Optimal Tax Treatment of Private Contributions for Public Goods with and without Warm Glow Preferences*

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Abstract

Tax-favored contributions for financing some public goods may be a useful part of optimal nonlinear income tax and expenditure policy. There are two sides to the potential gain from subsidized donations. First, for a given level of public good provision, higher private donations from high earners than low earners eases the incentive compatibility constraint for donors and so can raise social welfare. This follows since considering a lower-paid job includes a perception of a drop in public good provision. Second, private donation reduces consumption, easing the resource constraint. This paper explores optimal policy, using first a model with standard preferences and then a model with a warm glow of giving. In addition to showing the conditions for the level of public goods, the paper considers the pattern of optimal subsidization across earnings levels. Analysis of optimal taxation with warm glow preferences is sensitive to the choice of preferences that are relevant for a social welfare evaluation. After considering optimal rules with formulations of social welfare which do and do not include warm glow utility, the paper considers the choice of normative criterion. Like the earlier literature, this paper assumes that organizing private donations is costless while tax collection has a deadweight burden. Since private charitable fundraising is very

far from costless, the paper is an exploration of economic mechanisms, not a direct guide to policy.

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The US relies on tax-favored contributions for financing some public goods[1] as well as having direct government expenditures. There are a number of reasons why such reliance may be useful. From a political perspective, this approach shifts some decisionmaking from the legislative process to the decisions of individual donors (and the managers of charitable organizations). This includes religious organizations for which direct expenditures are constitutionally banned. From an economic perspective, this approach can be a useful part of optimal tax and expenditure policy. This paper explores the latter issue, using first a model with standard preferences and then a model with a warm glow of giving (Andreoni, 1990). While standard preferences are very distant from empirical reality, it seems useful to examine the economic mechanisms in this setting before examining a more realistic model with warm glow preferences.

The focus is on the interaction between an optimal nonlinear income tax and subsidized donations to finance public goods.[2] The model uses an additive preference structure so that the incentive compatibility constraint is not affected by the level of a publicly provided public good implying that

[1] This includes covering fixed costs for private goods with low marginal costs.

[2] This paper considers only resource-using public good provision. I plan to consider gifts to individuals and contributions to charities for the purpose of redistribution in a separate paper.

optimal public provision in the absence of donations satisfies the Samuelson rule. The optimality of the Samuelson rule does not generally carry over to optimal provision with subsidized donations. The paper also considers the pattern of optimal subsidization across earnings levels. While relevant for analysis of tax deductibility of charitable donations, the analysis does not get that far.

Analysis of optimal taxation with warm glow preferences is sensitive to the choice of preferences that are relevant for a social welfare evaluation. After considering optimal rules with formulations of social welfare which do and do not include warm glow utility, the paper considers the choice of normative criterion.

Conditions for the optimal level of publicly provided public goods have been studied with different types of taxation available (Atkinson and Stern (1974), Boadway and Keen (1993), Kaplow (1996)). Equilibrium with privately provided public goods has been studied by Bergstrom et al (1986). Analyses of tax expenditures for private contributions for public goods have been done by Feldstein (1980) and Roberts (1987). Closest to this analysis is that of Saez (2000b) that analyzed the tax treatment of voluntary donations in the case of a linear income tax. This paper focuses on private contributions with nonlinear income taxation. The issue arises in two contexts - as a substitute for public financing and when the government leaves the choice completely to the public.

There are two sides to the potential gain from subsidized donations. First, for a given level of public good provision, higher private donations by high earners than low earners eases the incentive compatibility constraint for donors and so can raise social welfare. This follows since considering a lower-paid job includes a perception of a drop in public good provision. Second, private donation reduces consumption, easing the resource constraint. While this observation is readily shown in the two-types model used by Boadway and Keen to analyze public provision of public goods, the analysis in the text uses instead a more tractable model of income taxation where the hours of work are fixed for any job rather than being a choice variable for workers (Diamond, 1980, Saez, 2000a).[3]

Briefly considered is a setting with multiple public goods that must receive uniform subsidization. For convenience, the notation is listed in Section 5. Like the earlier literature, this paper assumes that organizing private do-

[3] In addition to considering a model where the incentive compatibility constraint comes from imitating a worker with slightly lower skill, this alternative model allows the possibility that the best alternative to the equilibrium earnings level is withdrawal from the labor force.

nations is costless while tax collection has a deadweight burden. Since private charitable fundraising is very far from costless, the paper is an exploration of economic mechanisms, not a direct guide to policy.

1 Standard Preferences

1.1 Optimal Public Provision

To begin, we consider a variant of optimal income tax model in Diamond (1980) to which we add a single public good (as in Boadway and Keen (1993)). In this model, labor hours are not adjustable, implying that a job involves nonvarying levels of disutility for workers with different skills. To focus on the primary issue of the contrast between public and private provision, we assume that individual preferences are additive in the utility from the public good. A worker of type n can only work at a job that has productivity n or lower. Disutility from work at a job with productivity m for a worker of type n ($m \leq n$) is additive and denoted a_{mn}.[4] If a type n worker is

[4] In the Mirrlees model, a worker can provide any number of hours of labor at a fixed wage per hour, with the wage depending on skill. In practice, the opportunities in the labor market have a more complex structure. In the model in Diamond (1980) each individual has two opportunities - a particular job or no work at all. Here we effectively assume that each person has two job opportunities, just as Mirrlees assumes the next best alternative is to provide the effective labor of a slightly lower-skilled worker.

holding down a type m job, then utility is $u[c_m] - a_{mn} + b_n v[G]$, where c_m is the consumption from after-tax earnings, G is the level of public good, with u and v both increasing, concave and twice differentiable and b_n a weight that can vary with skill level but does not vary across people with the same skill. We assume that this parameter is the same for everyone, $b_n = b$, although we preserve the more general notation as a reminder of an effect that particularly matters when considering multiple public goods. We also assume the Inada condition for u. If not working, we simply set labor disutility equal to zero and assume the same utility functions of public and private good consumption. There may be a type-0 worker who has no productivity and does not work.

We assume that lower skilled jobs are less difficult: for $m_1 < m_2$ we have $a_{m_1 n} \leq a_{m_2 n}$. We assume that lower skilled workers find any job more difficult: for $n_1 < n_2$ we have $a_{mn_1} \geq a_{mn_2}$. We make these convenient assumptions even though neither is plausible once we consider the different nature of jobs with different productivities. We assume that the optimum allocates each type of worker to its own skill-level job (except in one section below). I conjecture that sufficient conditions for the optimum to have this character are that the difference in disutilities decline with skill - that is, the higher the skill the less the increase in disutility for any step up in job productivity,[5] and that there are similar relative numbers of different workers, and that the economy be poor enough to need this much work. Thus, if a worker chooses some skill level, no more highly skilled worker would choose a lower skill job. But this disutility condition alone does not rule out having no workers at some jobs and several types at others. With private donations, the analysis is potentially more complicated.

Restricting analysis to allocations where each type of workers is at the matching skill, social welfare maximization is:

$$\text{Maximize}_{c,G} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right]\right)$$

$$\text{subject to:} \quad E + pG + \sum N_n \left(c_n - n\right) \leq 0$$

$$u\left[c_n\right] - a_{nn} + b_n v\left[G\right] \geq u\left[c_m\right] - a_{mn} + b_n v\left[G\right] \;\text{ for } m < n \text{ for all } n \tag{1}$$

where E is other government expenditures and p is the cost per unit of the

[5] For $n' < n$, $m' < m$: $a_{mn'} - a_{m'n'} > a_{mn} - a_{m'n}$

public good. Forming a Lagrangian, we have

$$\begin{aligned} L &= \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right]\right) - \lambda \left(E + pG + \sum N_n \left(c_n - n\right)\right) \\ &+ \sum \sum \mu_{nm} \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right] - \left(u\left[c_m\right] - a_{mn} + b_n v\left[G\right]\right)\right) \end{aligned} \quad (2)$$

Given the additivity of utility in the public good, the FOC for public good provision is

$$\sum N_n b_n v'\left[G\right] = \lambda p \quad (3)$$

With the next best alternative for any worker being the next job down in productivity, the consumption of each type appears in two incentive compatibility constraints, except for the highest and lowest types. Thus, the FOC for consumption levels for all but these two types are

$$N_n \left(u'\left[c_n\right] - \lambda\right) = \left(\mu_{n+1n} - \mu_{nn-1}\right) u'\left[c_n\right] \quad (4)$$

with the same expression holding for the highest and lowest types without a Lagrangian for the nonexistent incentive compatibility constraint. Each of the incentive compatibility constraints is binding, implying rising consumption with skill (so that workers are willing to take the highest skill job for which they are able). That is, with public good utility dropping out of the incentive compatibility constraint, consumption rises with skill to offset the increase in labor disutility from a more difficult job:

$$u[c_n] - u[c_{n-1}] = a_{nn} - a_{n-1n} \tag{5}$$

In this setting, the pattern of marginal taxes can be very different from that in the Mirrlees model (Diamond, 1980, Saez, 2000b).

As shown by Boadway and Keen and Kaplow using the Mirrlees model, with this additive structure of benefits we obtain the Samuelson rule. The same holds here.

1.2 Increasing Social Welfare by Subsidized Private Provision

In the equilibrium occurring with optimal taxation and public provision, no worker would make a voluntary contribution to the public good. Since, by assumption, everyone has a nonnegative marginal utility from increases in public good provision, the Samuelson rule for public good provision ensures that no single individual would have a marginal rate of substitution large enough to warrant a voluntary contribution.[6] We begin by showing that social welfare can be improved from this allocation by inducing individuals with the highest productivity to make subsidized contributions. The gain from private donations has two sides. One is that a contribution to financing the public good by the highest type substitutes their consumption for government resources (without violating the incentive compatibility constraint), which is a gain for social welfare. The second is that a highest type worker considering switching to a lower paid job would then perceive a drop in public good provision. Thus, compared with completely public provision, there is a weakening in the incentive compatibility constraint. This can be used to tax the highest workers more heavily (net of contributions), freeing up valuable resources. Since no other workers can earn this much, this opportunity for the highest earners does not change the equilibrium for other workers. With preferences the same (apart from labor disutility) across skill levels, if those with the highest income are just willing to contribute a little to the public good, those with lower incomes are not willing to contribute, so we do not need the restriction that only the highest earners have access to subsidized

[6] We assume throughout the section without warm glow preferences that in the absence of subsidization there are no private donations, even though the Samuelson rule may not hold.

donations for this argument. The two sides of this effect are shown in two separate proofs of this welfare gain.

We begin by considering the level of private donations. For later use we use general notation, although in this subsection we restrict the subsidy to the highest earners. Denote by s_n the fraction of the public good contribution by a worker holding a job of skill n that is financed by the government. We denote earnings net of tax by x_n and the addition to the public good financed by the donation by g_n, so that we have

$$c_n = x_n - (1 - s_n)\, pg_n \tag{6}$$

Given the opportunity to contribute, a worker of type n choosing a job of type n makes the donation decision to maximize utility which can be written:

$$u\left[x_n - (1 - s_n)\, pg_n\right] - a_{nn} + b_n v\left[G_{\sim n} + g_n\right] \tag{7}$$

where we have introduced the notation $G_{\sim n}$ equal to the aggregate level of public good financed by contributions by other workers and the government as perceived by a worker of type n (including the donations by others of the same type). In equilibrium, own contribution plus perceived contributions of others will sum to the level of supply, G.

The contributions for workers who make positive contributions satisfy

$$(1-s_n)\,pu'\,[c_n] = (1-s_n)\,pu'\,[x_n-(1-s_n)\,pg_n] = b_n v'\,[G_{\sim n}+g_n] = b_n v'\,[G] \tag{8}$$

We turn now to an argument that the social welfare optimum with only public provision can be improved. This argument uses the assumption that all workers of the highest type have the same preferences, thereby making the gain from weakening the incentive compatibility constraint straightforward. Below we consider a more complex argument for the same result that extends to diverse public good preferences among the highest type. Starting with the equilibrium with the optimal taxes and government provided public good supply in Section 1.1 above, denoted by adding a * to variables, consider raising the after-tax income of the highest type enough to finance all of the public good given the subsidy rate needed for them to be willing to contribute to provide the same level of public good as at the optimum. This combination of transfer and donation subsidy leaves the real allocation totally unchanged but, as we will see, weakens the incentive compatibility constraint, thereby allowing a welfare improvement by increasing the taxation (lowering the consumption) of the highest type. Denote the skill level of the highest workers by n^H. In order to induce them to donate enough to provide the same level of public good while having the same consumption level, we must use a subsidy level that satisfies the individual first order condition:

$$\left(1 - s_{n^H}\right) p u' \left[c^*_{n^H}\right] = b_{n^H} v' \left[G^*\right] \tag{9}$$

Net of tax income must be sufficient to finance both consumption and donation:

$$x_{n^H} = c^*_{n^H} + \left(1 - s_{n^H}\right) g_{n^H} = c^*_{n^H} + \left(1 - s_{n^H}\right) G^*/N_{n^H} \tag{10}$$

To see that we have weakened the incentive compatibility constraint (and so have the potential to further raise welfare by increasing the taxation of high earners and decreasing the taxation of all lower earners) we can compare the incentive compatibility constraints with public and private provision. Note that by choosing x and s the government can select c and g. This is the place where we use the uniformity of preferences among the highest type workers. The incentive compatibility constraint with government provision

$$u\left[c^*_{n^H}\right] - a_{n^H n^H} + b_{n^H} v\left[G^*\right] \geq u\left[c_m\right] - a_{m n^H} + b_{n^H} v\left[G^*\right] \text{ for } m < n^H \tag{11}$$

changes to the following with subsidized private provision

$$u\left[c^*_{n^H}\right] - a_{n^H n^H} + b_{n^H} v\left[G^*\right] \geq u\left[c_m\right] - a_{mn^H} + b_{n^H} v\left[G^* - g_{n^H}\right] \text{ for } m < n^H \tag{12}$$

Thus the perceived drop in public good supply if switching jobs weakens the incentive compatibility constraint, allowing an increase in social welfare as long as this constraint is binding. The same argument can be seen from a dual perspective. With public provision, a worker can contribute to the public good, but faces a price p when doing so. With a subsidy, the price falls to $(1 - s_{n^H})\,p$, thereby raising utility if there is a positive donation. Switching to a lower job decreases income and therefore decreases the utility gain from the decline in the price of the public good since less (zero) will be donated at this price. Having different subsidies for different earnings levels permits the government to exploit this opportunity, although such differentiation in pricing is not needed for the argument used here.

The other side of the role of private donations can be seen in the following argument supporting the same conclusion that subsidized donations by the highest type can raise social welfare. This argument is more complex, but has the virtue of carrying over to settings where the public good preferences of the highest type might vary. As noted above, at the optimum with public provision that satisfies the Samuelson condition, no worker would make an unsubsidized donation to add to the public good. Consider the maximal subsidy for the highest type that results in no contribution, denoted s_{n^H}. Consider the impact on the Lagrangian expression from a small increase in this subsidy level without changing compensation levels. This adds to public good provision by the amount financed by donations (which reflects the response to both the subsidy increase and the donations of others). The impact on aggregated utility is the utility gain from the increase in the public good less the utility loss of the highest type from the consumption they give up for the unsubsidized share of the public good increase. This impacts the resource constraint only by the subsidized share of the cost of the increase in public good provision. In addition there is a weakening of the incentive compatibility constraint as above.

To see this formally, we rewrite the Lagrangian expression in section 1.1 to incorporate donations and evaluate the impact of subsidization at the point of zero donations and compensation and public good provision at the optimum above. We assume that the binding incentive compatibility constraints are that each worker consider the next lower skill job. We simplify the notation by writing μ_{nn-1} as μ_n. The Lagrangian expression in (2) above is now

$$\begin{aligned} L &= \sum N_n \left(u\left[x_n - (1-s_n)pg_n\right] - a_{nn} + b_n v\left[G\right]\right) \qquad (13) \\ &\quad -\lambda\left(E + pG + \sum N_n\left(x_n - (1-s_n)pg_n - n\right)\right) \\ &\quad +\sum \mu_n(u\left[x_n - (1-s_n)pg_n\right] - a_{nn} + b_n v\left[G\right] \\ &\quad -\left(u\left[x_{n-1} - (1-s_{n-1})pg_{n-1}\right] - a_{n-1n} + b_n v\left[G - g_n + g_{n-1}\right]\right)) \end{aligned}$$

A small increase in the subsidy level for the highest type, n^H, will induce small changes in their consumption, the public good level, and government resources, while leaving the consumption of others unchanged. If the public good level changes by $\frac{dG}{ds_{n^H}}$, then the consumption of each high type changes by $(1-s_{n^H})\,p\frac{dG}{ds_{n^H}}/N_{n^H}$, while the contribution financed by their donation changes by $\frac{dG}{ds_{n^H}}/N_{n^H}$, and the cost to the government budget is $s_{n^H}p\frac{dG}{ds_{n^H}}$. Examining the impact on the Lagrangian, we have

$$\begin{aligned} \frac{\partial L}{\partial s_{n^H}} &= \sum N_n b_n v'\left[G\right]\frac{dG}{ds_{n^H}} - \left(1-s_{n^H}\right)pu'\left(c_{n^H}\right)\frac{dG}{ds_{n^H}} - \lambda s_{n^H}p\frac{dG}{ds_{n^H}} \qquad (14) \\ &\quad +\mu_{n^H}\left(-\left(1-s_{n^H}\right)pu'\left(c_{n^H}\right)\left(\frac{1}{N_{n^H}}\right) + b_{n^H}v'\left[G\right] - b_{n^H}v'\left[G\right](1-\frac{1}{N_{n^H}}\right)\frac{dG}{ds_{n^H}} \end{aligned}$$

Simplifying and using the FOC for the consumption of the highest type, (9), and for public good provision, (3), we can write this as

$$\begin{aligned}
\frac{\partial L}{\partial s_{n^H}} &= \left(\lambda p - \left(1 - s_{n^H}\right) p \left(\lambda - \frac{\mu_{n^H}}{N_{n^H}} u'\left(c_{n^H}\right)\right) - \lambda s_{n^H} p\right) \frac{dG}{ds_{n^H}} \\
&\quad + \mu_{n^H} \left(- \left(1 - s_{n^H}\right) p u'\left(c_{n^H}\right) \left(\frac{1}{N_{n^H}}\right) + b_{n^H} v'\left[G\right] \left(\frac{1}{N_{n^H}}\right)\right) \frac{dG}{ds_{n^H}} \\
&= \frac{\mu_{n^H}}{N_{n^H}} b_{n^H} v'\left[G\right] \frac{dG}{ds_{n^H}} > 0
\end{aligned} \tag{15}$$

The positivity of this impact follows from the gain the the highest type perceive from their own contributions to the public good. Since they are the highest type, no other type has its incentive compatibility constraint weakened by the subsidization of donations by the highest type. From the social welfare FOC for consumption of the highest type, (4), and the individual FOC for donations, (9), this can be expressed in terms of the difference between the low marginal utility of their consumption as a consequence of their being the best paid workers in the economy and the Lagrangian on resources:

$$\frac{\partial L}{\partial s_{n^H}} = \frac{\mu_{n^H}}{N_{n^H}} \left(1 - s_{n^H}\right) p u'\left(c_{n^H}\right) \frac{dG}{ds_{n^H}} = -\left(1 - s_{n^H}\right) p \left(u'\left[c_{n^H}\right] - \lambda\right) \frac{dG}{ds_{n^H}} > 0 \tag{16}$$

That is, the gain can be written in terms of the amount of donations and the difference in value of donations in the hands of the donators as opposed to the hands of the government. Writing the expression in this form makes it clear that the argument does not need an assumption that all of the highest types value the public good the same. For example, in the model considered below, where some of each type do not value the public good at all the argument still carries over.

This argument rests heavily on the assumption that there is no cost of fund-raising. As stated in the introduction, this is an exploration of economic mechanisms, not a discussion that is directly policy relevant.

1.3 Optimal Subsidized Private Provision with Two Types of Workers

Next let us consider optimizing the allocation with the possibility of subsidizing the contributions of workers with different earnings at different rates. Since the choice of subsidy rate is equivalent to the choice of contribution level, we can let the contributions be the controls. Let us assume there are only two types. As we will see the optimum will have only the higher type contributing (or an equivalent allocation).

To set this up formally, we have

$$\text{Maximize}_{c,G,g} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right] \right)$$

$$\begin{aligned} \text{subject to:} \quad & E + pG + \sum N_n \left(c_n - n\right) \leq 0 \\ & u\left[c_2\right] - a_{22} + b_2 v\left[G\right] \geq u\left[c_1\right] - a_{12} + b_2 v\left[G - g_2 + g_1\right] \\ & G \geq \sum N_n g_n; \;\; g_n \geq 0 \text{ for all } n \end{aligned} \tag{17}$$

The optimum will have one of two forms - either both have the same consumption and (generically) the incentive compatibility constraint is not binding, or the optimum will have the incentive compatibility constraint binding. Both seem theoretically possible even when the constraint is binding with public provision.

Since individual donations enter only the incentive compatibility constraint, if that constraint is binding, the optimum will have only the higher type contributing $g_2 > 0, g_1 = 0$. In this case, the consumption allocation is similar to that in Section 1.1 above, with $c_2 > c_1$, although c_2 is lower relative to c_1 in the incentive compatibility constraint and the Samuelson rule may no longer hold. If the incentive compatibility constraint is not binding, then both skill types have the same consumption - higher skill workers are paid enough more to make their contributions to the public good. They are willing to undertake a more arduous job because of the increase in the public good that they perceive from the higher contribution they make when holding a higher paying job. That is, if the workers care enough about the public good it is not necessary to give higher consumption in order to induce employment at a more difficult job.

In both cases, we have an optimum with $g_1 = 0$ and $g_2 = G/N_2$. If the incentive compatibility constraint does not bind, other allocations of public good contributions are also optimal as long as the incentive compatibility constraint continues not to bind. In the allocation with $g_1 = 0$, allowing type 1 to contribute with the same subsidy rate as type 2 does not change the equilibrium since type 1 will not contribute (same v', equal or higher u' at $g_1 = 0$). Note that the allocation could have the property that the highest type receive a net-of-tax, gross-of-contribution income which exceeds their productivity.

Note that the subsidy rate for type 1 can exceed the subsidy rate for type 2 and still support the optimum. Using the maximal subsidy for type 1 that still leaves a zero contribution, we have:

$$\begin{aligned}(1-s_1)\,pu'[c_1] &= b_1 v'[G] &(18)\\ (1-s_2)\,pu'[c_2] &= b_2 v'[G] &(19)\end{aligned}$$

With $b_1 = b_2$, we have $s_1 \geq s_2$ since $c_1 \leq c_2$ with strict inequality if the incentive compatibility constraint is binding.

In the case that the incentive compatibility constraint continues to bind, the constraint becomes

$$u[c_2] - a_{22} + b_2 v[G] = u[c_1] - a_{12} + b_2 v[G - G/N_2] \quad (20)$$

Contrasting this with a binding incentive compatibility constraint without donations, we note that in the presence of donations c_2 is lower relative to c_1 because of the term $v[G - G/N_2]$ rather than $v[G]$ on the right hand side.

The FOC for the public good satisfies:

$$\sum N_n b_n v'[G] = \lambda p - \mu b_2 \left(v'[G] - v'[G(1-1/N_2)](1-1/N_2)\right) \quad (21)$$

This may or may not satisfy the Samuelson rule and can deviate in either direction depending on the shape of the public good utility function, that is the sign of $(v'[G] - v'[G(1-1/N_2)](1-1/N_2))$.

With more than two types there are different types of equilibria along the same lines, with the possibility of some incentive compatibility constraints binding and others not binding. In the Appendix we explore the case with three types of workers and the case with uniform subsidization.

In this setting it does not matter whether the government has the ability to directly contribute to the public good or not, but this does not extend to multiple public goods with uniform subsidization. If public good preferences vary across skills, we might now have lower skill workers contributing to different public goods than higher skill workers.

1.4 Extensions

1.4.1 Diverse Preferences with Two Types of Workers

We now drop the assumption that all workers of a given skill have the same preferences and assume that a fraction f_n of workers of type n have no utility from the public good. The second argument above that subsidized donations by the highest skill type can improve welfare above the optimum with only public provision by carries over to this case. Since those with and without public good concern receive the same pay gross of donations, there is a further constraint on the ability of the government to control both consumption and donations. The incentive compatibility constraints will differ for the workers with different preferences. A type-2 worker who does not value the public good will choose a type-2 job under the condition

$$u\left[x_2\right]-a_{22} \geq u\left[x_1\right]-a_{12} \tag{22}$$

The condition is exactly the same for a type-2 worker who does value the public good if there are no private donations. If there are private donations, then the constraint becomes

$$\text{Max } u\left[x_2-\left(1-s_2\right) p g_2\right]-a_{22}+b_2 v\left[G_{\text{-}2}+g_2\right] \geq \text{Max } u\left[x_1-\left(1-s_1\right) p g_1\right]-a_{12}+b_2 v\left[G_{\text{-}2}+g_1\right] \tag{23}$$

where we denote the perceived public good provided by everyone else but a worker of type 2 by $G_{\text{-}2}$. Assuming that type-1 workers do not receive a large enough subsidy to donate, this becomes

$$\text{Max } u\left[x_2-\left(1-s_2\right) p g_2\right]-a_{22}+b_2 v\left[G_{\text{-}2}+g_2\right] \geq u\left[x_1\right]-a_{12}+b_2 v\left[G_{\text{-}2}\right] \tag{24}$$

Subtracting the incentive compatibility constraint for those who do not value the public good from that of those who do, under these circumstances the former constraint will imply the latter provided

$$\text{Max } u\left[x_2 - (1 - s_2)\, pg_2\right] + b_2 v\left[G_{\sim 2} + g_2\right] \geq u\left[x_2\right] + b_2 v\left[G_{\sim 2}\right] \tag{25}$$

which is true as a maximization.

Given this structure of incentive compatibility constraints, in looking for an optimum there are two structures, both of which seem to be possible optima for different fractions of the highest skill types with the different preferences. In one possible optimum all workers of the highest skill work at the highest job, implying that the incentive compatibility constraint on the workers who donate does not bind (since it is strictly implied by the incentive compatibility constraint of nondonators). In the other possible optimum the workers of the highest skill who do not value the public good choose a lower-skilled job and the incentive constraint is binding on the remaining highest skill workers at the highest job. Presumably the allocation that is better depends on the relative size of the highest types who do and do not value the public good. In the Appendix, we present the FOC in both cases.[7]

1.4.2 Two Public Goods

Returning to the setting where all workers of a given skill have the same utility functions, we examine some complications arising from multiple public goods. We begin by considering the level of private donations. Given the opportunity to contribute, a worker of type n choosing a job of type n makes the donation decision to maximize utility which can be written:

$$u\left[x_n - \sum (1 - s_n)\, p g_{kn}\right] - a_{nn} + \sum b_{kn} v_k \left[G_{\tilde{}kn} + g_{kn}\right] \qquad (26)$$

where we have added a subscript k to distinguish public goods. We denote by s_n the fraction of the public good contribution financed by the government, which is assumed to be the same for all public goods. When different types of workers have different relative valuations of the different public goods, workers of the highest type will only contribute to one public good since they will not generally have the same marginal utility of the different public goods.

[7] Another approach to the trade-off between those who do and do not value the public good could be modeled by having a distribution of disutilities for each skill type (as in Diamond, 1980). Then there would be a marginal worker at each type with different margins for those with and without donations and so there would be some effect on labor supply from private contributions.

(This is one of the arguments for the empirical irrelevance of this model of donations.) Thus there may be a role for public contributions when preferences vary by good across skills since subsidized donations by lower-skilled workers weaken the incentive compatibility constraints for higher skilled workers.

1.4.3 No Government Contribution

The model has a slightly different character if the government lacks the ability to directly contribute to a public good. At the equilibrium level of private supply without subsidy (which might be zero), some subsidy will be worthwhile, with a corresponding adjustment of net-of-tax earnings. The only change in the formal statement of the maximization problem is to have an equality, rather than an inequality, in the definition of G, thereby ruling out public contributions. When the model has multiple public goods and diverse public good preferences across skill levels, then there is an argument for subsidization of donations by lower-skilled workers, which may overcome the impact on the incentive compatibility constraint in some cases. With a restriction to uniform subsidization across goods, the model would more closely resemble US tax policy.

2 Warm Glow Preferences

The failure of standard preferences to make sense of the pattern of donations has been widely noted (e. g., Margolis (1982), Morgan, Dye, and Hybels (1977)). In particular, many people spread their donations widely across charitable organizations even though their contributions are small relative to organizational budgets. Standard theory would direct all donations to the charity with the highest marginal utility of public good consumption until that had been reduced to equal the next highest. With small donations, such a move is unlikely to happen. To explain this pattern, a natural assumption is that of warm glow preferences (Andreoni (1990)). A realistic model of donations would pay close attention to the context of requests for donations as well as awareness of perceived social needs. This paper does not explore such modeling. Instead, we take the simple formulation of Andreoni and explore the implications of such a structure for optimal tax policy.

With warm glow preferences, behavior is modeled as if it maximized utility that depends not just on the final allocation of resources but also on the process that results in that allocation. That is, we add to preferences as above a concave utility of one's own donation, denoted $w[g]$, but no similar per se utility gain from the donations of others or public contributions.[8] In a context of multiple public goods and public subsidies, there are a variety of ways in which such preferences could be modeled, which we do not explore.[9] Thus the public good component of utility is $b_n v[G] + w[g_n]$. This additive structure is simple but not fully plausible. Presumably the warm glow utility also depends on the aggregate level of public good provision. We write the preferences so that behavior can be described by maximization of utility written as $u[c_n] - a_{nn} + b_n v[G] + w[g_n]$. Given this structure of preferences, the FOC for individual donations for workers who make positive contributions satisfies

$$\begin{aligned} (1-s_n)\,pu'[c_n] &= (1-s_n)\,pu'[x_n - (1-s_n)\,pg_n] \\ &= b_n v'[G_{\tilde{}n} + g_n] + w'[g_n] = b_n v'[G] + w'[g_n] \end{aligned} \tag{27}$$

That behavior is describable in this way does not necessarily imply that social welfare should be defined in the same way. That is, assume, as above,

[8] More complicated relative to the latter two assumptions seem the efforts of some to get others to donate or to lobby for more public provision. Thus the warm glow seems to come from activities to change public good provision, beyond just one's own donations.

[9] The pattern of giving across charities is complex. Donations to some charities are close warm glow substitutes for donations to similar charities, particularly when the donations are close in time. For example, preferences might be written as $u[c_n] - a_{nn} + \sum v_k[G_k] + V[\sum w_k[g_{nk}, G_k]]$

that the government switches from public provision to private provision, with the level of public good unchanged. Should the warm glows from private provision be part of a gain in social welfare from undertaking this change in financing? Perhaps there is resentment at the need to provide privately what is seen as a government obligation. Perhaps the "warm glow" comes from decreasing the disutility coming from pressure to donate, suggesting that the utility level from warm glow is negative, although with a positive derivative. This would matter in an equilibrium model where the degree of charitable solicitation depended on government policies as to public provision and subsidization of private donations. Below we will discuss the normative issues in this choice. First, we consider optimization with and without incorporation of warm glow in social welfare. For this purpose, we set up the optimization with a parameter, θ, that can be set to zero or one.

2.1 Social Welfare Optimization

Compared to the problem in Section 1, the objective function may or may not be the same, the resource constraint is unchanged, and the incentive compatibility constraints are changed by the presence of warm glow. Moreover, assuming that contributions can be subsidized but not taxed, there is an inequality constraint that they be at least as large as the endogenous level without any subsidy (rather than simply being constrained to be nonnegative).

The donation level with a zero subsidy (the minimum donation level) satisfies the individual FOC relating the marginal utility of private consumption to the marginal utility of consumption of the public good and the marginal warm glow. This implicit equation for the minimum donation can be expressed in terms of several different variables.

$$pu'[c_n] = pu'[x_n - pg_n] = b_n v'[G_{\sim n} + g_n] + w'[g_n] = b_n v'[G] + w'[g_n] \quad (28)$$

Depending on what control variables are used in the social welfare maximization, one would select the implicit equation in those variables for the minimum donation constraint. For example, if social welfare maximization is expressed in terms of private consumption and total public goods supply, one would have the constraints $g_n \geq g[c_n, G]$, with the minimum donation function given by the first and last expressions in (28). Note that it is the same function for all if b_n is the same for all, as we assume. Differentiating the implicit equation, we see that the minimum donation is increasing in private consumption and decreasing in total public good supply.

$$\frac{\partial g}{\partial c} = \frac{pu''}{w''} > 0, \frac{\partial g}{\partial G} = \frac{-bv''}{w''} < 0. \tag{29}$$

Subject to the minimum donation constraint, we write the social welfare maximization in terms of consumption, total public good supply and donations. Considering the two-types model with one public good, we can write a formulation allowing warm glow to enter ($\theta = 1$) and not enter ($\theta = 0$) social welfare.

$$\begin{aligned}
&\text{Maximize}_{c,G} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + bv\left[G\right] + \theta w\left[g_n\right]\right) \\
&\text{subject to:} \quad E + pG + \sum N_n \left(c_n - n\right) \leq 0 \\
&\qquad u\left[c_2\right] - a_{22} + bv\left[G\right] + w\left[g_2\right] \geq u\left[c_1\right] - a_{12} + bv\left[G - g_2 + g_1\right] + w\left[g_1\right] \\
&\qquad G \geq \sum N_n g_n; \;\; g_n \geq g\left[c_n, G\right] \text{ for all } n
\end{aligned} \tag{30}$$

Differentiating we have lots of FOC:

$$\frac{\partial L}{\partial c_1} = N_1 \left(u'\left[c_1\right] - \lambda\right) - \mu u'\left[c_1\right] - \xi_1 \frac{\partial g\left[c_1, G\right]}{\partial c} = 0 \tag{31}$$

$$\frac{\partial L}{\partial c_2} = N_2 \left(u' \left[c_2\right] - \lambda\right) + \mu u' \left[c_2\right] - \xi_2 \frac{\partial g \left[c_2, G\right]}{\partial c} = 0 \tag{32}$$

$$\frac{\partial L}{\partial G} = \sum \left(N_n b v' \left[G\right] - \xi_n \frac{\partial g \left[c_n, G\right]}{\partial G} \right) - \lambda p + \mu b \left(v' \left[G\right] - v' \left[G - g_2 + g_1\right]\right) + \nu = 0 \tag{33}$$

$$\frac{\partial L}{\partial g_1} = N_1 \theta w' \left[g_1\right] - \mu \left(b v' \left[G - g_2 + g_1\right] + w' \left[g_1\right]\right) - \nu N_1 + \xi_1 = 0 \tag{34}$$

$$\frac{\partial L}{\partial g_2} = N_2 \theta w' \left[g_2\right] + \mu \left(b v' \left[G - g_2 + g_1\right] + w' \left[g_2\right]\right) - \nu N_2 + \xi_2 = 0 \tag{35}$$

We consider the FOC separately for the values of θ of 0 and 1.

2.2 Warm Glow Preferences that do not enter Social Welfare

For warm glow preferences not entering the social welfare function, we set $\theta = 0$. This is the setting closest to that in Section 1 above. Warm glow affects the incentive compatibility constraint. Also the condition that donations not be subsidized may not be sufficient for them to equal zero. In this case, the impact of private consumption and public good level on donations will affect the optimal allocation. To consider this case, note that increasing the donation of the high type while lowering the donation of the low type weakens the incentive compatibility constraint while having no other effects, until the lower limit on g_1 is hit. Similarly, donations by the high type dominate public provision. Thus we know that $g_1 = g[c_1, G]$. Assuming that public good supply is less than optimal without any subsidization, we also have no binding minimum donation constraint for the high type, $\xi_2 = 0$. Thus we can write the FOC (31) and (32) for the case $\theta = 0$ as

$$\frac{\partial L}{\partial c_1} = N_1 (u'[c_1] - \lambda) - \mu u'[c_1] - \xi_1 \frac{\partial g[c_1, G]}{\partial c} = 0 \tag{36}$$

$$\frac{\partial L}{\partial c_2} = N_2 (u'[c_2] - \lambda) + \mu u'[c_2] = 0 \tag{37}$$

It is possible that the marginal warm glow utility at zero donations, $w'[0]$, is small enough that there can be positive subsidization and still no donation (as in Section 1). Provided this is not the case, then the gain from limiting the donations of the lower type, $\xi_1 \frac{\partial g[c_1,G]}{\partial c}$, enters the FOC. I have no simple statement of the effect of this term since this FOC interacts with the other FOC and the incentive compatibility constraint in determining the relative consumption levels of the different types.

Turning to the other FOC, with g_1 at its minimum, (33) and (35) become

$$\frac{\partial L}{\partial G} = bv'[G] \sum N_n - \xi_1 \frac{\partial g[c_1, G]}{\partial G} - \lambda p + \mu b(v'[G] - v'[G - g_2 + g_1]) + \nu = 0 \tag{38}$$

$$\frac{\partial L}{\partial g_2} = \mu(bv'[G - g_2 + g_1] + w'[g_2]) - \nu N_2 = 0 \tag{39}$$

It is plausible for all but very large donors that v'' is small enough that $v'[G] - v'[G - g_2 + g_1]$ is very small. Then the FOC for public good supply is approximately

$$\frac{\partial L}{\partial G} \approx bv'[G] \sum N_n - \xi_1 \frac{\partial g[c_1, G]}{\partial G} - \lambda p + \nu = 0 \tag{40}$$

Using (39), we can express the deviation from the Samuelson condition as

$$bv'[G] \sum N_n - \lambda p \approx \xi_1 \frac{\partial g[c_1, G]}{\partial G} - \nu = \xi_1 \frac{\partial g[c_1, G]}{\partial G} - \frac{\mu}{N_2}(bv'[G - g_2 + g_1] + w'[g_2]) \tag{41}$$

Using again the assumption of a small v'', and so approximate constancy of v', and the FOC for individual donations, we can write this as

$$bv'[G] \sum N_n - \lambda p = \xi_1 \frac{\partial g[c_1, G]}{\partial G} - \frac{\mu}{N_2}(1 - s_2) pu'[c_2] \tag{42}$$

From the FOC for consumption, this can be written as

$$bv'[G]\sum N_n - \lambda p \approx \xi_1 \frac{\partial g[c_1, G]}{\partial G} + (1 - s_2)\, p\,(u'[c_2] - \lambda) \tag{43}$$

or

$$bv'[G]\sum N_n \approx \xi_1 \frac{\partial g[c_1, G]}{\partial G} + s_2 p \lambda + (1 - s_2)\, p u'[c_2] \tag{44}$$

In contrast with a setting of purely public provision, public good supply tends to be increased because it eases the incentive compatibility constraint by lowering the minimal donation of the lower type and because some of the cost comes from consumption of the higher paid rather than from the government budget constraint.

Note that if the two types of workers care about different public goods, it remains the case that type 2 should be induced to contribute all of the public good provision that type 2 likes, while type 1 should not contribute above the minimum, with the public good valued by type 1 publicly provided. This follows from a uniformity of any subsidy at a given earnings level, independent of which public good is being supported.

2.3 Warm Glow Preferences that do enter Social Welfare

By having contributions enter the social welfare function as well as the incentive compatibility constraint, marginal private donations do not directly affect social welfare by the envelope theorem. In this case the FOC (31) - (35) become

$$\frac{\partial L}{\partial c_1} = N_1 \left(u' \left[c_1\right] - \lambda\right) - \mu u' \left[c_1\right] - \xi_1 \frac{\partial g \left[c_1, G\right]}{\partial c} = 0 \tag{45}$$

$$\frac{\partial L}{\partial c_2} = N_2 \left(u' \left[c_2\right] - \lambda\right) + \mu u' \left[c_2\right] - \xi_2 \frac{\partial g \left[c_2, G\right]}{\partial c} = 0. \tag{46}$$

$$\frac{\partial L}{\partial G} = \sum \left(N_n b v' \left[G\right] - \xi_n \frac{\partial g \left[c_n, G\right]}{\partial G} \right) - \lambda p + \mu b \left(v' \left[G\right] - v' \left[G - g_2 + g_1\right]\right) + \nu = 0 \tag{47}$$

$$\frac{\partial L}{\partial g_1} = N_1 w' \left[g_1\right] - \mu \left(b v' \left[G - g_2 + g_1\right] + w' \left[g_1\right]\right) - \nu N_1 + \xi_1 = 0 \tag{48}$$

$$\frac{\partial L}{\partial g_2} = N_2 w' \left[g_2\right] + \mu \left(b v' \left[G - g_2 + g_1\right] + w' \left[g_2\right]\right) - \nu N_2 + \xi_2 = 0 \tag{49}$$

It no longer appears that the donations of the low type should never be subsidized. Substituting donations by the low type for those of the high type when only the latter is subsidized is a way of raising social welfare (since they differ in marginal warm glow) that does not change resource use and is attractive for that reason, although it weakens the incentive compatibility constraint and is unattractive for that reason. The dominance of donations of the high type over public provision remains true.

If donations of both types are subsidized ($\xi_1, \xi_2 = 0$), the private consumption FOC (45) and (46) have the same form as with fully public provision.

$$\frac{\partial L}{\partial c_1} = N_1 \left(u' \left[c_1\right] - \lambda\right) - \mu u' \left[c_1\right] = 0 \tag{50}$$

$$\frac{\partial L}{\partial c_2} = N_2 \left(u' \left[c_2\right] - \lambda\right) + \mu u' \left[c_2\right] = 0 \tag{51}$$

The possibility that both types are subsidized allows an optimal consumption FOC without the term $\xi_1 \frac{\partial g[c_1,G]}{\partial c}$ which is necessarily present in the case where warm glow does not enter the social welfare function. Solving for marginal utilities while assuming that the incentive compatibility constraint is binding ($\mu > 0$), we have

$$\frac{u' \left[c_1\right]}{u' \left[c_2\right]} = \frac{1 + \mu/N_2}{1 - \mu/N_1} > 1 \tag{52}$$

If donations of both types are subsidized ($\xi_1, \xi_2 = 0$), the FOC for donations (48) and (49) become:

$$\frac{\partial L}{\partial g_1} = N_1 w'[g_1] - \mu\left(bv'[G - g_2 + g_1] + w'[g_1]\right) - \nu N_1 = 0 \tag{53}$$

$$\frac{\partial L}{\partial g_2} = N_2 w'[g_2] + \mu\left(bv'[G - g_2 + g_1] + w'[g_2]\right) - \nu N_2 = 0 \tag{54}$$

Solving for $w'[g_n]$ we have

$$\left(1 - \frac{\mu}{N_1}\right) w'[g_1] = \nu + \frac{\mu}{N_1} bv'[G - g_2 + g_1] \tag{55}$$

$$\left(1 + \frac{\mu}{N_2}\right) w'[g_2] = \nu - \frac{\mu}{N_2} bv'[G - g_2 + g_1] \tag{56}$$

Using (53), we can evaluate the total utility gain from a donation by the lower type

$$\begin{aligned} bv'[G] + w'[g_1] &= bv'[G] + \left(\nu + \frac{\mu}{N_1}\left(bv'[G - g_2 + g_1]\right)\right) / \left(1 - \frac{\mu}{N_1}\right) \qquad (57) \\ &= \left(bv'[G] + \nu + \frac{\mu}{N_1}\left(bv'[G - g_2 + g_1] - bv'[G]\right)\right) / \left(1 - \frac{\mu}{N_1}\right) \end{aligned}$$

From (27) and (57), we can evaluate the subsidy level.

$$(1 - s_1) = \frac{bv'[G] + w'[g_1]}{pu'[c_1]} = \frac{bv'[G] + \nu + \frac{\mu}{N_1}(bv'[G - g_2 + g_1] - bv'[G])}{p\lambda} \tag{58}$$

Similarly, for the high type

$$(1 - s_2) = \frac{bv'[G] + w'[g_2]}{pu'[c_2]} = \frac{bv'[G] + \nu - \frac{\mu}{N_2}(bv'[G - g_2 + g_1] - bv'[G])}{p\lambda} \tag{59}$$

Taking the ratio

$$\frac{1 - s_1}{1 - s_2} = \frac{bv'[G] + \nu + \frac{\mu}{N_1}(bv'[G - g_2 + g_1] - bv'[G])}{bv'[G] + \nu - \frac{\mu}{N_2}(bv'[G - g_2 + g_1] - bv'[G])} > 1 \tag{60}$$

Thus the high type receives a higher subsidy. If, as is plausible, v'' is small relative to individual donations, then both types have approximately the same rate of subsidization.

If donations of both types are subsidized ($\xi_1, \xi_2 = 0$), and also assuming v'' is small relative to individual donations, the FOC for public good level (33) becomes approximately:

$$bv'[G] \sum N_n - \lambda p + \nu \approx 0 \tag{61}$$

Consideration of the FOC for total supply parallels that in the case where warm glow does not enter social welfare, with the significant difference that the relevance of warm glow for social welfare reduces the apparent cost of the public good. Substituting from (54) and then (27) and then (51), we have.

$$\begin{aligned} bv'[G]\sum N_n &\approx \lambda p - \nu = \lambda p - w'[g_2] - \frac{\mu}{N_2}\left(bv'[G] + w'[g_2]\right) \qquad (62) \\ &= \lambda p - w'[g_2] - \frac{\mu}{N_2}(1 - s_2)\,pu'[c_2] \\ &= \lambda p - w'[g_2] + (1 - s_2)\,p\left(u'[c_2] - \lambda\right) \\ &= s_2\lambda p + (1 - s_2)\,pu'[c_2] - w'[g_2] \end{aligned}$$

If marginal warm glow utility, w', is much larger than marginal public good utility, v',then the last two terms approximately cancel and the FOC is as if the only cost of public goods were the marginal cost to the government of the marginal contribution by the high type. Similarly, the FOC can be expressed in terms of the marginal utilities of type 1.

$$bv'[G]\sum N_n \approx s_1\lambda p + (1 - s_1)\,pu'[c_1] - w'[g_1] \qquad (63)$$

2.4 Warm Glow Preferences and the Formulation of Social Welfare

Standard economic analysis views decisions as maximizing utility or optimizing preferences, with the preferences revealed by this maximization an accurate measure of consumer well-being. It is then plausible that these revealed preferences should also be the basis for normative analysis.[10] Indeed, this is the basis for the argument for consumer sovereignty. While it is standard to use a single utility measure for all three of consumer decisions, consumer well-being and normative evaluation, there has long been recognition of the inadequacy of this single measure in some circumstances.[11] One example is in legislated cooling-off periods, that allow consumers to cancel contracts within a few days of signing without any penalties. This recognizes the possibility of time-inconsistency and gives weight to the later (possibly less pressured) decision, not the earlier one, should they be in conflict. Similarly, in the realm of social security, policy appears to be driven in part by a sense that left to their own devices, many workers would not save enough for retirement.

[10] The discussion here is of the formulation of a social welfare function, not of Pareto comparisons. I am not convinced that these are necessarily congruent in all contexts.

[11] This focus on utility ignores other aspects of a good life.

Public pension reform proposals that replace traditional defined benefit systems with individual accounts generally include mandatory savings. Indeed this was the case with the Chilean reform. While one might argue that this is merely preventing free riding on minimum incomes for the elderly, this argument cannot explain policies that require savings well in excess of levels provided by such minimum income programs. Furthermore, in Chile, early access to retirement funds is available based on sufficient replacement rates, not solely absolute income levels.

Economists have recognized that some decisionmaking occurs with faulty or incomplete information.[12] Then, normative analyses should rely on more accurate probabilities if they are available. That is, policies should be evaluated based on the best available estimates of the probabilities of different outcomes, not the subjective probabilities of affected households that are missing pieces of information. Under this scenario, the evaluation is being based on what the households would want if they had the additional information available to the government. This approach leads to the provision

[12] Economists seem comfortable in treating decisions made in such a setting as reflecting incorrectly specified preferences, even though in basic Arrow-Debreu theory subjective probabilities, if they exist in a meaningful sense, are simply part of preferences. That there may not be meaningful subjective probabilities inherent in preferences has been noted in Yaari ().

of information as a useful (and sometimes possibly sufficient) policy intervention. However, provision of information may not be an option in some circumstances, and may not be needed for decisions by government. Further from standard analysis would be a recognition of failures to process information, rather than just faulty information or a lack thereof. Failures in combining alternative sources of information appear common. Recognizing failures to correctly implement Bayes rule leads one to consider substituting the correct information processing for the actual one. Compared with recognizing a gap in information, this is a step to closer to recognizing a failure to optimize relative to a normative standard for individual utility.

Behavioral economics has generated several descriptions of consumer behavior that have required a reexamination of normative considerations. When decisionmakers are not time consistent, there is an obvious need to modify standard normative practice. A standard vocabulary for describing the mathematical formulation of such decisionmaking is as a game among "different selves." Some have concluded that this justifies treating each of these separate decisionmakers as separate persons, each deserving of full normative treatment along the lines of standard analysis. I think this is a semantic confusion, transferring to a normative realm a vocabulary that is a useful shorthand in describing a mathematical model of behavior. There is a great deal shared by the "different selves" involved in a lifetime of decisions. It does not appear to make sense to give full weight to each of these selves as if they were totally different people. Such an approach would give much higher normative weight to those expecting to have longer lives, by counting each of these selves as a separate equally relevant unit for normative evaluation, even though they all care about past or anticipated consumption of the same person in the usual sense of person. Another approach in the literature is to ignore the extra weight given to current consumption in decisionmaking and do normative evaluations with the lifetime evaluation shared by all of these selves, apart from the salience of the present (different for each self). With the usual beta-delta formulation of quasi-hyperbolic preferences, this is equivalent to summing the utilities of contemporaneous consumption over a lifetime. This seems a more appealing approach to normative analysis, although it might be more difficult to sort out without the empirically implausible additive structure of decision utility. It should be considered a useful simple measure, not necessarily the final word on how to evaluate.

Another set of issues in behavioral economics arises from recognition of concern for others, whether through altruism, envy, or a sense of fairness.

Simply carrying the relationship between positive and normative analyses from conventional theory to this setting implies giving extra weight (possibly positive, possibly negative) to the objects of these feelings about others. The question raised by this practice is whether it is "double counting" in the sense that the use of a social welfare function as an evaluation of outcomes reflects precisely the sense of concern for others that also shows up in individual behavior in some circumstances. Hammond (1978) has made this point about altruism, that the use of a social welfare function incorporates the social interest inherent in the concern for others and inclusion of altruistic utility is then a form of double counting. That is, it is the role of a social welfare function as a guide to policy to reflect the concerns that people in a society have for others in that society. Indeed, Margolis (1982) has argued that we can think of individuals as having two sets of preferences, ones that are self-interested and ones that are group-interested. Behavior in different settings reflects the different preferences in different proportions. Voting is a setting where public concerns may affect behavior more than when making purchases. If the role of a social welfare function is to reflect the public selves of individuals, when they are being most reflective and open (possibly behind a veil of ignorance), then it would be double counting to use the aspect of their preferences that motivates the social welfare function as an argument of the social welfare function. A consequence would be to give much larger consumptions to those who are objects of positive feelings of others. It is not apparent that this is an attractive outcome, just as it is not apparent that it is attractive to give higher weight to the longer lived because of having more "selves."

When looking for a reprint to help with lecture preparation, I recently stumbled over a copy of a 1973 paper that Jim Mirrlees presented to the European Meetings of the Econometric Society. This paper considered the design of normative criteria in the presence of charitable contributions and went on to analyze optimal subsidization of charitable donations. The model used did not have endogenous labor supply, so the theoretical parts do not relate to this paper. However, the discussion of normative criteria does. Mirrlees recognized three models for individual donation decisions. In addition to the standard model, he noted what is now called a warm glow model, that utility includes the "increased pleasure or reduced pain brought about by the gift itself." And he noted that decisions are based on maximizing a function of own utility and of the welfare of society, as perceived by the decisionmaker. He examined policy under the latter formulation and under the standard model, which he described as “really rather odd.” He wrote utility in the nonstandard case as $V_h(u_h(x^h, Z), W_h(Z))$, where Z is the level of public good. Considering how to construct a “social valuation function,” he wrote: ”But if individual preferences are given by (1) , with the interpretation suggested above, there is no reason at all for specially considering a social valuation function that is a function of the V_h. There is more reason for assigning objectives that depend only on the W_h: in the special case, where all are identical, we might take W as the social valuation function, since everyone would agree that it ought to be.”

Still other considerations arise from the observation that the context of individual decisions has a strong influence on decisions (as with the cooling-off period after a purchase from a door-to-door salesperson). Whether this is the purchase of individual items (as in Thaler’s example of beer prices) or attitudes to individual gambles (as in loss aversion) it is clear that individuals make decisions that cannot be described as a maximization of preferences defined only over the final allocation of resources. Rather the process being used to reach a final allocation affects decisionmaking. This can be described as “preferences over process.” Recognizing a preference over process in both decisionmaking and the feeling of well-being when deciding does not automatically answer the question of how normative evaluation should be made. Should the normative evaluation depend on the process to the same extent that decisionmaking does? It seems to me it should not, for several reasons. Attitudes toward resource allocation processes depend on the perception of the process and this may not be stable. Think how easy it is to switch from thinking of oneself as generous to thinking of oneself as a sucker. Use of a normative evaluation that included concern about the process would call for altering the process to take advantage of possible utility gains as a result. But the attitude toward the process is likely to be affected by knowing that the process is itself an object of government design. That is, attitudes toward process are very dependent on how the process is framed.[13] Framing can be endogenous to attempts by agents to alter the frame (think how often this happens in the political process, for example, the conflicting labels of estate tax and death tax). Moreover, there may be an inherent element of deceit if the government is trying to frame the process in a way that has a positive influence on individual evaluations - this may not be an attractive part of democratic government. Any attempt to include process in the normative

[13] I am reminded of a Mad cartoon of a sodier on guard where each item – his uniform, gun, shelter, was labeled as to whose taxes had paid for that particular item.

evaluation should include all of the process, not just part. But that then includes the government decisionmaking as part of the process being evaluated, a backwards induction that seems impractical if not downright peculiar.

These issues can be put into a single framework if one follows the distinctions made by Kahneman (2000) between decision utility and experienced utility. (For this discussion, the important roles of retrospective and predicted utilities are not needed.) He has argued that an appropriate normative criterion is to integrate (without discounting) the moment-by-moment sense of well-being - the experienced utility. Insofar as decisions vary from optimizing this measure - which may (or may not) result in some preferences the maximization of which can explain decisions (decision utility) - then we can think of such decisionmaking as a mistake. This can be seen as paralleling the discussion of information gaps above.

The issue here can be presented in the following way. With any purchase there is experienced utility associated with the act of purchase - we feel good or bad (or neutral) about the act of purchasing. Second there is the experienced utility associated with the services provided by the object of a transaction. The experienced utility associated with the act of purchase should reflect (be related to) the anticipated experienced utility associated with the services provided by the object purchased. Purchase decisions reflect some combination of the utility associated with the purchase and the anticipated utility associated with the use of the object. The cooling-off period legislation reflects a judgment that in the purchase decision the experienced utility from the act of purchase may be overwhelming the experienced utility associated with future use. That is the judgment is that this is a situation where the deviation between decision utility and total experienced utility is likely to be large, and with a cooling-off period, this deviation will enter awareness, resulting in a possible reversal of a purchase decision. From this perspective, well-being at the moment of purchase is not totally irrelevant - feeling good while making a purchase counts, as would regret after having made a purchase of an object one ends up not using. But well-being at the moment of purchase gets no more normative weight than well-being later.

Taking this overall discussion to the issue of the warm glow associated with charitable donations has two dimensions. One is an underlying complex philosophical discussion. The other is a choice among simple tractable normative criteria in order to get on with analyses of policy issues. Such a choice needs to hope for robustness - more robustness than with alternative simple formulations. It seems to me best to omit warm glow from the normative evaluation as the simple tractable formulation most likely to lead to good policies.

Warm glow as modeled above is part of decision utility. There is also experienced utility associated with the act (the moment) of donation. This experienced utility should reflect the belief about the good that the donation will do. Given the evidence that some charitable donations are made with considerable ignorance of the use of the funds, some of this experienced utility of donation should be viewed as roughly similar to the experienced utility of purchases of objects that one does not enjoy using later. There is a difference in likelihood of learning the consequences of the expenditure. If the experienced utility from the act of purchase does not correlate well with the good the donation will do, the donor will frequently not be aware of this; there may not be later "regret" as can be the case with poor forecasts of how a private good purchase will be enjoyed. The clearest example of poor correlation is where a large percentage of some donations go to fundraisers who are paid on commission. If donors are reasonably content with their donations because they do not know what is done with the funds, we have the issue of whether the normative evaluation should be based on the experienced utility or what it would have been if the donor well well-informed.

Donors might prefer a government to pay attention to what they were trying to accomplish rather than what they thought they were accomplishing. This also points up the fragility of experienced utility in this case - information spread can change the experience greatly. Ignoring warm glow when deciding how much to subsidize charitable donations may be useful for balancing the scale of this deviation. Of course, many donations are made with detailed awareness of the use of the donated funds.

As noted above, experienced utility from the act of donation is plausibly related to the good the donation is expected to do. Does this make aggregation including all of this experienced utility partially double counting? Assume we have a group decision, where people are responsive to normative discussions as part of the joint decision. If people would make group decisions in this idealized setting in a way that is not consistent with adding up experienced utilities ignoring their sources, then it can be argued that a normative evaluation should do the same. This could arise with income transfers when some people care about the utility or expenditure or income of other people. These concerns may affect their individual behavior but may be adequately reflected, in their eyes, in a social welfare function that includes others and ignores the impact of their concern on their own well-being. That is, the experienced utility of donating to do good may double count the doing of good by the donations. A key part of the reason for a warm glow is the social interest inherent in the public good enjoyment of others. With the role of a social welfare function as a guide to policy being to reflect the concerns that people in a society have for others in that society, concern for others is present in public decisionmaking without including the warm glow from the same source.

If one were to include the utility of the resource allocation process, then all of the process should be included. With the standard general equilibrium model, one tracks all uses of resources. If one were only tracking some uses and ignoring others, then the welfare implications of a policy might be distorted by the pattern of inclusion and exclusion in the analysis. Thus valid partial equilibrium analyses include a numeraire good reflecting the uses of resources that are not modeled in detail in the partial equilibrium setting. That people get pleasure from donating to finance public goods is shown by their pattern of donations. Similarly, people do volunteer work for charities to induce other people to donate. There is a warm glow associated with the donations of others. But people also participate in the political process in a way that cannot be explained solely by economic self-interest. This includes some donations to political campaigns and, of course, extends to voting. If someone gets a warm glow from work on an election that results in an increase in some public good level, then such a warm glow seems to be on the same footing as that from a charitable donation. One can argue that the political warm glow is related to final good outcome, so an increase in subsidy rate would be as appealing as an increase in direct provision. But then, should the warm glow from giving also be related to final outcomes? If the government cuts back direct provision and increases subsidies, should the warm glow of someone who cares about the public good necessarily increase from this aspect of the process? Or does the warm glow come from the net effect? If the latter, substituting subsidized donations for direct provision would not raise welfare but it would be rated an improvement by inclusion of the warm glow of donations but not of the political process.[14]

The basic point here is that just as we need to pay attention to all of resource uses, so too if we want to count preferences based on process, we need to pay attention to all of the process, not just part of it. At present,

[14]Let me note that a similar issue arises with loss aversion. Insofar as apparent (framed) gains and losses are the basis for evaluation, then incomplete tracking of the entire process of allocation would give measured welfare gains from substituting some processes for others, even if the final allocation of resources were unchanged.

paying attention to none of it may be a more valid measure even for those who think utility from process ought to be on the same welfare footing as utility from final resource use.

Consideration of all of the process affects interpretation of warm glow since warm glow is very context dependent. That is, recognizing this source of welfare benefits would call for devoting resources to producing the context where warm glows would occur. But, being context dependent, the outcome becomes very sensitive to the framing. Recognizing that the government is creating an environment where one donates might remove much of the warm glow. Just as the reaction to making a particular gift depends on whether the gift is viewed as an act of generosity or a result of having been taken advantage of, so too is a possible unreliability of preservation of warm glows if the government is particularly involved in generating them. That is, it is unclear how preferences may be influenced by the process generating the opportunity/need for donations. I cannot fully credit the possibility of someone being happy that the government underprovided a public good so that the opportunity to donate was present. They may be happy if not aware of the government role in the process. But that takes us back to the issue of whether the government should base decisions on what people think is happening or what they would think if better informed.

I do not intend to argue that the process of resource allocation is irrelevant for social welfare analysis. Thus coin tosses for fairness in allocation of an indivisible good has been argued to raise social welfare (Diamond, 1967) and denial of the ability to donate can be viewed as a violation of a basic freedom, but choice of a subsidy rate based in part on warm glow utility seems to be in a different category.

The choice of a simple tractable approach to normative analysis requires care in its applications. The belief that consumers make lots of mistakes in purchasing decisions does not imply that the government should take over general purchasing for individuals - I don't think the government would do as well as individuals do, with all their flaws. And there is the argument about free choice - a consideration based on process. But one may conclude that individuals do a poor job in some particular realm and the government may usefully respond, for example, by banning products that fail some safety test or by requiring people to pay for some level of retirement consumption. Quite separate from direct interventions to change decisions that may be poorly made is whether recognition of systematic failures to optimize purchasing decisions should influence general policies. For example, we need to reconsider the measure of deadweight burdens of taxation in a context where money is spent with mixed success. If people decide on labor supply based on overvaluing expenditures from earnings, should this alter the normative evaluation of the impact of taxes on labor supply? Any reconsideration of deadweight burden measurement will have implications for income tax policy - an implication that does not involve drawing distinctions among different goods. Similarly, there are good reasons from following the political process determining public good expenditures to like the role of private donations. That still leaves the question of the level and pattern of subsidization that encourages such donations. The nature of donation decisions seems highly relevant for the choice of subsidies. The argument here is that the more appealing simple criterion would include the implications of warm glows in the description of behavior, but exclude it from the normative criterion.

3 Fundraising

In the models above, the government can leave room for private contributions in order to alter the effect of income taxation. As modeled here and in the previous literature cited above, the use of private donations does not have any resource cost (unlike the marginal deadweight burden from financing public contributions and the subsidies of private contributions). This is factually inaccurate. Considerable sums are spent on fund-raising by organizations that provide public goods. Such an additional source of deadweight burdens should affect the analysis but is not explored here. A model with an endogenous level of resource use for fund raising as a function of both public provision and the level and pattern of donation subsidies would be interesting. Going from analysis of models without fundraising costs to policy recommendations seems very premature.

The endogeneity of charitable solicitation raises another issue that matters when warm glow is counted in social welfare. The focus on warm glow has been on the marginal warm glow gain from an increase in donation. But some of this marginal warm glow is thought to come from decreasing the negative feelings associated with social pressure to donate (Keating, Pitts, and Appel (1981), Long (1976), Morgan, Dye, and Hybels, (1977)). In this setting, the role of government policy in determining the amount of social pressure becomes a very important issue. If donation lowers a utility loss from social pressure, public policy that lowers the pressure (for example by larger public provision and, perhaps, lower subsidies) may raise utility measured in this way.

4 Concluding Remarks

This paper has taken the standard optimal tax approach, not recognizing issues in the definition of income as having independent relevance. Taking the latter approach, one could argue that donations reduce income over which a consumer maintains control and therefore ought not to be part of taxable income taken as measuring the latter. This would parallel the view that to the extent that medical expenditures are beyond the choice of a consumer, medical expenses ought to be deducted from the measure of income over which an individual has control. The roles of income definition in terms of the philosophy or the political economy of taxation are separate issues.

5 Notation

n	skill index (equal to productivity)
N_n	number of workers of skill n
x_n	compensation at job of skill n
c_n	consumption of worker holding job of skill n
$u[c_n]$	utility of private good consumption
a_{mn}	disutility of labor for a worker of skill n holding a job of skill m
G	public good supply
$b_n v[G]$	utility of public good consumption
g_n	donation of worker holding a job of skill n
$w[g_n]$	warm glow utility
$G_{\sim n}$	public good provision net of donation of a worker holding a job of skill n
p	cost per unit of the public good
E	other public expenditures
s_n	donation subsidy rate for worker in job of skill n
f_n	fraction of workers of type n who value the public good
λ	Lagrangian on the resource constraint
μ_n	Lagrangians on the incentive compatibility constraints

ν Lagrangian on the adding up constraint for public good provision

ξ_n Lagrangians on the minimum donation constraints

θ index to distinguish SWF with and without warm glow utility

6 Appendix

6.1 Optimal Subsidized Private Provision with Three Types of Workers and Standard Preferences

With the standard model with only two types, relaxing the incentive compatibility constraint comes from having the public good contributions coming from the highest skilled type. With three types there is an issue of the allocation of contributions across the two higher types. We continue to analyze this in a setting where the degree of subsidy depends on income with complete flexibility. Since the choice of subsidy rate is equivalent to the choice of contribution level, we can let the contributions be the controls. Let us assume there are three types, with type 0 having no skill. We assume that the optimum has each type working at the job that matches the worker's skill and that the next best alternative for type 2 is to imitate type 1. We explore the allocation of donations between types 1 and 2.

$$\text{Maximize}_{c,G,g} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right]\right)$$

$$\begin{aligned}
\text{subject to:} \qquad & E + pG + \sum N_n \left(c_n - n\right) \leq 0 \\
& u\left[c_1\right] - a_{11} + b_1 v\left[G\right] \geq u\left[c_0\right] + b_1 v\left[G - g_1 + g_0\right] \\
& u\left[c_2\right] - a_{22} + b_2 v\left[G\right] \geq u\left[c_1\right] - a_{12} + b_2 v\left[G - g_2 + g_1\right] \\
& G \geq \sum N_n g_n; \;\; g_n \geq 0 \text{ for all } n
\end{aligned} \tag{64}$$

Since contributions by the lowest skilled can only hurt the incentive compatibility constraint, they are set to zero. Since contributions by the highest paid can only help the incentive compatibility constraint it follows that the government will not contribute and $G = \sum N_n g_n$. We assume that everyone has the same utility from the public good, the same value of b_n. To begin, we form a Lagrangian:

$$\begin{aligned}
L \;\; = \;\; & \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right]\right) - \lambda\left(E + pG + \sum N_n \left(c_n - n\right)\right) \\
& + \mu_1 \left(u\left[c_1\right] - a_{11} + b_1 v\left[G\right] - \left(u\left[c_0\right] + b_1 v\left[G - g_1\right]\right)\right) \\
& + \mu_2 \left(u\left[c_2\right] - a_{22} + b_2 v\left[G\right] - \left(u\left[c_1\right] - a_{12} + b_2 v\left[G - g_2 + g_1\right]\right)\right) \\
& + \nu\left(G - \sum N_n g_n\right) + \sum \xi_n g_n
\end{aligned} \tag{65}$$

Differentiating gives the FOC for consumption levels:

$$\frac{\partial L}{\partial c_0} = N_0 \left(u' \left[c_0 \right] - \lambda \right) - \mu_1 u' \left[c_0 \right] = 0 \tag{66}$$

$$\frac{\partial L}{\partial c_1} = N_1 \left(u' \left[c_1 \right] - \lambda \right) + \left(\mu_1 - \mu_2 \right) u' \left[c_1 \right] = 0 \tag{67}$$

$$\frac{\partial L}{\partial c_2} = N_2 \left(u' \left[c_2 \right] - \lambda \right) + \mu_2 u' \left[c_2 \right] = 0. \tag{68}$$

Differentiating gives the FOC for public good level and donations (using the observation that the lowest type does not contribute):

$$\frac{\partial L}{\partial G} = \sum N_n b_n v' \left[G \right] - \lambda p + \mu_1 b_1 \left(v' \left[G \right] - v' \left[G - g_1 \right] \right) + \mu_2 b_2 \left(v' \left[G \right] - v' \left[G - g_2 + g_1 \right] \right) + \nu = 0 \tag{69}$$

$$\frac{\partial L}{\partial g_1} = \mu_1 b_1 v' \left[G - g_1 \right] - \mu_2 b_2 v' \left[G - g_2 + g_1 \right] - \nu N_1 + \xi_1 = 0 \tag{70}$$

$$\frac{\partial L}{\partial g_2} = \mu_2 b_2 v' \left[G - g_2 + g_1 \right] - \nu N_2 + \xi_2 = 0 \tag{71}$$

Adding the last two equations and rearranging terms, we have

$$\mu_1 b_1 v' \left[G - g_1 \right] = \nu \left(N_1 + N_2 \right) - \xi_1 - \xi_2 \tag{72}$$

$$\mu_2 b_2 v' \left[G - g_2 + g_1 \right] = \nu N_2 - \xi_2 \tag{73}$$

This problem can have different optima, depending on which incentive compatibility constraints are binding. One possibility is that the public good is so important that none of the incentive compatibility constraints are binding. Then all workers have the same marginal utility of consumption and the contributions are in the ranges that support the labor allocation. Another possibility is that $g_1 = 0$ and $g_2 = G/N_2$. This can happen with several possibilities of binding constraints.

If the donations of both types 1 and 2 are positive ($\xi_1 = \xi_2 = 0$), then if the incentive compatibility constraint for either type is binding, so too is that for the other. From the FOC:

$$N_0\left(1 - \lambda/u'\left[c_0\right]\right) = \mu_1 = \frac{\nu\left(N_1 + N_2\right)}{b_1 v'\left[G - g_1\right]} \tag{74}$$

$$N_1\left(1 - \lambda/u'\left[c_1\right]\right) = \mu_2 - \mu_1 = \frac{\nu N_2}{b_2 v'\left[G - g_2 + g_1\right]} - \frac{\nu\left(N_1 + N_2\right)}{b_1 v'\left[G - g_1\right]} \tag{75}$$

$$N_2\left(1 - \lambda/u'\left[c_2\right]\right) = -\mu_2 = -\frac{\nu N_2}{b_2 v'\left[G - g_2 + g_1\right]} \tag{76}$$

Thus $\mu_2 \neq 0$ implies $\nu \neq 0$, implying $\mu_1 \neq 0$, and vice versa.

Further analysis of this case with positive donations by both higher types ($\xi_1 = \xi_2 = 0$) shows that $g_2 < 2g_1$. This follows from the condition to be shown that $v'[G-g_1] > v'[G-g_2+g_1]$. From (71) and (72), the ratio of marginal utilities satisfies

$$\frac{v'[G-g_1]}{v'[G-g_2+g_1]} = \frac{\mu_2(N_1+N_2)}{\mu_1 N_2} \tag{77}$$

Thus, it is sufficient to show that $\frac{\mu_1}{N_1+N_2} < \frac{\mu_2}{N_2}$. From (66) and (67), we have

$$\begin{aligned} \frac{\mu_1}{N_1+N_2} &= \frac{\mu_1-\mu_2}{N_1+N_2} + \frac{N_2}{N_1+N_2}\frac{\mu_2}{N_2} &(78)\\ &= -\frac{N_1}{N_1+N_2}(1-\lambda/u'[c_1]) - \frac{N_2}{N_1+N_2}(1-\lambda/u'[c_2]) &(79) \end{aligned}$$

$$\frac{\mu_2}{N_2} = -(1-\lambda/u'[c_2]) \tag{80}$$

Thus

$$\frac{\mu_2}{N_2} - \frac{\mu_1}{N_1+N_2} = \frac{N_1}{N_1+N_2}(\lambda/u'[c_2] - \lambda/u'[c_1]) \tag{81}$$

With the incentive compatibility constraint binding, we have the stated result.

6.2 Optimal Uniform Subsidized Private Provision with Standard Preferences

We turn now to a setting with uniform subsidization, sufficient to induce some private contributions. Since we have assumed that everyone has the same preferences (apart from the disutility of work) and that everyone faces the same subsidy rate, the population falls into two categories – those who contribute, all of whom have the same level of private good consumption, and those who do not contribute, and have lower private good consumption. That is, among all those contributing, we have

$$(1-s)\,pu'\left[x_n-(1-s)\,pg_n\right]=v'\left[G\right] \tag{82}$$

For those not contributing, we have

$$(1-s)\,pu'\left[x_n\right]>v'\left[G\right] \tag{83}$$

What drives the pattern of contributions is the pattern of compensation. Since higher jobs must pay more if they are to be held, it follows that there is a critical value of skill such that everyone with higher skill contributes and has the same consumption and everyone with lower skill does not contribute. The incentive compatibility constraint (compared with the next lowest skill) for a contributor would be

$$u\left[x_n - (1-s)\,pg_n\right] - a_{nn} + v\left[G\right] \geq u\left[x_{n-1} - (1-s)\,pg_{n-1}\right] - a_{n-1n} + v\left[G - g_n + g_{n-1}\right] \tag{84}$$

When the next lower skill is also contributing, since contributions continue until marginal utilities, and so private consumption levels, are equalized, we can write this as

$$u\left[x_n - (1-s)\,pg_n\right] - a_{nn} + v\left[G\right] \geq u\left[x_n - (1-s)\,pg_n\right] - a_{n-1n} + v\left[G - g_n + g_{n-1}\right] \tag{85}$$

or

$$-a_{nn} + v\left[G\right] \geq -a_{n-1n} + v\left[G - g_n + g_{n-1}\right] \tag{86}$$

Thus, where the incentive compatibility constraint is binding and donations are positive, we have a difference equation for contributions in equilibrium. (There are also equilibria where the incentive compatibility constraint does not bind.)

With everyone having the same utility of public and private consumption, we have a very peculiar equilibrium. Ranking people by after-tax income, we find that those with low incomes up to some level make no contributions to the public good. Above this level, contributions are so large that everyone has the same marginal utility of private consumption, and so the same level of private consumption.

6.3 Diverse Standard Preferences with Two Types of Workers

We now consider the model briefly discussed in section 1.4.1 where a fraction f_n of workers of type n have no utility from the public good. Given this structure of incentive compatibility constraints, there are two structures, both of which seem to be possible optima for different fractions of the highest skill types with different preferences. In one optimum all workers of the highest skill work at the highest job, implying that the incentive compatibility constraint on the workers who donate does not bind (since it is strictly implied by the incentive compatibility constraint of nondonators). In the other case the workers of the highest skill who do not value the public good drop down to a lower job and the incentive constraint is binding on the remaining highest skill workers at the highest job. We present the FOC in both cases after reviewing the optimum without donations.

With public provision we have a slightly changed optimization:

$$\text{Maximize}_{c,G} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + f_n b_n v\left[G\right]\right)$$

$$\text{subject to:} \quad E + pG + \sum N_n \left(c_n - n\right) \leq 0 \tag{87}$$

$$u\left[c_n\right] - a_{nn} \geq u\left[c_m\right] - a_{mn} \text{ for } m < n \text{ for all } n$$

Since we have assumed that choice of job does not affect the utility from the public good (whether positive or zero) the incentive compatibility constraint is not affected by the preference diversity. Thus, the allocation has the same structure as above, apart from the presence of f_n. That is, the allocation is as if everyone were the same with public good preferences $f_n b_n v\left[G\right]$.

We turn now to the optimum with possible donations, assuming two types of workers. Type 1 workers will work at type 1 jobs (assuming enough of a resource need) and will not make donations (as above). Type 2 workers who care about the public good will hold type-2 jobs (again assuming enough of a resource need) and will make donations. There are two possibilities for type 2 workers who do not value the public good - they can hold type-1 jobs or type-2 jobs. Thus one candidate for the best allocation is the one that can be achieved subject to all type-2 workers holding type-2 jobs. Another possibility is that social welfare could be higher by giving up the extra output from type-2 workers who don't value public goods by having them hold type-1 jobs and thereby having a weaker incentive compatibility constraint for type-2 workers who do value making donations. The two types of optima are both continuous in the fraction of type-2 workers who do not value the public good. Presumably, the choice between the two candidates for this optimum depends on the fraction of the highest workers who value the public good.

For completeness, we set out the expressions for social welfare in these cases.

The best allocation with no type-2 workers holding type-1 jobs is the solution to the following problem:

$$
\begin{aligned}
\text{Maximize}_{x,G,g,s} \quad & N_1 \left(u \left[x_1 \right] - a_{11} + f_1 b_1 v \left[G \right] \right) \\
& +N_2 \left(f_2 u \left[x_2 - \left(1 - s_2 \right) p g_2 \right] + f_2 b_2 v \left[G \right] + \left(1 - f_2 \right) u \left[x_2 \right] - a_{22} \right) \\
\text{subject to:} \quad & E + pG + N_1 \left(x_1 - n_1 \right) + N_2 \left(x_2 - n_2 \right) - f_2 N_2 \left(1 - s_2 \right) p g_2 \leq 0 \\
& u \left[x_2 \right] - a_{22} \geq u \left[x_1 \right] - a_{12} \\
& \left(1 - s_2 \right) p u' \left[x_2 - \left(1 - s_2 \right) p g_2 \right] = b_2 v' \left[G \right] \\
& G \geq \sum f_n N_n g_n; \;\; g_n \geq 0 \text{ for all } n
\end{aligned}
\tag{88}
$$

In this case resources are saved by having some type-2 workers lower their consumption to donate. There are three possibilities for the optimal donations - providing all of the public good, providing the amount that maximizes their forgone consumption, and providing enough to lower their consumption to the socially most valuable level. To see this let us consider a new variable, $z \equiv \left(1 - s_2 \right) p g_2$, equal to the amount of consumption given up for donations. It is only this variable that appears in the maximization problem, with the choice of subsidy rate and financed public good determined to equal this variable over an allowable range. One possibility is that the consumption given up is at the extreme of the range - that is as large as possible - either the maximum that can be induced or the maximum consistent with the total level of public good provision. Otherwise, z only appears in the following portion of the Lagrangian, $N_2 f_2 u[x_2 - z] + \lambda f_2 N_2 z$. Thus, with an interior solution, we have the FOC.

$$u'[x_2 - z] = \lambda \tag{89}$$

In contrast, the best allocation with some type-2 workers holding type-1 jobs is the solution to the following problem (setting g_1 equal to zero):

$$\begin{aligned}
\text{Maximize}_{x,G,g,s} \quad & N_1 (u[x_1] - a_{11} + f_1 b_1 v[G]) + (1 - f_2) N_2 (u[x_1] - a_{12}) \\
& + f_2 N_2 (u[x_2 - (1 - s_2) p g_2] - a_{22} + b_2 v[G]) \\
\text{subject to:} \quad & E + pG + (N_1 + (1 - f_2) N_2)(x_1 - n_1) + f_2 N_2 (x_2 - n_2 - (1 - s_2) p g_2) \leq 0 \\
& u[x_2 - (1 - s_2) p g_2] - a_{22} + b_2 v[G] \geq u[x_1] - a_{12} + b_2 v[G - g_2] \\
& u[x_1] - a_{12} \geq u[x_2] - a_{22} \\
& (1 - s_2) p u'[x_2 - (1 - s_2) p g_2] = b_2 v'[G] \\
& G \geq \sum f_n N_n g_n; \;\; g_n \geq 0 \text{ for all } n
\end{aligned} \tag{90}$$

When some type-2 workers hold type-1 jobs, the optimum can occur with the incentive compatibility constraint binding or not binding on the remaining type-2 workers.

Contrasting the two problems, we see that we have reversed the constraint for type-2 workers who do not value the public good and have different conditions applying to those who do care about the public good.[15]

7 References

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[15] Another approach to the trade-off between those who do and do not value the public good could be modeled by having a distribution of disutilities for each skill type (as in Diamond, 1980). Then there would be a marginal worker at each type with different margins for those with and without donations and so there would be some effect on labor supply from private contributions.

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