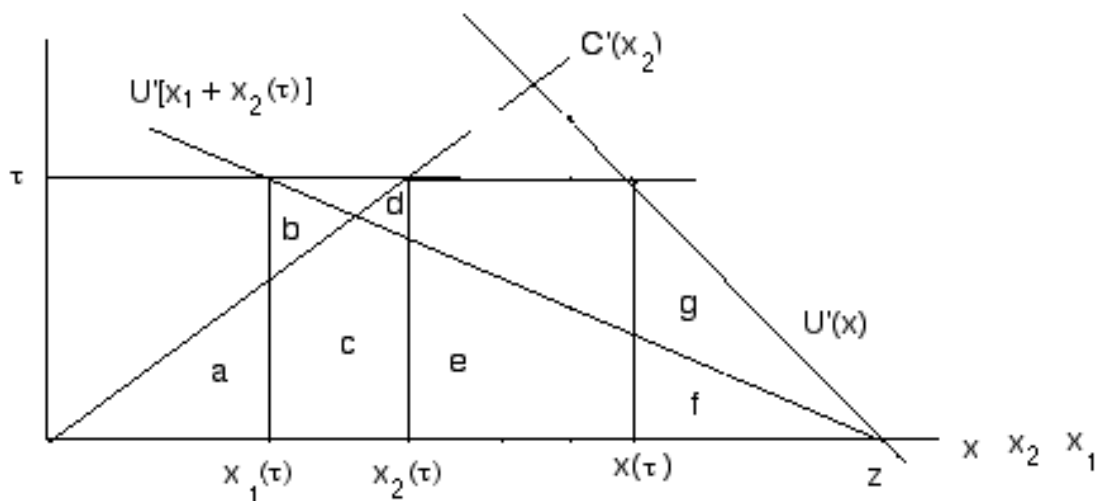


Figure 2

It thus seems possible that the area under the demand curve may be larger or smaller than the true welfare cost.

Claim 1 can be used for measuring a lower bound of the welfare cost. Aiyagari, Braun and Eckstein (1998) estimated the cost of creating demand deposits and credit cards and argued that this is about 0.5% of GDP. Humphrey, Pulley, and Vesala (HPV, 2000) estimate the social cost of a nation's payment system as 3% of its GDP. They argue that from the social point of view, debit card transactions are much cheaper than checks transactions and more generally electronic payment is cheaper than paper based payment (excluding cash which is assumed to have zero cost). But typically agents do not see the true social cost of making a payment because most banks do not charge for transactions if a minimum balance is held. In addition agents may prefer to make the payment by checks rather than by debit card because the floating period for checks is longer. It seems that the discrepancy between the social and the private cost will be eliminated once the Friedman rule of zero nominal

interest rate is adopted. In a world with zero nominal interest rates banks are expected to charge per transaction rather than requiring minimum balances and individuals will not care about the length of the float period. We may therefore expect that the cost of the payment system will go down with the nominal interest rate.

3. CROWDING OUT BORROWING AND LENDING

I now turn to the consumption-smoothing (asset) motive for holding money. I assume an overlapping generations economy. A new generation is born each period. Each generation lives for $T+1$ periods. I start with the case in which all agents born at the same time are identical and can be represented by a single agent. The representative agent consumes and receives income starting from the second period of his life: in periods $t = 1, \dots, T$. The income (endowment) at age t is Y_t units of the consumption good. I focus on the steady state in which only the age of the agent matters and therefore the calendar time at which the agent was born is suppressed.

There are two assets in the economy: money and indexed bonds. Money is an interest bearing government debt. We may think of a government run bank that pays interest on balances held in checking accounts (money) and on savings accounts (indexed bonds). The real interest rate on money is r_m and the real interest rate on indexed bonds is $r \geq r_m$.

As in Lucas and Stokey (1983), the representative agent may borrow real balances from the government (or the central bank) in the first period of his life. The amount that he may borrow must be less than L

units. The amount of the loan is a choice variable and is denoted by d .

The amount of real balances at age zero is thus:

$$(4) \quad m_0 = d \leq L.$$

The agent pays the interest $\{rd\}$ during his lifetime and the principle is paid at the end of his life. In addition to the loan the agent gets a transfer payment from the government of g_t units of consumption at age t .

I use b_t to denote the real value of bonds held at age t . As in the Baumol-Tobin model, there is a fixed cost of α units of consumption when going to the bank (the bond market). When the agent does not go to the bank at age t his bond holdings will be $b_t = (1+r)b_{t-1}$. The amount of bonds may be different if he goes to the bank. We may thus assume that the cost α occurs whenever $b_t \neq (1+r)b_{t-1}$ and therefore at age t the agent pays the cost of

$$(5) \quad TC_t(b_{t-1}, b_t) = \alpha \text{ if } b_t \neq (1+r)b_{t-1} \text{ and zero otherwise.}$$

The asset evolution equation is given by:

$$(6) \quad b_t + m_t = Y_t + (1+r_m)m_{t-1} + (1+r)b_{t-1} - rd - c_t - TC_t(b_{t-1}, b_t) + g_t$$

At the end of his life the agent returns the principle:

$$(7) \quad m_T = d.$$

The representative agent's utility function is given by the strictly monotone and strictly quasi-concave function: $U(c_1, \dots, c_T)$. His maximization problem is:

$$(8) \quad \max_{d, c_t, b_t, m_t} U(c_1, \dots, c_T)$$

$$\text{s.t. (4) - (7) and } b_0 = b_T = 0; \quad m_t \geq 0, \quad c_t \geq 0$$

The market clearing conditions are:

$$(9) \quad \sum_{t=1}^T c_t = \sum_{t=1}^T \{Y_t - TC_t(b_{t-1}, b_t)\}$$

$$(10) \quad \sum_{t=1}^T b_t = 0$$

$$(11) \quad \sum_{t=1}^T m_t = dT$$

A steady state equilibrium is a vector $(d, c_1, \dots, c_T, b_1, \dots, b_T, m_1, \dots, m_T; g_1, \dots, g_T; r, r_m)$ such that (a) given $(g_1, \dots, g_T; r, r_m)$, the magnitudes $(d, c_1, \dots, c_T, b_1, \dots, b_T, m_1, \dots, m_T)$ solve (8) and (b) the market clearing conditions (9) - (11) are satisfied.

Claim 2: In the steady state equilibrium:

$$(12) \quad (r - r_m)dT = \sum_{t=1}^T g_t$$

This says that the real value of the transfer must equal seigniorage revenues, where seigniorage is the nominal interest rate $R = r - r_m$ times real balances. In the steady state, the government uses the interest payments on the initial loan to finance the interest payments on money. Since the interest rate on money is lower than the interest rate on bonds there are some residual seigniorage revenues that are distributed as lump sum transfers.

The proofs of this and all other claims is in the Appendix.

I now turn to discuss equilibrium with $R = r - r_m = 0$ and a sufficiently large L . This choice of policy is "the Friedman rule" in our economy and I refer to it as the "Friedman rule equilibrium" (Note that when $R = r - r_m = 0$, (12) implies $\sum_{t=1}^T g_t = 0$).

Claim 3: Credit is not used ($b_t = 0$ for all t) in a Friedman rule equilibrium.

Using Claim 3 we substitute $b_t = 0$ in the asset evolution equation (6). By forward substitution we then get a budget constraint that can be expressed in present values terms:

$$(13) \quad \sum_{t=1}^T (1+r)^{-t} c_t = \sum_{t=1}^T (1+r)^{-t} Y_t$$

The budget constraint (13) and the market clearing condition (9) are satisfied when $r = 0$. Since in equilibrium the marginal rate of transformation is equal to the price, $r = 0$ is the interest rate in a Friedman rule equilibrium.

Welfare cost: I now turn to compare welfare between two equilibria: The "initial" equilibrium with strictly positive nominal interest rate and the Friedman rule equilibrium with zero nominal interest rate. I use $(c_1^i, \dots, c_T^i; b_1^i, \dots, b_T^i)$ to denote the consumption and bonds holdings in the "initial" equilibrium and (c_1^F, \dots, c_T^F) to denote the consumption in the Friedman rule equilibrium. Claim 3 and the market clearing condition (9) imply that under the Friedman rule aggregate consumption at time t is higher by aggregate spending on trading in the bonds market:

$$(14) \quad \sum_{t=1}^T (c_t^F - c_t^i) = \sum_{t=1}^T TC_t(b_{t-1}^i, b_t^i) = TC^i$$

To evaluate the welfare gains, I consider the following planner's problem:

$$(15) \quad V(x) = \max_{c_t} U(c_1, \dots, c_T) \text{ s.t. } \sum_{t=1}^T c_t = x.$$

The first order conditions for the planner's problem (15) are:

$$(16) \quad U_i(c_1, \dots, c_T) = U_1(c_1, \dots, c_T) \text{ for all } t,$$

where U_i denotes the partial derivative of U with respect to consumption at age t .

I now adapt the argument in Samuelson (1958) to show the following Claim.

Claim 4: In a Friedman rule equilibrium: $U(c_1^F, \dots, c_T^F) = V(Y = \sum_{t=1}^T Y_t)$.

Thus, the allocation in a Friedman rule economy is efficient. I now argue that in general, $U(c_1^i, \dots, c_T^i) < V(Y - TC^i)$. This says that the consumption allocation in the initial economy is not efficient relative to the resources that are devoted to consumption. The argument is an adaptation of Sargent and Wallace (1982): In a non-Friedman rule economy the marginal rate of substitution is different across agents of different ages.

I assume that as in the original Baumol-Tobin model, money may be used to smooth consumption between trips to the bank. The original model used continuous time. In our discrete model, money is used if the agent does not go every period to the bank and hold a strictly positive amount of money between trips to the bank. Formally, money is used if there are ages h and k ($h < k$) such that (a) the representative agent chooses to go to the bank at ages h and k but do not go to the bank in the period between these ages: At ages $h < t < k$ and therefore $b_t = (1+r)^{t-h} b_h$ for $h \leq t < k$ and $b_k \neq (1+r)^{k-h} b_h$; (b) he holds a strictly positive amount of money at some point during the above period: There exists an age index $h < j < k$ such that $m_j > 0$.

Claim 5: $U(c_1^i, \dots, c_T^i) \leq V(Y - TC^i)$ with strict inequality if money is used in the initial equilibrium.

Claims 4 and 5 imply:

Claim 6: $U(c_1^F, \dots, c_T^F) - U(c_1^i, \dots, c_T^i) \geq V(Y) - V(Y - TC^i)$ with strict inequality if money is used.

This says that from the planner's point of view, the utility gain from adopting the Friedman rule is greater than the utility gain from saving the resources spent on "going to the bank". Similarly to Claim 1, it implies that the saving of resources is a lower bound on the welfare gain.

4. CONTRACT ENFORCEMENT AND ADMINISTRATION COSTS

It was assumed that the government costlessly enforces loan contracts. I now turn to a more realistic case where contract enforcement is costly.

The case of imperfect enforcement of contracts is relevant for the important question of whether the government has an advantage over the banking sector in making loans. Banks spend resources on trying to minimize the probability of bankruptcy by verifying collaterals and the financial status of the borrower. From a social point of view some bankruptcies are transfers from the rich to the poor and therefore if income redistribution is desirable some of the resources spent on assessing the ability of the borrower to pay the loan are a "social waste". We will argue that even if the government does not want to redistribute wealth, it may still have an advantage in collecting loan payments because of its ability to levy taxes.

As before, I assume that agents live for $T+1$ periods but now agents are heterogeneous. There are N types of agents indexed by:

$h=1,\dots,N$. One agent of each type is born in every period. Thus, the distribution of the agents' characteristics is the same for all generations and as in the previous model, only the age matters. We will talk about agent (of type) h without specifying the calendar date of his birth.

As in the previous section the government offers loans to young people but here the loan payments are contingent on income. A consumer who earns in the current period Y units and owes the government d units will pay

$$(17) \quad p(d,Y) = \tau(Y)Y \text{ if } d > \tau(Y)Y \text{ and } d \text{ otherwise.}$$

Here $\tau(Y)$ is the fraction of income paid when the principle is not small. When the principle is small the agent pays it and becomes debt free.

The principle that consumer h owes the government evolves according to:

$$(18) \quad d_t^h = (1+r)(d_{t-1}^h - p(d_{t-1}^h, Y_t^h))$$

The government imposes income and consumption taxes. An income of Y is taxed at the average rate of $\theta(Y)$. The consumption tax rate does not depend on the level of the agent consumption and is denoted by κ .

There are administrative costs for collecting taxes and loan

payments.³ The cost of collecting income tax revenues of x units, loan payment revenues of y units and consumption tax revenues of z units from age t , type h agent is:

$$(19) \quad A_t^h(x, y, z),$$

where A_t^h is weakly increasing in all of its arguments. Mayshar (1991) assumes that the taxpayer devotes effort to paying taxes and sheltering income. Since income (effort) is exogenous in our framework, it does not matter who bears the cost. We should therefore think of (19) as reflecting the true social cost, including the effort spent by the individual agent in filing tax returns, hiring accountants and so on.

Both income tax and loan payments create incentives to under report income. But the incentives to avoid loan payments are less than the incentives to avoid income tax of the same rate. To make this point I consider a young individual who gets a salary that is reported to the IRS by his employer. In addition to his salary the individual has done a one time 10,000 dollars consulting job in the current year. Assume further that he pays on the margin 20% of his income as loan payments and 20% as income tax. If he does not report the consulting job income he will reduce his income tax payment by 2000 dollars and his loan payment by 2000 dollars. But his net wealth will go up by only 2000 dollars, if eventually he pays the loan in full. This will be the case for individuals who earns a relatively high salary or for individuals

³ See Slemrod and Yitzhaki (2002) for a survey of the literature on the administration costs of collecting taxes.

who expect their salary to go up in the future. The addition to net wealth will be 4000 dollars only if the individual is poor or plan to under report income consistently over time. The incentive to under report is thus less than in the case in which there are no loan payments but income tax is 40%. In this case the entire tax avoidance money represents an addition to wealth.

Since the administrative costs are likely to be larger when there are more incentives to under report, I assume:

$$(20) \quad A_t^h(x - \sigma, y + \sigma, z) \leq A_t^h(x, y, z) \text{ for all } x, y, z \geq 0 \text{ and } 0 \leq \sigma \leq x.$$

It seems that collecting consumption taxes from businesses is simpler than collecting income tax from individuals. This is especially true for value added tax because of the incentives for self-policing. See Alt (1983). I therefore assume:

$$(21) \quad A_t^h(x - \sigma, y, z + \sigma) \leq A_t^h(x, y, z) \text{ for all } x, y, z \geq 0 \text{ and } 0 \leq \sigma \leq x.$$

The assumptions (20) and (21) say that income tax is costly to collect relative to loan payments and consumption tax.

Finally, there are no lump sum transfers or taxes and the government spending is G units per period.

The consumer's asset evolution equation is now:

$$(22) \quad b_t^h + m_t^h = (1 - \theta(Y_t^h))Y_t^h + (1 + r_m)m_{t-1}^h + (1 + r)b_{t-1}^h - (1 + \kappa)c_t^h - TC_t(b_{t-1}^h, b_t^h) - p(d_{t-1}^h, Y_t^h)$$

And the consumer's problem is now:

$$(23) \quad \max_{d^h, c_t^h, b_t^h, m_t^h} U^h(c_1^h, \dots, c_T^h)$$

s.t. (4), (5), (17), (18), (22); $b_0^h = b_T^h = 0$; $m_t^h \geq 0$, $c_t^h \geq 0$

The market clearing conditions are now:

$$(24) \quad G + \sum_{h=1}^N \sum_{t=1}^T c_t^h = \sum_{h=1}^N \sum_{t=1}^T (Y_t^h - TC_t(b_{t-1}^h, b_t^h) - A_t^h [\theta(Y_t^h) Y_t^h, p(d_{t-1}^h, Y_t^h), \kappa c_t^h])$$

$$(25) \quad \sum_{h=1}^N \sum_{t=1}^T b_t^h = 0$$

$$(26) \quad \sum_{h=1}^N \sum_{t=1}^T m_t^h = T \sum_{h=1}^N d^h$$

claim 7: In a Friedman rule equilibrium with $r = r_m = 0$ we must have:

$$(27) \quad G + \sum_{h=1}^N d^h = \sum_{h=1}^N \sum_{t=1}^T (\theta(Y_t^h) Y_t^h + p(d_{t-1}^h, Y_t^h) + \kappa c_t^h)$$

This says that the government budget must always be balanced.

An Illustration: The above framework allows for "partial default" by poor people. The question is how to finance it. Here I consider an example in which a consumption tax is used both to finance the "partial default" by the "poor" and a reduction in the income tax paid by the "rich". In this example there is no wealth redistribution.

I assume two types of agents born in each period: "poor" and "rich". I start by assuming that the lowest annual earnings of the rich are higher than the highest annual earnings of the poor:

$$(31) \quad \min_t \{Y_t^R\} > \max_t \{Y_t^P\},$$

where the superscripts P and R denotes "poor" and "rich". Under this assumption the annual income of the poor is always below the median.

I assume $r = r_m = 0$ and $d^P = d^R = L$.⁴ The loan is equal to half of the total earnings of the poor:

$$(32) \quad L = 0.5 \sum_{t=1}^T Y_t^P$$

Loan payments are a flat rate of 25% of income so that the "poor" do not pay it in full. The rich pay their loan in full after ψT periods, where $0 < \psi < 1$ satisfies:

$$(33) \quad 0.25 \sum_{t=1}^{\psi T} Y_t^R = L$$

The "partial default" by the "poor" is financed by an increase in the consumption tax rate. The "rich" are compensated for this increase by an equal reduction in their income tax rate. I use Y^h for total income over the life cycle and $\theta(Y^h)$ for the average income tax rate:

$$(34) \quad Y^h = \sum_{t=1}^T Y_t^h, \quad \theta(Y^h) Y^h = \sum_{t=1}^T \theta(Y_t^h) Y_t^h$$

⁴ Since $r = r_m$, $d^h = L$ is a solution to the agents' problems.

I start with an "old" regime: $\{ \tau^O(Y^P) = \tau^O(Y^R) = 0, \theta^O(Y^R), \theta^O(Y^P), \kappa^O \}$.

The new regime is:

$$(35) \quad \begin{aligned} \tau^N(Y^P) = \tau^N(Y^R) = 0.25 \quad , \\ \theta^N(Y^R) = \theta^O(Y^R) - \varepsilon, \theta^N(Y^P) = \theta^O(Y^P), \quad \kappa^N = \kappa^O + \varepsilon. \end{aligned}$$

Since the rich pay their loans in full the loan payments do not create an incentive to under report income (see the discussion that led to [21]). But the reduction in the income tax rate will encourage the true reporting of income by the rich. The "poor" will treat the loan payments as an income tax and this has a discouraging effect on true reporting. On the whole, since the income tax paid by the "poor" is relatively small and since the cost of collecting consumption tax are relatively small, the total effect on the administrative costs is likely to be negative. In what follows I assume that there is no change in the total administrative costs.

From the IRS web site we find that in 2003 the average tax rate for the "poor" (bottom 50% of the income distribution) was only 3% and their adjusted gross income was: $Y^P = 879,735$ million dollars. I assume that over the life cycle consumption of the poor is roughly equal to their income. Since $L = 0.5Y^P$ and the poor pay only half of the loan, the consumption tax rate must be 25%.

In 2003 the US population age 20 - 24 was 20,594,710.⁵ Dividing this number by 5 we get that the number of 20 years old was:

⁵ U.S Census Bureau, International data base. Table 094 (Midyear Population by age and sex)

$N = 4,118,942$. Half of those are "poor". The amount of the loan per 20 years old is therefore: $L = \frac{0.5Y^p}{(\frac{1}{2})N} = 213,582$ dollars which is about

222,000 in terms of 2007 dollars.

A more realistic example that relaxes (31) will lead to a lower consumption tax rate. To illustrate this point, I assume three types of agents. A quarter of the population earns consistently above the median. A quarter earns consistently below the median and half are 50% of the time below the median and 50% of the time above the median. Assume further that only those who are consistently below the median do not pay their loan. In this case only a quarter of the population will "partially default" on their loan and the necessary consumption tax is only 12.5%.

6. SOME POLICY ISSUES

I now turn to discuss the effect of the loan program on labor supply and the relationship to social security.

Effect on labor supply: A loan payment has no effect on labor supply if it is returned in full. To see that note that the marginal loan payment for the rich in the above example is zero: If they earn an additional unit they will not increase the present value of their loan payments. There is also no wealth effect because the present value of the loan payments is equal to the amount of the loan.

Those who do not pay their loan in full will treat the loan payments as income tax. In our example, when a quarter of the population "partially default" those who default face a 37.5% marginal tax rate (25% loan payments and 12.5% consumption tax). In addition the "poor" in our example will experience an increase in wealth equal to the amount of the loan that is not paid.

In the absence of a change in productivity, this increase in the marginal tax rate and wealth will reduce the labor supply of the "poor". But the adoption of the Friedman rule is likely to increase labor productivity. Some will use the loan to get higher education or to accumulate other types of human capital. Some may buy a more reliable car and will come to work on time and so on.

Time spent on executing transactions is expected to go down with the adoption of the Friedman rule. For example, selling a house to a buyer that does not need a mortgage is likely to be simpler. And of course, you do not need to go to the bank to sell bonds for money.

Thus the "poor" will have more time to allocate between socially productive labor and leisure. This works in the direction of increasing labor supply.

For these reasons the effect on labor supply on the "poor" cannot be ascertained but the effect on the labor supply of the rich is likely to be positive because of the increase in productivity and the increase in the time available for socially productive activities.

Social security in reverse: The social security system has the following elements: (a) you pay during the working age and get the benefits in retirement; (b) the "poor" get a higher rate of return than the "rich"

and therefore the system as a whole may be viewed as transferring resources from the "rich" to the "poor". A Lucas-Stokey loan program that allows for "partial default" by poor people, also transfers resources from the "rich" to the "poor" but here you get the benefits at a young age and pay later.

There is an obvious tension between the social security program and the loan program discussed here. One of the main objectives of the social security system is to reduce the poverty level among the elderly. This is not the same as consumption smoothing. It may be argued that the social security program makes consumption smoothing more difficult, once the young are taken into account. Furthermore, the question of whether the social security achieves its objective in the most efficient way has been debated for a long time.

It is sometimes argued that we need social security because many people lack "self-control". The lack of self-control is certainly a problem for children. Traditional economics assumes that all children become adults and once they become adults they gain self-control. Behavioral economics argues that some people never become adults in this sense. It seems that self-control is endogenous to some degree. Parents that are over protective will tend to raise children that are less independent. Similarly, governments that are over protective may encourage myopic behavior.

The loan program discussed here is likely to encourage behavior that looks like "self control". For example, in the present system a young adult who drives an expensive car transmits a signal about the wealth of his parents or his own ability to make money. This signal may

help him with the other sex. Once everyone gets a loan, such a behavior may signal the lack of self-control.

In any case, the implementation of the Lucas-Stokey loan program may address the lack of self-control problem by offering options to buy annuities and other commitment devices.

7. CROWDING OUT

Implementing the Friedman rule by a government loan program will crowd out market activities in many sectors, especially in banking, rental, insurance and trade. In addition there will be savings on socially wasteful non-market activities. I now elaborate.

Banking: The average bank loan in 1988 was about 30,000 dollars.⁶

Therefore it seems that a government loan of say 100,000 dollars will crowd out most of the loan activities of banks.

The zero nominal interest policy will reduce banks activities on the liability side of their balance sheet. The commercial bank national average report (report, 1998) allocates expenses to various activities. According to the report 56% of the banks' non-interest expense are on the liability side (demand deposits, saving deposits and time deposits). These expenses are mostly on processing checks and other paper based obligations. They also spend resources on moving balances from checking accounts to savings type accounts and vice versa. I expect that in a

⁶ Calculated from the commercial bank national average report (report, 1998)

Friedman rule economy agents will have only one account (checking) and will use mostly debit cards and cash. There will be much less use of checks because zero nominal interest rate eliminates the benefit from the "float" period between the writing of a check and the actual payment.

Renting: Many people will use the loan to buy a house. Therefore, rental that is essentially used as substitute for loans will be crowded out.

Insurance: As pointed out by Bewley (1983) the initial loan may be used for self-insurance. Some insurance activity will be crowded out.

Wholesale and retail trade: The cost of trade will also be reduced because buyers will usually have enough money to carry out their desired transactions. This is the focus of Lagos and Wright (2005) who estimate the cost of 10% inflation to be between 3 to 5 percent of consumption.

8. CONCLUDING REMARKS

The question of the welfare cost of inflation is not a "straightforward extension of the welfare cost of an excise tax" as in Baily (1956). In the excise tax case, there is no ambiguity about the implementation of a zero tax policy. In the monetary case, zero seigniorage policy can be done by imposing taxes as in Friedman (1969) or by a loan program as in Lucas and Stokey (1983).

The literature has focused on the first tax alternative and considered the case in which only distortive taxes are possible. See for

example, Phelps (1973), Kimbrough (1986), Correia and Teles (1996) and Chari, Christiano and Kehoe (1996). Here I focus on the second loan alternative. Unlike a tax, a loan that is paid in full does not distort behavior.

In general we expect the optimal policy to crowd out activities that the government can do better than the private sector. Most monetary economists, Friedman being a prime example, believe that the private sector is in general more efficient than the government. But they still favor the government over the private sector in its money creation role. Here I extend this view and argue that the government may have an advantage over the private sector in making loans because it can levy taxes and because it may treat defaults made by "poor" people as part of its income redistribution policy.

We considered a Baumol-Tobin type model with two assets: indexed bonds and money. The difference between the two assets is in the ease in which one can affect their evolution. A trip to the bank is required for changing the evolution of bonds but the evolution of money can be changed without a trip to the bank. Since smoothing consumption requires changes in asset holdings, this gives money an advantage in smoothing consumption.

Both assets may be used when the rate of return on bonds is higher than the rate of return on money. But only money will be used when the two assets promise the same rate of return. This is the case of zero nominal interest or the Friedman rule. Under the Friedman rule the allocation is efficient. Moving from an economy with strictly positive nominal interest rate to an economy with zero nominal interest rate will increase consumption by the amount of resources spent on trips to the

bank. But in general welfare will increase by more than that because consumption smoothing is better under zero nominal interest rate.

We discussed implementation issues that are relevant for the question of whether the government has an advantage in making loans. In the example we worked out, consumption tax finances both "partial default" by "poor" people and a reduction in the income tax paid by the rich. In this example, the implementation of the Friedman rule by a loan program does not change the distribution of wealth. The effective marginal income tax rate equals the income tax rate when the loan is paid in full and the income tax rate plus the loan payment rate when the loan is not paid in full. In our example, the effective marginal income tax rate paid by the "rich" goes down while the effective marginal income tax rate paid by the "poor" goes up.

This move toward equality in the marginal tax rates is likely to be beneficial. Castaneda, Diaz-Gimenez and Rios-Rull (1998) estimated the benefits from moving to a proportional tax system in the US. They found that the benefits are large (increase in efficiency of 4.4% of output) but this comes at the expense of a large increase in wealth inequality. Our example suggests that a combination of loans that allows for "partial default" by poor people plus a consumption tax can be used to get a proportional tax system without the adverse effects on wealth distribution.

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APPENDIX

Proof of Claim 2: Summing the asset evolution equation (6) over t leads to:

$$(A1) \quad \sum_{t=1}^T b_t + m_t = \sum_{t=1}^T (Y_t + (1+r_m)m_{t-1} + (1+r)b_{t-1} - rd - c_t - TC_t(b_{t-1}, b_t) + g_t)$$

Substituting the market clearing conditions (9)-(11) in (A1) leads to (12). \square

Proof of Claim 3: Consider a plan $\{d, c_t, b_t, m_t\}$ that satisfies all the constraint in (8) and uses costly credit that leads to $TC_t > 0$ for some t . Under the Friedman rule the plan $\{d^* = L, c_t^* = c_t + TC_t, b_t^* = 0, m_t^* = m_t + b_t\}$ is also feasible. Since the "star" alternative has more consumption at each date the original plan cannot be optimal. We have thus shown that trading in the bonds market cannot be optimal under the Friedman rule. \square

Proof of Claim 4: The representative consumer's problem in a Friedman rule equilibrium is:

$$(A2) \quad \max_{c_t} U(c_1, \dots, c_T) \quad \text{s.t.} \quad (13).$$

The first order conditions for the problem (A2) require:

$$(A3) \quad \frac{U_t(c_1, \dots, c_T)}{U_1(c_1, \dots, c_T)} = (1+r)^{1-t} = 1 \quad \text{for all } t.$$

Thus, in a Friedman rule equilibrium the first order conditions (16) for the planner's problem are satisfied and therefore

$$U(c_1^F, \dots, c_T^F) = V(Y). \quad \square$$

Proof of Claim 5: When money is used in the initial equilibrium, there are indices $h < j < k$ such that $m_j > 0$, $b_t = (1+r)^{t-h} b_h$ for $h \leq t < k$ and $b_k \neq (1+r)^{k-h} b_h$. The first order conditions for the consumer's problem must be in this case:

$$(A4) \quad U_j(c_1^i, \dots, c_T^i) = (1+r_m)^{k-j} U_k(c_1^i, \dots, c_T^i);$$

$$(A5) \quad U_h(c_1^i, \dots, c_T^i) = (1+r)^{k-j} U_k(c_1^i, \dots, c_T^i)$$

To derive (A4) consider the following deviation from the optimal choice of the consumer in the initial equilibrium. The consumer reduces his consumption at age j by a unit, use it to increase his real balances and spend the accumulated amount of $(1+r_m)^{k-j}$ units at age k . The utility cost of reducing consumption at time j by a unit is $U_j(c_1^i, \dots, c_T^i)$. The utility gain of increasing consumption at age k by $(1+r_m)^{k-j}$ units is: $(1+r_m)^{k-j} U_k(c_1^i, \dots, c_T^i)$. Since at the optimum the agent cannot benefit from this deviation we must have:

$$(A6) \quad U_j(c_1^i, \dots, c_T^i) \geq (1+r_m)^{k-j} U_k(c_1^i, \dots, c_T^i)$$

Similarly, since $m_j > 0$ he can increase consumption at age j by a unit and reduce consumption at age k by $(1+r_m)^{k-j}$ units. Since at the optimum he cannot benefit from this deviation we must have:

$$(A7) \quad U_j(c_1^i, \dots, c_T^i) \leq (1+r_m)^{k-j} U_k(c_1^i, \dots, c_T^i)$$

Conditions (A6) and (A7) imply (A4). The first order conditions (16) to the planner's problem require: $U_h(c_1^i, \dots, c_T^i) = U_j(c_1^i, \dots, c_T^i) = U_k(c_1^i, \dots, c_T^i)$.

Since in the initial economy, $r_m < r$ the first order conditions (A4) and (A5) imply a violation of (16). Therefore a planner that can distribute $Y - TC^i$ units of consumption can improve on the initial equilibrium. \square

Proof of Claim 7: After forward substitution of (22) we get the budget constraint in terms of present values:

$$(A8) \quad d^h + \sum_{t=1}^T \left[(1 - \theta(Y_t^h)) Y_t^h - p(d_{t-1}^h, Y_t^h) - (1 + \kappa) c_t^h - TC_t(b_{t-1}^h, b_t^h) - (r - r_m) m_{t-1}^h \right] = 0$$

Summing over h leads to:

$$(A9) \quad \sum_{h=1}^N \left(d^h + \sum_{t=1}^T \left[(1 - \theta(Y_t^h)) Y_t^h - p(d_{t-1}^h, Y_t^h) - (1 + \kappa) c_t^h - TC_t(b_{t-1}^h, b_t^h) - (r - r_m) m_{t-1}^h \right] \right) = 0$$

Substituting $r = r_m$, $TC_t(b_{t-1}^h, b_t^h) = 0$ and the good market clearing condition in (A9) leads to:

$$(A10) \quad G + \sum_{h=1}^N \left(d^h - \sum_{t=1}^T \left[\theta(Y_t^h) Y_t^h + p(d_{t-1}^h, Y_t^h) + \kappa c_t^h - g_t^h \right] \right) = 0,$$

and to (27). \square